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Evaluating the Performance of Modified EWMA Control Schemes for Serially Correlated Observations

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Abstract

In this study, we propose the explicit formula of the average run length (ARL) for the autoregressive process with a quadratic trend on the modified exponentially weighted moving average (modified EWMA) control chart. The accuracy of the ARL from the explicit formula was compared with the ARL from the numerical integral equation (NIE) method derived by using the quadrature rule. The metrics of comparison were percentage accuracy and computational time. After that, the performance of the modified EWMA control chart is investigated in terms of the average run length, standard deviation of run length (SDRL), and median run length (MRL). In addition, the performance comparison on modified EWMA and the exponentially weighted moving average (EWMA) control charts was presented by using the relative mean index (RMI), the average extra quadratic loss (AEQL), and the performance comparison index (PCI) as the criteria. Furthermore, to determine the ability of the explicit formulas approach, the crude oil WTI price was applied, and it was shown that the modified EWMA control chart performed more significantly than the EWMA control chart under this condition.

Keywords: Average run length, numerical integral equation, autoregressive model, exponential white noise.

1. Introduction

Statistical process control (SPC) is a widely used tool in engineering and quality control to monitor and manage processes. SPC aims to monitor a process to make sure that processes are operating within acceptable limits and are capable of producing consistent, high-quality products. One tool for SPC is control charts. It has many applications and is commonly used in the manufacturing sector to monitor, control, and improve processes. Roberts (1959) proposed the Exponentially Weighted Moving Average (EWMA) control chart to detect small mean shifts in general processes. The EWMA control chart is widely used in industries such as manufacturing and many other areas such as business and public health (Wanga et al. 2021, Woodall 2006). Recently, the modified EWMA control scheme was proposed by Patel and Divecha (2011), which is effective at detecting small changes in a process parameter. Later, Khan et al. (2017) recently developed the modified EWMA statistic to improve its ability to rapidly monitor process shifts by including a constant value to the modified EWMA statistic. Recently, Aslam and Anwar (2020) proposed a new control chart, the Bayesian Modified-EWMA control chart, to monitor the location parameter in a process. A performance evaluation tool for comparing the performance is the average run length. The results represented that the Bayesian Modified-EWMA control chart performs better than the Bayesian EWMA control chart for monitoring small to moderate changes in the process.

The modified EWMA control chart is specifically designed for independent normally distributed observations and autocorrelated data, such as air pollution data, for which this control chart shows effective results (Supharakonsakun et al. 2020). In general, trend models are suitable for capturing long-term behavior. One approach to forecasting is to combine a deterministic time trend model with an autoregressive (AR(p)) model. The autoregressive with quadratic trend model is a time series forecasting model that combines autoregressive components with a quadratic trend component to capture non-linear patterns in time series data. This model is often used when there is evidence of a quadratic (parabolic) trend in the data. For this reason, this research will study the autoregressive process using the quadratic trend model. Additionally, a special case of white noise with an exponential distribution will be studied, as has been studied by Fellag and Ibazizen (2011) and Suriyakat et al. (2012). Previous research has shown that the ARL can be computed using various techniques. For instance, Areepong and Novikov (2008) proposed ARL using an explicit formula using the martingale approach for changes in exponential distribution. Suriyakat et al. (2012) proposed ARL using the explicit formula on an EWMA control chart for AR(1) observations with exponential white noise. Moreover, Sukparungsee and Areepong (2017) proposed the explicit analytical solution of the average run length of the EWMA control chart using an autoregressive model. They compared the results of ARL by using the numerical integral equation method. In addition, Sunthornwat et al. (2018) found the ARL with a practical investigation of estimating parameters of the EWMA control chart on the long memory AFRIMA process. Recently, Peerajit et al. (2018) studied a numerical integral equation method on a cumulative sum (CUSUM) control chart for a long-memory process with non-seasonal and seasonal ARFIMA models. Meanwhile, Supharakonsakun et al. (2020) found the exact solution of the average run length on a modified EWMA control chart for the first-order moving average process. Most recently, Phanthuna et al. (2022) derived explicit analytical solutions for the ARL of a Modified EWMA control chart with a Time Series Model with Fractionality and Integration with exponential white noise. However, no prior research has been done on the precise formulations of the ARL for the quadratic trend AR(p) model on the modified EWMA control chart. The goal of this research is to propose the ARL explicit formula on the modified EWMA control chart for AR(p) with a quadratic trend model. Moreover, a comparison of the performance of the modified EWMA and EWMA control charts has been presented.

2. The Modified EWMA Chart

Let $\{X_t; t = 1, 2, 3, ...\}$ be a sequence of the autoregressive model with target mean μ and constant variance σ^2. The modified EWMA statistic by Khan et al. (2017) was developed from a modified EWMA statistic (Patel and Divecha 2011). The modified EWMA statistic used on individual observations can be written as

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t + l(X_t - X_{t-1}), \tag{1}$$

where λ is an exponential smoothing parameter with $0 < \lambda \le 1$ and $Z_0 = \mu = X_0$ is an initial value. Moreover, $Z_t = (1-\lambda)Z_{t-1} + \lambda X_t$ is an EWMA statistic when $l = 0$. The mean and the asymptotic variance of the EWMA statistic are μ and $\left(\frac{\lambda}{2-\lambda}\right)\sigma^2$, respectively.

The upper and lower control limits of the EWMA control chart are

$$UCL / LCL = \mu \pm L_1 \sigma \sqrt{\frac{\lambda}{2-\lambda}}, \tag{2}$$

where L_1 is a suitable control width limit. The mean and the asymptotic variance of the modified EWMA statistic with l are μ and $\left(\frac{\lambda + 2\lambda l + 2l^2}{2-\lambda}\right)\sigma^2$, respectively. The upper and lower control limits of the modified EWMA control chart are

$$UCL / LCL = \mu \pm L_2 \sigma \sqrt{\frac{\lambda + 2\lambda l + 2l^2}{2-\lambda}}, \tag{3}$$

where L_2 is a suitable control width limit.

3. The AR(p) with Quadratic Trend Model

The equation of observations for a general autoregressive process or AR(p) with quadratic trend model in the case of exponential white noise is defined as

$$X_t = \eta + \gamma T_t + \nu T_t^2 + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t, \tag{4}$$

where η is a suitable constant, γ, ν are trend parameters and ϕ_i is an autoregressive coefficient and ε_t is the exponential white noise sequence of independent and identically distributed random variables such that $\varepsilon_t \sim Exp(\beta)$.

4. The Average Run Length of the Modified EWMA Control Chart

The average run length (ARL) is a widely used performance metric for control charts. The evaluation of the ARL can be conducted using several methods, including the utilization of an explicit formula, the numerical integral equation (NIE) method, the martingale approach, the Markov chain approach, or a Monte Carlo simulation.

4.1. The explicit formula method

This study introduces the explicit formula and the NIE approach as the effectiveness of the suggested control charts in terms of ARL. The first step involves incorporating (4) into (1) in order to compute the ARL for the autoregressive (AR) model with quadratic trend model on the modified EWMA control chart such that Z_t can be rewritten as

$$Z_t = (1-\lambda)Z_{t-1} + \lambda\left(\eta + \gamma T_t + \nu T_t^2 + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t\right) + l\left(\eta + \gamma T_t + \nu T_t^2 + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t - X_{t-1}\right). \tag{5}$$

The process is said to be out-of-control when Z_t falls outside the control limits. If a and b are the lower control limit (LCL) and the upper control limit (UCL) of Z_t for the in-control process, then $a < Z_t < b$. If the initial value of Z_t is u usually instead with the process mean, then $F(u)$ is defined for an initial value of the *ARL* as $F(u)=E_{\infty}(\tau_{a,b}),$ where $\tau_{a,b}=\inf\{t>0;\ Z_t\le a \text{ or } Z_t\ge b\}$. For the in-control process, an interval of the modified EWMA statistics Z_1 is

$$a<(1-\lambda)u+(l+\lambda)\sum_{i=1}^{p}\phi_i X_{p-i}-lX_0+(l+\lambda)\varepsilon_1+(l+\lambda)\eta+(l+\lambda)\gamma T_t+(l+\lambda)\nu T_t^2<b. \quad (6)$$

After that the interval of ε_1 can be found:

$$\frac{a-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2<\varepsilon_1<\frac{b-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2. \quad (7)$$

From Equation (7), the probability function of ε_1 is found as:

$$P(\text{LCL}<\varepsilon_1<\text{UCL})=\int_{\frac{a-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2}^{\frac{b-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2} f(\varepsilon_1)d\varepsilon_1.$$

For the modified EWMA control chart with the AR(p) with quadratic trend model, $F(u)$ can be rewritten as follows:

$$F(u)=1+\int_{\frac{b-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2}^{\frac{b-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2} F\left[\begin{array}{l}(1-\lambda)u-lX_0\\ +(l+\lambda)\sum_{i=1}^{p}\phi_i X_{p-i}\\ +(l+\lambda)y+(l+\lambda)\eta+(l+\lambda)\gamma T_t+(l+\lambda)\nu T_t^2\end{array}\right] f(y)dy. \quad (8)$$

Next step, Equation (8) is changed the variable of integration, then $F(u)$ is obtained as:

$$F(u)=1+\frac{1}{l+\lambda}\int_a^b L(k)f\left(\frac{k-(1-\lambda)u+lX_0}{(l+\lambda)}-\sum_{i=1}^{p}\phi_i X_{p-i}-\eta-\gamma T_t-\nu T_t^2\right)dk. \quad (9)$$

Since $f(k)=\frac{1}{\beta}e^{-\frac{k}{\beta}}$, where $\beta>0$.

$$F(u)=1+\frac{e^{\frac{(1-\lambda)u}{\beta(l+\lambda)}}\cdot e^{\frac{-lX_0}{\beta(l+\lambda)}}\cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma T_t+\nu T_t^2}{\beta}}}{\beta(l+\lambda)}\int_a^b F(k)\cdot e^{\frac{-k}{\beta(l+\lambda)}}dk. \quad (10)$$

Let $Q=\int_a^b F(k)\cdot e^{\frac{-k}{\beta(l+\lambda)}}dk$ and $C(u)=e^{\frac{(1-\lambda)u}{\beta(l+\lambda)}}\cdot e^{\frac{-lX_0}{\beta(l+\lambda)}}\cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma T_t+\nu T_t^2}{\beta}}$, where $a\le u\le b$ such that it can be rearranged as follows:

$$F(u)=1+\frac{C(u)}{\beta(l+\lambda)}\cdot Q. \quad (11)$$

For solving Q, we obtain

$$Q=\int_a^b F(k)\cdot e^{\frac{-k}{\beta(l+\lambda)}}dk$$

$$Q=\int_a^b e^{\frac{-k}{\beta(l+\lambda)}}dk+\int_a^b \frac{e^{\frac{(1-\lambda)k}{\beta(l+\lambda)}}\cdot e^{\frac{-jX_0}{\beta(l+\lambda)}}\cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma T_t+\nu T_t^2}{\beta}}}{\beta(l+\lambda)}\cdot Q\cdot e^{\frac{-k}{\beta(l+\lambda)}}dk$$

$$Q=\frac{-\beta(l+\lambda)\left[e^{\frac{-b}{\beta(l+\lambda)}}-e^{\frac{-a}{\beta(l+\lambda)}}\right]}{1+\frac{e^{\frac{-lX_0}{\beta(l+\lambda)}}\cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma T_t+\nu T_t^2}{\beta}}}{\lambda}\left[e^{\frac{-\lambda b}{\beta(l+\lambda)}}-e^{\frac{-\lambda a}{\beta(l+\lambda)}}\right]}.$$

After that Q and $C(u)$ are replaced in (11), then $F(u)$ can be found as

$$F(u)=1-\frac{\lambda\cdot e^{\frac{(1-\lambda)u}{\beta(l+\lambda)}}\left(e^{\frac{-b}{\beta(l+\lambda)}}-e^{\frac{-a}{\beta(l+\lambda)}}\right)}{\lambda\cdot e^{\frac{lX_0}{\beta(l+\lambda)}}\cdot e^{-\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma T_t+\nu T_t^2}{\beta}}+e^{\frac{-\lambda b}{\beta(l+\lambda)}}-e^{\frac{-\lambda a}{\beta(l+\lambda)}}}. \tag{12}$$

Finally, the solution of (12) is an explicit formula for ARL on the modified EWMA control chart for the AR(p) with quadratic trend model.

If the exponential parameter (β) is determined with β_0 before the start of the process, then the ARL is called ARL_0. Similarly, if the exponential parameter (β) is assigned to $\beta_1=(1+\delta)\beta_0$, where $\beta_1>\beta_0$ and δ is the shift sizes in an out-of-control process, then the ARL is called ARL_1.

According to Banach's fixed-point theorem, if an operator T is a contraction, and then the fixed-point equation $T(M(u))=M(u)$ has a unique solution. To show that Equation (10) exists and has a unique solution, theorem can be used as follows below.

Theorem 1: Banach's fixed-point theorem

Let (X,d) defined on a complete metric space and $T:X\to X$ satisfies the conditions of a contraction mapping with contraction constant $0\le r<1$ such that $\|T(F_1)-T(F_2)\|\le r\|F_1-F_2\|$, $\forall\, F_1,F_2\in X$. Then there exists a unique $F(\cdot)\in X$ such that $T(F(u))=F(u)$, i.e., a unique fixed-point in X.

Proof: Let T defined in (10) is a contraction mapping for $F_1,\ F_2\in K[a,b]$, such that

$$\|T(F_1)-T(F_2)\|\le r\|F_1-F_2\|,\ \forall M_1,M_2\in K[a,b]$$

with $0\le r<1$ under the norm $\|F\|_\infty=\sup_{u\in[a,b]}|F(u)|$, so

$$\|T(F_1)-T(F_2)\|_\infty=\sup_{u\in[a,b]}|F_1(u)-F_2(u)|$$

$$=\sup_{u\in[a,b]}\left|\frac{e^{\frac{(1-\lambda)u}{\beta(l+\lambda)}}\cdot e^{\frac{-lX_0}{\beta(l+\lambda)}}\cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma t+\nu t^2}{\beta}}}{\beta(l+\lambda)}\int_a^b(F_1(k)-F_2(k))\cdot e^{\frac{-k}{\beta(l+\lambda)}}dk\right|$$

$$\leq \sup_{u\in[a,b]}\left\{ e^{\frac{(1-\lambda)u+lX_0}{\beta(l+\lambda)}} \cdot e^{\frac{\sum_{i=1}^{p}\phi_i X_{p-i}+\eta+\gamma t+\nu t^2}{\beta}} \left(e^{\frac{-a}{\beta(l+\lambda)}} - e^{\frac{-b}{\beta(l+\lambda)}} \right) \right\} \left\| F_1 - F_2 \right\|_\infty$$

$$\leq r\left\| F_1 - F_2 \right\|_\infty .$$

Thus, $\left\| T(F_1)-T(F_2) \right\|_\infty \leq r\left\| F_1 - F_2 \right\|_\infty$ where a positive constant $r \in [0,1)$ and T is the contraction such that a contraction mapping can have at most one fixed point. Therefore, the Banach contraction principle is applied to testify a unique solution of the $F(u)$.

4.2. The numerical integral equation method

The NIE method can alternatively be used to approximate the ARL on the modified EWMA control chart for AR(p) with quadratic trend model. Let $F_{NIE}(u)$ be the estimated value of ARL with the $2m+1$ linear equation system for using the composite Simpson's quadrature rule which is effective as an explicit formula, then the solution becomes

$$F_{NIE}(u) \approx 1+\frac{1}{l+\lambda}\sum_{k=1}^{2m+1} w_k F\left(s_k\right) f\left(\frac{s_k-(1-\lambda)u+lX_0}{(l+\lambda)} - \sum_{i=1}^{p}\phi_i X_{p-i} - \eta - \gamma T_t - \nu T_t^2 \right), \tag{13}$$

where the set of points $s_{k+1} = kw_{k+1}$; $k = 0,1,2,...,2m,$ the weight assigned by the composite Simpson's formula is $w_{k+1} = \frac{1}{3}\left(\frac{b}{2m}\right)$; $k=0,2m,$ $w_{k+1} = \frac{4}{3}\left(\frac{b}{2m}\right)$; $k=1,3,...,2m-1,$ $w_{k+1} = \frac{2}{3}\left(\frac{b}{2m}\right)$; $k=2,4,...,2m-2,$ and $\varepsilon_t \sim Exp(\beta).$

5. Numerical Results

In this section, a simulation study comparing the accuracy of the explicit formulas $(F(u))$ and the NIE method $(F_{NIE}(u))$ for the ARL of AR(p) with quadratic trend process on the modified EWMA control chart are presented. The accuracy of the ARL is compared with the percentage accuracy which can be obtained from

$$\% Accuracy = 100 - \left| \frac{F(u)-F_{NIE}(u)}{F(u)} \right| \times 100\%. \tag{14}$$

In Table 1, the comparison of the ARL_1 values using the analytical explicit formula and numerical ARL methods on one-sided modified EWMA control chart for AR(1), AR(2) and AR(3) with quadratic trend model with $\eta=0.5, \gamma=1.5, \nu=2.5,$ $l=1,$ $\phi_1=0.1, \phi_2=0.2, \phi_3=0.3,$ $\text{ARL}_0=370$ is presented so that both methods are computed according to the percentage accuracy and computational time (CPU time). The results show that the ARL of each are very similar and the percentage accuracy equal to 100 such that they use for checking this precise explicit formula. However, the CPU time of the explicit formula is faster than the NIE method by around 10-11 seconds.

In addition, the Standard Deviation Run Length (SDRL) and Median Run Length (MRL) are performance measurements used to evaluate the effectiveness of control charts in detecting out-of-control conditions (Fonseca et al. 2021). For the in-control process, SDRL and MRL are calculated as follows.

$$ARL_0 = \frac{1}{\alpha}, SDRL_0 = \sqrt{\frac{1-\alpha}{\alpha^2}}, MRL_0 = \frac{\log(0.5)}{\log(1-\alpha)}, \tag{15}$$

where α represents type I error. In this study, ARL0 was fixed at 370 and it can be calculated as $SDRL_0$ and MRL_0 by (15) at approximately 370 and 256, respectively. On the other hand, for out-of-control situations, $SDRL_1$ and MRL_1 are calculated by substituting α with γ where γ represents type II error. The control chart's effectiveness in detecting various types of process variations can be evaluated and informed decisions about its performance. A lower the $SDRL_1$, MRL_1, and ARL_1 suggest better performance in quickly identifying shifts in the process mean.

In addition, the performance efficiency of the modified EWMA control chart is compared with the EWMA control chart by using the relative mean index (RMI) (Tang et al 2018). If the RMI is a small value, then overall, that control chart has a quick and robust performance for detecting shifts. RMI is defined as

$$RMI\,(c) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{ARL_i(c) - ARL_i(\mathrm{s})}{ARL_i(\mathrm{s})}\right], \tag{16}$$

where $ARL_i(c)$ is the ARL of the control chart for the shift size of row *i* and $ARL_i(s)$ is the smallest ARL of all control charts for the shift size of row *i*. In addition, the performance measurements can be used to assess a control chart's success throughout a variety of changes ($\delta_{\min} \le \delta \le \delta_{\max}$). Moreover, the average extra quadratic loss (AEQL) may refer to the average extra loss incurred due to an out-of-control condition. It could be calculated as the average difference between the observed values and the target or desired values during out-of-control periods. AEQL can be calculated as follows (Alevizakos et al. 2021)

$$AEQL = \frac{1}{\Delta}\sum_{\delta_i=\delta_{\min}}^{\delta_{\max}}\left(\delta_i^2 \times ARL(\delta_i)\right), \tag{17}$$

where δ represents the particular change in the process, and Δ represents the sum of number of divisions from $\delta_{\min}$ to $\delta_{\max}$. In this study, $\Delta = 10$ is determined from $\delta_{\min} = 0.001$ to $\delta_{\max} = 1.00$. The control chart with the lowest AEQL values perform the best. Additionally, the Performance Comparison Index (PCI) is a measurement used to compare the performance of different control charts. The PCI measurement is the ratio between the AEQL of the control chart and the most efficient control chart, which is shown as the lowest AEQL. The mathematical formula for the PCI is

$$PCI = \frac{AEQL}{AEQL_{lowest}}. \tag{18}$$

Table 1 Comparison of the ARL_1 values using the analytical and numerical ARL methods on one-sided modified EWMA control chart for AR(p) with quadratic trend model with $\eta = 0.5, \gamma = 1.5, \nu = 2.5,\ l = 1,\ \phi_1 = 0.1, \phi_2 = 0.2, \phi_3 = 0.3,\ \text{ARL}_0 = 370$

δ	λ	0.05			0.10		
	b	0.027194	0.022262	0.016489	0.027313	0.022357	0.016558
	Model	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.01	$F(u)$	45.7616	44.1824	41.9788	42.2207	40.7598	38.7259
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	45.7616	44.1824	41.9788	42.2207	40.7598	38.7259
	(Sec.)	10.641	10.547	10.813	10.500	10.547	10.593
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
0.03	$F(u)$	16.5458	15.921	15.0598	15.2321	14.6605	13.8732
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	16.5458	15.921	15.0598	15.2321	14.6605	13.8732
	(Sec.)	10.594	10.640	10.594	10.579	10.625	10.531
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
0.05	$F(u)$	10.0895	9.70138	9.16796	9.31816	8.96356	8.47654
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	10.0895	9.70138	9.16796	9.31816	8.96356	8.47654
	(Sec.)	10.656	10.687	10.703	10.562	10.359	10.515
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
0.10	$F(u)$	5.14877	4.95084	4.67983	4.80244	4.62143	4.3737
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	5.14877	4.95084	4.67983	4.80244	4.62143	4.3737
	(Sec.)	10.797	10.563	10.687	10.516	10.500	10.516
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
0.30	$F(u)$	1.9758	1.91311	1.82825	1.90114	1.84334	1.76512
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	1.9758	1.91311	1.82825	1.90114	1.84334	1.76512
	(Sec.)	10.641	10.688	10.578	10.547	10.515	10.500
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
0.50	$F(u)$	1.44304	1.40862	1.36258	1.41144	1.37951	1.3368
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	1.44304	1.40862	1.36258	1.41144	1.37951	1.3368
	(Sec.)	10.704	10.594	10.704	10.422	10.578	10.656
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
1.00	$F(u)$	1.13623	1.12238	1.10432	1.12762	1.11465	1.09774
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	1.13623	1.12238	1.10432	1.12762	1.11465	1.09774
	(Sec.)	10.719	10.703	10.625	10.532	10.625	10.500
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
1.50	$F(u)$	1.06664	1.05884	1.04886	1.06273	1.05539	1.046
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	1.06664	1.05884	1.04886	1.06273	1.05539	1.046
	(Sec.)	10.875	10.610	10.594	10.500	10.406	10.485
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00
2.00	$F(u)$	1.04038	1.03523	1.02873	1.03813	1.03327	1.02713
	(Sec.)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	F_{NIE}	1.04038	1.03523	1.02873	1.03813	1.03327	1.02713
	(Sec.)	11.078	10.828	10.938	10.672	10.734	10.719
	% Accuracy	100.00	100.00	100.00	100.00	100.00	100.00

Table 2 The ARL on one-sided modified EWMA control chart for AR(2) with quadratic trend model against EWMA control chart with $\eta = 0.05, \gamma = 0.5, \nu = 1.5,\ \phi_1 = 0.1, \phi_2 = 0.2,\ \text{ARL}_0 = 370$

λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	l=10	$l = 15$
		(h = 0.00000000987)	(b = 0.129193)	(b = 1.300860)	(b = 2.602380)	(b = 3.903930)
0.05	0.00	370.433	370.151	370.429	370.018	370.859
	0.001	362.275	313.133	204.58	196.057	193.456
	0.003	346.542	239.126	108.198	101.345	99.1789
	0.005	331.551	193.191	73.7109	68.5153	66.8694
	0.01	297.081	130.076	41.242	38.0966	37.1005
	0.03	193.59	55.2729	15.3409	14.1637	13.7911
	0.05	128.295	34.4374	9.66674	8.95883	8.73462
	0.10	49.2188	17.0954	5.29679	4.95824	4.85076
	0.30	3.05995	5.1554	2.34774	2.25754	2.22865
	0.50	1.19985	3.02263	1.76805	1.72369	1.70939
	1.00	1.00427	1.71144	1.351	1.33608	1.33121
	RMI	5.003	1.727	0.078	0.019	0.000
	AEQL	0.261	0.326	0.210	0.206	0.205
	PCI	1.273	1.591	1.026	1.006	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.000417)	(b = 0.131665)	(b = 1.30939)	(b = 2.61938)	(b = 3.92945)
0.10	0.00	370.731	370.366	370.381	370.576	370.939
	0.001	366.143	300.85	203.993	196.352	193.841
	0.003	357.161	218.522	107.718	101.497	99.4684
	0.005	348.434	171.395	73.3448	68.6175	67.085
	0.01	327.679	111.037	41.0186	38.1527	37.2293
	0.03	257.836	45.3619	15.2561	14.1835	13.8394
	0.05	204.74	28.0867	9.61525	8.9708	8.76407
	0.10	119.364	14.0078	5.27159	4.96416	4.86537
	0.30	21.2207	4.41922	2.3404	2.2594	2.23316
	0.50	6.43782	2.69247	1.76411	1.72475	1.71192
	1.00	1.61125	1.61223	1.34937	1.33657	1.33235
	RMI	9.047	1.341	0.070	0.017	0.000
	AEQL	0.711	0.295	0.210	0.206	0.205
	PCI	3.468	1.439	1.023	1.006	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	l=10	$l = 15$
		(h = 0.01683)	(b = 0.13799)	(b = 1.32677)	(b = 2.65394)	(b = 3.98145)
0.20	0.00	370.564	370.097	370.186	370.606	370.751
	0.001	359.512	277.086	202.845	196.658	194.532
	0.003	339.034	184.282	106.802	101.736	100.037
	0.005	320.473	137.959	72.6489	68.7971	67.5165
	0.01	280.905	84.5802	40.5956	38.261	37.4907
	0.03	181.757	32.9249	15.0961	14.2245	13.9385
	0.05	128.981	20.3029	9.51815	8.99589	8.82452
	0.10	67.2901	10.2717	5.2241	4.9767	4.89541
	0.30	14.6347	3.51436	2.32658	2.26335	2.24243
	0.50	6.13331	2.27856	1.75669	1.72701	1.71713
	1.00	2.14058	1.48349	1.34632	1.33762	1.33469
	RMI	6.058	0.833	0.057	0.014	0.000
	AEQL	0.619	0.257	0.209	0.206	0.205
	PCI	3.010	1.248	1.017	1.004	1.000

Table 3 The ARL on two-sided modified EWMA control chart for AR(2) with quadratic trend model against EWMA control chart with $\eta = 0.05, \gamma = 0.5, \nu = 1.5,\ \phi_1 = 0.1, \phi_2 = 0.2, a = 0.001,$ $ARL_0 = 370$

λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.00100001008)	(b = 0.129193)	(b = 1.30086)	(b = 2.60239)	(b = 3.90391)
0.05	0.00	370.822	370.822	370.429	370.553	370.122
	0.001	362.663	313.133	204.58	196.207	193.256
	0.003	346.927	239.126	108.198	101.385	99.1266
	0.005	331.933	193.191	73.7109	68.5334	66.8457
	0.01	297.451	130.076	41.242	38.1021	37.0934
	0.03	193.906	55.2729	15.3409	14.1644	13.7901
	0.05	128.551	34.4374	9.66674	8.9591	8.73426
	0.10	49.3574	17.0954	5.29679	4.95831	4.85066
	0.30	3.07166	5.1554	2.34774	2.25755	2.22864
	0.50	1.2014	3.02263	1.76805	1.72369	1.70938
	1.00	1.00432	1.71144	1.351	1.33608	1.33121
	RMI	5.015	1.727	0.077	0.019	0.000
	AEQL	0.261	0.325	0.210	0.206	0.204
	PCI	1.275	1.590	1.026	1.006	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.001421)	(b = 0.131667)	(b = 1.309389)	(b = 2.61938)	(b = 3.92945)
0.10	0.00	370.701	370.876	370.284	370.576	370.939
	0.001	366.116	301.187	203.963	196.352	193.841
	0.003	357.142	218.701	107.71	101.497	99.4684
	0.005	348.421	171.505	73.341	68.6175	67.085
	0.01	327.681	111.084	41.0174	38.1527	37.2293
	0.03	257.881	45.37	15.256	14.1835	13.8394
	0.05	204.809	28.09	9.61519	8.9708	8.76407
	0.10	119.45	14.0087	5.27158	4.96416	4.86537
	0.30	21.2615	4.41933	2.3404	2.2594	2.23316
	0.50	6.45409	2.69251	1.76411	1.72475	1.71192
	1.00	1.61406	1.61224	1.34937	1.33657	1.33235
	RMI	9.053	1.341	0.070	0.017	0.000
	AEQL	0.712	0.295	0.209	0.206	0.205
	PCI	3.474	1.439	1.023	1.005	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.017853)	(b = 0.137991)	(b = 1.32677)	(b = 2.653945)	(b = 3.98145)
0.20	0.00	370.118	370.433	370.186	370.863	370.751
	0.001	359.06	277.275	202.845	196.731	194.532
	0.003	338.575	184.365	106.802	101.755	100.037
	0.005	320.011	138.006	72.6489	68.8059	67.5165
	0.01	280.449	84.598	40.5956	38.2637	37.4907
	0.03	181.384	32.9277	15.0961	14.2249	13.9385
	0.05	128.69	20.304	9.51815	8.99602	8.82452
	0.10	67.1294	10.272	5.2241	4.97674	4.89541
	0.30	14.6055	3.5144	2.32658	2.26336	2.24243
	0.50	6.12432	2.27857	1.75669	1.72702	1.71713
	1.00	2.13941	1.48349	1.34632	1.33762	1.33469
	RMI	6.044	0.833	0.056	0.014	0.000
	AEQL	0.618	0.256	0.209	0.206	0.205
	PCI	3.006	1.248	1.017	1.004	1.000

Table 4 The ARL on one-sided modified EWMA control chart for AR(3) with quadratic trend model against EWMA control chart with $\eta = 0.05, \gamma = 0.5, \nu = 1.5,\ \phi_1 = 0.1, \phi_2 = 0.2, \phi_3 = 0.3$, $ARL_0 = 370$

λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.00000000731)	(b = 0.095526)	(b = 0.962)	(b =1.9245)	(b = 2.887)
0.05	0.00	370.338	370.202	370.164	370.728	370.855
	0.001	362.074	308.763	196.224	187.916	185.103
	0.003	346.144	231.537	101.42	94.9071	92.7793
	0.005	330.974	184.973	68.5405	63.6621	62.0864
	0.01	296.127	122.632	38.0688	35.1520	34.2194
	0.03	191.866	51.1068	14.0909	13.0129	12.6706
	0.05	126.458	31.6313	8.87751	8.23257	8.02789
	0.10	47.9094	15.5817	4.87402	4.56825	4.47106
	0.30	2.92167	4.66338	2.18516	2.10572	2.08027
	0.50	1.18079	2.74802	1.66246	1.62414	1.61178
	1.00	1.00367	1.59402	1.29266	1.2802	1.27614
	RMI	5.420	1.738	0.077	0.018	0.000
	AEQL	0.257	0.300	0.199	0.195	0.194
	PCI	1.319	1.540	1.023	1.005	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.0003085)	(b = 0.097235)	(b = 0.9667)	(b = 1.93378)	(b = 2.90091)
0.10	0.00	370.405	370.525	370.807	370.375	370.103
	0.001	365.706	295.779	195.636	187.762	185.072
	0.003	356.513	210.513	100.859	94.8360	92.8495
	0.005	347.587	163.218	68.1039	63.6161	62.1520
	0.01	326.379	104.184	37.7994	35.1277	34.2644
	0.03	255.283	41.8397	13.9883	13.0047	12.6888
	0.05	201.560	25.7630	8.81533	8.22773	8.03913
	0.10	115.971	12.7696	4.84372	4.56595	4.47667
	0.30	19.8247	4.01253	2.17647	2.1051	2.08198
	0.50	5.90903	2.46197	1.65785	1.62382	1.61273
	1.00	1.52503	1.51093	1.29079	1.28008	1.27656
	RMI	9.641	1.352	0.070	0.017	0.000
	AEQL	0.672	0.273	0.199	0.195	0.194
	PCI	3.449	1.402	1.021	1.005	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		(h = 0.012425)	(b = 0.101685)	(b = 0.97627)	(b = 1.95257)	(b = 2.92912)
0.20	0.00	370.078	370.715	370.384	370.023	370.121
	0.001	336.549	270.990	194.039	187.560	185.400
	0.003	336.549	176.064	99.6639	94.7281	93.0921
	0.005	316.985	130.296	67.2145	63.5428	62.331
	0.01	275.706	78.8272	37.2690	35.0873	34.3703
	0.03	174.782	30.2736	13.7910	12.9905	12.7282
	0.05	122.518	18.5953	8.69615	8.21931	8.06302
	0.10	62.7743	9.37263	4.78585	4.56196	4.48845
	0.30	13.2445	3.21271	2.15990	2.10403	2.08554
	0.50	5.50338	2.10329	1.64906	1.62328	1.61471
	1.00	1.96235	1.40306	1.28723	1.27989	1.27743
	RMI	6.274	0.845	0.057	0.014	0.000
	AEQL	0.566	0.239	0.198	0.196	0.195
	PCI	2.901	1.229	1.015	1.003	1.000

Table 5 The ARL on two-sided modified EWMA control chart for AR(3) with quadratic trend model against EWMA control chart with $\eta = 0.05, \gamma = 0.5, \nu = 1.5,\ \phi_1 = 0.1, \phi_2 = 0.2, \phi_3 = 0.3, a = 0.001,$ $\text{ARL}_0 = 370$

λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		($h = 0.001000007452$)	($b = 0.096538$)	($b = 0.96301$)	($b = 1.9255$)	($b = 2.888$)
0.05	0.00	370.057	370.472	370.149	370.016	370.365
	0.001	361.807	308.867	196.203	187.725	184.975
	0.003	345.902	231.500	101.405	94.8543	92.7446
	0.005	330.755	184.889	68.5299	63.6366	62.0697
	0.01	295.961	122.529	38.0626	35.1429	34.2135
	0.03	191.832	51.0429	14.0886	13.0110	12.6694
	0.05	126.482	31.5894	8.87615	8.23164	8.02728
	0.10	47.9591	15.5613	4.87338	4.56787	4.47081
	0.30	2.92909	4.6589	2.18499	2.10563	2.08021
	0.50	1.18186	2.74617	1.66238	1.62409	1.61175
	1.00	1.00371	1.59356	1.29264	1.28019	1.27613
	RMI	5.421	1.736	0.077	0.018	0.000
	AEQL	0.257	0.299	0.199	0.195	0.194
	PCI	1.320	1.539	1.023	1.005	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		($h = 0.0013115$)	($b = 0.098253$)	($b = 0.96772$)	($b = 1.934799$)	($b = 2.90193$)
0.10	0.00	370.428	370.528	370.852	370.319	370.110
	0.001	366.343	295.696	195.632	187.740	185.069
	0.003	357.140	210.385	100.850	94.8264	92.8460
	0.005	348.203	163.090	68.0961	63.6100	62.1493
	0.01	326.971	104.079	37.7943	35.1246	34.2627
	0.03	255.787	41.7897	13.9863	13.0037	12.6882
	0.05	201.989	25.7318	8.81412	8.22712	8.03876
	0.10	116.260	12.7550	4.84315	4.56567	4.47649
	0.30	19.8959	4.00937	2.17632	2.10503	2.08193
	0.50	5.93232	2.46066	1.65778	1.62379	1.61271
	1.00	1.52837	1.51061	1.29077	1.28007	1.27655
	RMI	9.666	1.350	0.070	0.017	0.000
	AEQL	0.674	0.273	0.199	0.195	0.194
	PCI	3.459	1.401	1.021	1.005	1.000
λ	Shift size (δ)	EWMA	Modified EWMA			
			$l = 0.5$	$l = 5$	$l = 10$	$l = 15$
		($h = 0.013447$)	($b = 0.102715$)	($b = 0.97731$)	($b = 1.95361$)	($b = 2.93017$)
0.20	0.00	370.811	370.520	370.547	370.060	370.603
	0.001	358.943	270.809	194.070	187.563	185.516
	0.003	337.084	175.924	99.6654	94.7255	93.1190
	0.005	317.418	130.184	67.2122	63.5402	62.3420
	0.01	275.952	78.7548	37.2661	35.0854	34.3729
	0.03	174.739	30.2450	13.7895	12.9897	12.7282
	0.05	122.419	18.5783	8.69523	8.21885	8.06291
	0.10	62.6873	9.36489	4.78540	4.56174	4.48835
	0.30	13.2255	3.21105	2.15979	2.10397	2.08551
	0.50	5.49774	2.10261	1.64901	1.62326	1.61469
	1.00	1.96178	1.40290	1.28721	1.27988	1.27742
	RMI	6.283	0.843	0.057	0.014	0.000
	AEQL	0.565	0.239	0.198	0.196	1.195
	PCI	2.898	1.228	1.015	1.003	1.000

Table 6 The ARL on one-sided modified EWMA control chart for AR(2) with quadratic trend model against EWMA control chart with $\lambda = 0.05, \eta = 0, \gamma = 5.769, \nu = -0.094,$ $\phi_1 = 1.079, \phi_2 = -0.33,\ \text{ARL}_0 = 370$

Control Chart		EWMA	Modified EWMA				
δ		(h = 0.13947)	l = 0.5	l = 2	l = 5	l = 10	l = 15
			(b = 1.80041)	(b = 6.7779)	(b = 16.733)	(b = 33.3247)	(b = 49.9165)
0.001	ARL_1	366.9080	340.1950	334.6510	333.9310	333.113000	333.1000
	$SDRL_1$	366.4077	339.6946	334.1506	333.4306	332.612624	332.5996
	MRL_1	253.9745	235.4585	231.6157	231.1166	230.549590	230.5406
0.003	ARL_1	360.6840	292.2880	280.7650	278.5250	277.360000	277.1490
	$SDRL_1$	360.1837	291.7876	280.2646	278.0246	276.859549	276.6485
	MRL_1	249.6604	202.2518	194.2647	192.7120	191.904520	191.7583
0.005	ARL_1	354.6600	256.2350	241.8640	238.9300	237.635000	237.3340
	$SDRL_1$	354.1596	255.7345	241.3635	238.4295	237.134473	236.8335
	MRL_1	245.4848	177.2618	167.3005	165.2668	164.369213	164.1606
0.01	ARL_1	340.4120	195.9120	179.7320	176.3770	175.080000	174.7180
	$SDRL_1$	339.9116	195.4114	179.2313	175.8763	174.579284	174.2173
	MRL_1	235.6089	135.4490	124.2338	121.9083	121.009304	120.7584
0.03	ARL_1	292.9320	101.1570	88.96210	86.48440	85.6159000	85.34300
	$SDRL_1$	292.4316	100.6558	88.46069	85.98295	85.1144314	84.84153
	MRL_1	202.6982	69.76954	61.31660	59.59917	58.9971675	58.80801
0.05	ARL_1	256.6450	68.36800	59.33320	57.5176	56.8946000	56.69440
	$SDRL_1$	256.1445	67.86616	58.83108	57.01541	56.3923834	56.19218
	MRL_1	177.5460	47.04166	40.77908	39.52058	39.0887337	38.94996
0.10	ARL_1	194.8690	38.01360	32.66310	31.59930	31.2403000	31.12290
	$SDRL_1$	194.3684	37.51027	32.15921	31.09528	30.7362334	30.61882
	MRL_1	134.7260	26.00091	22.29197	21.55453	21.3056731	21.22429
0.30	ARL_1	95.41880	14.14920	12.18670	11.79900	11.6696000	11.62670
	$SDRL_1$	94.91748	13.64004	11.67600	11.28793	11.1584033	11.11546
	MRL_1	65.79209	9.456671	8.095658	7.826755	7.73700260	7.707247
0.50	ARL_1	60.69010	8.961690	7.789770	7.558030	7.48078000	7.455170
	$SDRL_1$	60.18802	8.446905	7.272602	7.040297	6.96285067	6.937174
	MRL_1	41.71964	5.858364	5.044950	4.884059	4.83042215	4.812640
RMI		3.045	0.145	0.032	0.010	0.002	0.000
AEQL		2.962	0.464	0.402	0.390	0.386	0.385
PCI		7.691	1.207	1.046	1.014	1.003	1.000

Generally, the PCI value equal to 1 indicates better performance in terms of quickly detecting out-of-control conditions and minimizing false alarms.

According to Tables 2-5, the comparison of the ARL on one-sided and two-sided modified EWMA control charts for AR(2) and AR(3) with quadratic trend model against EWMA control chart with $\eta = 0.05, \gamma = 0.5, \nu = 1.5,\ \phi_1 = 0.1, \phi_2 = 0.2,\ \text{ARL}_0 = 370$ given $\lambda = 0.05, 0.1$ are presented. The ARL of the modified EWMA control chart are almost lower than the EWMA chart for all λ. Therefore, the modified EWMA control chart have a higher performance than the EWMA control chart. Moreover, the performance of the modified EWMA control chart is better when the l increases for all λ. In addition, the modified EWMA control chart for all l is more effective for a higher λ. Moreover, the RMI, AEQL and PCI values obtained from each control chart to see the performance of each chart. It was found that the modified EWMA control chart had the best performance because it gave the lowest RMI, AEQL and PCI equal to 1. Therefore, it also can be concluded that the modified EWMA control chart performs better than the EWMA control chart.

6. Application

The explicit formulas for the ARL on the modified EWMA control chart are applied and compared performance with the EWMA control chart using the monthly crude oil West Texas Intermediate (WTI) price from January 2020 to May 2023. Based on the model estimation through the maximum likelihood estimation, the coefficient parameters of AR(2) with a quadratic trend model are obtained as follows: with $\gamma = 5.769$, $\nu = -0.094$, $\phi_1 = 1.079, \phi_2 = -0.33$, and the in-control parameter equal to 7.0751. Through the parameter of this prediction model it can be assigned as follows:

$$\hat{X}_t = 5.769T_t - 0.094T_t^2 + 1.079X_{t-1} - 0.33X_{t-2}$$

The ARL values for AR(2) with quadratic trend model on the EWMA and modified EWMA control charts are compared in terms of ARL using the explicit formula method, the results of which are summarized in Table 6 and Figure 1; it can be seen that the results are obviously in agreement with those in Tables 2 and 3. When considering the SDRL and MRL values, the results are the same as the ARL values. From the table, it was found that the modified EWMA control chart has the lowest RMI, AEQL, and PCI of all levels, as shown in Figure 2. To sum up, the explicit formula approach is a good alternative for practical applications in detecting changes in process mean on the modified control chart.

Furthermore, Figure 3 displays the modified EWMA and EWMA statistics for the crude oil West Texas Intermediate (WTI) price, which have been fitted to the autoregressive (AR) model with a quadratic trend. The findings suggest that the modified EWMA control chart is capable of detecting a shift in the process at the first observation, but the EWMA scheme only identifies the change at the fourth observation. Therefore, the findings indicate that the modified exponentially weighted moving average (EWMA) control chart exhibits superior effectiveness compared to the EWMA control chart.

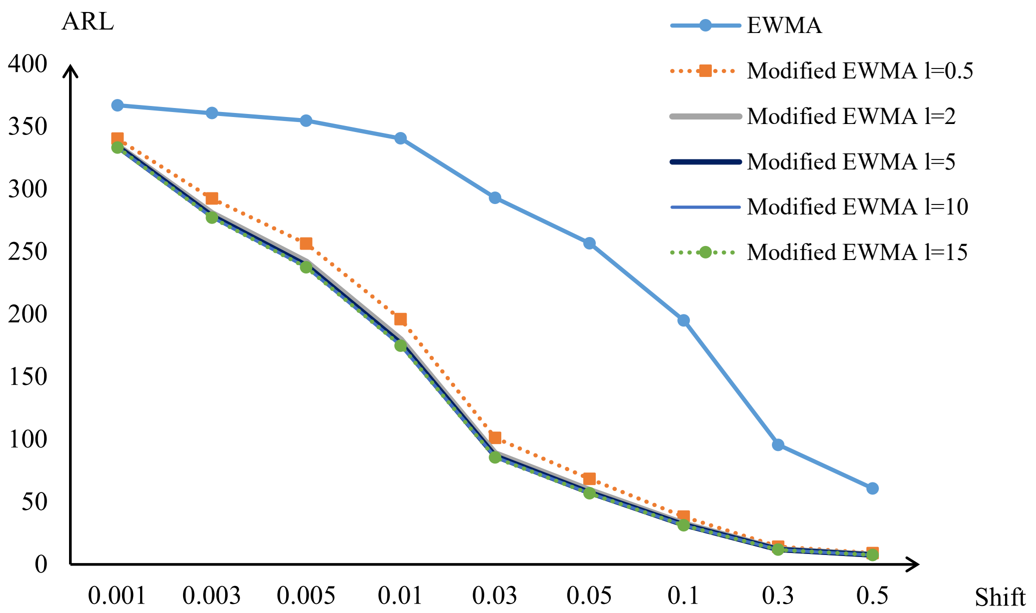


Figure 1 Comparison the ARL values between modified EWMA and EWMA control charts

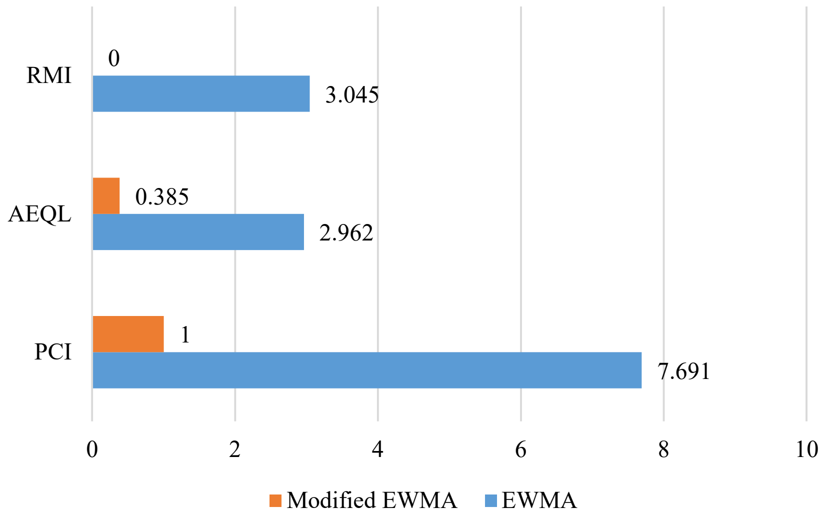
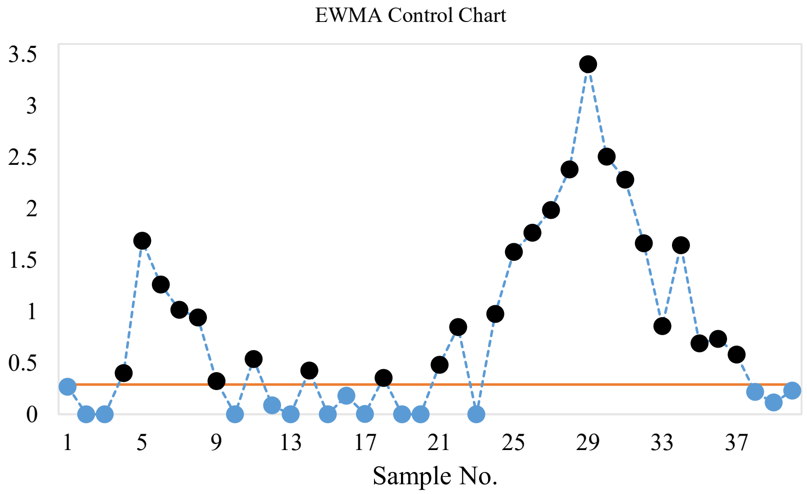
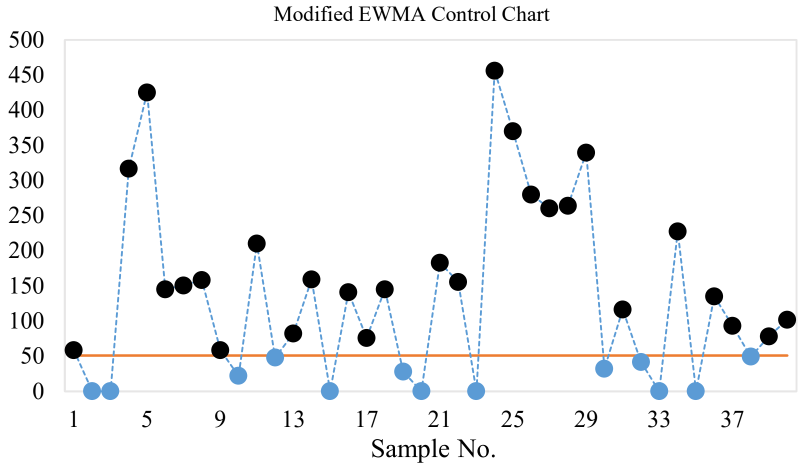


Figure 2 Comparison the RMI, AEQL and PCI values between modified EWMA and EWMA control charts



(a)



(b)

Figure 3 The crude oil WTI price dataset fitted to AR(2) with quadratic trend model process running on $\lambda = 0.1$ (a) EWMA control chart and (b) modified EWMA control chart

7. Discussion and Conclusions

In this research, the ARL explicit formulas on the modified EWMA control chart for AR(p) with the quadratic trend model were derived. The explicit formula is a method for finding the exact value of the ARL and is very useful in terms of decreased computational time. The numerical integral equation (NIE) method is used to compare the explicit formula by measuring the percentage accuracy (%). The results show that the explicit formula is as accurate as the NIE method but is computed in much less time. The modified EWMA and EWMA control charts were presented to assess their effectiveness in detecting process shifts to compare the performance of the control charts. The results found that the modified EWMA control chart had the best performance because it gave the lowest RMI, AEQL values, and PCI values equal to 1. Furthermore, the modified EWMA charts for all present better results than the EWMA control chart, and the modified EWMA chart is more effective for increased. From the study of both simulation and its application to real data; it is known that the proposed ARL explicit formulas can be used as a criterion to measure the efficiency of control charts accurately, and quickly, and to give results in the same direction. In future research, it is also possible to develop formulas for ARL values on modified EWMA control charts for new control charts or other interesting models.

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