Thailand Statistician
July 2024; 22(3): 674-687
http://statassoc.or.th
Contributed paper

Estimation of AQL, LQL and Quality Regions for Group Chain Sampling Plan with Binomial Distribution

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Received: 17 April 2022
Revised: 10 July 2022
Accepted: 26 July 2022

Abstract

Acceptance sampling is a technique for ensuring that both producers and consumers are satisfied with the quality of a product. This paper considers a group chain sampling plan (GChSP) using the binomial distribution. The probability of lot acceptance, $L(p)$, is determined by satisfying the producer's and consumer's risks under the specified design parameters. This paper proves that the proportion of defective decreases when the value of design parameters such as $g, r, i, \beta,$ and α increase. In this paper for specified values of producer's and consumer's risks, four different quality regions are estimated. The findings suggest that for the same values of design parameters the GChSP gives less proportion of defectives than the existing Bayesian group chain sampling plan (BGChSP). Therefore, the GChSP is better equipped for lot inspection in the manufacturing industry, especially those involved with destructive testing of high-quality products.

Keywords: Acceptance sampling, consumer's risk, producer's risk, proportion of defective, probability of lot acceptance

1. Introduction

Acceptance sampling is a technique that falls between zero inspection and 100% inspection. Acceptance sampling is the process of inspecting the sample of items from a production lot. Meanwhile, items are accepted without being inspected at all in zero inspection, whereas each item is inspected before being accepted in 100% inspection. Hence, acceptance sampling was constructed as an alternative for 100% inspection. This is because 100% inspection is impractical, particularly when the testing is destructive or expensive. The purpose of acceptance sampling is to assist the manufacturer in accepting or rejecting the batch, neither to estimate nor improve the quality of the batch Montgomery (2009). The acceptance sampling can indirectly encourage manufacturers to improve quality, thereby reducing the chance of batch rejection. Acceptance sampling is often employed as a quality control measure of raw materials and components, work-in-progress, or finished goods in various industries.

The development of several sampling plans that consider customer's risk, producer's risk, and sample size can be used to track the evolution of acceptance sampling. The probability of rejecting a good lot is defined as the producer's risk, while the probability of accepting a bad lot is identified as the consumer's risk by Sankar and Jeganathan (2019). The development of a sampling plan has the purpose of determining the minimum number of samples that will be inspected. Many researchers have proposed various combinations of sampling plans with different distributions to obtain the minimum sample size (Dobbah et al. 2018, Hafeez et al. 2022a, 2022b).

The single sampling plan (SSP) was first proposed by Dodge and Romig (1941), they consider only one item to decide about the lot under inspection. Epstein (1954), extend SSP to the chain sampling plan (ChSP-1) by addressing the weaknesses in SSP with zero acceptance number. In ChSP-1, the decision is based on cumulative sample results instead of the only current lot as in SSP. Ramaswamy and Jayasri (2014, 2015) have developed ChSP-1 with the generalized Rayleigh distribution and Weibull distribution. For ChSP-1, the current lot under inspection will be accepted if only one defective item is detected in the sample, and no further defective items are detected in the following lots. While compared to the SSP and ChSP-1 has been shown to have a higher probability of lot acceptance when the acceptance number is zero.

Consequently, the group acceptance sampling plan (GASP) is proposed for inspecting several items at the same time. Researchers such as Aslam and Jun (2009) and Aslam et al. (2011) have developed GASP using different distributions. Later group chain sampling plan (GChSP) was proposed by Mughal (2018), by considering the idea of GASP and ChSP-1. In GChSP the decision is based on cumulative results and this plan has ability to do multiple inspections at once Teh, Aziz and Zain (2021). For Marshall Olkin extended Lomax distribution new group chain sampling plan and two sided group chain acceptance sampling plan was considered by Aziz et al. (2022a, 2022b).

All these plans estimate one point and decide whether the lot under inspection will be accepted or rejected at that point. In this paper, first-time quality regions will be estimated for GChSP to satisfy both consumer and producer at the same time. These quality regions will provide a range of acceptable quality based on consumer's and producer's risks. Two points will be estimated for each quality region which is called acceptable quality level (AQL) and limiting quality level (LQL). For all possible combinations of specified design parameters, four quality regions will be estimated namely, probabilistic quality region (PQR) denoted by R_1, quality decision region (QDR) denoted by R_2, limiting quality region (LQR) denoted by R_3, and indifference quality region (IQR) denoted by R_4.

1.1. Glossary of Symbols

g : Number of groups

i : Number of preceding lots

r : Group size (available number of testers)

n : Sample size

d : Number of defective items

α : Producer's risk (Probability of rejecting a good lot)

β : Consumer's risk (Probability of accepting a bad lot)

$L(p)$: Probability of lot acceptance

T : Operating ratio between PQR and QDR

T_1 : Operating ratio between PQR and LQR

T_2 : Operating ratio between PQR and IQR.

2. Methodology

2.1. Operating procedure

The operational procedure of GChSP is described below:

(i) Select an ideal sample of size n and divide it into g groups. Such as each group contains r items and the required sample size $n = r \times g$.

(ii) Start the inspection and count the number of defectives d.

(iii) If in the current sample no defective is found, i.e., $d = 0$, accept the lot

(iv) If in the current sample more than one defective found, i.e., $d > 1$, reject the lot.

(v) If in the current sample one defective is found, i.e., $d = 1$, and in the immediately preceding i sample have no defective, accept the lot.

By addition law of probability, the probability of acceptance for zero and one defective using GChSP can be written as

$$L(p) = P(d = 0) + \{P(d = 1) / P(d = 0)_i\}. \quad (1)$$

This procedure is illustrated through a tree diagram for $i = 2$ in Figure 1. The defective and non-defective products are denoted by D and $\bar{D}$, respectively.

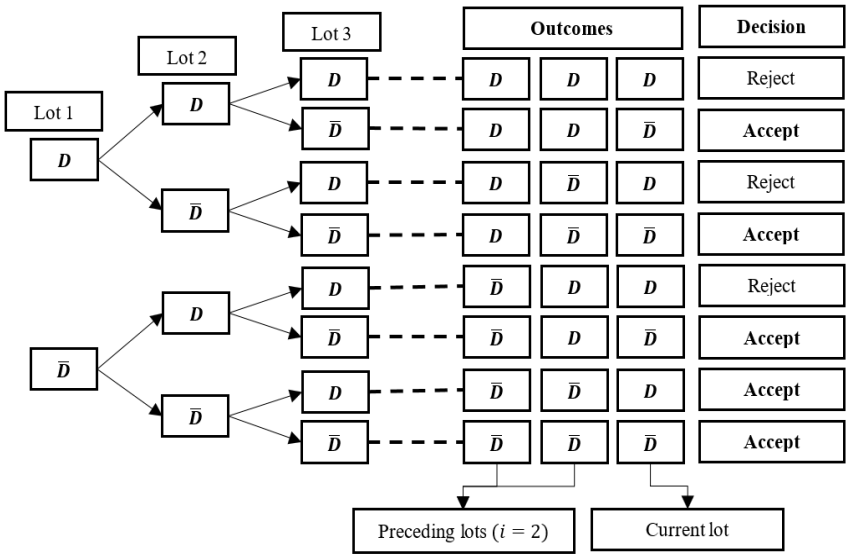

Figure 1 Tree diagram of GChSP for lot inspection when $i = 2$

Based on Figure 1, when $i = 2$ the following outcomes meet the acceptance criteria $\{DD\bar{D}, D\bar{D}\bar{D}, \bar{D}D\bar{D}, \bar{D}\bar{D}D, \bar{D}\bar{D}\bar{D}\}$.

2.2. Probability defective and probability of lot acceptance

By using group chain probability of lot acceptance, the possible outcomes can be written in the form of an equation:

$$L(p)_{GChSP} = P_{0,(r*g)} + P_{0,(r*g)}P_{0,(r*g)}P_{1,(r*g)}, \quad (2)$$

$$L(p)_{GChSP} = P_{0,(r*g)} + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2. \quad (3)$$

Finally, the general expression is the operating characteristic (OC) function of ChSP-1 provided by Dodge (1955).

$$L(p)_{GChSP} = P_{0,(r*g)} + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^{i}. \tag{4}$$

By considering binomial distribution to achieve the probability of lot acceptance for zero and one defective product. Here the binomial distribution is applicable because the product fulfils all four properties of the binomial experiment. This is applicable when a lot consists of identical and independent trails, the inspection outcomes are categorized into two mutually exclusive and independent outcomes Hafeez and Aziz (2022a, 2022b). So, the probability of lot acceptance can be written as

$$L(p) = \sum_{c=0}^{1}\binom{r*g}{c}p^{c}\left(1-p\right)^{r*g-c}, \tag{5}$$

where p is the probability of defective. After simplifying Equation (5) for zero and one defective product by replacing $c=0$ and $c=1,$ we get their corresponding probabilities:

$$P_0 = \left(1-p\right)^{r*g}, \tag{1}$$

$$P_1 = (r*g)p\left(1-p\right)^{r*g-1}. \tag{2}$$

For GChSP the probability of lot acceptance after replacing Equations (6) and (7) in (4) is (Mughal 2018)

$$L(p) = \left(1-p\right)^{r*g} + (r*g)p\left(1-p\right)^{r*g-1}\left(1-p\right)^{r*g*i}. \tag{3}$$

For GChSP the probability of lot acceptance based on the binomial distribution, we can rewrite Equation (8) as the OC function of the binomial model.

$$L(p) = \left(1-p\right)^{r\times g} + (r\times g)p\left(1-p\right)^{r*g(i+1)-1}. \tag{4}$$

For specified design parameters, the fraction of defectives is estimated from Equation (9) and presented in Table 1. Here Newton's approximation is used in Equation (9), where p is used as a control point by reducing $L(p)$.

2.3. Construction of quality region

For a sampling plan, a method of quality interval derived based on the range of quality instead of a point-wise description is called the quality region. This method can be adopted in the elementary production process, where the stipulated quality level is advised to be fixed at a later stage and provides a new concept for designing sampling plans through quality levels. For the product acceptance to meet the present product quality requirements, the sampling plans provide the decision rules for producers and consumers. Suppliers require high-quality products with a very low fraction of defectives due to rapid advancements in manufacturing technology. Unfortunately, in some situations, the traditional method may fail to discover minute defectives among the products. Quality interval sampling plans are introduced to overcome such problems. This idea provides a higher probability of acceptance, which depends on the quality measure of the AQL and LQL.

A quality region is based on two points AQL and LQL. Resembling the conventional OC curve, the AQL refers to the producer's risk $\alpha,$ and LQL refers to the consumer's risk $\beta,$ which needs to be minimized. The range of values between these two points, AQL and LQL, is called the quality region (QR).

(i) Probabilistic Quality Region (PQR)

In this interval, the product is accepted in PQR with the highest probability of 0.95 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL β. It is clear that PQR R_1 is exactly the standard setting of AQL $= p_1$ and LQL $= p_2$. In Figure 2, it is indicated that PQR lies between two points $p_1 \le R_1 \le p_2$. From the specified design parameters, this region considers $\alpha = \beta = 0.05$ and the range of PQR is based on $R_1 = p_2 - p_1$, where the values of R_1 are given in Table 2.

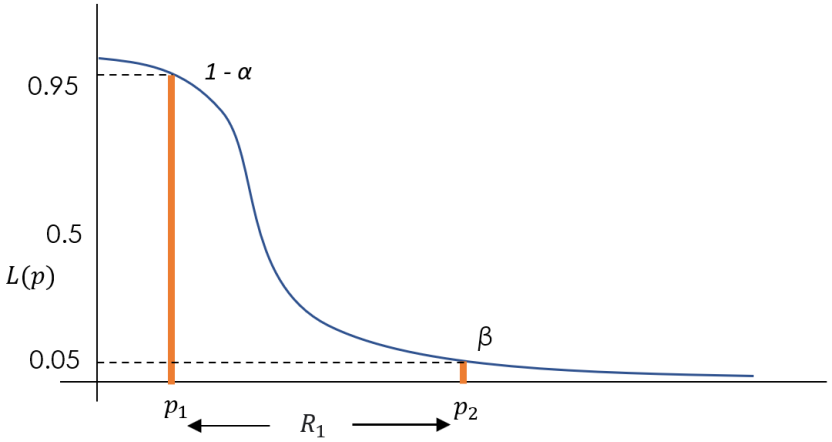

Figure 2 Probabilistic quality region (PQR)

(ii) Quality Decision Region (QDR)

In this interval, the product is accepted in QDR with the highest probability of 0.95 and the lowest probability of 0.25. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL β. Where QDR R_2 is exactly the standard setting of AQL $= p_1$ and LQL $= p_\beta$. It is also presented in Figure 3, that QDR lies between $p_1 \le R_2 \le p_\beta$. This region considers consumer's risk $\alpha = 0.05$ and producer's risk $\beta = 0.25$. In Table 2, the range of QDR $R_2 = p_\beta - p_1$ is given.

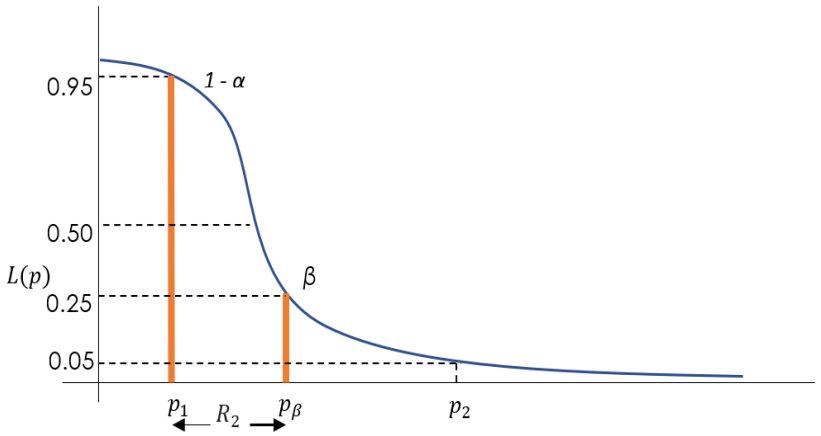

Figure 3 Quality decision region (QDR)

(iii) Limiting Quality Region (LQR)

In this interval, the product is accepted in LQR with the highest probability of 0.75 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL β. Where LQR R_3 is exactly the standard setting of AQL $= p_\alpha$ and LQL $= p_2$. It is shown in Figure 4 that LQR lies between $p_\alpha \le R_3 \le p_2$ points on the x-axis. This region considers $\alpha = 0.25$ and $\beta = 0.05$. Thus, the range of LQR is $R_3 = p_2 - p_\alpha$ presented in Table 2.

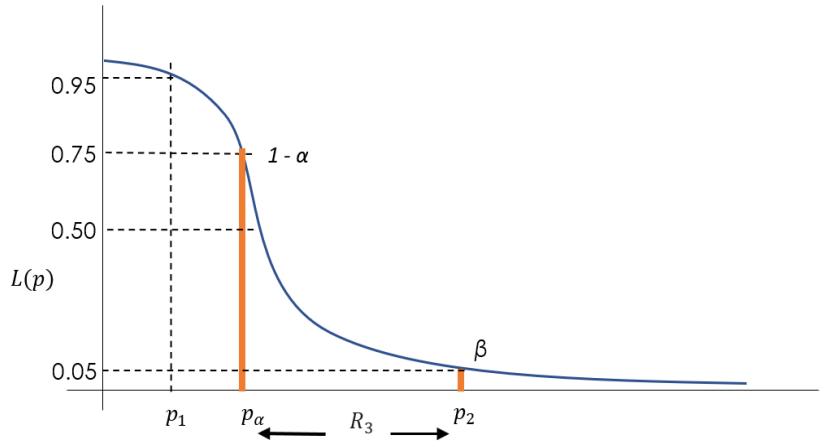

Figure 4 Limiting Quality Region (LQR)

(iv) Indifference Quality Region (IQR)

In this interval, the product is accepted in IQR with the highest probability of 0.5 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL β. Where IQR R_4 is exactly the standard setting of AQL $= p_*$ and LQL $= p_2$. IQR lies between two points $p_* \le R_4 \le p_2$, that are highlighted on x-axes in Figure 5. This region considers producer's risk $\alpha = 0.5$ and consumer's risk $\beta = 0.05$, also estimated values of the range of IQR $R_4 = p_2 - p_*$ are given in Table 2.

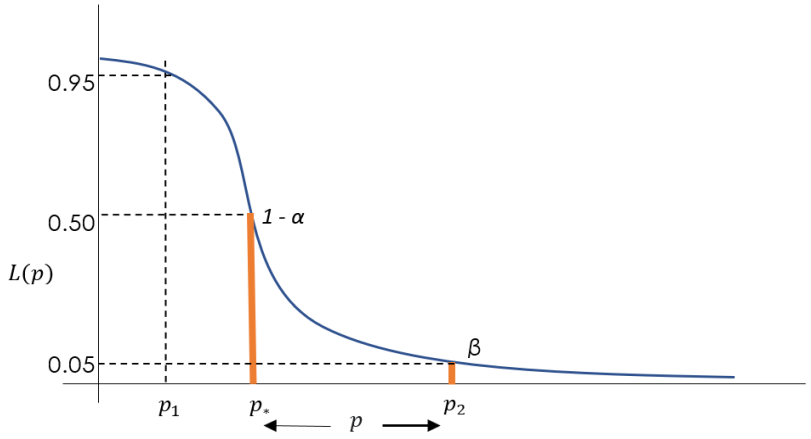

Figure 5 Indifference Quality Region (IQR)

2.4. Selection of Sampling Plan for GChSP

For any given value of PQR (R_1), QDR (R_2), LQR (R_3), IQR (R_4), We can find the operating ratio $T=\frac{R_1}{R_2}, T_1=\frac{R_1}{R_3}$ and $T_2=\frac{R_1}{R_4}$. Find the value which is approximately equal to the required ratio under the column of T, T_1 and T_2 in Table 2. From this operating ratio the corresponding design parameters g, r and i can be determined in Table 2. By using these design parameters for GChSP, the values of quality regions can be assessed from Table 2, and required AQL and LQL can be obtained from Table 1.

3. Results

3.1. Numerical examples

1) Specified PQR and QDR

Assume a manufacturer produces defectives in the PQR and QDR regions 0.25% and QDR 0.12% respectively, these values are based on the manufacturer's requirement he can change these values according to his need. Then $R_1=0.0020, R_3=0.0017$ and the determined operating ratio is 2.0833. In Table 2, find a T value that is approximately equal to the specified ratio, which is found to be $T=2.0798$, with corresponding design parameters $g=3, r=3$ and $i=4$. For this operating ratio, the ranges of PQR and QDR are $R_1=0.2694$ and $R_2=0.1295$ respectively in Table 2. Hence the required design parameters for GChSP are found $g=3, r=3$ and $i=4,,$ with $p_1=0.0138, p_\beta=0.1433$ and $p_2=0.2831$ from Table 1.

Table 1 Certain p values in GChSP for specified g, r, i and $L(p)$

g	r	i	$L(p)$								
			0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	2	1	0.0460	0.1068	0.1559	0.2654	0.4202	0.5896	0.73200	0.8049	0.9075
		2	0.0347	0.0817	0.1208	0.2119	0.3522	0.5259	0.6901	0.7783	0.9001
		3	0.0291	0.0693	0.1035	0.1863	0.3233	0.5072	0.6845	0.7765	0.9000
		4	0.0256	0.0617	0.0930	0.1712	0.3088	0.5019	0.6838	0.7764	0.9000
	3	1	0.0300	0.0704	0.1037	0.1807	0.2972	0.4388	0.5759	0.6564	0.7915
		2	0.0228	0.0543	0.0809	0.1447	0.2486	0.3896	0.5409	0.6331	0.7846
		3	0.0192	0.0462	0.0695	0.1273	0.2279	0.3754	0.5364	0.6317	0.7846
		4	0.0170	0.0412	0.0625	0.1170	0.2176	0.3715	0.5359	0.6316	0.7846
	4	1	0.0222	0.0525	0.0777	0.1370	0.2297	0.3482	0.4712	0.5484	0.6900
		2	0.0170	0.0407	0.0608	0.1098	0.1919	0.3085	0.4418	0.5284	0.6838
		3	0.0144	0.0347	0.0524	0.0967	0.1759	0.2972	0.4381	0.5272	0.6838
		4	0.0127	0.0309	0.0471	0.0888	0.1678	0.2940	0.4377	0.5271	0.6838
2	2	1	0.0222	0.0525	0.0777	0.1370	0.2297	0.3482	0.4712	0.5484	0.6900
		2	0.0170	0.0407	0.0608	0.1098	0.1919	0.3085	0.4418	0.5284	0.6838
		3	0.0144	0.0347	0.0524	0.0967	0.1759	0.2972	0.4381	0.5272	0.6838
		4	0.0127	0.0309	0.0471	0.0888	0.1678	0.294	0.4377	0.5271	0.6838
	3	1	0.0146	0.0348	0.0517	0.0923	0.1578	0.2459	0.3437	0.4093	0.5408
		2	0.0113	0.0271	0.0407	0.0741	0.1317	0.2174	0.3217	0.3940	0.5359
		3	0.0095	0.0231	0.035	0.0652	0.1207	0.2093	0.3190	0.3931	0.5358
		4	0.0085	0.0206	0.0315	0.0599	0.1151	0.2071	0.3187	0.3931	0.5358
	4	1	0.0109	0.0260	0.0388	0.0696	0.1202	0.1899	0.2699	0.3254	0.4417
		2	0.0084	0.0203	0.0305	0.0559	0.1003	0.1677	0.2525	0.3131	0.4377
		3	0.0072	0.0174	0.0263	0.0492	0.0918	0.1614	0.2504	0.3124	0.4377
		4	0.0063	0.0155	0.0237	0.0452	0.0876	0.1597	0.2501	0.3123	0.4377
3	2	1	0.0146	0.0348	0.0517	0.0923	0.1578	0.2459	0.3437	0.4093	0.5408
		2	0.0113	0.0271	0.0407	0.0741	0.1317	0.2174	0.3217	0.3940	0.5359
		3	0.0095	0.0231	0.035	0.0652	0.1207	0.2093	0.3190	0.3931	0.5358
		4	0.0085	0.0206	0.0315	0.0599	0.1151	0.2071	0.3187	0.3931	0.5358
	3	1	0.0097	0.0231	0.0345	0.0620	0.1074	0.1704	0.2437	0.2950	0.4042
		2	0.0075	0.0180	0.0272	0.0498	0.0896	0.1505	0.2279	0.2838	0.4006
		3	0.0063	0.0154	0.0234	0.0439	0.0820	0.1449	0.2260	0.2832	0.4005
		4	0.0056	0.0138	0.0211	0.0403	0.0782	0.1433	0.2258	0.2831	0.4005
	4	1	0.0073	0.0173	0.0259	0.0467	0.0813	0.1304	0.1886	0.2302	0.3217
		2	0.0056	0.0135	0.0204	0.0375	0.0678	0.1150	0.1763	0.2215	0.3187
		3	0.0048	0.0116	0.0176	0.0330	0.0621	0.1107	0.1748	0.2209	0.3187
		4	0.0042	0.0103	0.0158	0.0303	0.0593	0.1095	0.1746	0.2209	0.3187
4	2	1	0.0109	0.026	0.0388	0.0696	0.1202	0.1899	0.2699	0.3254	0.4417
		2	0.0084	0.0203	0.0305	0.0559	0.1003	0.1677	0.2525	0.3131	0.4377
		3	0.0072	0.0174	0.0263	0.0492	0.0918	0.1614	0.2504	0.3124	0.4377
		4	0.0063	0.0155	0.0237	0.0452	0.0876	0.1597	0.2501	0.3123	0.4377
	3	1	0.0073	0.0173	0.0259	0.0467	0.0813	0.1304	0.1886	0.2302	0.3217
		2	0.0056	0.0135	0.0204	0.0375	0.0678	0.1150	0.1763	0.2215	0.3187
		3	0.0048	0.0116	0.0176	0.033	0.0621	0.1107	0.1748	0.2209	0.3187
		4	0.0042	0.0103	0.0158	0.0303	0.0593	0.1095	0.1746	0.2209	0.3187
	4	1	0.0054	0.0129	0.0194	0.0351	0.0615	0.0992	0.1448	0.1780	0.2525
		2	0.0042	0.0101	0.0153	0.0282	0.0513	0.0875	0.1353	0.1712	0.2501
		3	0.0036	0.0087	0.0132	0.0249	0.0469	0.0842	0.1342	0.1708	0.2501
		4	0.0032	0.0077	0.0119	0.0228	0.0447	0.0833	0.1341	0.1707	0.2501

Table 2 For some specified values of design parameters, the range of quality regions and operating ratios for GChSP

g	r	i	R_1	R_2	R_3	R_4	T	T_1	T_2
1	2	1	0.6981	0.4827	0.5395	0.3847	1.4462	1.2940	1.8145
		2	0.6966	0.4442	0.5664	0.4261	1.5682	1.2298	1.6349
		3	0.7072	0.4379	0.5901	0.4532	1.6149	1.1983	1.5603
		4	0.7147	0.4402	0.6052	0.4676	1.6235	1.1810	1.5285
	3	1	0.5861	0.3685	0.4757	0.3592	1.5905	1.2321	1.6315
		2	0.5788	0.3353	0.4884	0.3845	1.7265	1.1851	1.5053
		3	0.5854	0.3292	0.5044	0.4038	1.7785	1.1608	1.4499
		4	0.5904	0.3303	0.5146	0.414	1.7877	1.1473	1.4261
	4	1	0.4959	0.2958	0.4114	0.3187	1.6767	1.2055	1.5559
		2	0.4877	0.2678	0.4186	0.3365	1.8213	1.1652	1.4495
		3	0.4925	0.2625	0.4305	0.3514	1.8765	1.1439	1.4017
		4	0.4962	0.2631	0.4383	0.3593	1.8861	1.1321	1.3811
2	2	1	0.4959	0.2958	0.4114	0.3187	1.6767	1.2055	1.5559
		2	0.4877	0.2678	0.4186	0.3365	1.8213	1.1652	1.4495
		3	0.4925	0.2625	0.4305	0.3514	1.8765	1.1439	1.4017
		4	0.4962	0.2631	0.4383	0.3593	1.8861	1.1321	1.3811
	3	1	0.3745	0.2112	0.317	0.2515	1.7735	1.1816	1.4893
		2	0.3669	0.1903	0.3198	0.2622	1.9277	1.1471	1.3991
		3	0.3700	0.1862	0.3279	0.2724	1.9871	1.1284	1.3580
		4	0.3724	0.1864	0.3331	0.2779	1.9975	1.1180	1.3399
	4	1	0.2994	0.1638	0.2557	0.2052	1.8271	1.1705	1.4589
		2	0.2928	0.1474	0.2572	0.2129	1.9873	1.1385	1.3757
		3	0.295	0.1441	0.2631	0.2206	2.0474	1.1212	1.3375
		4	0.2968	0.1442	0.2671	0.2247	2.0581	1.1114	1.3208
3	2	1	0.3745	0.2112	0.317	0.2515	1.7735	1.1816	1.4893
		2	0.3669	0.1903	0.3198	0.2622	1.9277	1.1471	1.3991
		3	0.3700	0.1862	0.3279	0.2724	1.9871	1.1284	1.3580
		4	0.3724	0.1864	0.3331	0.2779	1.9975	1.1180	1.3399
	3	1	0.2719	0.1473	0.2330	0.1876	1.8455	1.1670	1.449
		2	0.2658	0.1324	0.2340	0.1943	2.0073	1.1359	1.3683
		3	0.2677	0.1294	0.2393	0.2012	2.0687	1.1188	1.3309
		4	0.2694	0.1295	0.2428	0.2049	2.0798	1.1092	1.3145
	4	1	0.2129	0.1131	0.1836	0.1489	1.8829	1.1599	1.4301
		2	0.2079	0.1015	0.1839	0.1536	2.0484	1.1305	1.3537
		3	0.2094	0.0991	0.1879	0.1589	2.1122	1.1143	1.3181
		4	0.2106	0.0992	0.1906	0.1617	2.1234	1.105	1.3026
4	2	1	0.2994	0.1638	0.2557	0.2052	1.8271	1.1705	1.4589
		2	0.2928	0.1474	0.2572	0.2129	1.9873	1.1385	1.3757
		3	0.295	0.1441	0.2631	0.2206	2.0474	1.1212	1.3375
		4	0.2968	0.1442	0.2671	0.2247	2.0581	1.1114	1.3208
	3	1	0.2129	0.1131	0.1836	0.1489	1.8829	1.1599	1.4301
		2	0.2079	0.1015	0.1839	0.1536	2.0484	1.1305	1.3537
		3	0.2094	0.0991	0.1879	0.1589	2.1122	1.1143	1.3181
		4	0.2106	0.0992	0.1906	0.1617	2.1234	1.1050	1.3026
	4	1	0.165	0.0863	0.1429	0.1165	1.9125	1.1551	1.4165
		2	0.161	0.0773	0.1429	0.1199	2.0818	1.1265	1.3431
		3	0.1621	0.0755	0.1459	0.1239	2.1459	1.111	1.3087
		4	0.163	0.0756	0.1479	0.126	2.1571	1.102	1.2936

2) Specified PQR and LQR

Assume a manufacturer produces defectives in the PQR and LQR regions 0.20% and LQR 0.17% respectively. Then $R_1 = 0.0020, R_3 = 0.0017$, and the determined operating ratio is 1.1765. In Table 2, for the specified ratio, the value is found to be $T_1 = 1.1737,$ with parallel design parameters $g = 2, r = 4$ and $i = 1.$ Therefore, for this operating ratio, the ranges of PQR and LQR are $R_1 = 0.2994$ and $R_3 = 0.2551$ respectively from Table 2. Hence for GChSP required design parameters are found to be $g = 2, r = 4$ and $i = 1$ with $p_1 = 0.026, p_\alpha = 0.0696,$ and $p_2 = 0.3254$ from Table 1.

3) Specified PQR and IQR

Assume a manufacturer produces defectives in the PQR and IQR regions of 0.20% and 0.15% respectively. Then $R_1 = 0.0020,$ $R_4 = 0.0015$ and the determined operating ratio is 1.3333. For the specified ratio, the value is found to be $T_2 = 1.3375$ with parallel design parameters $g = 2, r = 4$ and $i = 3,$ in Table 2. Therefore, the ranges of PQR and IQR for this operating ratio are $R_1 = 0.295$ and $R_4 = 0.2206$ respectively from Table 2. For GChSP the parameters $g = 2, r = 4$ and $i = 3$ are required, with $p_1 = 0.0174, p_* = 0.0918$ and $p_2 = 0.3124$ from Table 1.

3.2. Performances comparison

In this section for the specified values of design parameters, the probability of defective by all four quality regions is compared with BGChSP suggested by Hafeez and Aziz (2019).

Table 3 Comparison between quality regions for GChSP and BGChSP for $g = 2, r = 3$ and $i = 4$

Quality region	BGChSP			GChSP
	$s = 1$	$s = 2$	$s = 3$	
R_1	0.7506	0.5895	0.5160	0.3724
R_2	0.3299	0.2548	0.2308	0.1864
R_3	0.7028	0.5456	0.4736	0.3331
R_4	0.6115	0.4723	0.4062	0.2779

Table 4 Comparison between quality regions for GChSP and BGChSP for $g = 4, r = 3$ and $i = 2$

Quality region	BGChSP			GChSP
	$s = 1$	$s = 2$	$s = 3$	
R_1	0.6287	0.4042	0.3282	0.2079
R_2	0.2141	0.1517	0.1337	0.1015
R_3	0.6001	0.3779	0.3026	0.1839
R_4	0.5452	0.3357	0.2644	0.1536

From Tables 3 and 4, it can be observed that for all quality regions, GChSP gives the lowest probability of defective than BGChSP. From Table 2, it can be observed that as the value of g, r increases, the range of quality regions decreases. Also, for GChSP as the value of i increases the range of quality regions increases for PQR, LQR and IQR, but decreases for QDR.

From these results, we conclude that in industry GChSP and BGChSP are used on the same product for inspection under the same conditions. Then GChSP will accept fewer defective products than the BGChSP for the same values of design parameters.

4. Application

For the real-life application of GChSP the data set is taken from Walpole et al. (2007), where large steel plates are being manufactured. Every hour a sample size of 50 is collected, and the number of defectives is noted in each sample. From manufacturing lots 20 samples are selected in which the number of defectives are found; $4,2,1,3,0,1,2,2,3,1,4,5,3,2,2,4,3,2,1,$ and 3. Suppose the design parameters, preceding lots $i=3$ and the available number of testers $r=5$. Hence the sample size $n=50$ is divided into $g=10$ groups, i.e., $n=r\times g=5\times 10=50$.

When the experimenter set up the GChSP plan according to the above-mentioned specifications. According to PQR considering consumer's risk 0.05 and producer's risk 0.05, we use Equation (9) and follow the same procedure as for Table 1. Then the estimated values for AQL $(p_1=0.002563)$, LQL $(p_2=0.097491)$ and the range of PQR is $R_1=p_2-p_1=0.094928$.

Now, for QDR the approximate value for AQL is $p_1=0.002563$, LQL is $p_\beta=0.036068$ and the range of QDR is $R_2=0.033505$. Based on PQR and QDR, the operating ratio $T=\frac{R_1}{R_2}=2.833$. Similarly, for LQR the estimated values for AQL is $p_\alpha=0.008249$, LQL is $p_2=0.097491$ and the range of LQR is $R_3=0.089242$. Based on PQR and LQR, the operating ratio $T_1=\frac{R_1}{R_3}=1.064$. Likewise, for IQR the estimated values for AQL is $p_*=0.017462$, LQL is $p_2=0.097491$ and the range of IQR is $R_4=0.080029$. Based on PQR and IQR, the operating ratio $T_2=\frac{R_1}{R_4}=1.186$.

5. Conclusions

In this paper, by using binomial distribution the probability of lot acceptance is obtained and quality regions are estimated for GChSP. These quality regions provide an acceptable range of quality for both producer and consumer. The results have shown that the proportion of defective decreases when the value of design parameters such as g,r and i increase. The comparison has exposed that GChSP provides a smaller number of defectives than the BGChSP, while still having a higher probability of lot acceptance. We can suggest that GChSP with these quality regions has the ability to reduce inspection, operating costs, and a lower risk of item damage due to mishandling. In conclusion, we suggest that GChSP is a better alternative option for manufacturers to use for production lot inspection.

Acknowledgements

All authors express their gratitude to the reviewers for their valuable suggestions, as these helped to enhance the manuscript.

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Appendix A

R program of group chain sampling plan to generate Table 1.

```
g=c(rep(c(rep(1,12),rep(2,12),rep(3,12),rep(4,12))))
r=c(rep(c(rep(2,4),rep(3,4),rep(4,4)),4))
i=c(rep(rep(c(1:4),12)))
P <- list()
for(gc in  1:9){
  if(gc==1){
    pbar=c(0.99);
  }else if(gc==2){
    pbar=c(0.95);
  }else if(gc==3){
    pbar=c(0.9);
  }else if(gc==4){
    pbar=c(0.75);
  }else if(gc==5){
    pbar=c(0.5);
  }else if(gc==6){
    pbar=c(0.25);
  }else if(gc==7){
    pbar=c(0.1);
  }else if(gc==8){
    pbar=c(0.05);
  }else if(gc==9){
    pbar=c(0.01);
  }
  Ptemp = paste('P',gc,sep="")
  Ptemp=c(gc)

  for(w in  1:length(g)){
    f <- function(p)  (pbar-(((1-p)^(r[w]*g[w]))+r[w]*g[w]*p*(1-p)^(r[w]*g[w]*(i[w]+1)-1)))
    Ptemp[w]=uniroot(f, lower=0, upper=1)$root;
  }
  P[gc] <- list(Ptemp)
}
####Table 1####
table=cbind(g,r,i,P[[1]],P[[2]],P[[3]],P[[4]],P[[5]],P[[6]],P[[7]],P[[8]],P[[9]])
table1=round(table, digits = 4)
print(table1)
```

Appendix B

R program to construct quality regions for group chain sampling plan to generate Table 2.

```
###PQR 0.05 - 0.95###
R1=P[[8]]-P[[2]]
###QDR 0.25 - 0.95###
R2=P[[6]]-P[[2]]
###LQR 0.05 - 0.75###
R3=P[[8]]-P[[4]]
###IQR 0.05 - 0.5###
R4=P[[8]]-P[[5]]

###Operating ratios###
T=R1/R2
T1=R1/R3
T2=R1/R4

####Table2####
tabl2=cbind(g,r,i,R1,R2,R3,R4,T,T1,T2)
table2=round(tabl2, digits = 4)
print(table2)
```