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# Explicit Formulas of Average Run Lengths of Moving Average-Range Control Chart

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**Abstract**

Based on the statistical principles, the control chart detects and controls the production process to meet the required quality. This research aimed to present the explicit formula of average run length for moving average based on range $(MA_R)$ control chart for detecting a change of variation. In addition, the efficiency of the change detection of the $MA_R$ control chart and R chart at different levels of the parameter change are compared. The criteria used for measuring the efficiency included the average run length for the control process (ARL). The research showed that the processes' results were under normal distribution. The performance of the control chart shows that the $MA_R$ chart has lower $ARL_1$ values than the R chart for all change levels. The adaptation results of the proposed control chart to two sets of actual data corresponded to the research results.

**Keywords**: Closed-form formulas, variation, moving average control chart, average run length, range.

## 1. Introduction

Control charts are commonly used tools for controlling process quality, reducing variation, and improving processes to ensure efficiency. Process changes that often occur include average or standard deviation, etc. Therefore, control charts can follow an ongoing production process to monitor data change trends until changes outside of control limits are detected. Control charts are also indicators of production process efficiency and are used to identify causes of data change. They can also be used to standardize product configurations to meet manufacturer and consumer standards. The quality control charts can be divided into two types: the control chart for variables consists of the average control chart (x-bar chart), the standard deviation chart (S chart), and the range control chart (R chart). Both the S and R charts measure subgroup variability. The S chart uses the standard deviation to represent the data spread, and the R chart uses the range. Use the S chart when subgroup sizes are nine or more excellent. S chart uses all the data to calculate the subgroup process standard deviation. They use an

S chart for processes with a high production rate or when data collection is quick and inexpensive. Use the R chart when subgroup sizes are eight or fewer. The second type is control charts for attributes, which are control charts used for detecting the number of defects or nonconformities, which is counting data and is an integer. An example of this type of control chart is the defect proportion control chart (p chart), the number of defect control chart (np chart), and the number of nonconforming products per unit control chart (u chart), etc. (Montgomery 2009).

In 1923, Shewhart (1923) proposed the Shewhart control chart, an effective control chart for detecting significant average changes. However, the small mean change could not be detected. Subsequently, other quality control charts have been developed to detect small changes more efficiently than Shewhart's control charts. Later, in 2004, Khoo developed a moving average control chart (MA) using a simple idea to calculate the MA statistics by giving a width of average ($w$). This control chart is easy to calculate and implement as well as its efficiency suits for small to moderate shifts (see Areepong and Sukparungsee (2013), Chananet et al. (2015), Sukparungsee et al. (2020), Taboran et al. (2020), Saengsura et al. (2022)). In 2016, Adeoti and Olaomi (2016) developed an MA control chart for the standard deviation, so-called the MA-S control chart as a mixed control chart (see also Phantu and Sukparungsee (2020) and Sukparungsee et al. (2021)) and proposed the explicit formulas to determine the average run length (ARL) (see Raweesawat and Sukparungsee 2024) and compare the results in detecting variance changes with the S chart. Recently, Chananet et al. (2024) designed a moving average based on range $(MA_R)$ control chart for process variation based on range value. It was suitable for magnitudes of small sizes. Such research shows that control charts are often used with mean and variance measures such as range and standard deviation. Process variance measurements, such as process consistency checks, are more critical than process averages in some situations. Therefore, a process variability control chart must be developed to restore the process as smoothly as possible.

The most commonly used control chart performance check is ARL, which is divided into two states: in control average run length ($ARL_0$) and out of control average run length ($ARL_1$). The methodology for calculating the ARL used, Monte Carlo simulation (MC), estimates the ARL from a simulation under given circumstances. It is a simple and convenient way to validate the results obtained by other methods. However, such methods have limitations in processing results that are time-consuming. Subsequently, an explicit formula method took less time to calculate. Nevertheless, it may not be found in all cases of the study.

In this research, we have intensively extend from Chananet et al. 2024, which derive a proof of explicit formula of average run length (ARL) for the $MA_R$ control chart to reduce the time-consuming from the previous work. In addition, the performance of the $MA_R$ control chart is compared with the R chart for detecting process variations and applying them to real data. The control chart gives the lowest value $ARL_1$, indicating that the control chart is most effective in detecting variation changes.

## 2. Research Methodology

This research aims to propose the explicit formulas for the ARL of the $MA_R$ control chart. The efficiency of detection of process variation is compared with the classical range chart (R chart), in which the proposed explicit formulas are less time-consuming compared with other methods. The control chart had the lowest $ARL_1$, so detecting the variation changes was the most efficient. This section will explain the methodology in the following details.

### 2.1. Control charts and their properties

#### 2.1.1 R chart

A range chart is a statistical process control (SPC) tool that displays the variation within a data set. It tracks the variation in a process over time and helps identify any changes in the process variance. It plots the range of the data in each subgroup, where the range is calculated from the difference between the highest and lowest values in each subgroup over time. The R chart is suitable if the sample sizes $(n)$ are negligible $(n \leq 10)$. For developing a quality control chart, it is essential to always consider this R chart in conjunction with the x-bar chart, which can be calculated to find the average of the range $(\bar{R})$ as follows:

$$\bar{R} = \frac{\sum_{j=1}^{m} R_j}{m}, \tag{1}$$

where the value of from (1) is the difference between the highest value and the lowest value in sample $j$. The calculation of the upper control limit (UCL) and lower control limit (LCL) is divided into 2 cases: known and unknown parameters $\sigma$. For the latter case, the parameter must be estimated. Montgomery (2009) stated that in the process variability, an unbiased estimator of $\sigma$, is $\hat{\sigma} = \bar{R} / d_2$ for the R chart. Consequently, the control limits are as follows:

1) Known $\sigma$

$$\begin{aligned} UCL &= d_2\sigma + 3d_3\sigma = D_2\sigma, \\ CL &= d_2\sigma, \\ LCL &= d_2\sigma - 3d_3\sigma = D_1\sigma, \end{aligned} \tag{2}$$

where the values from (2), $D_1 = (d_2 - 3d_3)$ and $D_2 = (d_2 + 3d_3)$, are factors of control limits and depend on the sample size $(n)$. In addition, the values of $d_2$ and $d_3$ are also the factors of control limits which the tables of the factor of control limits are addressed in several quality control textbooks.

2) Unknown $\sigma$, then an estimate $\hat{\sigma} = \bar{R} / d_2$,

$$\begin{aligned} UCL &= \bar{R} + 3\frac{d_3}{d_2}\bar{R} = D_4\bar{R}, \\ CL &= \bar{R}, \\ LCL &= \bar{R} - 3\frac{d_3}{d_2}\bar{R} = D_3\bar{R}. \end{aligned} \tag{3}$$

Then, the values from (3), $D_3 = \left(1 - 3\frac{d_3}{d_2}\right)$ and $D_4 = \left(1 + 3\frac{d_3}{d_2}\right)$. In addition, their values are a constant found in the factor of control limits as well as $D_1$ and $D_2$.

#### 2.1.2 Moving average control chart

In the moving average control chart, the width $(w)$ and the statistics of the MA control chart (Khoo 2004) at $i$ are calculated from the moving average at each $w$. There are two cases as follows:

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + \ldots + X_1}{i} & ; i < w \\ \dfrac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{w} & ; i \geq w, \end{cases} \tag{4}$$

where $w$ is the width of the MA control chart, and the mean and variance of statistics $MA_i$ are shown in Equations (5) and (6):

$$E\left(MA_i\right) = \mu_0, \tag{5}$$

and

$$Var\left(MA_i\right) = \begin{cases} \dfrac{\sigma^2}{i}, & i < w \\ \dfrac{\sigma^2}{w}, & i \geq w. \end{cases} \tag{6}$$

Therefore, the control limits of the MA control chart are as follows:

$$UCL / LCL = \begin{cases} \mu_0 \pm \dfrac{3\sigma}{\sqrt{i}} & ; \; i < w \\ \mu_0 \pm \dfrac{3\sigma}{\sqrt{w}} & ; \; i \geq w, \end{cases} \tag{7}$$

where $\mu_0$ is the mean, and $\sigma$ is the standard deviation of the process from (7) when it is under control.

### 2.1.3 Moving average-range control chart

The moving average based on range $(MA_R)$ control chart can be used to detect a change in the process mean and process variability (i.e., Chananet et al. 2024). The $MA_R$ control chart is implemented to detect a change in process variation based on the range value, which depends on the sample size $(n)$. The $MA_R$ statistic of width $w$ at times $i$ is calculated as (8)

$$MA_{R_i} = \begin{cases} \dfrac{R_i + R_{i-1} + R_{i-2} + \ldots}{i} & ; i < w \\ \dfrac{R_i + R_{i-1} + \ldots + R_{i-w+1}}{w} & ; i \geq w, \end{cases} \tag{8}$$

where $R_j$ is the range of each sample number. The $MA_R$ statistics from (4) can be rewritten as follows:

$$MA_{R_i} = \begin{cases} \dfrac{\sum_{j=1}^{i} R_j}{i} & ; i < w \\ \dfrac{\sum_{j=i-w+1}^{i} R_j}{w} & ; i \geq w. \end{cases} \tag{9}$$

The expectation of the $MA_R$ statistics when $i < w,$ is presented in (10),

$$E(MA_{R_i}) = E\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) = \frac{1}{i}\sum_{j=1}^{i} E\left(R_j\right) = d_2\sigma. \tag{10}$$

Also, the expectation of the $MA_R$ statistics, when $i \geq w,$ shown in (11),

$$E(MA_{R_i}) = E\left(\frac{1}{w}\sum_{j=i-w+1}^{w} R_j\right) = \frac{1}{w}\sum_{j=i-w+1}^{w} E\left(R_j\right) = d_2\sigma. \tag{11}$$

The variance of the $MA_R$ statistics, when $i < w,$ is presented in (12),

$$Var(MA_{R_i}) = Var\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) = \frac{1}{i^2}\sum_{j=1}^{i} Var\left(R_j\right) = \frac{d_3^2\sigma^2}{i}. \tag{12}$$

Also, the variance of the $MA_R$ statistics, when $i \geq w,$ shown in (13),

$$Var(MA_{R_i}) = Var\left(\frac{1}{w}\sum_{j=i-w+1}^{i} R_j\right) = \frac{1}{w^2}\sum_{j=i-w+1}^{i} Var\left(R_j\right) = \frac{d_3^2\sigma^2}{w}. \tag{13}$$

Therefore, the upper control limit (UCL) and lower control limit (LCL) of the $MA_R$ chart can be calculated in two cases following:

1) Known $\sigma$

1.1) when $i < w,$

$$UCL = d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma + \frac{3d_3\sigma}{\sqrt{i}} = D_6^*\sigma$$

$$CL = d_2\sigma$$

$$LCL = d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma - \frac{3d_3\sigma}{\sqrt{i}} = D_5^*\sigma,$$

where $D_5^* = \left(d_2 - 3\frac{d_3}{\sqrt{i}}\right)$ and $D_6^* = \left(d_2 + 3\frac{d_3}{\sqrt{i}}\right),$ they are the control limits factor.

1.2) when $i \geq w,$

$$UCL = d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma + \frac{3d_3\sigma}{\sqrt{w}} = D_8^*\sigma$$

$$CL = d_2\sigma$$

$$LCL = d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma - \frac{3d_3\sigma}{\sqrt{w}} = D_7^*\sigma,$$

where $D_7^* = \left(d_2 - \frac{3d_3}{\sqrt{w}}\right)$ and $D_8^* = \left(d_2 + \frac{3d_3}{\sqrt{w}}\right),$ they are the factor of control limits from the proposed chart.

2) Unknown $\sigma$

2.1) when $i < w,$

$$UCL = \overline{R} + 3\overline{R}\frac{d_3}{d_2\sqrt{i}} = D_{10}^*\overline{R}$$

$$CL = d_2\sigma$$

$$LCL = \overline{R} - 3\overline{R}\frac{d_3}{d_2\sqrt{i}} = D_9^*\overline{R},$$

where $D_9^* = \left(1 - 3\frac{d_3}{d_2\sqrt{i}}\right)$ and $D_{10}^* = \left(1 + 3\frac{d_3}{d_2\sqrt{i}}\right).$

2.2) when $i \geq w$,

$$UCL = \overline{R} + 3\overline{R}\frac{d_3}{d_2\sqrt{w}} = D_{12}^{*}\overline{R}$$

$$CL = \overline{R}$$

$$LCL = \overline{R} - 3\overline{R}\frac{d_3}{d_2\sqrt{w}} = D_{11}^{*}\overline{R},$$

where $D_{11}^{*} = \left(1 - 3\frac{d_3}{d_2\sqrt{w}}\right)$ and $D_{12}^{*} = \left(1 + 3\frac{d_3}{d_2\sqrt{w}}\right)$.

### 2.2. Performance measurement

Control chart performance is traditionally quantified regarding the Average Run Length (ARL). Run Length is defined as the number of observations in the process from the start of the control to the first out-of-control signal. After this observation, the counting process is stopped, and the calculation of the run length is recommended for the following in-control observation. Accordingly, ARL is the mean of the run length of the realized control chart. They were divided into in-control processors ($ARL_0$) and out of control processors ($ARL_1$). The minimum $ARL_1$ indicated the most efficient control chart. The $ARL_0$ can be calculated as in (14), and the $ARL_1$ can be calculated as in (15).

$$ARL_0 = \frac{1}{\alpha}, \tag{14}$$

where $\alpha$ is the probability that a process is found out of the control limit when the process does not change and

$$ARL_1 = \frac{1}{1-\beta}, \tag{15}$$

where $\beta$ is the probability that the process is found to be in control state limit when the process changes.

## 3. Explicit formulas for determining ARL for $MA_R$

In this section, the explicit formula of the $MA_R$ control chart is proposed. The performance for control charts is ARL, which consists of 2 types: $ARL_0$ and $ARL_1$. The explicit formulas for evaluating $ARL_0$ and $ARL_1$ can be analytically derived by the central limit theorem. Given o.o.c. is an out-of-control limit and

Let $ARL = n$,

$$\frac{1}{ARL} = \left(\frac{1}{n}\right)\text{P(o.o.c signal at time } i < w) + \left(\frac{n-w+1}{n}\right)\text{P(o.o.c signal at time } i \geq w),$$

$$= \frac{1}{n}\left[\sum_{i=1}^{w-1}\left(\text{P}\left(\frac{1}{i}\sum_{j=1}^{i}R_j > UCL_i\right) + \text{P}\left(\frac{1}{i}\sum_{j=1}^{i}R_j < LCL_i\right)\right)\right] +$$

$$\left(\frac{n-w+1}{n}\right)\left[\text{P}\left(\frac{1}{w}\sum_{j=i-w+1}^{i}R_j > UCL_w\right) + \text{P}\left(\frac{1}{w}\sum_{j=i-w+1}^{i}R_j < LCL_w\right)\right].$$

Assume that $Z_1 = \dfrac{\sum_{j=1}^{i} R_j - \bar{R}}{\dfrac{d_3\sigma}{\sqrt{i}}}$ and $Z_2 = \dfrac{\sum_{j=1}^{i} R_j - \bar{R}}{\dfrac{d_3\sigma}{\sqrt{w}}}$. Therefore,

$$\frac{1}{ARL} = \frac{1}{n}\left[\sum_{i=1}^{w}\left(\mathrm{P}\left(Z_1 > i\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right) + \mathrm{P}\left(Z_1 < i\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right)\right)\right] +$$

$$\left(\frac{n-w+1}{n}\right)\left[\mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right) + \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right)\right].$$

Then, given $A = \sum_{i=1}^{w}\left(\mathrm{P}\left(Z_1 > i\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right) + \mathrm{P}\left(Z_1 < i\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right)\right)$,

and $B = \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right) + \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right)$.

Therefore,

$$\frac{1}{n} = \left(\frac{1}{n}\right)(A) + \left(\frac{n-w+1}{n}\right)(B),$$

$$\frac{1-A}{n} = \left(\frac{n-w+1}{n}\right)(B),$$

$$(1-A)B^{-1} = \left(\frac{n-w+1}{n}\right)n,$$

$$n = (1-A)B^{-1} + w - 1,$$

$$ARL = (1-A)B^{-1} + w - 1,$$

$$ARL = n = \left\{1 - \sum_{i=1}^{w}\left(\mathrm{P}\left(Z_1 > i\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right) + \mathrm{P}\left(Z_1 < i\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{i}}}\right)\right)\right)\right\} \times$$

$$\left\{\mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma + \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right) + \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma - \frac{3d_3\sigma}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma}{\sqrt{w}}}\right)\right)\right\}^{-1} + w - 1.$$

**Proposition I.** The control process is given parameter $\sigma = \sigma_0$. The explicit formula for the ARL0 of the $MA_R$ control chart is as follows:

$$ARL_0 = n = \left\{1 - \sum_{i=1}^{w}\left(\mathrm{P}\left(Z_1 > i\left(\frac{d_2\sigma_0 + \frac{3d_3\sigma_0}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma_0}{\sqrt{i}}}\right)\right) + \mathrm{P}\left(Z_1 < i\left(\frac{d_2\sigma_0 - \frac{3d_3\sigma_0}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma_0}{\sqrt{i}}}\right)\right)\right)\right\} \times$$

$$\left\{\mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma_0 + \frac{3d_3\sigma_0}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma_0}{\sqrt{w}}}\right)\right) + \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma_0 - \frac{3d_3\sigma_0}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma_0}{\sqrt{w}}}\right)\right)\right\}^{-1} + w - 1. \quad (16)$$

**Proposition II.** The process is in an out of control state with parameter $\sigma = \sigma_1$ and $\sigma_1 = \delta\sigma_0$. Where $\delta$ is a magnitude of the shift, the explicit formula for ARL1 of the $MA_R$ control chart can be written as follows:

$$ARL_1 = n = \left\{1 - \sum_{i=1}^{w}\left(\mathrm{P}\left(Z_1 > i\left(\frac{d_2\sigma_1 + \frac{3d_3\sigma_1}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma_1}{\sqrt{i}}}\right)\right) + \mathrm{P}\left(Z_1 < i\left(\frac{d_2\sigma_1 - \frac{3d_3\sigma_1}{\sqrt{i}} - \bar{R}}{\frac{3d_3\sigma_1}{\sqrt{i}}}\right)\right)\right)\right\} \times$$

$$\left\{\mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma_1 + \frac{3d_3\sigma_1}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma_1}{\sqrt{w}}}\right)\right) + \mathrm{P}\left(Z_2 > w\left(\frac{d_2\sigma_1 - \frac{3d_3\sigma_1}{\sqrt{w}} - \bar{R}}{\frac{3d_3\sigma_1}{\sqrt{w}}}\right)\right)\right\}^{-1} + w - 1. \quad (17)$$

## 4. Numerical Results

This section presents the numerical results of the ARL of the $MA_R$ chart obtained using the ARL approximation previously proposed on (16) and (17). Assumed that the process in control is normally distributed with $N(\mu, \sigma^2)$, and when it is out of control, it is normally distributed as $N(\mu, \delta\sigma^2)$ for $n = 5,10$ and 15. The magnitude of shift values is given as $\delta = \sigma_1 / \sigma_0$ when $\delta$ is 1.00, 1.05, …, 3.00. It is assumed that $\mu = 0$ and $\sigma_0^2 = 1$. Let in-control process, the ARL0 of the control chart are approximately 200, 370, and 500. The span $(w)$ values are 2, 3, 4, 5, 10 and 15. A comparison of the performance of the $MA_R$ and R chart is divided into 3 cases as follows: case 1 considers the ARL0 equal to 200 for sample size $(n)$ equal to 5, 10, and 15, as shown in Tables 1-3, respectively. On Table 1, the $MA_R$ control chart with small subgroup $n = 5$ outperform to R chart for all magnitudes of change as well as the case of medium $n = 10$ and large subgroups $n = 15$ as shown on Tables 2 and 3. Next, case 2 determines the ARL0 equal to 370 for sample size $(n)$ equal to 5, 10, and 15, as shown in Tables 4-6, respectively. In addition, the performance of $MA_R$ control chart will increase as the span $w$ decrease when the magnitudes of change are increased while the sizes of subgroup is not effected to the performance of the proposed chart. The comparison of that point of view can show as

Table 1 vs Table 4 when $n$ = 5, Table 2 vs Table 5 when $n$ = 10, and Table 3 vs Table 6. Finally, case 3, given $ARL_0$ equal to 500 for sample size $(n)$ equal to 5, 10, and 15, is shown in Tables 7-9, respectively. On the other hand, the variation studies of the performance of the $MA_R$ control chart as $ARL_0$ = 500, they do not matter to the performance of monitoring in change of process variation as shown on Tables 7, 8 and 9 when $n$ = 5, 10 and 15, respectively. The $MA_R$ control chart has lower $ARL_1$ values than the R chart for all change levels. When the process change is small, the span $(w)$ values must be significant to maintain $ARL_1$. On the other hand, when the process changes large, the value of span $(w)$ values should be small to make $ARL_1$ the lowest.

**Table 1** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 200 and $n$ = 5

| Shift $(\delta)$ | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 |
| 1.05 | 140.3230 | 128.9800 | 119.9020 | 112.2770 | 105.7180 | 82.8149 | **69.2185** |
| 1.10 | 85.7641 | 69.0309 | 58.1944 | 50.4868 | 44.7215 | 29.7648 | **24.3516** |
| 1.15 | 52.3257 | 38.0813 | 30.2019 | 25.2166 | 21.8402 | 14.9467 | **13.9421** |
| 1.20 | 33.4372 | 22.7953 | 17.5546 | 14.5328 | 12.6635 | **9.8979** | 10.6325 |
| 1.25 | 22.5744 | 14.8071 | 11.3299 | 9.4963 | 8.4774 | **7.7561** | 9.1353 |
| 1.50 | 6.0559 | 4.0426 | 3.4852 | **3.3876** | 3.4806 | 4.5542 | 5.3930 |
| 1.75 | 3.0648 | 2.3127 | **2.2437** | 2.3489 | 2.4997 | 3.0542 | 3.2039 |
| 2.00 | 2.0960 | **1.7519** | 1.7911 | 1.8955 | 1.9949 | 2.1997 | 2.2178 |
| 2.25 | 1.6724 | **1.4943** | 1.5512 | 1.6252 | 1.6799 | 1.7511 | 1.7533 |
| 2.50 | 1.4518 | **1.3512** | 1.4024 | 1.4503 | 1.4788 | 1.5046 | 1.5049 |
| 2.75 | 1.3233 | **1.2623** | 1.3034 | 1.3337 | 1.3486 | 1.3587 | 1.3588 |
| 3.00 | 1.2425 | **1.2031** | 1.2349 | 1.2542 | 1.2623 | 1.2665 | 1.2665 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 2** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 200 and $n$ = 10

| Shift $(\delta)$ | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 |
| 1.05 | 128.4350 | 111.3620 | 98.7281 | 88.8339 | 80.8394 | 56.5641 | **44.8931** |
| 1.10 | 68.1749 | 49.2543 | 38.6542 | 31.9023 | 27.2972 | 17.5576 | **15.5777** |
| 1.15 | 37.2796 | 24.0558 | 17.8840 | 14.4638 | 12.4167 | **9.6097** | 10.4868 |
| 1.20 | 22.0938 | 13.4479 | 9.9361 | 8.2297 | 7.3642 | **7.1777** | 8.7067 |
| 1.25 | 14.1825 | 8.4610 | 6.4154 | 5.5712 | **5.2570** | 6.0446 | 7.4954 |
| 1.50 | 3.5765 | 2.5001 | **2.3747** | 2.4728 | 2.6250 | 3.1402 | 3.2432 |
| 1.75 | 1.9217 | **1.6227** | 1.6762 | 1.7584 | 1.8177 | 1.8853 | 1.8866 |
| 2.00 | 1.4264 | **1.3288** | 1.3744 | 1.4057 | 1.4192 | 1.4262 | 1.4262 |
| 2.25 | 1.2264 | **1.1913** | 1.2159 | 1.2260 | 1.2287 | 1.2294 | 1.2294 |
| 2.50 | 1.1314 | **1.1177** | 1.1298 | 1.1330 | 1.1336 | 1.1337 | 1.1337 |
| 2.75 | 1.0815 | **1.0757** | 1.0816 | 1.0827 | 1.0828 | 1.0828 | 1.0828 |
| 3.00 | 1.0533 | **1.0506** | 1.0536 | 1.0540 | 1.0540 | 1.0540 | 1.0540 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 3** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 200 and $n$ = 15

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w = 2$ | $w = 3$ | $w = 4$ | $w = 5$ | $w = 10$ | $w = 15$ |
| 1.00 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 | 200.0410 |
| 1.05 | 120.5710 | 100.5020 | 86.4464 | 75.9280 | 67.7508 | 44.8350 | **35.1310** |
| 1.10 | 58.6265 | 39.8219 | 30.1293 | 24.3246 | 20.5696 | 13.6154 | **13.0242** |
| 1.15 | 30.1843 | 18.3292 | 13.3012 | 10.7294 | 9.3173 | **8.0954** | 9.4647 |
| 1.20 | 17.2241 | 9.9867 | 7.3652 | 6.2388 | **5.7685** | 6.3548 | 7.8600 |
| 1.25 | 10.8078 | 6.2612 | 4.8571 | 4.3942 | **4.3174** | 5.3785 | 6.4869 |
| 1.50 | 2.7211 | 2.0253 | **2.0199** | 2.1313 | 2.2433 | 2.4530 | 2.4649 |
| 1.75 | 1.5549 | **1.4046** | 1.4559 | 1.4957 | 1.5137 | 1.5233 | 1.5233 |
| 2.00 | 1.2273 | **1.1913** | 1.2135 | 1.2211 | 1.2228 | 1.2231 | 1.2231 |
| 2.25 | 1.1057 | **1.0964** | 1.1038 | 1.1051 | 1.1053 | 1.1053 | 1.1053 |
| 2.50 | 1.0536 | **1.0510** | 1.0534 | 1.0537 | 1.0537 | 1.0537 | 1.0537 |
| 2.75 | 1.0292 | **1.0283** | 1.0292 | 1.0292 | 1.0292 | 1.0292 | 1.0292 |
| 3.00 | 1.0169 | **1.0166** | 1.0169 | 1.0169 | 1.0169 | 1.0169 | 1.0169 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 4** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 370 and $n$ = 5

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w = 2$ | $w = 3$ | $w = 4$ | $w = 5$ | $w = 10$ | $w = 15$ |
| 1.00 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 |
| 1.05 | 246.5320 | 224.1790 | 206.5610 | 191.9180 | 179.4200 | 136.2650 | **110.6810** |
| 1.10 | 140.6390 | 110.7480 | 91.8105 | 78.4999 | 68.6041 | 42.8262 | **32.8674** |
| 1.15 | 80.7148 | 57.1306 | 44.3198 | 36.2611 | 30.7928 | 19.1580 | **16.5504** |
| 1.20 | 49.0197 | 32.3575 | 24.2480 | 19.5584 | 16.6145 | **11.6604** | 11.7999 |
| 1.25 | 31.7130 | 20.0591 | 14.8641 | 12.0837 | 10.4813 | **8.6854** | 9.8610 |
| 1.50 | 7.3995 | 4.7131 | 3.9215 | **3.7212** | 3.7676 | 4.8641 | 5.8177 |
| 1.75 | 3.4773 | 2.5209 | **2.3970** | 2.4899 | 2.6455 | 3.2733 | 3.4612 |
| 2.00 | 2.2788 | **1.8511** | 1.8769 | 1.9865 | 2.0965 | 2.3392 | 2.3633 |
| 2.25 | 1.7716 | **1.5534** | 1.6093 | 1.6907 | 1.7533 | 1.8397 | 1.8427 |
| 2.50 | 1.5128 | **1.3909** | 1.4448 | 1.4987 | 1.5320 | 1.5636 | 1.5641 |
| 2.75 | 1.3640 | **1.2910** | 1.3355 | 1.3700 | 1.3876 | 1.4000 | 1.4001 |
| 3.00 | 1.2713 | **1.2247** | 1.2596 | 1.2818 | 1.2914 | 1.2966 | 1.2966 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 5** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0 = 370$ and $n = 10$

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 |
| 1.05 | 223.0820 | 190.2430 | 166.4390 | 148.0490 | 133.3290 | 89.0911 | **67.6091** |
| 1.10 | 109.3030 | 76.6795 | 58.8120 | 47.5431 | 39.8763 | 23.2628 | **18.9805** |
| 1.15 | 55.9383 | 34.7694 | 25.0410 | 19.6470 | 16.3748 | **11.2359** | 11.5515 |
| 1.20 | 31.4098 | 18.2777 | 12.9750 | 10.3514 | 8.9534 | **7.9011** | 9.3389 |
| 1.25 | 19.2733 | 10.9193 | 7.9111 | 6.6058 | **6.0409** | 6.5095 | 8.0308 |
| 1.50 | 4.2011 | 2.7758 | **2.5629** | 2.6399 | 2.7960 | 3.3927 | 3.5298 |
| 1.75 | 2.0990 | **1.7124** | 1.7586 | 1.8497 | 1.9197 | 2.0060 | 2.0079 |
| 2.00 | 1.4998 | **1.3738** | 1.4234 | 1.4607 | 1.4777 | 1.4870 | 1.4870 |
| 2.25 | 1.2632 | **1.2178** | 1.2461 | 1.2585 | 1.2620 | 1.2630 | 1.2630 |
| 2.50 | 1.1520 | **1.1343** | 1.1486 | 1.1525 | 1.1533 | 1.1534 | 1.1534 |
| 2.75 | 1.0939 | **1.0864** | 1.0935 | 1.0949 | 1.0950 | 1.0951 | 1.0951 |
| 3.00 | 1.0612 | **1.0577** | 1.0614 | 1.0619 | 1.0619 | 1.0619 | 1.0619 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 6** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0 = 370$ and $n = 15$

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 | 370.3980 |
| 1.05 | 207.8520 | 169.8180 | 143.7900 | 124.5930 | 109.8100 | 68.7062 | **50.9066** |
| 1.10 | 92.7534 | 60.9126 | 44.8579 | 35.3197 | 29.1445 | 17.1815 | **15.1219** |
| 1.15 | 44.5737 | 25.9068 | 18.0931 | 14.0702 | 11.8022 | **9.1108** | 10.2252 |
| 1.20 | 24.0492 | 13.2192 | 9.2974 | 7.5509 | **6.7426** | 6.8718 | 8.4217 |
| 1.25 | 14.3998 | 7.8480 | 5.7848 | 5.0321 | **4.8117** | 5.7689 | 7.0067 |
| 1.50 | 3.1229 | 2.1993 | **2.1518** | 2.2637 | 2.3888 | 2.6491 | 2.6666 |
| 1.75 | 1.6632 | **1.4644** | 1.5188 | 1.5663 | 1.5893 | 1.6026 | 1.6027 |
| 2.00 | 1.2691 | **1.2208** | 1.2470 | 1.2567 | 1.2590 | 1.2595 | 1.2595 |
| 2.25 | 1.1248 | **1.1122** | 1.1213 | 1.1231 | 1.1233 | 1.1233 | 1.1233 |
| 2.50 | 1.0632 | **1.0596** | 1.0627 | 1.0630 | 1.0631 | 1.0631 | 1.0631 |
| 2.75 | 1.0344 | **1.0332** | 1.0343 | 1.0344 | 1.0344 | 1.0344 | 1.0344 |
| 3.00 | 1.0198 | **1.0194** | 1.0198 | 1.0198 | 1.0198 | 1.0198 | 1.0198 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 7** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 500 and $n$ = 5

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 |
| 1.05 | 324.2970 | 293.3570 | 269.1610 | 249.1630 | 232.1650 | 173.8850 | **139.5040** |
| 1.10 | 178.8650 | 139.4200 | 114.6960 | 97.4259 | 84.6333 | 51.3661 | **38.3112** |
| 1.15 | 99.6701 | 69.6520 | 53.4946 | 43.3703 | 36.5061 | 21.7447 | **18.0908** |
| 1.20 | 59.0699 | 38.4183 | 28.4341 | 22.6650 | 19.0303 | 12.6810 | **12.4361** |
| 1.25 | 37.4386 | 23.2871 | 17.0032 | 13.6280 | 11.6610 | **9.1938** | 10.2294 |
| 1.50 | 8.1643 | 5.0850 | 4.1573 | **3.8967** | 3.9146 | 5.0112 | 6.0186 |
| 1.75 | 3.7004 | 2.6301 | **2.4748** | 2.5595 | 2.7163 | 3.3787 | 3.5869 |
| 2.00 | 2.3747 | **1.9015** | 1.9193 | 2.0307 | 2.1456 | 2.4075 | 2.4349 |
| 2.25 | 1.8228 | **1.5829** | 1.6377 | 1.7225 | 1.7890 | 1.8834 | 1.8868 |
| 2.50 | 1.5439 | **1.4106** | 1.4655 | 1.5223 | 1.5580 | 1.5928 | 1.5932 |
| 2.75 | 1.3846 | **1.3051** | 1.3511 | 1.3877 | 1.4067 | 1.4204 | 1.4205 |
| 3.00 | 1.2857 | **1.2354** | 1.2718 | 1.2953 | 1.3057 | 1.3115 | 1.3115 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 8** Comparison $ARL_1$ of R and $MA_R$ control chart when given $ARL_0$ = 500 and $n$ = 10

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 |
| 1.05 | 291.8340 | 246.9160 | 214.6880 | 189.9630 | 170.2690 | 111.4930 | **82.9969** |
| 1.10 | 137.5570 | 95.2035 | 72.2635 | 57.8765 | 48.1137 | 26.8594 | **21.0466** |
| 1.15 | 68.1988 | 41.6678 | 29.5788 | 22.8893 | 18.8197 | 12.1786 | **12.1259** |
| 1.20 | 37.3141 | 21.2675 | 14.8210 | 11.6179 | 9.8858 | **8.2867** | 9.6488 |
| 1.25 | 22.4033 | 12.3912 | 8.7867 | 7.1983 | **6.4795** | 6.7410 | 8.2827 |
| 1.50 | 4.5495 | 2.9240 | **2.6600** | 2.7231 | 2.8790 | 3.5146 | 3.6703 |
| 1.75 | 2.1934 | **1.7581** | 1.7993 | 1.8942 | 1.9694 | 2.0659 | 2.0681 |
| 2.00 | 1.5380 | **1.3963** | 1.4475 | 1.4879 | 1.5067 | 1.5174 | 1.5174 |
| 2.25 | 1.2821 | **1.2310** | 1.2611 | 1.2747 | 1.2787 | 1.2799 | 1.2799 |
| 2.50 | 1.1626 | **1.1426** | 1.1580 | 1.1624 | 1.1632 | 1.1634 | 1.1634 |
| 2.75 | 1.1003 | **1.0918** | 1.0995 | 1.1010 | 1.1012 | 1.1012 | 1.1012 |
| 3.00 | 1.0652 | **1.0614** | 1.0653 | 1.0659 | 1.0659 | 1.0659 | 1.0659 |

Note: The bold number on each row is the smallest $ARL_1$ for each shift level.

**Table 9** Comparison ARL$_1$ of R and $MA_R$ control chart when given ARL$_0$ = 500 and n = 15

| Shift ($\delta$) | R chart | $MA_R$ control chart | | | | | |
|---|---|---|---|---|---|---|---|
| | | $w=2$ | $w=3$ | $w=4$ | $w=5$ | $w=10$ | $w=15$ |
| 1.00 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 | 500.1140 |
| 1.05 | 270.9330 | 219.2750 | 184.3210 | 158.7310 | 139.1230 | 84.9390 | **61.4257** |
| 1.10 | 116.0120 | 75.0161 | 54.5720 | 42.4875 | 34.6765 | 19.3757 | **16.3487** |
| 1.15 | 53.9418 | 30.7262 | 21.0858 | 16.1229 | 13.3052 | **9.6753** | 10.6124 |
| 1.20 | 28.3301 | 15.1915 | 10.4498 | 8.3165 | 7.2983 | **7.1335** | 8.6871 |
| 1.25 | 16.5834 | 8.7830 | 6.3166 | 5.3874 | **5.0787** | 5.9561 | 7.2502 |
| 1.50 | 3.3446 | 2.2910 | **2.2182** | 2.3283 | 2.4591 | 2.7455 | 2.7664 |
| 1.75 | 1.7206 | **1.4947** | 1.5499 | 1.6013 | 1.6269 | 1.6423 | 1.6423 |
| 2.00 | 1.2909 | **1.2357** | 1.2638 | 1.2747 | 1.2773 | 1.2780 | 1.2780 |
| 2.25 | 1.1347 | **1.1202** | 1.1303 | 1.1322 | 1.1325 | 1.1325 | 1.1325 |
| 2.50 | 1.0682 | **1.0641** | 1.0675 | 1.0679 | 1.0679 | 1.0679 | 1.0679 |
| 2.75 | 1.0370 | **1.0357** | 1.0369 | 1.0370 | 1.0370 | 1.0370 | 1.0370 |
| 3.00 | 1.0213 | **1.0209** | 1.0213 | 1.0214 | 1.0214 | 1.0214 | 1.0214 |

Note: The bold number on each row is the smallest ARL$_1$ for each shift level.

## 5. Practical Applications

### 5.1. Application I

To demonstrate the application of the R and $MA_R$ control chart. The simulated data for the 25 subgroups with fourth sizes are shown in Table 10. The first fifteen samples are assumed to be in control, and the following ten subgroups are out of control. The S, R and $MA_R$ control chart for $w$ equal to 3 are plotted in Figures 1(a), 1(b) and 1(c), respectively. The results show that the R chart has no point plot outside the control limits in this sample and fails to detect a shift in the process variability when the shift occurs.

**Table 10** The simulated data for the 25 subgroups with fourth sizes

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| 64.0800 | 27.3816 | 141.6149 | 143.7999 |
| −40.3340 | −55.9834 | 50.2346 | 135.6725 |
| −7.9538 | 46.6545 | 41.1491 | −38.0159 |
| −44.6047 | 177.5491 | −122.7990 | −103.7390 |
| −59.8107 | 49.5423 | 35.7507 | −150.3102 |
| −3.4794 | 45.9422 | 248.7460 | 100.0977 |
| −20.1378 | −42.0193 | 22.9045 | −174.4091 |
| 12.5464 | −63.5503 | 159.2997 | −68.8055 |
| 210.9037 | 58.5432 | 67.6875 | 92.7382 |
| 6.9895 | −43.9235 | −51.6511 | −8.3742 |
| −83.0418 | 72.9149 | 22.8306 | 390.3523 |
| 51.5989 | 303.3984 | −185.8505 | 63.2211 |
| 153.2237 | −103.3403 | 231.9195 | −94.5058 |
| 190.9359 | 216.0095 | 24.9871 | 54.7366 |

**Table 10** (Continued)

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---:|---:|---:|---:|
| 70.1551 | 54.5970 | 1.9787 | −110.4029 |
| 2.7016 | −44.5989 | 61.7838 | 120.7181 |
| 26.3848 | −118.4138 | −66.8903 | −40.3953 |
| 14.9689 | 52.2048 | −20.0342 | −126.9771 |
| 406.8810 | 77.7828 | −66.0924 | 64.5313 |
| −135.2496 | 103.1199 | −129.0047 | 97.9938 |
| −83.5145 | 47.2105 | 67.5735 | 144.2719 |
| 52.7670 | 148.6692 | 132.9198 | 70.5915 |
| 11.6681 | −5.0409 | 152.6231 | −61.8640 |
| −11.3519 | 144.7597 | 134.8395 | 259.6313 |
| −13.7595 | 165.0084 | 46.7414 | −38.9697 |

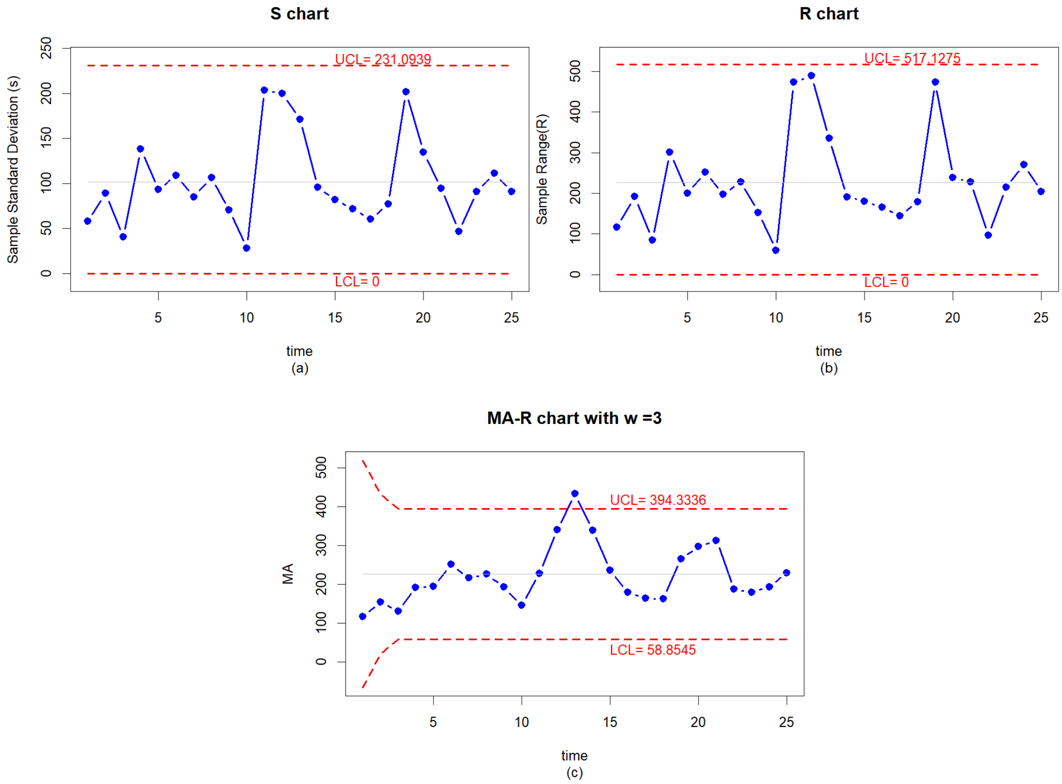


**Figure 1** Compare three control charts: (a) S-chart, (b) R-chart, and (c) $MA_R$ control chart for simulation data

## 5.2. Application II

This example consists of 20 observations for the flow width measurement in the hard-bake process, each of size five wafers and assumed to be a normal distribution for a slight shift in the process variability (see example from Montgomery 2009). The first ten samples are assumed to be in control.

The process is assumed to follow an N(10,1) distribution, and the next ten subgroups are generated from $N(10,2)$. The S, R and $MA_R$ control chart for w = 4 are plotted in Figures 2(a), 2(b) and 2(c), respectively. The result shows that the R chart has no point plot outside the control limits in this sample and fails to detect a shift in the process variability when the shift occurs.

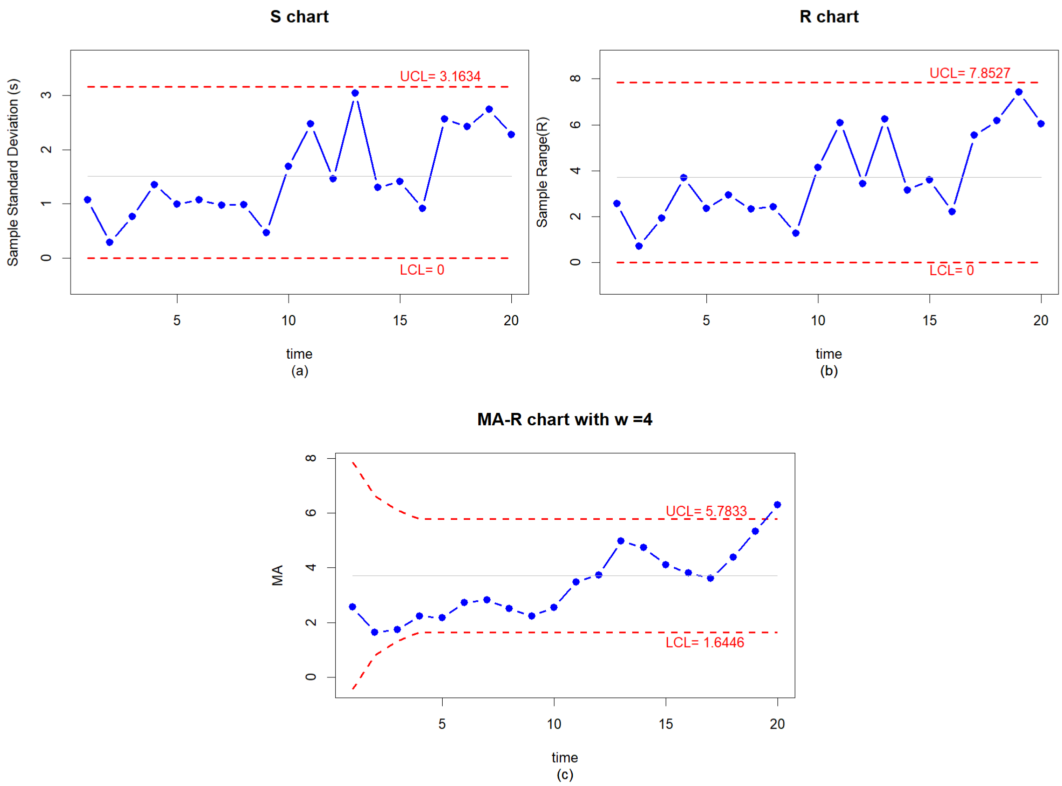


**Figure 2** Compare three control charts: (a) S-chart, (b) R-chart, and (c) $MA_R$ control chart for real application II

## 6. Conclusions and Recommendations

In this study, the efficiency of the $MA_R$ control chart was compared with R charts using $ARL_1$ as the evaluation criteria under the normality assumption. The numerical result of the $MA_R$ chart was obtained from the explicit formula for both cases of known and unknown parameter $\sigma$, with different values of sample size ($n$) and width ($w$). The performance comparison of the $MA_R$ chart versus the $R$ chart using two applications consists of simulated data and flow width measurement data. The comparison shows that the proposed chart is superior to the R chart with both application I for simulated data and application II for real application data in hard bake process. This is guaranteed to the performance of the proposed control chart that is very useful and easy to implement with the proposed explicit formulas. Also, the $MA_R$ control chart performs better for small and large sample sizes for both small and moderate shifts in process variability. Therefore, the $MA_R$ control chart is an effective alternative to $R$ due to the more straightforward calculation and interpretation. This proposed control chart might be robust to the non-normal assumption which the authors are currently investigating into the several case studies.

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