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Study of Merger and Acquisition (M&A) with Spline function in AR Time Series Model Under Bayesian Framework

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Abstract

In this paper, we use an autoregressive model to investigate the behavior of mergers and acquisitions. It studies a non-linear time trend, which is approximately converted into a linear time trend using the spline function, which divides the series into piecewise linear segments between the knots. These knots are the change points where the trend pattern gets changed. The major aim of this study is to offer a merged autoregressive spline (M-ARS) model that can be used to analyze the influence of the merger on the parameters as well as model behavior. First, we obtained an estimation setup based on the well-known classical least square method and posterior distributions under the Bayesian approach with different loss functions. Then, the effect on the series based on the merger variable is significantly determined by the Bayes factor. The applicability of the proposed model is illustrated based on a simulation study and real application in the Indian banking sector.

Keywords: Autoregressive model, posterior distribution, loss function, merger, spline function, linear and non-linear time trend.

1. Introduction

A time series is a method for analyzing and modelling chronological data. When there is an association between past observations, then an autoregressive (AR) model is a plausible model to forecast future behavior based on previous information (Box and Jenkins 1970, Newbold 1983). The AR time series model has attracted a large number of researchers in both econometrics and statistics for several decades (Kumar and Kumar 2019). In AR series, sometimes associated series are involved along with dependent series, which affect the process (Kumar et al. 2017). Though these associated series do not remain with the series for a long time, they get merged with dependent series after some time. So, few series under study are terminated after a certain period in the observed series. This process is known as merger and acquisition (M&A). Recently, such type of modelling had done by Kumar and Agiwal (2020).

In terms of mergers and acquisitions Hossain (2021) and Alarco (2018), discussed about the significant changes in the international landscape over the last several years. Deregulation of the financial sector has resulted in the introduction of new players and goods with advanced technology, globalization of the financial markets, changing consumer behavior, broader services at lower prices, shareholder wealth demands, and so forth (Paul 2017, Aljadani 2019). Thus, M&A is a powerful technique in the globalization of the economy's growth and expansion. Khan (2011) discussed about the major motivation for M&A is to create synergy, which means that two plus two is more than four, and this logic tempts organizations to merge during difficult circumstances.

In recent decades, researchers have taken the inferences to perform research in the area of merger concepts for the growth of companies and examined the effects and performance after mergers. There is a lot of literature available on the M&A process like Lubatkin (1983) addressed merger problems and showed benefits to the acquiring organization. Resende (1999) studied the M&A series using Markov switching modes and observed that merger presence and endogenous shocks had a significant effect. Diaz et al. (2004) found bank performance when banking and non-banking organizations merged and acquired in the European Union and got efficient profit. DeLong and DeYong (2007) carried out a study on 50 of the biggest US bank mergers between 1979 and 1984 to analyze cash flow performance and observed that the operational performance of the merged organizations improved significantly. Agiwal and Kumar (2021) proposed a merged autoregressive (M-AR) model for analysis of the M&A concept in univariate mobile banking series.

When the time trend comes non-linear it's difficult to predict. So, to overcome this problem spline function is a plausible function to M&A (Kumar et. al 2020) and the trend pattern of the structure shows the data's non-linearity. Hence, the spline function can be used to approximate this non-linear trend into piece-wise modelling. With advancements on both the theoretical and computational fronts, spline function has become a well-established technique in statistical analysis. Eubank (1999) discussed that a spline function is the smoothest possible piecewise polynomial that retains a segment nature. Hurley et al. (2006) called spline as lines or curves function which is usually required to be continuous and smooth. In particular, it is frequently used to build explanatory models in time series and economics. This is used to describe smooth functions of interest, including non-linear effects, in many new methodological developments in the current time series.

In this paper, we are focused on developing the M-ARS time series model with the help of the AR model and some associated series. These associated series are merged into the observed series after a considerable period. In this newly developed model, the spline function is used as a trend converter. Therefore, an extended time series model is proposed to manage the non-linear time trend and to understand the effects of mergers and acquisitions. Section 2 describes the expanded form of the merger autoregressive model through the spline function. The classical and Bayesian methodologies under different loss functions are discussed in Section 3 for the proposed model and also defined the testing procedure to show the merged effect. Sections 4 and 5 respectively consider the simulation and real data analysis to show the methodology's adequacy and appropriateness in a real application. The brief conclusion is defined in Section 6.

2. Merger Autoregressive Spline (M-ARS) Model

This section considers the merged autoregressive (M-AR) model with the inclusion of spline function for controlling the non-linear trend pattern through piecewise models. Up to merger time T_m, the observed series follows AR(p1) process with k time-dependent associated variables and a linear spline function. These associate series also follow the AR model with different orders (r_h; $h = 1, 2, \ldots, k$). After the merger time point, associate series are merged into the observed AR series with a different order p_2. This shows that observations of the associate series are not recorded due to being merged into the acquired series. But this may change the structure of the series which is controlled by linear spline function only. The proposed model is called a merged autoregressive spline(M-ARS) model. Finally, the structure of the M-ARS model will be in this form.

$$y_t = \begin{cases} \theta_1 + \sum_{i=1}^{p_1} \Phi_{1i} y_{t-j} + \sum_{h=1}^{k} \sum_{j=1}^{r_h} \delta_{hj} z_{h,t-j} + \sum_{n=1}^{A_1} \Psi_{1n} S_n(t) + \varepsilon_t & ;\ t \le T_m \\ \theta_2 + \sum_{i=1}^{p_2} \Phi_{2i} y_{t-j} + \sum_{n=1}^{A_2} \Psi_{2n} S_n(t) + \varepsilon_t & ;\ t > T_m, \end{cases} \tag{1}$$

where $\{y_t, t = 1,2,...,T\}$ is an observed series, θ_1 and θ_2 are the intercept terms, A_1 is the number of knots before merger time that contain locations of knots $(t_{11}, t_{12}, ..., t_{1A_1})$ and A_2 is the number of knots after merger time that contains locations of knots $(t_{21}, t_{22}, ..., t_{2A_2})$. Ψ_{in} is the coefficient of n^{th} knot, where i = 1 and 2, and by Equation (2), we define $S_n(t)$ which is a spline function, described as a linear polynomial form which is as follows.

$$S_n(t) = (t - t_i)^+ = \begin{cases} t - t_i;\ t > t_i, \\ 0;\ t \le t_i, \end{cases} \tag{2}$$

ε_t is assumed to be i.i.d. (independently identically distributed) normal random variable with mean zero and unknown variance σ^2 and the merger coefficient of the h^{th} series is denoted by δ_h. Model (1) can be defined in matrix notation as shown below.

$$Y_{T_m} = \theta_1 l_{T_m} + \Phi_1 X_{T_m} + \delta_1 Z_{T_m} + \Psi_1 S_{T_m} + \varepsilon_{T_m} \tag{3}$$

$$Y_{T-T_m} = \theta_2 l_{T-T_m} + \Phi_2 X_{T-T_m} + \Psi_2 S_{T-T_m} + \varepsilon_{T-T_m}. \tag{4}$$

Moreover, the final model in matrix form has been written by combining Equations (3) and (4),

$$\begin{pmatrix} Y_{T_m} \\ Y_{T-T_m} \end{pmatrix} = \begin{pmatrix} l_{T_m} & 0 \\ 0 & l_{T-T_m} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} X_{T_m} & 0 \\ 0 & X_{T-T_m} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} + \begin{pmatrix} Z_{T_m} \\ 0 \end{pmatrix} \begin{pmatrix} \delta^1 \\ 0 \end{pmatrix} + \begin{pmatrix} S_{T_m} & 0 \\ 0 & S_{T-T_m} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{T_m} \\ \varepsilon_{T-T_m} \end{pmatrix} \tag{5}$$

We can also write the Equation (5) in this form,

$$Y = L\theta + X\beta + Z\delta + S\Psi + \varepsilon, \tag{6}$$

where

$$Z_{T_m}^m = \begin{pmatrix} z_{h,0} & z_{h,-1} \cdots & z_{h,1-r_h} \\ z_{h,1} & z_{h,0} \cdots & z_{h,2-r_h} \\ \vdots & & \\ z_{h,T-1} & z_{h,T-2} \cdots & z_{h,T-r_h} \end{pmatrix}, \quad Z_{T_m} = (Z_{T_m}^1, Z_{T_m}^2, \cdots Z_{T_m}^k), \quad Z = \begin{pmatrix} Z_{T_m} \\ 0 \end{pmatrix},$$

$$\Phi_1 = \left(\Phi_{11}\ \Phi_{12} \cdots \Phi_{1p_1}\right)', \quad \Phi_2 = \left(\Phi_{21}\ \Phi_{22} \cdots \Phi_{2p_2}\right)', \quad \beta = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

$$S_{T-T_m} = \begin{pmatrix} S_1(T_m+1) & S_2(T_m+1) \cdots S_{A_2}(T_m+1) \\ S_1(T_m+2) & S_2(T_m+2) \cdots S_{A_2}(T_m+2) \\ \vdots & \\ S_1(T) & S_2(T) \quad \cdots S_{A_2}(T) \end{pmatrix}, \quad S_{T_m} = \begin{pmatrix} S_1(1) & S_2(1) & \cdots S_{A_1}(1) \\ S_1(2) & S_2(2) & \cdots S_{A_1}(2) \\ \vdots & & \\ S_1(T_m) & S_2(T_m) & \cdots S_{A_1}(T_m) \end{pmatrix},$$

$$X_{T_m} = \begin{pmatrix} y_0 & y_{-1} & \cdots & y_{1-p_1} \\ y_1 & y_0 & \cdots & y_{2-p_1} \\ \vdots & & \ddots & \\ y_{T_m-1} & y_{T_m-2} & \cdots & y_{T_m-p_1} \end{pmatrix}, \quad X_{T-T_m} = \begin{pmatrix} y_{T_m} & y_{T_m-1} & \cdots & y_{T_m+1-p_2} \\ y_{T_m+1} & y_{T_m} & \cdots & y_{T_m+2-p_1} \\ \vdots & & \ddots & \\ y_{T-1} & y_{T-2} & \cdots & y_{T-p_2} \end{pmatrix},$$

$$L = \begin{pmatrix} l_{T_m} & 0 \\ 0 & l_{T-T_m} \end{pmatrix}, \quad X = \begin{pmatrix} X_{T_m} & 0 \\ 0 & X_{T-T_m} \end{pmatrix}, \quad S = \begin{pmatrix} S_{T_m} & 0 \\ 0 & S_{T-T_m} \end{pmatrix}, \quad \delta^1 = (\delta_1 \ \ \delta_2 \cdots \ \delta_k)',$$

$$\delta = \begin{pmatrix} \delta^1 \\ 0 \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_{T_m} \\ \varepsilon_{T-T_m} \end{pmatrix}, \quad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \quad \Phi = (\Phi_1 \ \ \Phi_2)', \quad \Psi = (\Psi_1 \ \ \Psi_2)',$$

$$\delta_h = (\delta_{h1} \ \ \delta_{h2} \cdots \delta_{hr_h})',$$

The proposed AR model is used to analyze the non-linear trend components when the merger is taking place. This merger has several impacts to change the structure of the observed series which only be analyzed by putting the spline function that makes the model as a piecewise linear AR model with merging observations. Thus, the spline function can be used as a very important tool to convert the non-linear trend pattern into a linear trend pattern in the presence of the M&A process. The inference of the proposed model is described under both classical and Bayesian points of view in the next section.

3. Inference for the Study

The primary conclusion of any research problem is to understand the data generating process based on the available information. The time series models are used to develop a future forecasting mechanism based on current and previous situations. In time series, one may be interested in drawing inferences about the structure of the model through estimation as well as concluding the model through hypothesis testing. Hence, the purpose of this section is to discuss the estimation and testing techniques to determine the M-ARS model's performance.

3.1. Classical estimators

Maximum likelihood (ML) and least squared (LS) techniques are commonly used for parameter estimation in regression-based models because it gains a closed functional form or simply transfer in matrix form (Ghosh. et al. 2007). For the M-ARS model, parameters of interest are $\theta, \beta, \delta,$ and Ψ by using LS and its associated sum of square residuals (SSR)define in the Equations (7) and (8) for a given time series model in Equation (6) is given below,

$$\hat{\Theta} = \begin{pmatrix} \hat{\theta} \\ \hat{\beta} \\ \hat{\delta} \\ \hat{\Psi} \end{pmatrix} = (E'E)^{-1} E'Y, \tag{7}$$

where $\mathrm{E} = \left(\mathrm{L} \ \ \mathrm{X} \ \ \mathrm{Z} \ \ \mathrm{S}\right)$ and

$$\mathrm{SSR} = (\mathrm{Y} - \mathrm{E}(\mathrm{E}'\mathrm{E})^{-1}\mathrm{E}'\mathrm{Y})'(\mathrm{Y} - \mathrm{E}(\mathrm{E}'\mathrm{E})^{-1}\mathrm{E}'\mathrm{Y}). \tag{8}$$

3.2. Bayesian estimation

In the Bayesian framework, prior information is about the unknown parameters which are equally important as the likelihood function of the model. To determine the posterior probability, the prior function is needed. For all parameters of the M-ARS model, we consider the informative conjugate priors function and adopt a multivariate normal (MVN) distribution with a different means and common variance for intercept, autoregressive, merging coefficients, and spline coefficients. Assume an inverted gamma prior where “ a ” and “ b ” are the hyper parameters of gamma prior. These priors are chosen the same as the priors taken in the paper Kumar and Agiwal (2020). Which are as follows,

$$\theta \sim N(\mu, I_2\sigma^2), \quad \mu \in \mathbb{R}^2, \sigma^2 > 0, \tag{9}$$

$$\beta \sim N(\gamma, I_{P_1+P_2}\sigma^2), \quad \gamma \in \mathbb{R}^{P_1+P_2}, \sigma^2 > 0, \tag{10}$$

$$\delta \sim N(\alpha, I_R\sigma^2), \quad \alpha \in \mathbb{R}^R, \sigma^2 > 0, \tag{11}$$

where $R = \sum_{h=1}^{k} r_h / 2$,

$$\Psi \sim N(c, I_{A_1+A_2}\sigma^2), \quad c \in \mathbb{R}^{A_1+A_2}, \sigma^2 > 0, \tag{12}$$

$$\sigma^2 \sim IG(a,b), \quad a,b > 0. \tag{13}$$

By using the priors, which are defined in Equations (9) to (13), we obtain the joint prior distribution defined in Equation (14).

$$\begin{aligned} \Pi(\Theta) &= f(\theta)f(\beta)f(\delta)f(\psi)f(\sigma^2) \\ &= \frac{1}{(2\pi)^{(p_1+p_2+R+A_1+A_2+2)/2}}.(\sigma^2)^{-(\frac{R+P_1+P_2+A_1+A_2}{2}+a+2)}.\frac{b^a}{\Gamma a}.\exp[-\frac{1}{2\sigma^2}\{(\theta-\mu)' I_2^{-1}(\theta-\mu) \\ &\quad +(\beta-\gamma)' I_{p_1+p_2}^{-1}(\beta-\gamma)+(\delta-\alpha)' I_R^{-1}(\delta-\alpha)+(\Psi-c)' I_{A_1+A_2}^{-1}(\Psi-c)+2b\}], \end{aligned} \tag{14}$$

The proposed model’s likelihood function is written as in Equation (15),

$$L(\Theta|y) = \frac{(\sigma^2)^{-T/2}}{(2\pi)^{T/2}} \exp\left[-\frac{1}{2\sigma^2}\{(Y - L\theta - X\beta - Z\delta - S\Psi)'(Y - L\theta - X\beta - Z\delta - S\Psi)\}\right]. \tag{15}$$

The posterior distribution is determined by combining the information from the observed series with the joint prior distribution. The posterior distribution for the proposed model is of the form in the Equation (16).

$$\begin{aligned} &\Pi(\Theta|\mathrm{y}) \propto \Pi(\Theta).\mathrm{L}(\Theta|\mathrm{y}) \\ &\propto (\sigma^2)^{-(\frac{p_1+p_2+R+A_1+A_2+T}{2}+a+2)} \exp[-\frac{1}{2\sigma^2}\{(\theta-\mu)' I_2^{-1}(\theta-\mu)+(\beta-\gamma)' I_{p_1+p_2}^{-1}(\beta-\gamma)+ \\ &\qquad (\delta-\alpha)' I_R^{-1}(\delta-\alpha)+(\Psi-c)' I_{A_1+A_2}^{-1}(\Psi-c)+(Y-L\theta-X\beta-Z\delta-S\Psi)' \\ &\qquad (Y-L\theta-X\beta-Z\delta-S\Psi)\}]. \end{aligned} \tag{16}$$

For parameter estimation under the Bayesian approach, a loss function must be specified based on decision theory. Several symmetric and asymmetric loss functions can be used to select the most favorable explanation of the generated samples, which are decreasing the associated risk in the simulated samples. However, there is no method for deciding on the loss function. Hence, we have optimized the presented model's inferences by using the following loss functions: (1) squared error loss function (SELF), (2) absolute loss function (ALF), and (3) entropy loss function (ELF).

$$\phi_{\mathrm{SELF}}(\Theta \mid \mathrm{Y}) = \mathrm{E}_\pi(\phi(\Theta) \mid \mathrm{Y}) = \mathrm{K}\int_\Theta \phi(\Theta)\pi(\Theta \mid \mathrm{Y})d\Theta,$$

$$\phi_{\mathrm{ELF}}(\Theta \mid \mathrm{Y}) = [\mathrm{E}_\pi(\phi^{-1}(\Theta) \mid \mathrm{Y})]^{-1} = \left(\mathrm{K}\int_\Theta \phi^{-1}(\Theta)\pi(\Theta \mid \mathrm{Y})d\Theta\right)^{-1},$$

$$\phi_{ALF}(\Theta \mid Y) = E_{\pi}\left|\Theta - \hat{\Theta}\right| = K\int_{\Theta}\left|\Theta - \hat{\Theta}\right|\pi(\Theta \mid Y)d\Theta.$$

It should be noted that evaluating the ratio of multiple integrals, a closed expression is difficult to derive analytically. This is a big challenge in implementing the Bayes technique. To obtain posterior samples from the posterior distribution, we use the MCMC algorithm. The conditional posterior distribution of respective parameters is obtained and given below in the equations from (17) to (21).

$$\theta \mid \beta,\delta,\Psi,\sigma^2,Y \sim MVN\left(\left((Y - X\beta - Z\delta - S\Psi)' L + \mu I_2^{-1}\right)\left(L'L + I_2^{-1}\right)^{-1}, \left(L'L + I_2^{-1}\right)^{-1}\sigma^2\right), \quad (17)$$

$$\beta \mid \theta,\delta,\Psi,\sigma^2,Y \sim MVN\left(\left((Y - L\theta - Z\delta - S\Psi)' X + \gamma I_{p_1+p_2}^{-1}\right)\left(X'X + I_{p_1+p_2}^{-1}\right)^{-1}, \left(X'X + I_{p_1+p_2}^{-1}\right)^{-1}\sigma^2\right), \quad (18)$$

$$\delta \mid \theta,\beta,\Psi,\sigma^2,Y \sim MVN\left(\left((Y - L\theta - X\beta - S\Psi)' Z + \alpha I_R^{-1}\right)\left(Z'Z + I_R^{-1}\right)^{-1}, \left(Z'Z + I_R^{-1}\right)^{-1}\sigma^2\right), \quad (19)$$

$$\Psi \mid \theta,\beta,\delta,\sigma^2,Y \sim MVN\left(\left((Y - L\theta - X\beta - Z\delta)' S + cI_{A_1+A_2}^{-1}\right)\left(S'S + I_{A_1+A_2}^{-1}\right)^{-1}, \left(S'S + I_{A_1+A_2}^{-1}\right)^{-1}\sigma^2\right), \quad (20)$$

$$\Pi(\sigma^2 / Y,\theta,\beta,\delta,\Psi,\Psi) \sim IG\left(\frac{(R + p_1 + p_2 + A_1 + A_2 + T)}{2} + a + 1, K\right), \quad (21)$$

where $K = \frac{1}{2}[(Y - L\theta - X\beta - Z\delta - S\Psi)'(Y - L\theta - X\beta - Z\delta - S\Psi) + (\theta - \mu)' I_2^{-1}(\theta - \mu) + (\beta - \gamma)' I_{p_1+p_2}^{-1}(\beta - \gamma) + (\delta - \alpha)' I_R^{-1}(\delta - \alpha) + (\Psi - c)' I_{A_1+A_2}^{-1}(\Psi - c) + 2b].$

Here, we use the Gibbs sampling algorithm to obtain posterior samples from the specified conditional posterior distribution because all conditional posterior distributions are coming in a closed and standard form.

3.3. Significance test for merger coefficient

From a Bayesian perspective, we are presented a procedure using Bayes factors for testing the impact of merger/acquire series on models, intended to analyze the impact on the model as associate series may affect the model (Van de et al. 2021). The merger may have favorable or unfavorable consequences. This testing procedure is completed with the help of 4 different hypothetical procedures. Here, we test some possible hypothesis against the H_1 hypothesis.

H1 : both structural break and merger coefficient are effective,

$$\theta_1 \neq \theta_2,\ \Phi_1 \neq \Phi_2,\ \Psi_1 \neq \Psi_2 \text{ and } \delta \neq 0.$$

Here, we consider that, break occurred in all parameters of the model, then the model reduces to

$$H_1 : Y = L\theta + X\beta + Z\delta + S\Psi + \varepsilon.$$

H2 : structural break is present but merger coefficients are not effective,

$$\theta_1 \neq \theta_2,\ \Phi_1 \neq \Phi_2, \Psi_1 \neq \Psi_2 \text{ and } \delta = 0.$$

Here, "Structural Break" means, break occurred in the parameters rather than merger parameter δ, and merger parameter $\delta = 0$, means that merger is not effective model reduces to

$$H_2 : Y = L\theta + X\beta + S\Psi + \varepsilon.$$

H3 : No structural break but merger coefficients are effective,

$$\theta_1 = \theta_2, \Phi_1 = \Phi_2, \Psi_1 = \Psi_2 \text{ and } \delta \neq 0.$$

Here, "No Structural Break" means, no break occurred in the parameters rather than merger parameter δ, and merger parameter $\delta \neq 0$, means that, merger is effective. Model reduces to

$$H_3 : Y = L_{T_m}\theta_1 + X_{T_m}\Phi_1 + Z_{T_m}\delta_1 + S_{T_m}\Psi_1 + \varepsilon_{T_m}.$$

H4 : Both structural break and merger coefficients are not effective,

$$\theta_1 = \theta_2,\ \Phi_1 = \Phi_2,\ \Psi_1 = \Psi_2 \text{ and } \delta = 0.$$

Here, we consider that, no break occurred in all parameters of the model, then model reduces to

$$H_4 : Y = L_{T_m}\theta_1 + X_{T_m}\Phi_1 + S_{T_m}\Psi_1 + \varepsilon_{T_m}.$$

Posterior probability under the hypothesis (H_1) is defined in the Equation (22),

$$P(Y|H_1) = \frac{b^a |D_1|^{\frac{-1}{2}} |D_2|^{\frac{-1}{2}} |D_3|^{\frac{-1}{2}} |D_4|^{\frac{-1}{2}} \Gamma\left(\frac{T}{2}+a\right)}{(2\pi)^{\frac{T}{2}} \Gamma a \left(\frac{N_1}{2}\right)^{\frac{T}{2}+a}}. \quad (22)$$

Similarly, posterior probabilities under the alternative hypothesis H_2, H_3 and H_4 are defined as

$$P(Y|H_2) = \frac{b^a |D_1|^{\frac{-1}{2}} |D_2|^{\frac{-1}{2}} |D_3|^{\frac{-1}{2}} \Gamma\left(\frac{T}{2}+a\right)}{(2\pi)^{\frac{T}{2}} \Gamma a \left(\frac{N_0}{2}\right)^{\frac{T}{2}+a}}, \quad P(Y|H_3) = \frac{b^a |E_1|^{\frac{-1}{2}} |E_2|^{\frac{-1}{2}} |E_3|^{\frac{-1}{2}} |E_4|^{\frac{-1}{2}} \Gamma\left(\frac{T}{2}+a\right)}{(2\pi)^{\frac{T}{2}} \Gamma a \left(\frac{Q_2}{2}\right)^{\frac{T}{2}+a}},$$

$$P(Y|H_4) = \frac{b^a |E_1|^{\frac{-1}{2}} |E_2|^{\frac{-1}{2}} |E_3|^{\frac{-1}{2}} \Gamma\left(\frac{T}{2}+a\right)}{(2\pi)^{\frac{T}{2}} \Gamma a \left(\frac{Q_1}{2}\right)^{\frac{T}{2}+a}}.$$

Under the H_1 hypothesis, we find out the Bayes factors $(BF_{12}, BF_{13}, BF_{14})$ by using posterior probability, BF_{12} means that the ratio of the probabilities of $P(Y|H_1)$ and $P(Y|H_2)$, and same did for BF_{12} and BF_{13},

$$BF_{12} = \frac{P(Y|H_1)}{P(Y|H_2)}, \quad BF_{13} = \frac{P(Y|H_1)}{P(Y|H_3)}, \quad BF_{14} = \frac{P(Y|H_1)}{P(Y|H_4)}.$$

And all Bayes factors will be in this form,

$$BF_{12} = |D_4|^{-\frac{1}{2}} \left(\frac{N_0}{N_1}\right)^{\frac{T}{2}+a}, \quad BF_{13} = \frac{|D_1|^{\frac{-1}{2}} |D_2|^{\frac{-1}{2}} |D_3|^{\frac{-1}{2}} |D_4|^{\frac{-1}{2}} (Q_2)^{\frac{T}{2}+a}}{|E_1|^{\frac{-1}{2}} |E_2|^{\frac{-1}{2}} |E_3|^{\frac{-1}{2}} |E_4|^{\frac{-1}{2}} (N_1)^{\frac{T}{2}+a}},$$

$$BF_{14} = \frac{|D_1|^{\frac{-1}{2}} |D_2|^{\frac{-1}{2}} |D_3|^{\frac{-1}{2}} |D_4|^{\frac{-1}{2}} (Q_1)^{\frac{T}{2}+a}}{|E_1|^{\frac{-1}{2}} |E_2|^{\frac{-1}{2}} |E_3|^{\frac{-1}{2}} (N_1)^{\frac{T}{2}+a}}.$$

where

$$D_1=(L'L+I_2^{-1}),$$
$$D_2=(X'X+I_{p_1+p_2}^{-1}-X'L'D_1^{-1}LX),$$
$$D_3=Z'Z+I_R^{-1}-Z'LD_1^{-1}L'Z-(Z'X-Z'LD_1^{-1}L'X)'D_2^{-1}(Z'X-Z'LD_1^{-1}L'X),$$
$$D_4=S'S+I_{A_1+A_2}^{-1}-S'LD_1^{-1}L'S-(ZS'-S'LD_1^{-1}L'Z)+(S'X-S'LD_1^{-1}L'X)'D_2^{-1}(S'X-S'LD_1^{-1}L'X)D_3^{-1}$$
$$((S'Z-S'LD_1^{-1}L'Z)+(S'X-S'LD_1^{-1}\ell L'X)D_2^{-2}(S'X-S'LD_1^{-1}L'X)-$$
$$(S'X-S'LLD_1^{-1}L'X)'D_2^{-1}(S'X-S'LD_1^{-1}L'X)),$$
$$B_4=(YS+CI_{A_1+A_2}^{-1}-(y'L+\mu'I_2^{-1})D_1^{-1}L'S-B_{22}{}'D_3^{-1}((S'Z-S'L'D_1^{-1}LZ)+$$
$$(S'X-S'LD_1^{-1}L)'D_2^{-1}(S'X-S'LD_1^{-1}L)B_{12}^{-1}D_2^{-1}(S'X-S'L'D_1^{-1}L'X)),$$
$$B_{22}'=(ZY'+I_R^{-1}\alpha-(LY'+\mu I_2^{-1})'D_1^{-1}L'Z-((XY'+\gamma I_{p_1+p_2}^{-1}-(Ly'+\mu I_2^{-1})'LD_1^{-1}X)'(X-LD_1^{-1}L'X)D_2^{-1}Z),$$
$$B_{12}'=((XY'+\gamma I_{p_1+p_2}^{-1}-(Ly'+\mu I_2^{-1})'LA_1^{-1}X),$$
$$N_0=Y'Y+\mu'I_2^{-1}\mu+\gamma'I_{P_1+P_2}^{-1}\gamma+\alpha'I_R^{-1}\alpha+2b-(LY+\mu I_2^{-1})'A_1^{-1}(L'Y+\mu I_2^{-1})-B_{22}'D_3'B_{22}-B_{12}'D_2^{-1}B_{12},$$
$$N_1=N_0+C'I_{A_1+A_2}^{-1}C+B_4'D_4^{-1}B_4,$$
$$E_1=(L_{T_m}'L_{T_m}+I_2^{-1}),$$
$$E_2=(X_{T_m}'X_{T_m}+I_{p_1}^{-1}-X_{T_m}'L_{T_m}'E_1^{-1}L_{T_m}X_{T_m}),$$
$$E_3=Z_{T_m}'Z_{T_m}+I_R^{-1}-Z_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'Z_{T_m}-(Z_{T_m}'X_{T_m}-Z_{T_m}'L_{T_m}A_1^{-1}L_{T_m}'X_{T_m})'E_2^{-1}(Z'X_{T_m}-Z_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m}),$$
$$E_4=S_{T_m}'S_{T_m}+I_{A_1}^{-1}-S'L_{T_m}E_1^{-1}L_{T_m}'S_{T_m}-(Z_{T_m}S_{T_m}'-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'Z_{T_m})+$$
$$(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})'E_2^{-1}(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})E_3^{-1}((S_{T_m}'Z_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'Z_{T_m})$$
$$+(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})E_2^{-2}(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})$$
$$-(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})'E_2^{-1}(S_{T_m}'X_{T_m}-S_{T_m}'S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})),$$
$$F_4=(YS_{T_m}+CI_{A_1}^{-1}-(Y'L_{T_m}+\mu'I_2^{-1})E_1^{-1}L_{T_m}'S_{T_m}-B_{22}{}'E_3^{-1}((S_{T_m}'Z_{T_m}-S_{T_m}'L_{T_m}E_1^{-1}L_{T_m}'Z_{T_m})+$$
$$(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}'E_1^{-1}L_{T_m})'E_2^{-1}(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}'E_1^{-1}L_{T_m})B_{12}^{-1}E_2^{-1}(S_{T_m}'X_{T_m}-S_{T_m}'L_{T_m}'E_1^{-1}L_{T_m}X_{T_m})),$$
$$F_{22}'=(Z_{T_m}Y'+I_R^{-1}\alpha_{T_M}-(L_{T_m}Y'+\mu_{T_m}I^{-1})'E_1^{-1}L_{T_m}'Z_{T_m}-$$
$$((X_{T_m}Y'+\gamma_{T_m}I_{p_1+p_2}^{-1}-(L_{T_m}Y'+\mu_{T_m}I_2^{-1})'L_{T_m}E_1^{-1}X_{T_m})'(X_{T_m}-L_{T_m}E_1^{-1}L_{T_m}'X_{T_m})E_2^{-1}Z_{T_m}),$$
$$F_{12}'=((X_{T_m}Y'+\gamma_{T_m}I_{p_1+p_2}^{-1}-(L_{T_m}Y'+\mu_{T_m}I_2^{-1})'L_{T_m}E_1^{-1}X_{T_m}),$$
$$Q_1=Y'Y+\mu_{T_m}'I^{-1}\mu_{T_m}+\gamma_{T_m}'I_{P_1}^{-1}\gamma_{T_m}+\alpha_{T_m}'I_R^{-1}\alpha_{T_m}+2b_{T_m}-(L_{T_m}'Y+\mu_{T_m}I^{-1})'E_1^{-1}(L_{T_m}'Y+\mu_{T_m}I^{-1})$$
$$-F_{22}'E_3'F_{22}-F_{12}'E_2^{-1}F_{12},$$
$$Q_2=Q_1+C_{T_m}'I_{A_1}^{-1}C_{T_m}+F_4'E_4^{-1}F_4,$$

The Bayes factor makes it simple to decide whether a hypothesis should be accepted or rejected (Williams et al. 2017). When the value of BF_{12}, BF_{13} and BF_{14} is too much high, then we reject the hypothesis H_1, H_2 and H_3, and vice-versa.

4. Simulation Study

The main aim of statistics is to use appropriate statistical theories to make significant inferences for a given model. Simulation is a flexible technique for analyzing the behavior of a proposed study and comparing the best estimate. In the simulation, a sample of random data is generated in such a manner that it properly analyses the problem and presents the results. Based on simulated samples with various sample sizes, we observe the behavior of the AR model in the presence of merger and spline function. We do this by simulating a series of sizes $T = (100, 200)$ with merger locations (T/4, T/2, 3T/4) and consider different knot locations including before and after merger under the condition that the number of knots is known in advance. The response series Y 's initial value is assumed 10. With the help of the initial value of Y, we easily generate complete series of the autoregressive model. We also assume that the merger series follows the AR model with different orders (r_h). Before and after the merger, the process follows AR(1) and AR(2) model, respectively.

The average estimates (AE) and related mean square errors (MSE) of the Bayes estimators of the model parameters are reported after obtaining the estimators of the parameters of the proposed model for each generated sample. We use the Gibbs sampling technique for 10000 iterations and burn-in 1000 replications to get the posterior samples from the conditional posterior. For the different sizes of the series with varying merger time and knot locations (T_{11} and T_{12} are knots before the merger, T_{21} knot after the merger), AE of the estimators of the parameters have been summarized in Tables 1-6. Figure 1 to Figure12 show the comparison between the estimation methods.

Table 1 Average estimated values at merger time $(T_m) = T / 4$

Parameters	T=100, T_{11}=15, T_{12}=20, T_{21}=30				T=200, T_{11}=30, T_{12}=40, T_{21}=60			
	OLS	SELF	ELF	ALF	OLS	SELF	ELF	ALF
θ_1 (0.25)	0.2682	0.2679	0.2677	0.2676	0.2499	0.2497	0.0249	0.2495
θ_2 (0.35)	0.3615	0.3609	0.3674	0.3614	0.3392	0.3497	0.0349	0.3491
Φ_{11}(0.4)	0.4112	0.4155	0.4124	0.4112	0.4053	0.4012	0.4015	0.4012
Φ_{12} (0.7)	0.7184	0.7171	0.7163	0.7159	0.7125	0.7089	0.7115	0.7098
Φ_{22} (0.2)	0.2154	0.2121	0.2146	0.2118	0.2123	0.2018	0.2031	0.2002
δ_{11} (0.2)	0.2143	0.2143	0.2147	0.2143	0.2026	0.2036	0.2065	0.2031
δ_{21} (0.3)	0.3195	0.3187	0.3184	0.3183	0.3145	0.3032	0.3016	0.3025
δ_{22} (0.2)	0.2164	0.2194	0.2213	0.2153	0.2123	0.2136	0.2125	0.2112
Ψ_{11} (1.2)	1.2352	1.2386	1.2334	1.2318	1.2154	1.2178	1.2045	1.2027
Ψ_{12} (1.7)	1.7248	1.7273	1.7208	1.7204	1.7012	1.7065	1.7032	1.7012
Ψ_{21} (0.2)	0.2138	0.2139	0.2138	0.2108	0.2101	0.2098	0.2085	0.1998
σ^2 (0.5)	0.5116	0.5191	0.5132	0.5128	0.5025	0.5011	0.5036	0.5021

Table 2 Average estimated values at merger time $(T_m) = T/2$

Parameters	T=100, T11=25, T12=40, T21=70				T=200,T11=70, T12=90, T21=150			
	OLS	SELF	ELF	ALF	OLS	SELF	ELF	ALF
θ_1 (0.25)	0.2682	0.2672	0.2673	0.2672	0.2418	0.2415	0.2414	0.2411
θ_2 (0.35)	0.3612	0.3606	0.3654	0.3611	0.3472	0.3481	0.3415	0.3442
Φ_{11}(0.4)	0.4195	0.4155	0.4122	0.4198	0.3977	0.3911	0.3925	0.3922
Φ_{12} (0.7)	0.7182	0.7171	0.7163	0.7158	0.6991	0.6978	0.6978	0.6985
Φ_{22} (0.2)	0.2152	0.2123	0.2143	0.2113	0.1981	0.1993	0.1965	0.1987
δ_{11} (0.2)	0.2141	0.2142	0.2144	0.2141	0.1965	0.192	0.1925	0.1911
δ_{21} (0.3)	0.3175	0.3185	0.3183	0.3182	0.2995	0.2931	0.2965	0.2941
δ_{22} (0.2)	0.2162	0.2192	0.2198	0.2151	0.1998	0.1982	0.1984	0.1991
Ψ_{11} (1.2)	1.2348	1.2362	1.2312	1.2315	1.1956	1.1952	1.1924	1.1924
Ψ_{12} (1.7)	1.7224	1.7255	1.7205	1.7202	1.6971	1.6962	1.6951	1.6932
Ψ_{21} (0.2)	0.2125	0.2115	0.2125	0.2105	0.1971	0.1981	0.1965	0.1912
σ^2 (0.5)	0.5101	0.5146	0.5126	0.5116	0.4932	0.4957	0.4953	0.4905

Table 3 Average estimated values at merger time $(T_m) = 3T/4$

Parameters	T=100, T11=45, T12=70, T21=90				T=200, T11=110, T12=140, T21=190			
	OLS	SELF	ELF	ALF	OLS	SELF	ELF	ALF
θ_1 (0.25)	0.2411	0.2412	0.2409	0.2405	0.2411	0.2412	0.2409	0.2405
θ_2 (0.35)	0.3462	0.3451	0.3414	0.3412	0.3462	0.3451	0.3414	0.3412
Φ_{11}(0.4)	0.3962	0.3902	0.3913	0.3918	0.3962	0.3902	0.3913	0.3918
Φ_{12} (0.7)	0.6975	0.6966	0.6961	0.6948	0.6975	0.6966	0.6961	0.6948
Φ_{22} (0.2)	0.1919	0.1939	0.1932	0.1925	0.1919	0.1939	0.1932	0.1925
δ_{11} (0.2)	0.1958	0.1917	0.1916	0.1908	0.1958	0.1917	0.1916	0.1908
δ_{21} (0.3)	0.2946	0.2922	0.2952	0.2935	0.2946	0.2922	0.2952	0.2935
δ_{22} (0.2)	0.1981	0.1974	0.1971	0.1981	0.1981	0.1974	0.1971	0.1981
Ψ_{11} (1.2)	1.1949	1.194	1.1912	1.1913	1.1949	1.194	1.1912	1.1913
Ψ_{12} (1.7)	1.6963	1.6953	1.6941	1.6924	1.6963	1.6953	1.6941	1.6924
Ψ_{21} (0.2)	0.1964	0.1972	0.1961	0.1877	0.1964	0.1972	0.1961	0.1877
σ^2 (0.5)	0.4915	0.4931	0.4941	0.4901	0.4915	0.4931	0.4941	0.4901

Table 4 Testing of the hypothesis with varying time, knot, and merger locations

T	T_m	Location of the knots			BF_{12}	BF_{13}	BF_{14}
		T_{11}	T_{12}	T_{21}			
100	T/4=25	15	20	30	1.81E+115	2.40E+208	7.35E+195
	T/2=50	30	40	60	1.30E+115	3.45E+115	3.58E+195
	3T/4=75	45	70	90	3.48E+195	2.86E+166	6.29E+174
200	T/4=50	30	40	60	1.89E+115	3.86E+208	8.73E+195
	T/2=100	70	90	150	1.59E+115	3.76E+115	3.87E+195
	3T/4=150	110	140	190	5.65E+195	4.46E+166	8.78E+174

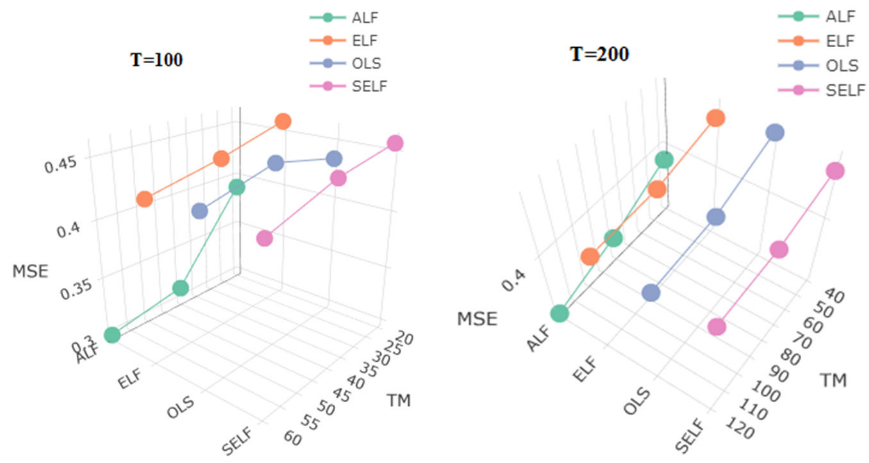

Figure 1 MSE for the parameters θ_1 with varying T and T_m

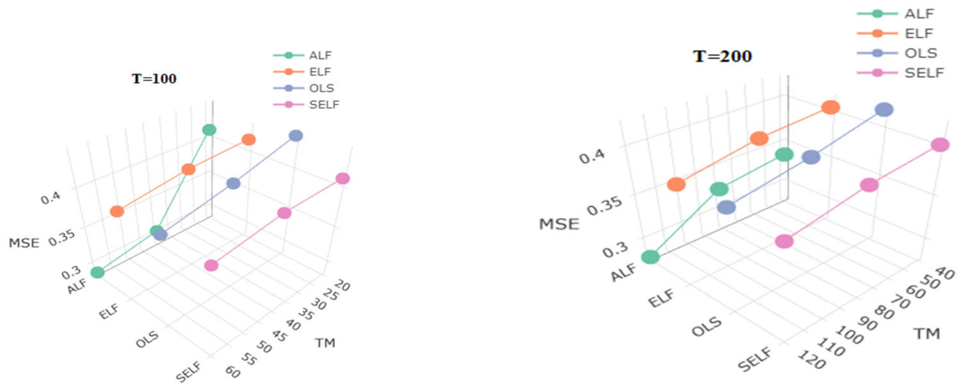

Figure 2 MSE for the parameters θ_2 with varying T and T_m

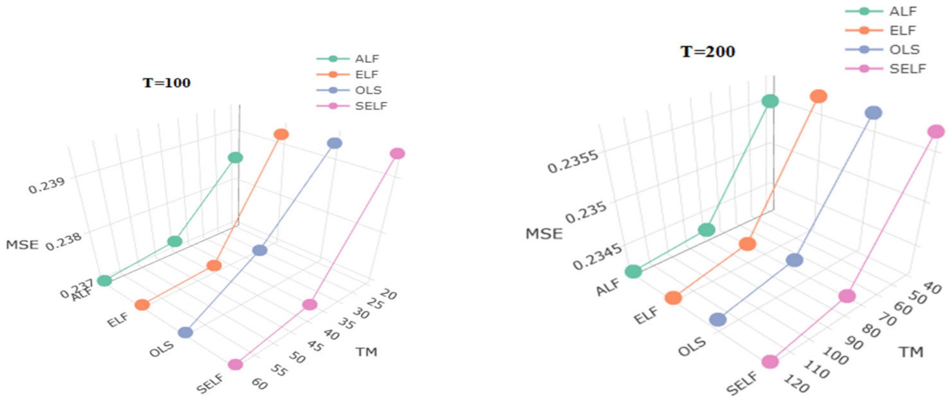

Figure 3 MSE for the parameters Φ_{11} with varying T and T_m

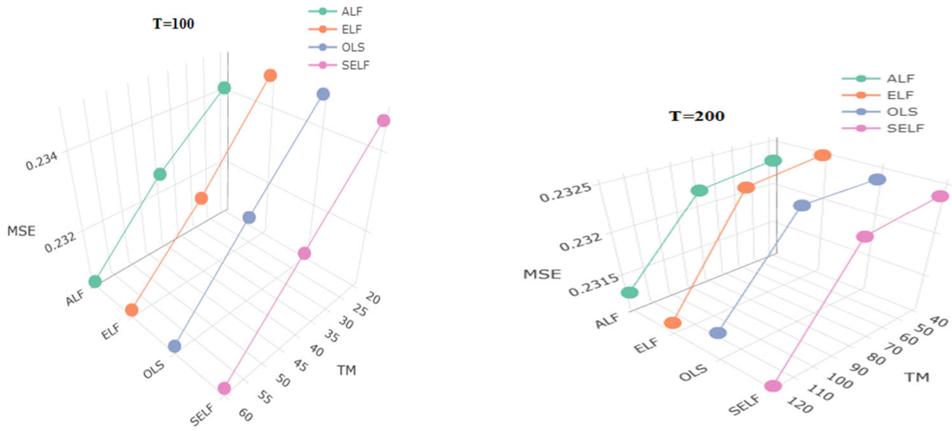

Figure 4 MSE for the parameters Φ_{12} with varying T and T_m

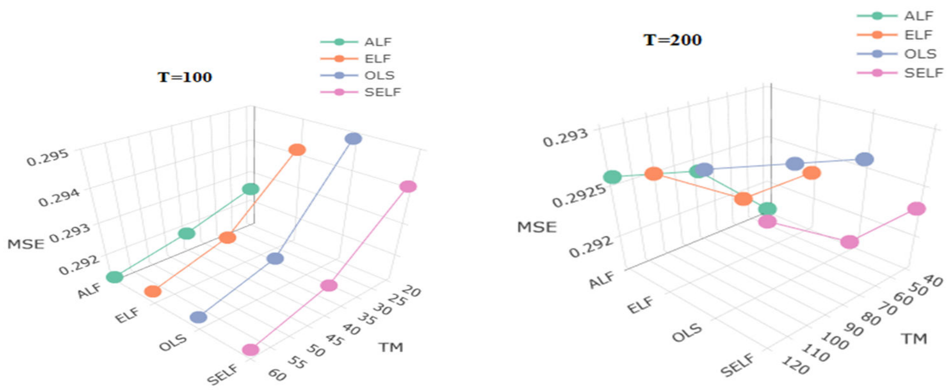

Figure 5 MSE for the parameters Φ_{22} with varying T and T_m

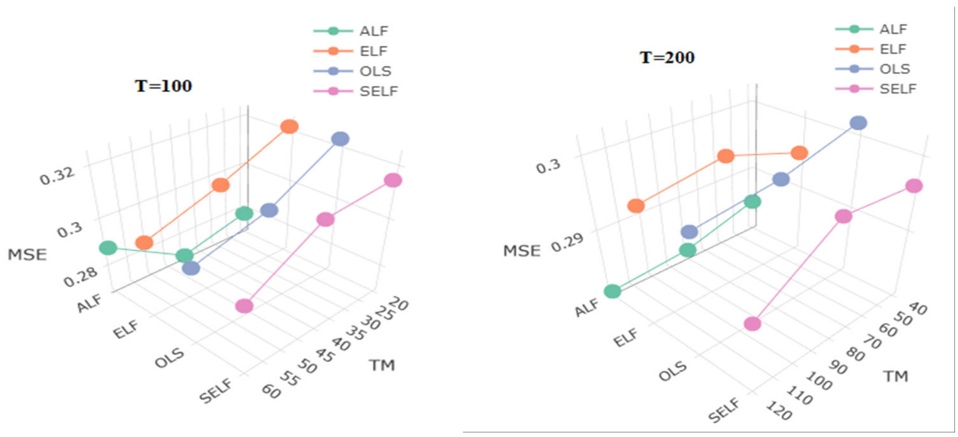

Figure 6 MSE for the parameters δ_{11} with varying T and T_m

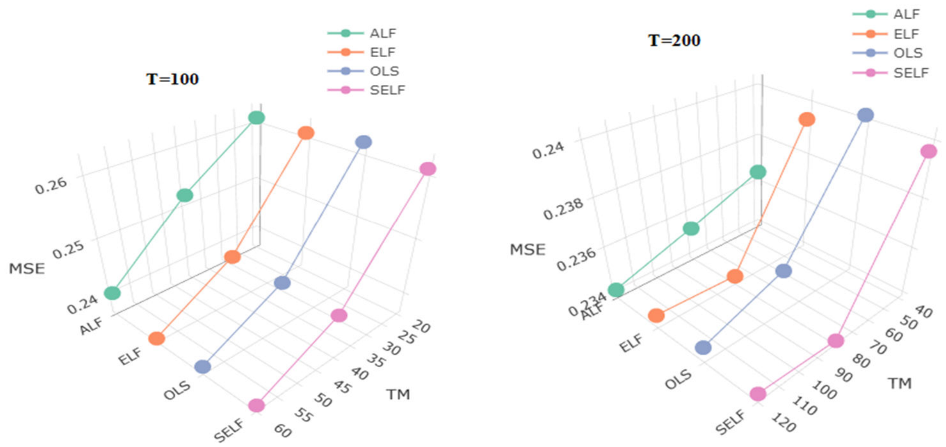

Figure 7 MSE for the parameters δ_{21} with varying T and T_m

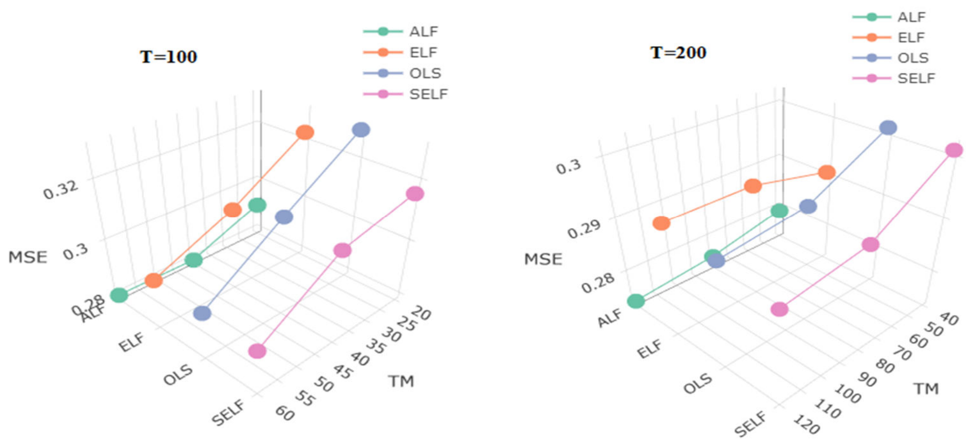

Figure 8 MSE for the parameters δ_{22} with varying T and T_m

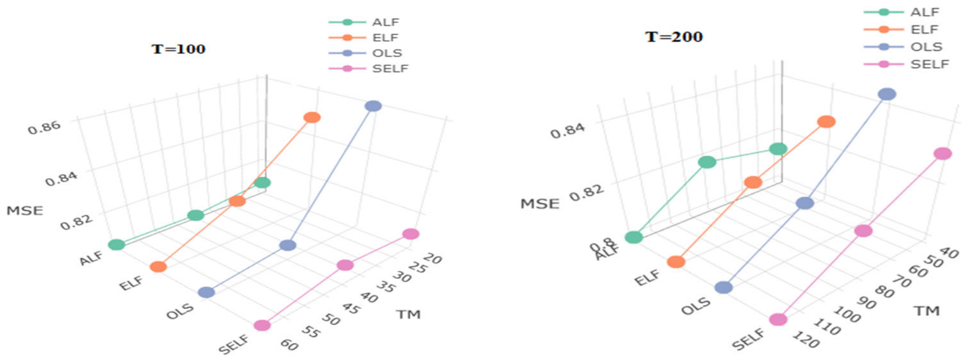

Figure 9 MSE for the parameters Ψ_{11} with varying T and T_m

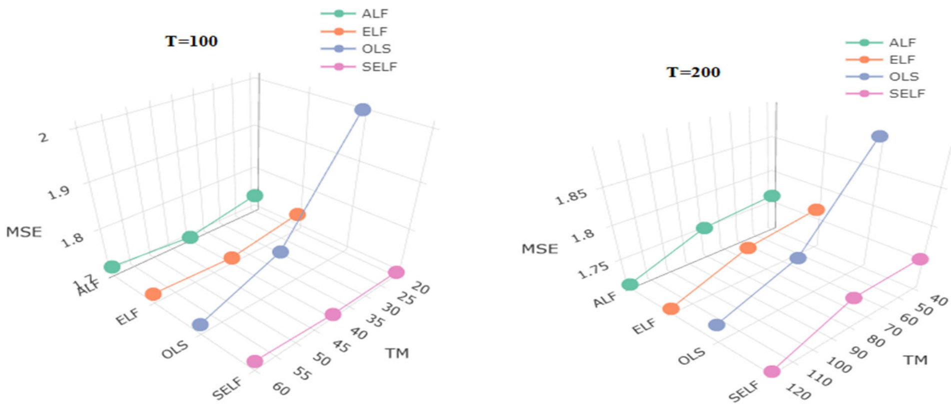

Figure 10 MSE for the parameters Ψ_{12} with varying T and T_m

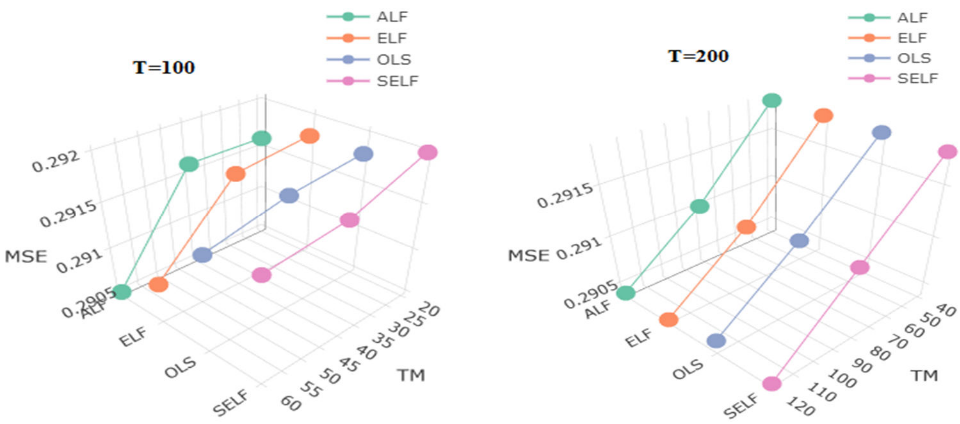

Figure 11 MSE for the parameters Ψ_{21} with varying T and T_m

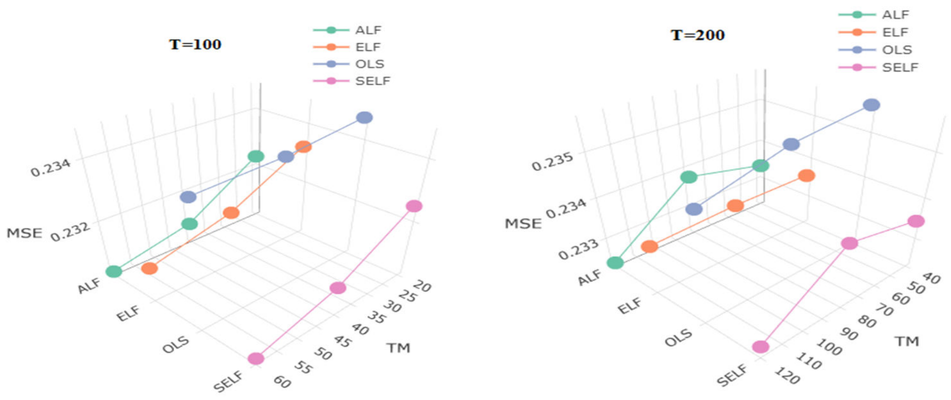

Figure 12 MSE for the parameters σ^2 with varying T and T_m

The performance of the Bayes estimator is compared with OLS. The estimated values of the parameters are obtained by taking the average over all the cycles of the respective parameters and are recorded in Tables 1-3 for different sizes of the series at different merger times with different knot locations, values of MSE of the parameters over all the cycles are also calculated and are compared by Figure 1 to Figure 12. Tables 1-3 and Figure1 to Figure12 observed the following.

(i) The average estimates of the parameters are close to the true values of the parameter. When the series size increases, the average estimated values move closer to the true values and thus provide more efficient estimates for all the model parameters.

(ii) All estimators are efficient because we noted that as the size of the series increases, the MSE of the estimator decreases.

(iii) For all the parameters, the MSE of OLS is higher than Bayes estimators, and also, for most of the parameters, the MSE of ALF is least compared to the remaining estimators. Therefore, ALF under the Bayes estimator outperforms the SELF, ELF and OLS in terms of because ALF considered equal weight to under and over estimation.

(iv) So, we conclude in terms of known prior distribution that the Bayesian procedure provides better performance in comparison to OLS because it contains additional information about the model parameters.

(v) In Table 4, we observed that there is strong evidence to support the presence of mergers in the series. Because of the Bayes factor, as we increased the size of series at different merger times and different knot locations got the increasing Bayes factor values. So after the analysis, we reject the alternative hypothesis and conclude that the merger affects the series positively.

5. Real Data Analysis

Over the previous two decades, the entire Indian banking sector has been consolidated to reap the benefits of mergers and acquisitions**.** Banks' primary and ultimate responsibility is to accelerate the country's economic growth and provide capital for investment**.** The State Bank of India **(**SBI**)** is the country's largest bank**.** SBI combined with five of its associate bank's names, State bank of Hyderabad **(**SBH**)**, State bank of Bikaner & Jaipur **(**SBBJ**)**, State bank of Mysore **(**SBM**)**, State bank of Patiala **(**SBP**)**, State bank of Travancore (SBT**)** including Bharatiya Mahila bank on April 1, 2017**.** The only way to help these banking units is to merge SBI associates into SBI**.** It would not only help their associates' profits, but it will also provide SBI with a good chance to grow its market position in India's untapped market. Payments can be made in a variety of ways in Indian banking. One of the ways to send money is using NEFT. NEFT is an electronic payment system created by the Reserve Bank of India to make it easier for consumers to transfer money from one bank to another in India. It is a secure and efficient way to transfer funds between banks. For real data analysis, we are used the monthly NEFT data series of SBI and its associated banks from March 2010 to October 2021.

In the present study, we are used primary data and taken a data series in the form of value/volume every month for model analysis, where value is the number of money transactions every month (in millions) and volume are the total number of transactions every month in this data set merger time is at 90. We change the data for the merger banks into payment per transaction for data analysis and took the NEFT series as autoregressive and perform the analysis to see how associated bank's mergers influenced the SBI series. First, we fit an autoregressive model for NEFT banking service to find out the order (lag) of SBI and its associated merger banks and then study the inferences. The descriptive statistics and lag of the AR model are shown in Table 5.

To find out the knot in the series, we have made the combination of knots on the basis that some number of knots occurred before the merger and some will occur after the merger. Here, we are considered a maximum of six (3 (before the merger), 3 (after the merger)) knots in the series and then find AIC and BIC values of the series. Based on minimum AIC and BIC values, we identify the number of knots present in the series. As seen in Table 6, one-one knot is present before and after the merger. This shows that the series is having a linear time trend model. Then, we find out the location of knots using AIC and BIC. The most suitable location of the knot before and after the merger is T_{11} = 30 and T_{21} = 95, respectively. It is concluded that the M-ARS model satisfies the merger situation because Bayes factors are too much high, i.e., 3.32E+80, 3.46E+76, and 2.12E+195 to reject the alternative models. After obtaining the lag (order) of each related series, number and location of knots, we use the M-ARS model to estimate the model parameters using the OLS and Bayesian approaches and observe that the estimated value may vary when considering the merger and spline function in the series. From Table 7, we observed that there is a positive impact on the SBI series due to the merger of associated banks. To know the impact of the associated series, the presence of the merged series is tested using the proposed hypotheses and reported in Table 8. Table 8 explains the relationship between associated banks and SBI and shows that bank mergers have a significant impact on the SBI series because the Bayes factor is so much high to reject the alternative hypothesis and conclude that the proposed model is well suitable for this banking series. In Table 7, the performance of the Bayes estimator is compared with OLS. Bayes estimators perform better than OLS, because the estimated value for most of the parameters is much closed to initial estimates and ALF perform better in comparison of other Bayes estimators.

Table 5 Descriptive statistics and order of the NEFT series

Series	Mean	St. deviation	Skewness	Kurtosis	Order
SBI	0.0627	0.0178	1.8360	6.6610	4
SBH	0.0529	0.0104	1.0806	5.9354	2
SBBJ	0.0491	0.0203	3.5720	4.0752	3
SBM	0.0497	0.0111	1.5882	8.8719	3
SBP	0.0707	0.0197	1.4653	5.9649	1
SBT	0.0414	0.0114	4.0519	7.6312	3
M-SBI	0.2972	0.2392	0.0697	1.0399	1

Table 6 Knot selection using information criteria

No. of Knots		Selection criteria	
Before merger	After merger	AIC	BIC
1	1	86.1662	121.1296
1	2	86.6106	121.4739
1	3	86.4363	121.2997
3	3	86.4769	121.2912
3	2	86.6019	121.4601
3	1	86.7576	121.6209
2	3	86.4277	121.2915
2	1	86.7576	121.6209
2	2	86.6020	121.4653

Table 7 OLS and Bayes estimates based on NEFT series

Parameters	SELF	ALF	ELF	OLS
θ_1	0.1103	0.0760	0.1558	0.2663
θ_2	0.0628	0.0691	0.2395	0.2746
Φ_{11}	0.0636	-0.0313	0.6392	-0.1212
Φ_{21}	0.3266	0.3669	0.4618	0.4622
Φ_{22}	0.2281	0.2395	0.4493	0.5635
δ_{11}	0.0080	0.0658	0.3804	-0.9276
δ_{21}	-0.0133	-0.0057	0.2966	-0.7008
δ_{22}	-0.0142	-0.0098	0.3643	0.2889
δ_{23}	-0.0959	-0.1011	0.3441	-0.5287
δ_{24}	0.1735	0.2122	0.4180	4.5956
Ψ_{11}	-0.0012	-0.0018	0.0074	-0.0034
Ψ_{12}	0.0050	0.0053	0.0161	0.0086
Ψ_{21}	0.0017	0.0030	0.0220	0.0147
σ^2	0.0595	0.0656	0.0625	0.0203

Table 8 Testing the hypothesis for NEFT series

T_m (Time of merger)	Spline Knots		BF_{12}	BF_{13}	BF_{14}
	T_{11}	T_{21}			
90	30	95	3.32E+80	3.46E+76	2.12E+195

6. Conclusions

In this paper, a time series model with a polynomial-time trend approximated by a spline function is proposed to describe the merger and acquisition situation. The spline function has the property of approximating non-linear time series with an appropriate degree of polynomial-time trend model. Classical and Bayesian estimation approaches are used to record the estimated values of the M-ARS model parameters. The testing procedure is used to observe the existence of merged series in the acquisition series in presence of knot points. As we know, SBI associate banks have merged into SBI to improve Indian banking performance. We used the NEFT banking data of SBI for analysis purposes. The simulation and empirical analysis verify the model's applicability and purposes. The methodology suggested that under absolute loss function, model perform better as compared to other estimators. This may be happened because ALF considered both under and over estimation equally. This work may be extended for the case of multiple mergers with spline function in both AR and panel merger models. This model may also be useful in various applications when series having non-linear nature with a merger was taken place in the study series.

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