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Combined Quality Control Scheme for Monitoring Autocorrelated Process

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Abstract

In statistical process control, the control chart helps to diagnose the presence of variation due to assignable causes so that the process can achieve statistical control. There is no doubt that the process exhibiting autocorrelation degrades the functioning of control chart by producing incessant false signals or responding gradually to out-of-control state. The inefficiency of Shewhart control chart to spot small displacements leads to the application of alternate charting techniques like cumulative sum (CUSUM) and exponentially weighted moving average (EWMA). Both CUSUM and EWMA are helpful in detecting small to moderate displacements in the process. A mixed EWMA-CUSUM (MEC) chart was also proposed to improve the detection ability against the smaller shifts. This paper proposed a combined EWMA-MEC quality control scheme to detect small, moderate and large shifts. We fitted an autoregressive process to the autocorrelated observation and applied the charting technique directly to the residuals. Performance measure average run length (ARL) is used to assess the impact of the proposed scheme. We have evaluated ARL of the proposed scheme and compared it with the ARL of MEC, CUSUM and EWMA control charts. The results indicate that the proposed scheme is more sensitive to detecting small to moderate shifts than the previous schemes. We have also discussed the performance of the proposed scheme for the misdesigned charts, i.e., if the shift is different than the anticipated shift, and found that the proposed scheme performs better for the misdesigned cases than the traditional charts.

Keywords: EWMA, mixed EWMA-CUSUM, autocorrelation, average run length, average run length ratio, combined EWMA-MEC.

1. Introduction

The amalgamation of seven major statistical process control (SPC) tools, generally known as “the magnificent seven”, plays a crucial role in every production process. These tools aid in quality improvement and boost the productivity of a process by keeping an eye on the variations in the process parameters. Based on SPC literature, causes of variation in any production procedure can be categorized into two parts: i) Variations by common cause, ii) Variations by special (assignable) cause.

In any production process, if the source of variation is a common cause only, the process is said to be statistically in control. However, behavior goes statistically out of control when the special cause enters the process. Also, common causes have random behavior which cannot be controlled while special causes have unnatural variation which should be controlled by taking corrective measures and actions. The factors responsible for this unnatural variation include unskilled operators, faulty machine parts, etc. Initially, Shewhart (1920) came up with the first control chart known as the Shewhart chart. After that, many researchers developed several control chart techniques for monitoring the process, but Shewhart (1920) technique has remained widespread in applications. The reason behind this is the simplicity of its implementation and the low cost, time, and resources required. But Stombous et al. (2000) quantified that "such simple charts are usually far from optimal and may even be inappropriate". This can be explained in such a way that Shewhart charts are inefficient in detecting small-sized sustained shifts in the process. This is due to its limitation of utilizing process information from the very recent observation only and ignoring any other information observed by the whole sequence of process data points. To overcome this drawback, the Western Electric Supplementary Run rules were introduced, but it has been found that these run rules are not effective as they increase the number of false alarms by dramatically reducing the average run length (ARL) when the process is already in statistical control. The quality control engineers need an alternative to the Shewhart chart for small shifts. The cumulative sum (CUSUM) chart suggested by Page (1954) and the exponentially weighted moving average (EWMA) chart suggested by Roberts (1959) are two key techniques for monitoring small shifts in the process. In many processes, the primary assumption of independently and identically distributed observations is not always fulfilled. This assumption is violated when the process of generating data points is autocorrelated. This autocorrelation is inherent in nature and disrupts the behavior of control chart schemes by increasing the incessant number of false indications. Several articles on efficient handling of the impact of autocorrelation have been discussed previously. We can offset the impact of autocorrelation by sampling process observation less frequently. However, this approach has a downside due to the availability of less information, which makes the control scheme less efficient in detecting any changes in the process. Therefore, the phenomenon of autocorrelation should deal with some other impactful strategies like the use of residual control schemes, modified control charts, skip sampling strategy, etc. The article presented here deals with the application of residual control charts. In the residual scheme, a time series model is fitted to the autocorrelated data, and charting techniques are applied directly to the residuals. This is because the residuals are expected to be uncorrelated. Therefore, this approach renovates the existing chart methodology into residual chart methodology. Also, we must have proper knowledge of fitting the time series model to the autocorrelated observation. It is quite challenging to choose an appropriate time series model and estimate the parameters of the chosen model to make this charting technique more efficient.

According to Harris and Ross (1991), "the effect of autocorrelation on the performance of the EWMA and CUSUM charts also concluded that serious error may arise if the problem of autocorrelation is not considered". Alwan and Roberts (1988) handled the autocorrelation problem by developing the Shewhart residual control chart. Wardell (1992) used ARMA(1,1) model for modeling autocorrelated observation sequence and compared the Shewhart and exponentially weighted moving average chart with the special cause charts and common cause charts developed by Alwan and Roberts. Karaoglan and Bayban (2011) performed a case study using real-life (vegetable oil) data from industry by fitting trend AR(1) process models and analyzing the efficiency of control charts. Lu and Reynolds (1999) explored the ability of CUSUM residual and EWMA residual control charts to perform using AR(1) plus a random error term model. Zhang (1998) introduced an exponentially weighted moving average control chart for the stationary process (EWMAST) and compared its performance with modified residual Shewhart for AR(1), AR(2) and ARMA(1,1) models. Recently, Abbas et al. (2010) and Zaman et al. (2015) suggested “the mixed EWMA-CUSUM (MEC) and mixed CUSUM-EWMA (MCE) control chart techniques for monitoring the normally distributed process observations”. However, these charts cannot offset the influence of autocorrelation to some extent. So, Abbasi et al. (2017) studied “the impact of autocorrelation using the mixed EWMA-CUSUM (MEC) and mixed CUSUM-EWMA (MCE) residual control chart techniques. Ali and Lone (2021) article present “deviation based exponentially weighted moving average control charts and observed significant improvements using the new proposal to detect out-of-control situations”. Tyagi and Yadav (2021) presented, “a combination of EWMA and CUSUM charting techniques supplementing modifications in the control limits which is found reasonably well for detecting particularly smaller displacements in the autocorrelated process”. Khusna et al. (2021) propose a Max-MCUSUM control chart based on the residual of multioutput least square support vector regression (MLS-SVR) and is more sensitive to detect mean vector shift. The performing ability of these control charts is compared with the existing traditional and modified control charts and found to be effective in detecting small shifts in the process”. Hawkins and Wu (2015) quantify that “the CUSUM scheme outperforms EWMA at the shift for which each was designed. If the actual shift is smaller than that used in the design, the EWMA scheme performs more efficiently than CUSUM”. In this article, we propose the combination of EWMA-MEC quality control charts for residuals, intending to enhance the detection ability of the control chart structure. We have designed our proposed scheme for detecting shifts of different sizes, as discussed by Hawkins and Wu (2015). The rest of the paper is organized as follows. Section 2 considers the modeling of autocorrelated data. Section 3 illustrates the design structure of the CUSUM, EWMA and MEC residual control chart. Section 4 and Section 5 deal with the proposed scheme and its performance evaluation respectively. Lastly, the conclusions of this article are presented in Section 6.

2. Modelling of Autocorrelated Data for Residual

In residual chart methodology, a suitable time series model is fitted to the process data to accommodate the autocorrelation problem. The obtained residuals e_t are expected to be uncorrelated. Now, we apply the control chart techniques to the residuals directly. We have used AR(1) time series model in this article. The AR(1) is given by:

$$X_t = \mu + \phi(X_{t-1} - \mu) + \varepsilon_t \tag{1}$$

where X_t is observed time series at time t, ϕ indicates autoregressive parameter ($\phi < 1$), μ denotes the mean of the process data and ε_t indicates the white noise term which is distributed independently and normally with 0 mean and variance σ_ε^2 (i.e., $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$). For allowing shifts in the process mean, a time-varying mean is included in the in equation (1). Therefore,

$$X_t = \mu_t + \phi(X_{t-1} - \mu_{t-1}) + \varepsilon_t \tag{2}$$

For modeling assignable causes, a mean shift of magnitude δ is inserted in equation (2) such that the mean shifts to $\mu + \delta$ from μ. We obtained the residuals which are as follows:

$$e_t = X_t - \hat{X}_t, \tag{3}$$

$$\begin{aligned} e_t &= \varepsilon_t + \delta,\ t = 1 \\ e_t &= \varepsilon_t + (1-\phi)\delta,\ t > 1, \end{aligned} \tag{4}$$

where $\hat{X}_t = \mu(1-\phi) + \phi X_{t-1}$ following the assumptions that all the estimates of the coefficients are accurate. $\hat{X}_t$ are the forecast values of X_t. Subsequently, residuals (e_t) are considered original observations, for which we apply control chart techniques instead of the original process information X_t.

3. Design Structure of Residual Control charts

A brief discussion of the residual control chart's structure has been done in this section.

3.1. CUSUM residual control chart

The traditional CUSUM scheme efficiently senses small displacements in the process's mean. To neutralize the impact of autocorrelation, the traditional CUSUM scheme was altered to the CUSUM residual scheme. The two-sided CUSUM residual statistic are given by

$$C_t^+ = \max(0,\ e_t - \mu - a_0 + C_{t-1}^+),\ \ C_t^- = \max(0,\ \mu - e_t - a_0 + C_{t-1}^-). \tag{5}$$

Here, $a_0 = a*\sigma_e$ and $b_0 = b*\sigma_e$ are the parameters of the CUSUM chart known as reference value and decision interval respectively; the value of a is taken as 0.5 as it tunes the CUSUM chart sensitive for small and moderate shifts experienced by the process. The process remains in-control until any of the CUSUM statistics exceeds the decision interval b_0,

$$C_t^+ > b_0,\ \ C_t^- > b_0. \tag{6}$$

3.2. EWMA residual control chart

The EWMA control scheme was introduced by Roberts (1959). The EWMA control scheme is best noted for dealing with the problem of small and moderate shifts in the process mean. As mentioned, this is due to its characteristic of accumulating very recent information. The mass involved with the process data declines exponentially as the process observation becomes less recent. The EWMA residual control chart is used in case of autocorrelated data for which the test statistic is defined as

$$Z_t = (1-\lambda)Z_{t-1} + \lambda e_t, \tag{7}$$

where λ is an invariable smoothing parameter satisfying $(0 < \lambda \le 1)$. The primary value Z_0 is taken as the process target i.e., the mean of the prior information is occasionally used as the preliminary value of the EWMA. The variance for EWMA statistics is given by

$$Var(Z_t) = \sigma_{z_t}^2 = \sigma_e^2 = \left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]. \tag{8}$$

The process gives indications of out-of-control if Z_t falls outside the control limits given by

$$\begin{cases} UCL = \mu + L\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]}, \\ LCL = \mu - L\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]}. \end{cases} \tag{9}$$

Here σ_e^2 denotes the standard deviation of residuals and L represents the width of the control limit.

3.3. MEC residual control chart

This section presents the design structure of MEC residual chart. In MEC chart CUSUM statistics of the EWMA statistic are plotted on control charts. The MEC residual charting statistics are defined as

$$\begin{cases} MEC_t^+ = \max(0, Z_t - \mu - a_{0_t} + MEC_{t-1}^+), \\ MEC_t^- = \max(0, \mu - Z_t - a_{0_t} + MEC_{t-1}^-). \end{cases} \tag{10}$$

where the Z_t is the EWMA statistic given in Equation (7). The parameter a_{0_t} signifies the reference value and another parameter, say b_{0_t} against which the CUSUM statistics is plotted is known as decision interval for MEC residual chart. The process remains under control until the statistics MEC_t^+ and MEC_t^- are plotted outside the decision interval b_{0_t}. It can be observed that if these statistics plotted above the decision interval b_{0_t}, the process mean is shifted above or below the target value. The value of the decision interval is chosen according to a prefixed in-control ARL. The parameters a_{0_t} and b_{0_t} are time-varying and their values are given by

$$\begin{cases} a_{0_t} = a\sqrt{Var(Z_t)} = a\sigma_e\sqrt{\left(\dfrac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]}, \\ b_{0_t} = b\sqrt{Var(Z_t)} = b\sigma_e\sqrt{\left(\dfrac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]}. \end{cases} \tag{11}$$

4. Proposed Combined EWMA-MEC Quality Control Scheme

This section presents the proposed control chart scheme, an assortment of the EWMA residual and MEC residual control chart schemes. Therefore, the technique becomes very sensitive for detecting small shifts. In this technique, the process is considered to be out of control when the EWMA or MEC statistic goes beyond the EWMA control limits or decision interval b_{0_t} respectively i.e., when

$$\begin{cases} Z_t > UCL = \mu + L\sigma_e\sqrt{\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right]} \\ Z_t < LCL = \mu - L\sigma_e\sqrt{\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right]} \\ MEC_t^+ > b_{0_t} \\ MEC_t^- > b_{0_t}. \end{cases} \tag{12}$$

The proposed scheme is also designed separately to detect particularly small, moderate, and large shifts. Here, we consider parameters for the CUSUM and EWMA as suggested by Hawkins and Wu (2014).

Table 1 Parameters for CUSUM and EWMA

Shift	δ	k	λ
Small	0.5	0.25	0.047
Moderate	1	0.5	0.134
Large	2	1	0.364

The constant a and b resembles the constant k and h respectively of the classical CUSUM scheme. The proposed scheme designed for detecting small shifts by considering the value of

$k = 0.25$ and $\lambda = 0.047$. Similarly, for moderate shifts, the values of k and λ are taken as 0.5 and 0.134 respectively, and for large shifts, these values are taken as 1 and 0.364 respectively which are already shown in Table 1.

We have simulated data with the help of R-software. The autocorrelated observations are generated from the series of normally distributed observations with mean zero and unit standard deviation. After that AR(1) model is fitted to the autocorrelated observations and residuals are obtained. The control chart strategies are now directly applied to the residuals instead of process observations. The ARLs of the proposed control chart and others are computed at different levels of autocorrelation ($\phi = 0.25, 0.5, 0.75$ and 0.9) for the process shifts in the mean ranging from 0.0 to 2.0. The out-of-control ARLs are calculated by keeping the in-control ARL = 370 approximately and control limits are adjusted so that the scenario of in-control ARL i.e., 370 is sustained. The final tabulated ARLs are the average one thousand ARL values for each shift in the process mean.

Performance Evaluation of Combined EWMA-MEC Control Chart Scheme

The section evaluates the performance of the proposed combined EWMA-MEC scheme and compares it with the MEC, EWMA and CUSUM schemes. The ARL values of the proposed scheme are given in Table 2, 3, and 4. The performance of any control chart scheme is usually measured in terms of ARL. ARL means an average number of observations considered before a signal occurs, showing that the state of the process is out of control. We have used the ARL ratio to compare our proposed scheme with others. The ARL ratio is given by

$$\text{ARL Ratio} = \frac{ARL_{MEC} \,/\, ARL_{EWMA} \,/\, ARL_{CUSUM}}{ARL_{combined\ EWMA-MEC}}. \tag{13}$$

The ARL of the proposed scheme will be better if the above ratio exceeds one. If the ratio is less than one, the existing scheme outperforms the proposed one. All these charting techniques are designed for different sized-shifts.

Table 2 ARL of proposed combined EWMA-MEC quality control scheme designed for small shifts at $a = 0.25$, $b = 72.8$, $\lambda = 0.047$ and $L = 2.67$

δ	$\phi = 0$	$\phi = 0.25$	$\phi = 0.5$	$\phi = 0.75$	$\phi = 0.9$
0.0	371.61	371.76	370.21	371.38	370.72
0.1	188.93	234.79	293.19	344.25	363.74
0.2	91.99	127.78	187.74	296.77	352.12
0.3	53.52	79.46	128.22	237.02	334.38
0.4	34.84	54.00	91.58	190.50	316.05
0.5	24.29	38.66	69.19	155.39	294.70
0.6	17.78	28.93	53.95	126.97	270.18
0.7	13.68	22.30	42.89	107.98	247.51
0.8	10.90	17.84	34.90	92.10	226.67
0.9	8.95	14.55	28.95	79.44	205.96
1.0	7.46	12.17	24.32	68.78	189.05
1.1	6.37	10.37	20.63	60.80	175.70
1.2	5.51	8.93	17.77	53.65	160.35
1.3	4.83	7.82	15.53	48.14	147.55
1.4	4.28	6.89	13.67	43.15	138.88

Table 2 (Continued)

δ	$\phi=0$	$\phi=0.25$	$\phi=0.5$	$\phi=0.75$	$\phi=0.9$
1.5	3.84	6.14	12.16	38.81	126.60
1.6	3.46	5.52	10.90	34.93	120.02
1.7	3.15	5.00	9.82	31.66	110.95
1.8	2.89	4.55	8.94	28.94	103.72
1.9	2.66	4.17	8.15	26.42	97.69
2.0	2.47	3.84	7.48	24.18	91.70

Table 3 ARL of proposed combined EWMA-MEC quality control scheme designed for moderate shifts at $a=0.5$, $b=33.2$, $\lambda=0.134$ and $L=2.945$

δ	$\phi=0$	$\phi=0.25$	$\phi=0.5$	$\phi=0.75$	$\phi=0.9$
0.0	370.56	372.84	369.35	372.66	371.77
0.1	213.63	260.36	308.60	351.43	368.19
0.2	99.89	143.99	214.77	314.14	356.74
0.3	56.83	85.33	141.73	261.99	352.94
0.4	37.91	56.97	99.76	216.17	328.52
0.5	27.63	41.90	73.59	174.91	311.74
0.6	21.01	32.11	57.10	142.78	291.35
0.7	16.51	25.69	45.95	117.55	269.56
0.8	13.20	21.01	38.00	100.00	253.69
0.9	10.78	17.43	32.28	85.03	236.20
1.0	8.96	14.69	27.68	72.96	215.79
1.1	7.59	12.52	23.97	64.50	196.16
1.2	6.53	10.80	20.97	57.01	183.59
1.3	5.70	9.36	18.52	50.90	168.83
1.4	5.03	8.24	16.49	45.95	155.59
1.5	4.48	7.30	14.70	41.76	141.68
1.6	4.02	6.52	13.16	38.06	132.90
1.7	3.65	5.88	11.91	34.86	123.75
1.8	3.33	5.34	10.80	32.08	114.12
1.9	3.06	4.87	9.80	29.72	106.81
2.0	2.82	4.48	8.99	27.51	99.52

Table 4 ARL of proposed combined EWMA-MEC quality control scheme designed for large shifts at $a=1$, $b=7.35$, $\lambda=0.364$ and $L=3.1$

δ	$\phi=0$	$\phi=0.25$	$\phi=0.5$	$\phi=0.75$	$\phi=0.9$
0.0	370.25	370.17	371.81	371.11	370.20
0.1	279.27	316.02	343.14	360.53	366.11
0.2	160.00	212.24	283.17	346.82	365.83
0.3	88.36	136.75	219.35	314.68	359.99
0.4	52.37	88.64	160.25	281.29	354.13
0.5	33.70	58.77	118.32	250.22	343.19
0.6	23.51	41.41	88.41	216.55	326.85

Table 4 (Continued)

δ	$\phi=0$	$\phi=0.25$	$\phi=0.5$	$\phi=0.75$	$\phi=0.9$
0.7	17.63	30.72	66.91	186.81	320.97
0.8	13.82	23.66	52.01	159.86	308.01
0.9	11.30	18.91	41.42	137.58	298.67
1.0	9.47	15.53	33.75	119.48	285.99
1.1	8.13	13.11	27.93	101.95	267.81
1.2	7.09	11.29	23.51	88.99	255.70
1.3	6.26	9.90	20.20	76.43	241.80
1.4	5.57	8.79	17.58	67.63	227.75
1.5	5.00	7.85	15.53	58.95	216.31
1.6	4.51	7.08	13.81	52.01	203.34
1.7	4.09	6.45	12.47	46.58	191.21
1.8	3.74	5.89	11.28	41.39	181.02
1.9	3.43	5.41	10.31	37.35	169.98
2.0	3.16	4.99	9.49	33.56	158.05

4.1. Proposed EWMA-MEC versus MEC

The ARL values of the MEC scheme designed for detecting small, moderate, and large shifts in the process and their corresponding ARL ratio against our proposed scheme are given in Tables 5, 6 and 7, respectively. Also, the results for all types of shifts are shown graphically in Figure 1. After observing the values of ARL ratio for small shifts, we found that it is greater than one for all the process shifts. So, our proposed scheme performs more efficiently than the MEC scheme.

The ARL ratio for moderate-sized shifts in the process gives us a clear view that for weakly autocorrelated data our proposed scheme is effective for detecting moderate and large-sized shifts. Figure 1(a) and Figure 1(b) show that the proposed scheme performs better for small and moderate shifts till the process observations are moderately autocorrelated. ARL ratio for large shifts signifies that for weakly autocorrelated processes the proposed scheme performs more efficiently than MEC scheme. Figure 1(c) also shows that as the autocorrelation increases in the observation, the proposed scheme performance is approximately same as the MEC scheme.

Some interesting facts that can be derived from Figure 1 are as follows:

• If proposed scheme is designed for small shifts $(\delta=0.5)$ but in the process moderate or large shift $(\delta>0.5)$ occurs, then up to moderate autocorrelation the misdesigned proposed scheme gives far more efficient results than the misdesigned MEC.

• If the proposed scheme is designed for moderate shifts $(\delta=1)$ but large one occurs $(\delta>1)$ in the process, then for $\phi\leq 0.5$ the misdesigned proposed scheme gives much better results than the misdesigned MEC.

Table 5 ARL ratios and ARL of MEC control chart scheme designed for small shifts at $a = 0.25$, $b = 92$ and $\lambda = 0.047$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	371.30	1.00	371.76	1.00	370.58	1.00	371.77	1.00	372.43	1.00
0.1	193.91	1.03	240.96	1.03	298.64	1.02	348.48	1.01	368.73	1.01
0.2	102.93	1.12	137.15	1.07	196.78	1.05	298.11	1.00	352.75	1.00
0.3	70.65	1.32	92.49	1.16	136.58	1.07	242.78	1.02	339.56	1.02
0.4	54.96	1.58	70.62	1.31	103.49	1.13	196.16	1.03	321.38	1.02
0.5	45.96	1.89	57.95	1.50	83.82	1.21	162.08	1.04	296.26	1.01
0.6	40.02	2.25	49.93	1.73	70.66	1.31	137.05	1.08	273.54	1.01
0.7	35.80	2.62	44.22	1.98	61.53	1.43	118.37	1.10	252.56	1.02
0.8	32.59	2.99	40.00	2.24	55.07	1.58	103.80	1.13	233.37	1.03
0.9	30.08	3.36	36.72	2.52	49.99	1.73	92.16	1.16	215.89	1.05
1.0	28.02	3.75	34.06	2.80	45.96	1.89	83.45	1.21	196.16	1.04
1.1	26.32	4.14	31.90	3.08	42.74	2.07	76.36	1.26	181.12	1.03
1.2	24.88	4.51	30.08	3.37	39.99	2.25	70.63	1.32	168.34	1.05
1.3	23.66	4.90	28.48	3.64	37.71	2.43	65.67	1.36	156.75	1.06
1.4	22.57	5.27	27.13	3.93	35.75	2.61	61.62	1.43	145.70	1.05
1.5	21.62	5.63	25.94	4.22	34.06	2.80	58.08	1.50	137.02	1.08
1.6	20.77	6.00	24.89	4.51	32.56	2.99	54.98	1.57	128.77	1.07
1.7	20.03	6.36	23.94	4.79	31.24	3.18	52.39	1.66	121.07	1.09
1.8	19.33	6.70	23.09	5.08	30.07	3.36	49.88	1.72	114.98	1.11
1.9	18.72	7.04	22.33	5.35	29.00	3.56	47.86	1.81	108.67	1.11
2	18.15	7.36	21.63	5.63	28.05	3.75	45.98	1.90	103.31	1.13

Table 6 ARL ratios and ARL of MEC control chart scheme designed for small shifts at $a = 0.25$, $b = 27.5$ and $\lambda = 0.134$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	368.83	1.00	368.39	0.99	367.47	0.99	371.07	1.00	371.36	1.00
0.1	207.51	0.97	254.05	0.98	310.11	1.00	358.57	1.02	369.64	1.00
0.2	96.78	0.97	137.21	0.95	207.49	0.97	308.50	0.98	356.07	1.00
0.3	57.24	1.01	83.23	0.98	137.23	0.97	255.78	0.98	347.30	0.98
0.4	39.99	1.05	57.42	1.01	96.75	0.97	206.97	0.96	330.25	1.01
0.5	31.08	1.12	43.48	1.04	72.63	0.99	166.65	0.95	308.77	0.99
0.6	25.66	1.22	34.96	1.09	57.20	1.00	137.56	0.96	287.98	0.99
0.7	22.03	1.33	29.52	1.15	47.30	1.03	114.52	0.97	265.68	0.99
0.8	19.49	1.48	25.63	1.22	40.03	1.05	96.37	0.96	246.26	0.97
0.9	17.57	1.63	22.83	1.31	35.00	1.08	83.16	0.98	224.01	0.95
1.0	16.07	1.79	20.65	1.41	31.05	1.12	72.28	0.99	208.57	0.97
1.1	14.86	1.96	18.96	1.51	28.04	1.17	64.00	0.99	188.98	0.96
1.2	13.88	2.13	17.57	1.63	25.64	1.22	57.29	1.01	174.97	0.95
1.3	13.05	2.29	16.40	1.75	23.67	1.28	51.67	1.02	159.68	0.95

Table 6 (Continued)

δ	$\phi=0$ ARL	ARL Ratio	$\phi=0.25$ ARL	ARL Ratio	$\phi=0.5$ ARL	ARL Ratio	$\phi=0.75$ ARL	ARL Ratio	$\phi=0.9$ ARL	ARL Ratio
1.4	12.34	2.46	15.43	1.87	22.03	1.34	47.07	1.02	149.67	0.96
1.5	11.73	2.62	14.60	2.00	20.67	1.41	43.40	1.04	136.21	0.96
1.6	11.20	2.79	13.88	2.13	19.47	1.48	40.13	1.05	127.15	0.96
1.7	10.73	2.94	13.25	2.25	18.44	1.55	37.31	1.07	118.18	0.95
1.8	10.30	3.10	12.69	2.38	17.55	1.63	34.96	1.09	110.39	0.97
1.9	9.92	3.24	12.18	2.50	16.75	1.71	32.97	1.11	103.24	0.97
2.0	9.58	3.39	11.74	2.62	16.05	1.79	31.06	1.13	96.74	0.97

Table 7 ARL ratios and ARL of MEC control chart scheme designed for large shifts at $a=1$, $b=6.32$ and $\lambda=0.364$

δ	$\phi=0$ ARL	ARL Ratio	$\phi=0.25$ ARL	ARL Ratio	$\phi=0.5$ ARL	ARL Ratio	$\phi=0.75$ ARL	ARL Ratio	$\phi=0.9$ ARL	ARL Ratio
0.0	371.66	1.00	370.37	1.00	367.75	0.99	373.16	1.01	368.96	1.00
0.1	271.05	0.97	304.45	0.96	334.31	0.97	362.14	1.00	363.90	0.99
0.2	147.29	0.92	202.68	0.95	273.77	0.97	340.37	0.98	358.33	0.98
0.3	81.03	0.92	125.84	0.92	202.84	0.92	305.76	0.97	354.52	0.98
0.4	48.17	0.92	80.78	0.91	147.13	0.92	271.31	0.96	350.91	0.99
0.5	31.67	0.94	54.38	0.93	109.21	0.92	235.09	0.94	344.49	1.00
0.6	22.52	0.96	38.60	0.93	80.16	0.91	202.78	0.94	327.27	1.00
0.7	17.14	0.97	28.91	0.94	61.71	0.92	172.40	0.92	314.09	0.98
0.8	13.73	0.99	22.58	0.95	48.29	0.93	147.23	0.92	300.91	0.98
0.9	11.43	1.01	18.30	0.97	38.92	0.94	126.97	0.92	285.49	0.96
1.0	9.80	1.03	15.27	0.98	31.71	0.94	108.43	0.91	275.55	0.96
1.1	8.59	1.06	13.06	1.00	26.46	0.95	93.18	0.91	258.82	0.97
1.2	7.70	1.09	11.43	1.01	22.53	0.96	80.45	0.90	242.69	0.95
1.3	6.99	1.12	10.16	1.03	19.50	0.97	70.31	0.92	227.86	0.94
1.4	6.41	1.15	9.16	1.04	17.21	0.98	61.55	0.91	214.06	0.94
1.5	5.95	1.19	8.36	1.07	15.26	0.98	54.31	0.92	200.82	0.93
1.6	5.56	1.23	7.70	1.09	13.70	0.99	48.35	0.93	190.43	0.94
1.7	5.22	1.28	7.15	1.11	12.46	1.00	43.04	0.92	177.30	0.93
1.8	4.94	1.32	6.69	1.14	11.42	1.01	38.66	0.93	166.71	0.92
1.9	4.69	1.37	6.29	1.16	10.55	1.02	34.96	0.94	157.89	0.93
2.0	4.47	1.42	5.95	1.19	9.80	1.03	31.65	0.94	147.12	0.93

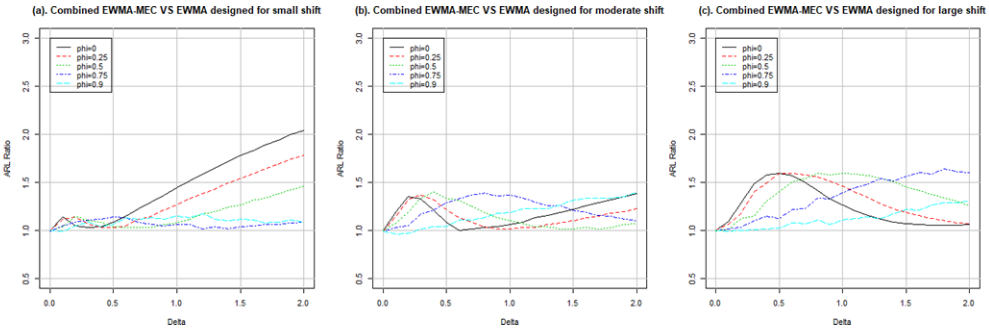

Figure 1 ARL ratio graphs between proposed combined EWMA-MEC and MEC scheme

4.2. Proposed combined EWMA-MEC versus EWMA

The ARL values of the EWMA scheme designed for detecting small, moderate and large shifts in the process and their corresponding ARL ratio against our proposed scheme are given in Tables 8, 9 and 10, respectively. Also, the results for all types of shifts are shown graphically in Figure 2. The design structure of our proposed scheme exceptionally outperforms the EWMA scheme for small, moderate and large-sized shifts. Figure 2(a) shows that as the shift increases from moderate to large, the proposed scheme performs outstanding than EWMA scheme. Observing the ARL ratio values in Table 9 and Figure 2(b), we can say that the design structure of our proposed scheme for moderate shifts performs exceptionally well for moderate-sized shifts for moderate and large autocorrelation. This particular design structure of the proposed scheme also performs efficiently for small and large-sized sifts. The values in Table 10 and Figure 2(c) reveal that the proposed scheme for large shifts outperforms for large shifts when ϕ is also large and when ϕ is small and moderate, its performance is more efficient than the performance of EWMA for small and moderate shifts.

Some important points to be concluded in the Figure 2 are as follows:

• If proposed scheme is aimed for small shifts $(\delta = 0.5)$ but large shift $(\delta > 1)$ is experienced by the process, then for moderate autocorrelation the misdesigned proposed scheme gives much better results than the misdesigned EWMA chart.

• If our proposed scheme is aimed to detect large shifts $(\delta = 2)$ but small or moderate shifts experienced by the process, then for moderately autocorrelated, the misdesigned proposed scheme gives more efficient results than the misdesigned EWMA chart.

Table 8 ARL Ratios and ARL of EWMA control chart scheme designed for small shifts at $\lambda = 0.047$ and $L = 2.47$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	370.41	1.00	370.93	1.00	370.12	1.00	369.51	0.99	369.84	1.00
0.1	216.00	1.14	262.14	1.12	303.67	1.04	360.35	1.05	359.35	0.99
0.2	96.27	1.05	146.21	1.14	215.98	1.15	322.69	1.09	369.03	1.05
0.3	55.37	1.03	84.81	1.07	141.52	1.10	263.96	1.11	364.60	1.09
0.4	36.30	1.04	55.72	1.03	99.73	1.09	213.92	1.12	328.34	1.04
0.5	26.44	1.09	39.78	1.03	72.22	1.04	176.99	1.14	311.21	1.06
0.6	20.44	1.15	30.47	1.05	55.75	1.03	143.63	1.13	304.38	1.13
0.7	16.81	1.23	24.88	1.12	44.04	1.03	117.19	1.09	277.85	1.12

Table 8 (Continued**)**

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	370.41	1.00	370.93	1.00	370.12	1.00	369.51	0.99	369.84	1.00
0.1	216.00	1.14	262.14	1.12	303.67	1.04	360.35	1.05	359.35	0.99
0.2	96.27	1.05	146.21	1.14	215.98	1.15	322.69	1.09	369.03	1.05
0.3	55.37	1.03	84.81	1.07	141.52	1.10	263.96	1.11	364.60	1.09
0.4	36.30	1.04	55.72	1.03	99.73	1.09	213.92	1.12	328.34	1.04
0.5	26.44	1.09	39.78	1.03	72.22	1.04	176.99	1.14	311.21	1.06
0.6	20.44	1.15	30.47	1.05	55.75	1.03	143.63	1.13	304.38	1.13
0.7	16.81	1.23	24.88	1.12	44.04	1.03	117.19	1.09	277.85	1.12
0.8	14.19	1.30	20.51	1.15	35.78	1.03	97.75	1.06	256.69	1.13
0.9	12.28	1.37	17.66	1.21	30.70	1.06	83.37	1.05	230.70	1.12
1.0	10.85	1.45	15.44	1.27	26.35	1.08	73.18	1.06	216.79	1.15
1.1	9.68	1.52	13.76	1.33	23.13	1.12	64.49	1.06	198.20	1.13
1.2	8.75	1.59	12.30	1.38	20.72	1.17	54.47	1.02	187.05	1.17
1.3	7.99	1.65	11.21	1.43	18.71	1.20	49.91	1.04	165.10	1.12
1.4	7.37	1.72	10.28	1.49	16.92	1.24	44.18	1.02	152.42	1.10
1.5	6.83	1.78	9.44	1.54	15.48	1.27	40.39	1.04	141.93	1.12
1.6	6.35	1.83	8.75	1.59	14.26	1.31	36.65	1.05	133.18	1.11
1.7	5.96	1.89	8.21	1.64	13.14	1.34	33.45	1.06	119.42	1.08
1.8	5.59	1.94	7.70	1.69	12.38	1.38	30.80	1.06	113.23	1.09
1.9	5.31	2.00	7.26	1.74	11.55	1.42	28.50	1.08	108.75	1.11
2.0	5.04	2.04	6.86	1.78	10.89	1.46	26.36	1.09	99.55	1.09

Table 9 ARL ratios and ARL of EWMA control chart scheme designed for moderate shifts at $\lambda = 0.134$ and $L = 2.77$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	370.12	1.00	370.74	0.99	370.65	1.00	369.83	0.99	369.21	0.99
0.1	255.56	1.20	299.06	1.15	332.81	1.08	362.01	1.03	351.66	0.96
0.2	135.01	1.35	190.68	1.32	256.10	1.19	331.27	1.05	346.07	0.97
0.3	75.35	1.33	117.25	1.37	189.23	1.34	306.46	1.17	359.85	1.02
0.4	45.81	1.21	75.11	1.32	139.78	1.40	261.49	1.21	340.79	1.04
0.5	30.20	1.09	51.03	1.22	98.77	1.34	225.62	1.29	324.21	1.04
0.6	21.10	1.00	36.42	1.13	74.92	1.31	189.37	1.33	322.66	1.11
0.7	16.86	1.02	28.02	1.09	57.64	1.25	160.60	1.37	298.12	1.11
0.8	13.57	1.03	21.89	1.04	45.33	1.19	139.06	1.39	287.65	1.13
0.9	11.17	1.04	17.84	1.02	36.58	1.13	115.66	1.36	277.65	1.18
1.0	9.54	1.06	14.97	1.02	30.52	1.10	99.81	1.37	256.61	1.19
1.1	8.30	1.09	12.88	1.03	25.59	1.07	86.46	1.34	242.15	1.23
1.2	7.37	1.13	11.16	1.03	21.91	1.04	74.16	1.30	226.03	1.23
1.3	6.60	1.16	9.94	1.06	19.18	1.04	64.33	1.26	208.15	1.23
1.4	5.99	1.19	8.90	1.08	16.79	1.02	57.72	1.26	197.56	1.27

Table 9 (Continued)

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
1.5	5.49	1.22	8.06	1.10	14.96	1.02	50.66	1.21	186.18	1.31
1.6	5.07	1.26	7.37	1.13	13.52	1.03	45.16	1.19	177.63	1.34
1.7	4.70	1.29	6.79	1.15	12.20	1.02	40.54	1.16	165.71	1.34
1.8	4.40	1.32	6.28	1.18	11.17	1.03	36.83	1.15	153.33	1.34
1.9	4.13	1.35	5.86	1.20	10.36	1.06	33.22	1.12	144.03	1.35
2.0	3.90	1.38	5.50	1.23	9.59	1.07	30.15	1.10	138.88	1.40

Table 10 ARL ratios and ARL of EWMA control chart scheme designed for large shifts at $\lambda = 0.364$ and $L = 2.95$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	370.17	1.00	370.22	1.00	370.13	1.00	370.76	1.00	371.13	1.00
0.1	306.08	1.10	335.56	1.06	348.44	1.02	365.83	1.01	363.78	0.99
0.2	206.29	1.29	249.87	1.18	319.71	1.13	357.23	1.03	365.30	1.00
0.3	131.43	1.49	191.43	1.40	251.43	1.15	347.09	1.10	362.53	1.01
0.4	82.60	1.58	132.13	1.49	209.59	1.31	323.18	1.15	360.65	1.02
0.5	53.87	1.60	93.58	1.59	166.59	1.41	280.89	1.12	351.82	1.03
0.6	36.98	1.57	66.20	1.60	133.38	1.51	263.91	1.22	353.96	1.08
0.7	26.41	1.50	48.61	1.58	103.27	1.54	229.64	1.23	344.34	1.07
0.8	19.58	1.42	36.86	1.56	82.91	1.59	213.22	1.33	341.63	1.11
0.9	15.01	1.33	28.67	1.52	65.41	1.58	182.39	1.33	318.73	1.07
1.0	11.98	1.27	22.72	1.46	53.81	1.59	165.98	1.39	318.17	1.11
1.1	9.78	1.20	18.36	1.40	44.32	1.59	147.32	1.44	301.56	1.13
1.2	8.18	1.15	15.17	1.34	36.91	1.57	132.20	1.49	291.64	1.14
1.3	6.97	1.11	12.63	1.28	31.10	1.54	117.62	1.54	271.78	1.12
1.4	6.06	1.09	10.78	1.23	26.59	1.51	102.32	1.51	267.97	1.18
1.5	5.34	1.07	9.34	1.19	22.60	1.46	92.21	1.56	263.02	1.22
1.6	4.78	1.06	8.19	1.16	19.53	1.41	83.36	1.60	246.25	1.21
1.7	4.31	1.06	7.26	1.13	17.11	1.37	73.92	1.59	242.21	1.27
1.8	3.93	1.05	6.50	1.10	15.06	1.33	67.88	1.64	233.61	1.29
1.9	3.61	1.05	5.85	1.08	13.48	1.31	60.03	1.61	218.71	1.29
2.0	3.35	1.06	5.34	1.07	11.97	1.26	53.77	1.60	206.25	1.30

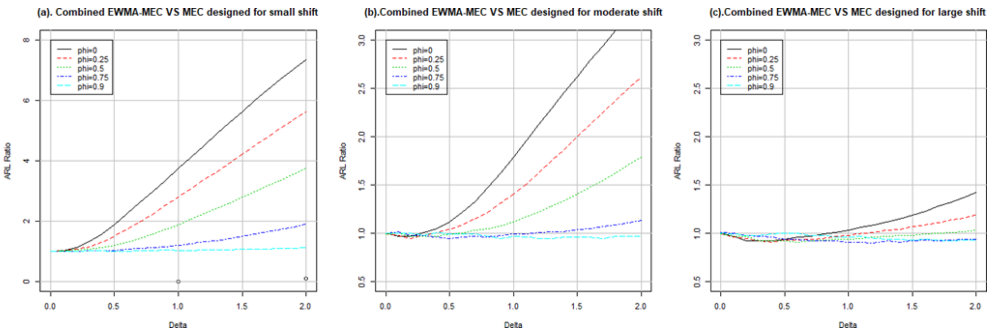

Figure 2 ARL ratio graphs between proposed combined EWMA-MEC and EWMA scheme.

4.3. Proposed combined EWMA-MEC versus CUSUM

The ARL values of the CUSUM scheme designed for detecting small, moderate and large shifts in the process and their corresponding ARL ratio against our proposed scheme are given in Tables 11, 12 and 13, respectively. Also, the results for all types of shifts are shown graphically in Figure 3. Observing the values in Tables 11 and 12, we can conclude that the proposed scheme for small and moderate shifts performs outstandingly well for small, moderate and large shifts. The design structure of our proposed scheme for large shifts performs more efficiently than the CUSUM scheme for large as well as moderate shifts.

Figure 3(a) also shows that as the shift increases from moderate to large then for moderately autocorrelated data, the proposed scheme performs much better. Ratio curves in Figure 3(b) indicate the performance of our proposed scheme for moderate shifts and show that it is more efficient for moderate shifts for moderate and large autocorrelation. Figure 3(c) shows that proposed scheme for large shifts performs outstanding for large shifts for strong autocorrelated data. When autocorrelation is weak and moderate, it performs better than CUSUM for moderate shifts. Looking at the ratio curves in Figure 3 some interesting conclusions can be derived which are as follows:

- If our proposed scheme is planned to detect small $(\delta = 0.5)$ shifts but a large $(\delta > 1)$ shift arises in the process, then for moderate autocorrelation the misdesigned proposed scheme will give drastically improved results compared to the misdesigned CUSUM chart.
- If our proposed scheme is designed for large shifts $(\delta = 2)$ but a moderate shift is experienced by the process, then for $\phi \le 0.5$ the misdesigned proposed scheme gives more efficient results than the misdesigned CUSUM chart.

Table 11 ARL ratios and ARL of CUSUM control chart scheme designed for small shifts at $a = 0.25$ and $b = 8.01$

δ	$\phi = 0$ ARL	ARL Ratio	$\phi = 0.25$ ARL	ARL Ratio	$\phi = 0.5$ ARL	ARL Ratio	$\phi = 0.75$ ARL	ARL Ratio	$\phi = 0.9$ ARL	ARL Ratio
0.0	370.79	1.00	370.31	1.00	369.02	1.00	371.88	1.00	369.84	1.00
0.1	235.50	1.25	284.55	1.21	323.43	1.10	358.51	1.04	368.82	1.01
0.2	115.53	1.26	166.54	1.30	237.64	1.27	324.70	1.09	361.09	1.03
0.3	63.34	1.18	99.01	1.25	163.98	1.28	282.45	1.19	353.17	1.06
0.4	40.36	1.16	63.44	1.17	115.75	1.26	238.05	1.25	333.73	1.06
0.5	28.80	1.19	44.54	1.15	83.42	1.21	200.35	1.29	325.61	1.10
0.6	22.13	1.24	33.62	1.16	63.12	1.17	165.44	1.30	311.10	1.15

Table 11 (Continued)

δ	$\phi=0$ ARL	ARL Ratio	$\phi=0.25$ ARL	ARL Ratio	$\phi=0.5$ ARL	ARL Ratio	$\phi=0.75$ ARL	ARL Ratio	$\phi=0.9$ ARL	ARL Ratio
0.7	17.98	1.31	26.74	1.20	49.86	1.16	136.84	1.27	288.92	1.17
0.8	15.08	1.38	22.19	1.24	40.41	1.16	116.61	1.27	274.41	1.21
0.9	12.98	1.45	18.84	1.30	33.67	1.16	97.39	1.23	256.31	1.24
1.0	11.40	1.53	16.35	1.34	28.77	1.18	84.15	1.22	239.44	1.27
1.1	10.16	1.60	14.47	1.40	25.01	1.21	72.49	1.19	219.93	1.25
1.2	9.18	1.66	12.97	1.45	22.19	1.25	63.56	1.18	203.91	1.27
1.3	8.37	1.73	11.77	1.51	19.81	1.28	55.73	1.16	191.79	1.30
1.4	7.69	1.79	10.77	1.56	17.94	1.31	49.68	1.15	178.05	1.28
1.5	7.12	1.85	9.91	1.61	16.37	1.35	44.67	1.15	164.11	1.30
1.6	6.63	1.91	9.19	1.67	15.08	1.38	40.35	1.16	153.64	1.28
1.7	6.21	1.97	8.56	1.71	13.94	1.42	36.71	1.16	143.07	1.29
1.8	5.84	2.02	8.02	1.76	12.99	1.45	33.70	1.16	132.39	1.28
1.9	5.51	2.07	7.53	1.81	12.14	1.49	31.11	1.18	122.20	1.25
2.0	5.22	2.12	7.12	1.85	11.42	1.53	28.78	1.19	116.16	1.27

Table 12 ARL ratios and ARL of CUSUM control chart scheme designed for moderate shifts at $a=0.5$ and $b=4.77$

δ	$\phi=0$ ARL	ARL Ratio	$\phi=0.25$ ARL	ARL Ratio	$\phi=0.5$ ARL	ARL Ratio	$\phi=0.75$ ARL	ARL Ratio	$\phi=0.9$ ARL	ARL Ratio
0.0	370.89	1.00	370.20	0.99	370.81	1.00	371.04	1.00	370.48	1.00
0.1	285.27	1.34	317.98	1.22	345.40	1.12	361.00	1.03	364.47	0.99
0.2	160.72	1.61	218.24	1.52	279.78	1.30	341.75	1.09	363.90	1.02
0.3	90.70	1.60	139.77	1.64	217.87	1.54	310.76	1.19	358.72	1.02
0.4	54.49	1.44	90.91	1.60	162.60	1.63	281.91	1.30	355.01	1.08
0.5	35.30	1.28	61.34	1.46	120.18	1.63	250.28	1.43	341.39	1.10
0.6	24.61	1.17	42.97	1.34	92.05	1.61	220.42	1.54	326.74	1.12
0.7	18.33	1.11	31.88	1.24	70.41	1.53	189.15	1.61	328.14	1.22
0.8	14.41	1.09	24.59	1.17	54.45	1.43	162.60	1.63	311.55	1.23
0.9	11.76	1.09	19.62	1.13	43.28	1.34	140.05	1.65	294.77	1.25
1.0	9.92	1.11	16.18	1.10	35.31	1.28	120.58	1.65	284.26	1.32
1.1	8.55	1.13	13.66	1.09	29.26	1.22	104.43	1.62	272.76	1.39
1.2	7.53	1.15	11.79	1.09	24.65	1.18	90.77	1.59	257.38	1.40
1.3	6.70	1.18	10.31	1.10	21.05	1.14	78.86	1.55	246.37	1.46
1.4	6.05	1.20	9.20	1.12	18.37	1.11	69.61	1.51	231.55	1.49
1.5	5.52	1.23	8.27	1.13	16.17	1.10	61.51	1.47	218.87	1.54
1.6	5.07	1.26	7.51	1.15	14.43	1.10	54.45	1.43	208.63	1.57
1.7	4.70	1.29	6.89	1.17	12.98	1.09	48.63	1.40	195.03	1.58
1.8	4.37	1.31	6.36	1.19	11.75	1.09	43.43	1.35	184.61	1.62
1.9	4.10	1.34	5.90	1.21	10.78	1.10	39.03	1.31	172.85	1.62
2.0	3.85	1.37	5.52	1.23	9.90	1.10	35.14	1.28	162.54	1.63

Table 13: ARL Ratios and ARL of CUSUM control chart scheme designed for large shifts at $a=1$ and $b=2.51$.

δ	$\phi=0$ ARL	ARL Ratio	$\phi=0.25$ ARL	ARL Ratio	$\phi=0.5$ ARL	ARL Ratio	$\phi=0.75$ ARL	ARL Ratio	$\phi=0.9$ ARL	ARL Ratio
0	369.54	1.00	369.19	1.00	369.44	0.99	369.82	1.00	369.81	1.00
0.1	320.60	1.15	337.99	1.07	348.55	1.02	364.02	1.01	360.21	0.98
0.2	236.37	1.48	284.38	1.34	320.83	1.13	353.88	1.02	360.45	0.99
0.3	159.25	1.80	214.38	1.57	282.51	1.29	339.32	1.08	356.76	0.99
0.4	104.03	1.99	156.89	1.77	238.78	1.49	324.28	1.15	358.18	1.01
0.5	68.45	2.03	114.91	1.96	194.91	1.65	302.60	1.21	344.36	1.00
0.6	46.19	1.96	84.06	2.03	158.03	1.79	284.05	1.31	344.18	1.05
0.7	32.43	1.84	62.21	2.02	128.69	1.92	260.68	1.40	348.42	1.09
0.8	23.46	1.70	46.50	1.97	103.10	1.98	235.86	1.48	330.35	1.07
0.9	17.51	1.55	35.31	1.87	84.12	2.03	216.18	1.57	326.86	1.09
1	13.52	1.43	27.36	1.76	68.36	2.03	195.20	1.63	318.19	1.11
1.1	10.75	1.32	21.73	1.66	55.76	2.00	173.94	1.71	311.96	1.16
1.2	8.75	1.23	17.54	1.55	46.29	1.97	159.67	1.79	308.80	1.21
1.3	7.34	1.17	14.37	1.45	38.66	1.91	141.67	1.85	295.22	1.22
1.4	6.27	1.13	12.03	1.37	32.50	1.85	128.02	1.89	288.43	1.27
1.5	5.44	1.09	10.19	1.30	27.49	1.77	115.98	1.97	280.33	1.30
1.6	4.80	1.07	8.75	1.24	23.42	1.70	104.48	2.01	273.22	1.34
1.7	4.30	1.05	7.64	1.19	20.20	1.62	93.99	2.02	261.70	1.37
1.8	3.88	1.04	6.76	1.15	17.50	1.55	84.62	2.04	252.53	1.40
1.9	3.54	1.03	6.03	1.12	15.36	1.49	76.47	2.05	246.46	1.45
2	3.25	1.03	5.45	1.09	13.50	1.42	68.79	2.05	239.51	1.52

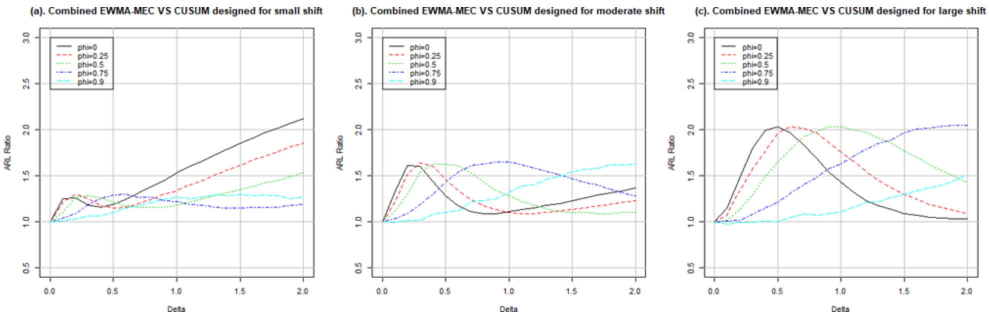

Figure 3 ARL ratio graphs between proposed combined EWMA-MEC and CUSUM scheme**.**

5. Empirical Study

Example 1. Simulated data

Here we simulated 100 process observations of an AR(1) model with $\phi=0.5$ and standardized the residuals obtained from the model. At observation 84, a shift of $1\sigma_e$ was introduced into the observations. We choose various parameters that give in-control ARL = 370. Figure 4 shows the graphical display of different residual charts discussed in this article. After the shift included in the observations, the CUSUM and proposed combined EWMA-MEC chart signaled at 92th observation with 7 and 9 total signals respectively. The EWMA chart signaled at 91th observation with total 5 signals whereas the MEC chart signaled at 94th observation with total 7 signals. The proposed scheme was slow in detecting first signal as compared to EWMA; but, in all, it detected the most signals. So, this example concludes the proposed combined EWMA-MEC scheme as the best-performing scheme.

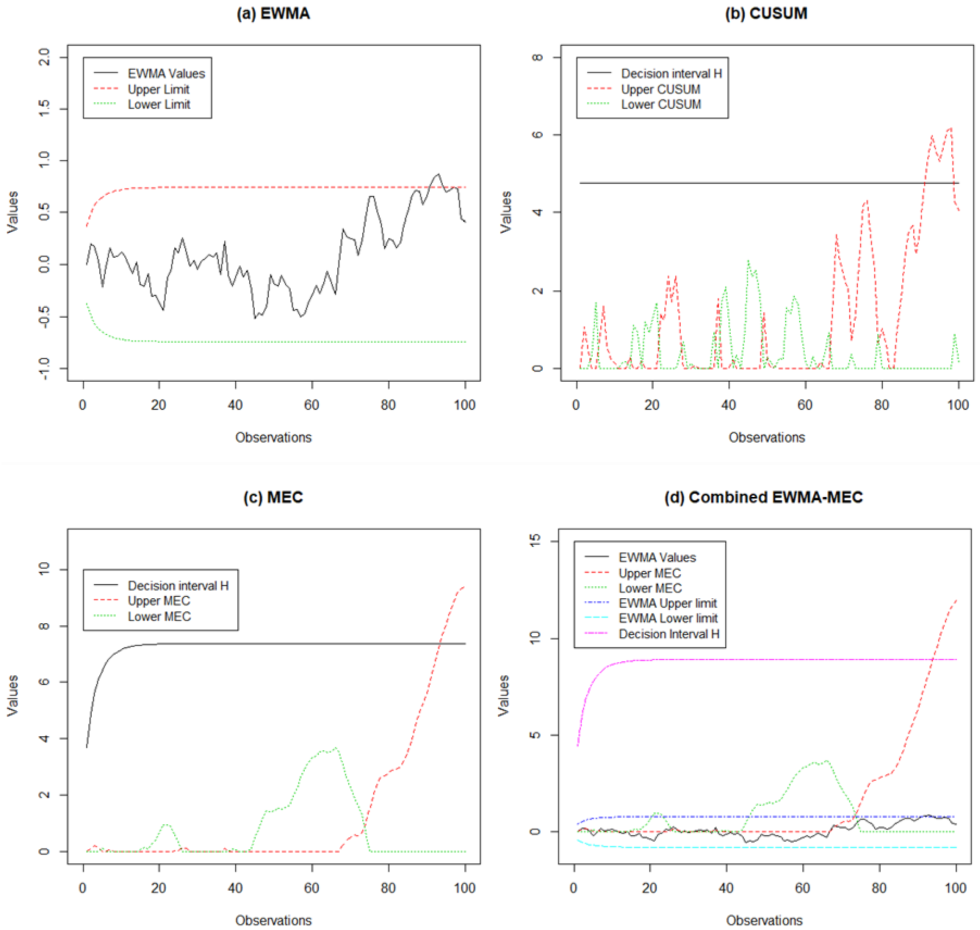

Figure 4 Residual plots for simulated observations

Example 2. Tensile Strength (Real-life) data

The following example considers a real data set to show the performance of the charts. Here we have used the dataset from Shewhart (1931) which giving measurements of tensile strength with two others quality characteristic for 60 specimen of a certain aluminum die casting. This illustration used data on tensile strength quality characteristic. An ideal fit for AR(1) model to the tensile strength data is determined at $\phi = 0.22$ After fitting the model, the obtained residuals are then standardized. At observation 41, a shift of $1.5\sigma_e$ was introduced into the observations. Figure 5 shows the output of different residual charts discussed in this article. After 40th observations, the EWMA, CUSUM and proposed Combined EWMA-MEC chart signaled at 45th observation with 11, 13 and 14 total signals respectively. The MEC chart signaled at 48th observation with total 13 signals. So, we can conclude our proposed scheme efficient as it gives two signals more than EWMA and one signal more than CUSUM and MEC.

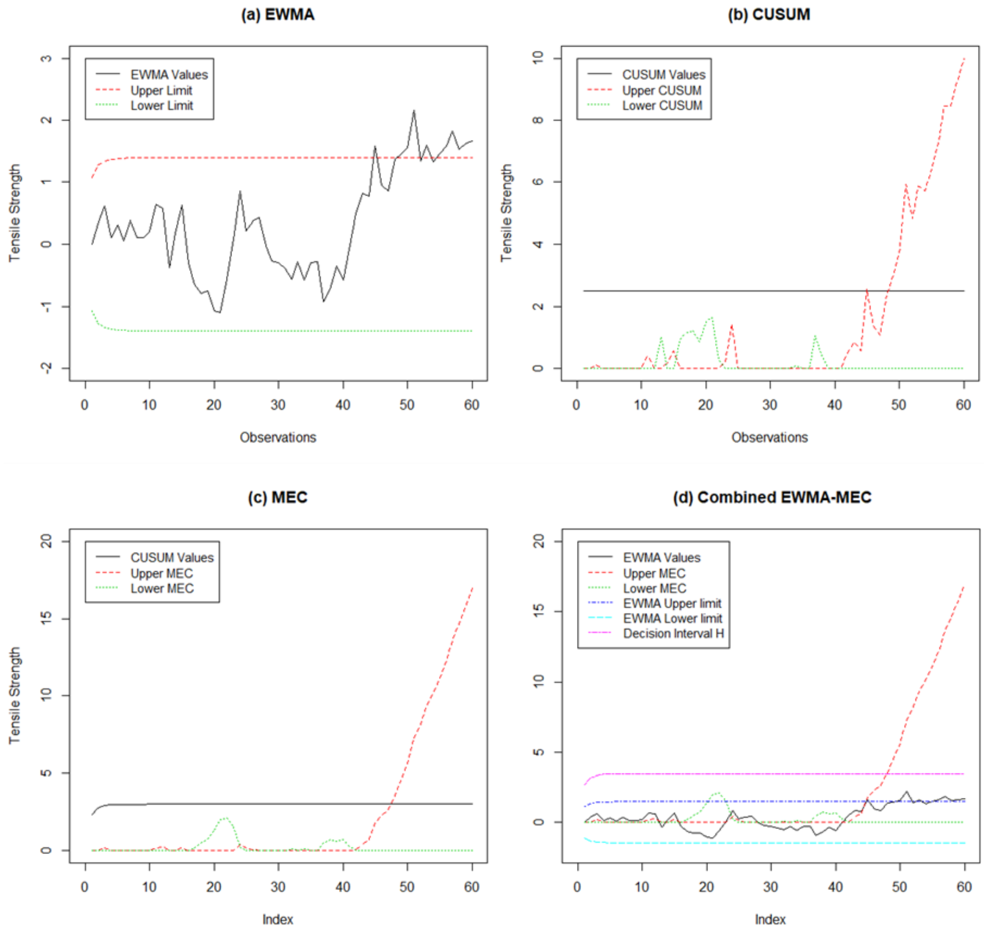

Figure 5: Residual plots for real tensile strength data

6. Conclusions

The major concern emerges in the manufacturing and industrial operations when the process observation inhales autocorrelation. This is because the autocorrelation destructs the detecting ability of control charts by increasing the false signals frequently. In this article, we have presented the Combined EWMA-MEC residual chart for AR(1) process designed separately for giving signals for small, moderate and large shifts occurring in the process and ARL values with desired parameters is shown in Tables 2-4. The proposed scheme is then compared with the MEC, EWMA and CUSUM schemes. The comparison revealed that our proposed scheme designed for detecting small, moderate and large displacements in the process outperforms the MEC, EWMA and CUSUM schemes for all types of displacements for designed as well as in misdesigned case.

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