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Exponential Estimator in the Absence and Presence of Non-Response using Double Sampling

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Abstract

This paper suggests modified exponential estimators for estimating the population mean of the study variable by utilizing the information on the auxiliary variable in double sampling scheme under two situations: i) when there is full information available on the study as well as auxiliary variable, ii) when there is a non-response on the study as well as auxiliary variable. A theoretical study is carried out when the population mean of the auxiliary variable is not known. The expressions of the biases and mean squared errors (MSE's) have been obtained under large sample approximation under double sampling. Theoretical comparison of proposed estimator with already existing estimators has been studied and the favourable conditions have been obtained for which proposed estimator is better than existing estimators. An empirical study has also been done in the support of theoretical results. Moreover, these theoretical findings are supported by simulation study.

Keywords: Study variable, auxiliary variable, bias, mean squared error (MSE), percent relative efficiency (PRE), simulation.

1. Introduction

Generally, the auxiliary information is used to improve the efficiency of the estimators. Cochran (1940) and Murthy (1964) studied the ratio and product estimators for estimating the population mean of the study variable using auxiliary variable. Hansen et al. (1953) proposed the difference estimator which was subsequently modified to provide the linear regression estimator for the population mean or total. Various researchers like Srivastava (1971), Singh and Kumar (2009) and others have proposed efficient estimators of population mean or total over usual per unit estimator.

Sometimes, information on auxiliary variable may be readily available for all the units of population: for example, Tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation and number of beds available in different hospitals may be known well in advance in health care surveys. If such information lacks, it is sometimes, relatively cheap to take large preliminary sample where auxiliary variable alone is measured, such practice is applicable in double sampling. Neyman (1938) pioneered the concept of double sampling for estimating the population parameters. Sukhatme (1962), Singh and Vishwakarma (2007), Singh and Choudhury (2012), Yadav et al. (2016), Zaman and Dunder (2020), Zaman (2020, 2021) and Zaman and Kadilar (2019, 2021a, 2021b) suggested different estimators to estimate population mean by using auxiliary variable(s) in double sampling.

In human surveys, the problem of non-response is inevitable because the information cannot be obtained from all the sample units selected in the survey due to various reasons, viz. lack of information, absence at the time of survey and refusal of the respondent are the main causes of non-response. Hansen and Hurwitz (1946) were the first who introduced the estimation method to deal with the problem of non-response. They developed an unbiased estimator for the population mean in the presence of non-response by dividing the population into two groups, viz. response group and non-response group and suggested for taking a sub-sample of the non-responding units to avoid bias due to non-response. Sodipo and Obisesan (2007) have considered the problem of non-response, in sample survey with full response of an auxiliary variable. Further, other authors like Cochran (1977), Okafor and Lee (2000), Khare and Rehman (2014), Singh et al. (2010), Chaudhary and Kumar (2015), Khan and Al-Hossain (2016), Sanaullah et al. (2018), Guha and Chandra (2019), Kumar and Sharma (2020), Murtaza et al. (2020), Mehta and Tailor (2020), Misra et al. (2021) have studied the problem of non-response under double sampling.

In the present study, we propose a modified exponential estimator for estimating the population mean of study variable by utilizing the information on auxiliary variable using double sampling scheme under two situations:
i) when there is full information available on the study as well as auxiliary variable.
ii) when there is a non-response on the study as well as auxiliary variable.

2. Some Existing Estimators

We consider the important estimator in literature under double sampling as follows:
Sukhatme (1962) suggested a ratio estimator in double sampling as

$$T_{Rd} = \frac{\bar{y}}{\bar{x}}\bar{x}',$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is the sample mean of y and $\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the sample means of x for the first phase and the second phase samples, respectively.

Singh and Vishwakarma (2007) developed an exponential estimator to double sampling as

$$T_{BTd} = \bar{y}\ exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right).$$

Following estimators can be extended to double sampling as ???

Yadav and Kadilar (2013) exponential type estimator extend in double sampling as:

$$T_{YKd} = k\bar{y}\ exp\left(\frac{(a\bar{x}' + b) - (a\bar{x} + b)}{(a\bar{x}' + b) + (a\bar{x} + b)}\right),$$

where a and b are the choosen constants or the function of parameters.

Singh and Pal (2015) exponential estimator extend in double sampling as

$$T_{SPd} = \bar{y}\left(\frac{a\bar{x}' + b}{a\bar{x} + b}\right) exp\left(\frac{a(\bar{x}' - \bar{x})}{a(\bar{x}' + \bar{x}) + 2b}\right).$$

Cochran (1977) classical regression estimator extended in double sampling as

$$T_{regd} = \bar{y} + b(\bar{x}' - \bar{x}),$$

where b is the regression coefficient of y on x.

The mean squared error (MSE) of the above estimator to the first degree of approximation in double sampling are as follows

$$MSE(T_{Rd}) = \bar{Y}^2\left(\lambda C_y^2 + \lambda^*\left(C_x^2 - 2C_{yx}\right)\right), \tag{1}$$

$$MSE(T_{BTd}) = \bar{Y}^2\left(\lambda C_y^2 + \lambda^*\left(\frac{C_x^2}{4} - C_{yx}\right)\right), \tag{2}$$

$$MSE_{min}(T_{YKd}) = \bar{Y}^2\left(1 - \frac{\left(\lambda^*\left(2\xi^2 C_x^2 - \xi C_{yx}\right)+1\right)^2}{\left(\lambda C_y^2 + \lambda'\left(5\xi^2 C_x^2 - 4\xi C_{yx}\right)+1\right)}\right), \tag{3}$$

$$MSE(T_{SPd}) = \bar{Y}^2\left(\lambda C_y^2 + \frac{3\lambda^*\theta C_x^2}{4}\left(3\theta - 4\rho_{yx}\frac{C_y}{C_x}\right)\right), \tag{4}$$

$$MSE(T_{regd}) = \bar{Y}^2 C_y^2\left(\lambda + \lambda^*\rho_{yx}^2\right), \tag{5}$$

where $\lambda = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\lambda' = \left(\frac{1}{n'} - \frac{1}{N}\right)$, $\lambda^* = \lambda - \lambda' = \left(\frac{1}{n} - \frac{1}{n'}\right)$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_{yx} = \rho_{yx}C_yC_x$, $\xi = \frac{a\bar{X}}{2(a\bar{X}+b)}$, $\theta = \frac{a\bar{X}}{a\bar{X}+b}$, $\bar{Y}$ is the population mean of the study variable and ρ_{yx} is the population correlation coefficient between the study and the auxiliary variables.

In most of the survey, the required information on different variables may not be obtained correctly due to various reasons in practice. For example, some of the respondents may avoid answering the questionnaire or provide incomplete information, sometimes it is hard to find respondents to participate questionnaire, etc. [Muneer, Shabbir and Khalil (2018)]. This situation is named as non-response.

Hansen and Hurwitz (1946) first to deal with this problem and introduced a new technique of sub-sampling the non-respondents. In this method, suppose that $S = (S_1, S_2, ..., S_N)$ consists of N units. From S, a sample of size n is drawn without replacement (SRSWOR) and (y_i, x_i) are the values of the study and auxiliary variables for the i^{th} unit $(i = 1, 2, ..., N)$ of the population, respectively. Population of size N is composed of N_1 and N_2 belonging to the responding units and non-responding units, respectively. Similarly, sample of size n is divided in two parts as n_1 responding units and n_2 non-responding units. A sub-sample of size $r = n_2/h(h > 1)$ units is randomly drawn from n_2 where h is the inverse sampling rate. Note that $W_1 = \frac{N_1}{N}$ and $W_2 = \frac{N_2}{N}$ are the proportions of the responding and the non-responding for the population, respectively.

Then, Hansen and Hurwitz (1946) were the first to propose the unbiased estimator for estimating the population mean in the presence of non-response as follows

$$\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_{2(r)},$$

where $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$ indicate responding and non-responding proportions for the sample, respectively. In addition, y_1 and $y_{2(r)}$ represent the sample means of the study variable depending on n_1 and r units, respectively.

The variance of the estimator $\bar{y}^*$ is given by

$$V(\bar{y}^*) = \bar{Y}^2\left(\lambda C_y^2 + \frac{W_2(h-1)}{n}C_{y(2)}^2\right),$$

where $C_y^2 = S_y^2/\bar{Y}^2$ and $C_{y(2)}^2 = S_{y(2)}^2/\bar{Y}^2$.

Similarly, one can define unbiased estimator of the auxiliary variable as

$$\bar{x}^* = w_1\bar{x}_1 + w_2\bar{x}_{2(r)},$$

where x_1 and $x_{2(r)}$ represent the sample means of the study variable depending on n_1 and r units, respectively.

The variance of the estimator $\bar{x}^*$ is given by

$$V(\bar{x}^*) = \bar{X}^2\left(\lambda C_x^2 + \frac{W_2(h-1)}{n}C_{x(2)}^2\right),$$

where $C_x^2 = S_x^2/\bar{X}^2$ and $C_{x(2)}^2 = S_{x(2)}^2/\bar{X}^2$.

When non-response exists only on the study variable and the population mean of the auxiliary variable is known, Rao (1986) modified the ratio and the regression estimators introduced by Cochran can be extended in double sampling as

$$T_{Rd}^{*} = \frac{\bar{y}^{*}}{\bar{x}}\bar{x}',$$
$$T_{regd}^{*} = \bar{y}^{*} + b^{*}(\bar{x}' - \bar{x}),$$

where $\bar{y}^{*}$ represents the sample mean of the study variable in the case of non-response and $b^{*} = S_{yx}^{*}/S_{x}^{2}$; $S_{yx}^{*} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}), S_{x}^{2} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$.

The expressions of the MSE for the estimators T_{Rd}^{*} and T_{regd}^{*} are given as

$$MSE(T_{Rd}^{*}) = \bar{Y}^{2}\left(\lambda C_{y}^{2} + \lambda^{*}\left(C_{x}^{2} - 2C_{yx}\right) + \frac{W_2(h-1)}{n}C_{y(2)}^{2}\right), \tag{6}$$

$$MSE(T_{regd}^{*}) = \bar{Y}^{2}\left(C_{y}^{2}\left(\lambda + \lambda^{*}\rho_{yx}^{2}\right) + \frac{W_2(h-1)}{n}C_{y(2)}^{2}\right), \tag{7}$$

where $C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$, ρ_{yx} is the correlation coefficient between y and x of responding units.

Singh et al. (2009) proposed an exponential type estimator when non-response occurs only on the study variable as

$$T_{BTd}^{*} = \bar{y}^{*} exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right).$$

The MSE of the estimator T_{BTd}^{*}, up to the first order of approximation, is given by

$$MSE(T_{BTd}^{*}) = \bar{Y}^{2}\left(\lambda C_{y}^{2} + \lambda^{*}\left(\frac{C_{x}^{2}}{4} - C_{yx}\right) + \frac{W_2(h-1)}{n}C_{y(2)}^{2}\right). \tag{8}$$

When non-response occurs on both the study and auxiliary variables as well and the population mean of the auxiliary variable is known then Cochran (1977) suggested ratio and regression estimators in double sampling as

$$T_{Rd}^{**} = \frac{\bar{y}^{*}}{\bar{x}^{*}}\bar{x}',$$
$$T_{regd}^{**} = \bar{y}^{*} + b^{*}(\bar{x}' - \bar{x}^{*}).$$

The MSE of T_{Rd}^{**} and T_{regd}^{**} are

$$MSE(T_{Rd}^{**}) = \bar{Y}^{2}\left(\lambda C_{y}^{2} + \lambda^{*}\left(C_{x}^{2} - 2C_{yx}\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^{2} + C_{x(2)}^{2} - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}\right)\right), \tag{9}$$

$$MSE(T_{regd}^{**}) = \bar{Y}^{2}\left(C_{y}^{2}\left(\lambda + \lambda^{*}\rho_{yx}^{2}\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^{2} + \rho_{yx}^{2}\frac{C_{y}^{2}}{C_{x}^{2}}C_{x(2)}^{2} - 2\rho_{yx}\frac{C_{y}}{C_{x}}C_{yx(2)}\right)\right). \tag{10}$$

Singh et al. (2009) defined exponential type estimator in double sampling as

$$T_{BTd}^{**} = \bar{y}^{*} exp\left(\frac{\bar{x}' - \bar{x}^{*}}{\bar{x}' + \bar{x}^{*}}\right),$$

whose MSE is given by

$$MSE(T_{BTd}^{**}) = \bar{Y}^{2}\left(\lambda C_{y}^{2} + \lambda^{*}\left(\frac{C_{x}^{2}}{4} - C_{yx}\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^{2} + \frac{C_{x(2)}^{2}}{4} - C_{yx(2)}\right)\right). \tag{11}$$

Unal and Kadilar (2019) suggested exponential family of estimators in two situations in double sampling as

Situation 1: When non-response occurs only on the study variable and the population mean of the auxiliary variable is unknown, then the families of estimators in double sampling are as follows

$$T_{1,id} = k\bar{y}^* \left(\frac{a_i \bar{x}' + b_i}{a_i \bar{x} + b_i} \right)^{\alpha} exp\left(\frac{a_i(\bar{x}' - \bar{x})}{a_i(\bar{x}' + \bar{x}) + 2b_i} \right);\ i = 1, 2, ..., 10,$$

where $k(\neq 0)$ and α is a suitably chosen constant, (a_i, b_i) are either real numbers or the parameters of the auxiliary variable.

The expression for bias and MSE of the estimator to the first degree of approximation are

$$Bias(T_{1,id}) = \bar{Y}(k-1) + \bar{Y}\lambda^* \left\{ C_x^2 \frac{k\theta_i^2}{2} \left(\frac{3}{4} + \alpha^2 \right) - C_{yx} k\theta_i \left(\frac{1}{2} + \alpha \right) \right\},$$

and

$$MSE(T_{1,id}) = \bar{Y}^2 \left\{ (k-1)^2 + k^2 \lambda C_y^2 + \lambda^* \left(\theta_i^2 C_x^2 \left(k^2 + 2k^2\alpha^2 - k\alpha^2 + \alpha k^2 - \frac{3}{4}k \right) + \theta_i C_{yx} \left(k + 2k\alpha - 2k^2 - 4k^2\alpha \right) \right) \right\} + \frac{W_2(k-1)}{n} C_{y(2)}^2,$$

under the optimal value of k as

$$k^* = \frac{A_1}{A_2},$$

where $A_1 = \lambda^* \left(C_x^2 \theta_i^2 \left(\alpha^2 + \frac{3}{4} \right) - C_{yx} \theta_i \left(1 + 2\alpha \right) \right) + 2$ and

$A_2 = \lambda 2C_y^2 + \lambda^* \left(2\theta_i^2 C_x^2 + 4\alpha^2 \theta_i^2 C_x^2 + 2\alpha \theta_i^2 C_x^2 - 4\theta_i C_{yx} + 8\alpha \theta_i C_{yx} \right) + 2\frac{W_2(h-1)}{n} C_{y(2)}^2 + 2.$

The minimum mean squared error of $T_{(1,id)}$ can be obtained as

$$MSE_{min}(T_{1,id}) = \bar{Y}^2 \left(1 - \frac{A_1^2}{2A_2} \right). \tag{12}$$

Situation 2: When non-response occurs on both the study and the auxiliary variables and population mean of the auxiliary variable is unknown, then the family of estimators are

$$T_{2,id} = k\bar{y}^* \left(\frac{a_i \bar{x}' + b_i}{a_i \bar{x}^* + b_i} \right)^{\alpha} exp\left(\frac{a_i(\bar{x}' - \bar{x}^*)}{a_i(\bar{x}' + \bar{x}^*) + 2b_i} \right);\ i = 1, 2, ..., 10.$$

The expression for bias and MSE of the estimator to the first degree of approximation in double sampling are as follows

$$Bias(T_{2,id}) = \bar{Y} \left\{ (k-1) + \left(\lambda^* \left(C_x^2 \frac{k\theta_i^2}{2} \left(\frac{3}{4} + \alpha^2 \right) - C_{yx} k\theta_i \left(\frac{1}{2} + \alpha \right) \right) + \frac{W_2(h-1)}{n} \left(C_{x(2)}^2 - \rho_{yx(2)} C_{y(2)} C_{x(2)} \right) \right) \right\};\ i = 1, 2, ..., 10.$$

$$MSE(T_{2,id}) = \bar{Y}^2 \left\{ (k-1)^2 + k^2 \lambda C_y^2 + \lambda^* \left(k\theta_i^2 C_x^2 \left(k - \frac{3}{4} - \alpha^2 + 2k\alpha^2 + k\alpha \right) + k\theta_i C_{yx} \left(1 + 2\alpha - 2k - 4k\alpha \right) \right) + \frac{W_2(h-1)}{n} \left(C_{y(2)}^2 + C_{x(2)}^2 + C_{yx(2)} \right) \right\},$$

is optimal, when the value of k is

$$k^{**} = \frac{A_3}{A_4},$$

where $A_3 = \lambda^*\left(C_x^2\theta_i^2\left(\alpha^2+\frac{3}{4}\right)-C_{yx}\theta_i\left(1+2\alpha\right)\right)+\frac{W_2(h-1)}{n}\left(C_{x(2)}^2-C_{yx(2)}\right)+2$ and

$A_4 = \lambda 2C_y^2+\lambda^*\left(2\theta_i^2C_x^2\left(2\alpha^2+\alpha+1\right)-\theta_iC_{yx}\left(4+8\alpha\right)\right)+\frac{W_2(h-1)}{n}\left(C_{y(2)}^2+C_{x(2)}^2-C_{yx(2)}\right)+2.$

Thus the minimum mean squared error of $T_{2,id}$ is given by

$$MSE_{min}(T_{2,id}) = \bar{Y}^2\left(1-\frac{A_3^2}{2A_4}\right);\ i=1,2,...,10. \tag{13}$$

In the present study, we propose a new class of modified exponential estimator in double sampling with full response and in the presence of non-response for the estimation of population mean of the study variable and studied their properties in Section 3. Theoretical and numerical comparisons are made between the proposed and existing estimators in Section 4 and Section 5, respectively. Finally in Section 6, some concluding remarks are given.

3. The Proposed Family of Estimators

In the given sampling scheme we have proposed a modified exponential estimators for estimating the population mean of study variable by utilizing the information on auxiliary variable using double sampling scheme under two situations

i) when there is full information available on the study as well as on auxiliary variable.

ii) when there is a non-response on the study as well as on auxiliary variable.

Situation1: When there is complete information on study as well as on auxiliary variable and the population mean of auxiliary variable is unknown, we propose the following class of estimator as

$$\tau_{1,id} = \delta\bar{y}\left(\frac{(1-c_i)\bar{x}+c_i\bar{x}'}{c_i\bar{x}+(1-c_i)\bar{x}'}\right)^{\eta} exp\left(\frac{c_i(\bar{x}-\bar{x}')}{(c_i\bar{x}+d_i)+(a_i\bar{x}'+d_i)}\right),$$

where $\delta(\neq 0)$ and η are suitably chosen constants, and c_i, d_i are either real numbers or the functions of the known parameters of the auxiliary variable, such as the coefficient of variation, coefficient of kurtosis, correlation coefficients, etc. Various number of estimators can be generated by taking suitable choices of constants δ, η, c_i, d_i, respectively. Some of the estimators are presented in the Table 1.

Table 1 Members of the proposed estimators for different values of $c_i's$ and $d_i's$

S.No.	Values		Estimator(s)
	c_i	d_i	
1	1	1	$\tau_{1,1} = \delta\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\eta} exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'+2}\right)$
2	1	C_x	$\tau_{1,2} = \delta\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\eta} exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'+2C_x}\right)$
3	1	ρ_{yx}	$\tau_{1,3} = \delta\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\eta} exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'+2\rho_{yx}}\right)$
4	1	0	$\tau_{1,4} = \delta\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\eta} exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'}\right)$
5	C_x	ρ_{yx}	$\tau_{1,5} = \delta\bar{y}\left(\frac{\bar{x}+C_x(\bar{x}'-\bar{x})}{\bar{x}'+C_x(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{C_x(\bar{x}-\bar{x}')}{C_x(\bar{x}+\bar{x}')+2\rho_{yx}}\right)$
6	ρ_{yx}	C_x	$\tau_{1,6} = \delta\bar{y}\left(\frac{\bar{x}+\rho_{yx}(\bar{x}'-\bar{x})}{\bar{x}'+\rho_{yx}(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{\rho_{yx}(\bar{x}-\bar{x}')}{\rho_{yx}(\bar{x}+\bar{x}')+2C_x}\right)$
7	ρ_{yx}	1	$\tau_{1,7} = \delta\bar{y}\left(\frac{\bar{x}+\rho_{yx}(\bar{x}'-\bar{x})}{\bar{x}'+\rho_{yx}(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{\rho_{yx}(\bar{x}-\bar{x}')}{\rho_{yx}(\bar{x}+\bar{x}')+2}\right)$
8	1	$\beta_2(x)$	$\tau_{1,8} = \delta\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\eta} exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'+2\beta_2(x)}\right)$
9	$\beta_2(x)$	1	$\tau_{1,9} = \delta\bar{y}\left(\frac{\bar{x}+\beta_2(x)(\bar{x}'-\bar{x})}{\bar{x}'+\beta_2(x)(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{\beta_2(x)(\bar{x}-\bar{x}')}{\beta_2(x)(\bar{x}+\bar{x}')+2}\right)$
10	$\beta_2(x)$	C_x	$\tau_{1,10} = \delta\bar{y}\left(\frac{\bar{x}+\beta_2(x)(\bar{x}'-\bar{x})}{\bar{x}'+\beta_2(x)(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{\beta_2(x)(\bar{x}-\bar{x}')}{\beta_2(x)(\bar{x}+\bar{x}')+2C_x}\right)$
11	C_x	$\beta_2(x)$	$\tau_{1,11} = \delta\bar{y}\left(\frac{\bar{x}+C_x(\bar{x}'-\bar{x})}{\bar{x}'+C_x(\bar{x}-\bar{x}')}\right)^{\eta} exp\left(\frac{C_x(\bar{x}-\bar{x}')}{C_x(\bar{x}+\bar{x}')+2\beta_2(x)}\right)$

To obtain the Bias and MSE of $\tau_{1,id}$; $i = 1,2,...,11$, we assume

$$\bar{y} = \bar{Y}(1+\varepsilon_o); \bar{x} = \bar{X}(1+\varepsilon_1); \bar{x}' = \bar{X}(1+\varepsilon_1').$$

then we have, $E(\varepsilon_o) = E(\varepsilon_1) = E(\varepsilon_1') = 0$; $E(\varepsilon_o^2) = \lambda C_y^2$; $E(\varepsilon_1^2) = \lambda C_x^2$; $E(\varepsilon_1'^2) = \lambda' C_x^2$ and $E(\varepsilon_o\varepsilon_1) = \lambda\rho_{yx}C_yC_x$; $E(\varepsilon_1\varepsilon_1') = \lambda' C_x^2$; $E(\varepsilon_o\varepsilon_1') = \lambda'\rho_{yx}C_yC_x$.

Now, the family of estimators $\tau_{1,id}$; $i = 1,2,\ldots,11$ can be expressed in terms of $\varepsilon's$ to the first degree of approximation, we get

$$\tau_{1,id} = \delta\bar{Y}(1+\varepsilon_o)\left(\frac{(1-c_i)\bar{X}(1+\varepsilon_1)+c_i\bar{X}(1+\varepsilon_1')}{c_i\bar{X}(1+\varepsilon_1)+(1-c_i)\bar{X}(1+\varepsilon_1')}\right)^{\eta} exp\left(\frac{c_i(\bar{X}(1+\varepsilon_1)-\bar{X}(1+\varepsilon_1'))}{(c_i\bar{X}(1+\varepsilon_1)+d_i)+(c_i\bar{X}(1+\varepsilon_1')+d_i)}\right).$$

$$\left(\tau_{1,id}-\bar{Y}\right) = \bar{Y}\left(\left(\delta-1\right)+\delta A_i\varepsilon_1-\delta A_i\varepsilon_1'+\delta B_i\varepsilon_1^2+\delta C_i\varepsilon_1'^2+\delta D_i\varepsilon_1\varepsilon_1'+\delta\varepsilon_o+\delta A_i\varepsilon_o\varepsilon_1-\delta A_i\varepsilon_o\varepsilon_1'\right), \quad (14)$$

where $A_i = \omega_i - 2\eta c_i + \eta; \omega_i = \dfrac{c_i\bar{X}}{2(c_i\bar{X}+d_i)}$,

$$B_i = \eta\omega_i - \frac{\omega_i^2}{2} - 2\eta\omega_i c_i + 2\eta^2 c_i^2 - 2\eta^2 c_i + \eta c_i + \frac{\eta(\eta-1)}{2},$$

$$C_i = \frac{3}{2}\omega_i^2 + \eta\omega_i - 2\eta\omega_i c_i + 2\eta^2 c_i^2 - 2\eta^2 c_i - \eta c_i + \frac{\eta(\eta+1)}{2},$$

and $D_i = 4\eta\omega_i c_i - \omega_i^2 - 2\eta\omega_i + 2\eta c_i^2 + 4\eta^2 c_i - \eta^2 - 2\eta^2 c_i^2$.

Taking expectation on both sides of (14), we get the bias of $\tau_{1,id}; i = 1,2,\ldots,11$ as

$$Bias(\tau_{1,id}) = \bar{Y}\left((\delta-1) + \delta\lambda B_i C_x^2 + \delta\lambda' C_x^2\left(C_i + D_i\right) + \delta\rho_{yx}C_yC_x(\lambda-\lambda')\right).$$

Squaring both sides of (14) and then taking expectation, we get the MSE of $\tau_{1,id}$ as

$$MSE(\tau_{1,id}) = \bar{Y}^2\left((\delta-1)^2 + \delta^2\lambda C_y^2 + \delta^2 A_i^2 C_x^2(\lambda-\lambda') + 2\delta^2 A_i\rho_{yx}C_yC_x(\lambda-\lambda')\right); \quad i = 1,2,...,11. \tag{15}$$

Differentiating the $MSE(\tau_{1,id})$ with respect to δ and equating it to zero, we obtain the optimum value of δ as

$$\delta = \frac{1}{1 + \lambda C_y^2 + A_i^2 C_x^2(\lambda-\lambda') + 2A_i\rho_{yx}C_yC_x(\lambda-\lambda')}.$$

$$\delta = \frac{1}{1+A} = \delta_o(say), \tag{16}$$

where $A = (\lambda C_y^2 + A_i^2 C_x^2(\lambda-\lambda') + 2A_i\rho_{yx}C_yC_x(\lambda-\lambda'))$.

Replacing the value of δ from (16) in (15), we get the minimum MSE of the proposed family of estimators as

$$MSE_{min}(\tau_{1,id}) = \bar{Y}^2\left(\frac{A}{1+A}\right);\ i = 1,2,...,11. \tag{17}$$

One can obtain the minimum MSE of the estimators defined in Table 1 by substituting the suitable values of c_i and d_i.

Situation 2: When there is non-response on study variable as well as auxiliary variable and population mean of the auxiliary variable is known, we propose the following class of estimator as follows

$$\tau_{2,id} = \delta\bar{y}^*\left(\frac{(1-c_i)\bar{x}^* + c_i\bar{x}'}{c_i\bar{x}^* + (1-c_i)\bar{x}'}\right)^{\eta} exp\left(\frac{c_i(\bar{x}^* - \bar{x}')}{(c_i\bar{x}^* + d_i) + (a_i\bar{x}' + d_i)}\right).$$

Similarly as in Table 1, one can also write the members of the proposed family of estimators in (34) under the case of non-response existing both on the study variable and the auxiliary variable for different values of c_i and d_i, respectively.

To obtain the Bias and MSE of $\tau_{1,i}; i = 1,2,\ldots,11$, we assume

$$\bar{y}^* = \bar{Y}(1+\varepsilon_o^*);\ \bar{x}^* = \bar{X}(1+\varepsilon_1^*);\ \bar{x}' = \bar{X}(1+\varepsilon_1').$$

then we have,

$E(\varepsilon_o^*) = E(\varepsilon_1^*) = E(\varepsilon_1') = 0$; $E(\varepsilon_o^{*2}) = \lambda C_y^2 + \theta C_{y(2)}^2$; $E(\varepsilon_1^{*2}) = \lambda C_x^2 + \theta C_{x(2)}^2$; $E(\varepsilon_1'^2) = \lambda' C_x^2$ and $E(\varepsilon_o^*\varepsilon_1^*) = \lambda C_{yx} + \theta C_{yx(2)}$; $E(\varepsilon_1^*\varepsilon_1') = \lambda' C_x^2$; $E(\varepsilon_o^*\varepsilon_1') = \lambda' C_{yx}$.

where $\theta = \frac{W_2(h-1)}{n}$ and $W_2 = \frac{N_2}{N}$; $h > 1$.

The expression of bias and MSE of $\tau_{2,id};\ i=1,...,11$ to the first degree of approximation are obtained as

$$Bias(\tau_{2,id})=\bar{Y}\left((\delta-1)+\delta B_i(\lambda C_x^2+\theta C_{x(2)}^2)+\delta\lambda' C_x^2(C_i+D_i)+\delta A_i(\lambda C_{yx}+\theta C_{yx(2)})+\delta\lambda' C_{yx}\right),$$

and

$$MSE(\tau_{2,id})=\bar{Y}^2\big((\delta-1)^2+\delta^2(\lambda C_y^2+\theta C_{y(2)}^2)+\delta^2A_i^2(\lambda C_x^2+\theta C_{x(2)}^2)-\delta^2\lambda' A_i^2C_x^2 -2\delta^2\lambda' C_{yx}+2\delta^2A_i(\lambda C_{yx}+\theta C_{yx(2)})\big);\ \ i=1,2,...,11, \tag{18}$$

which is optimum when

$$\delta=\frac{1}{1+C+A_iD-\lambda' A_i^2C_x^2-2\lambda' A_iC_{yx}+2A_iE},$$

where $C=\lambda C_y^2+\theta C_{y(2)}^2$; $D=\lambda C_x^2+\theta C_{x(2)}^2$ and $E=\lambda C_{yx}+\theta C_{yx(2)}$.

The optimal value of δ is given as

$$\delta=\frac{1}{1+B}=\delta_{oo}(say), \tag{19}$$

where $B=(C+A_iD-\lambda' A_i^2C_x^2-2\lambda' A_iC_{yx}+2A_iE)$.

Substituting the optimum value of δ from (19) in (18), we get the minimum MSE of $\tau_{2,id}$ as

$$MSE_{min}(\tau_{2,id})=\bar{Y}^2\left(\frac{B}{1+B}\right);\ i=1,2,...,11. \tag{20}$$

We would like to note that the values of min. MSE are also different for each estimator $\tau_{2,1},\tau_{2,2},...,\tau_{2,11}$ as in Situation 1.

4. Efficiency Comparisons

Situation 1: In this section, we compare the MSE equation of the proposed family of estimators $\tau_{1,id};\ i=1,2,...11$ with the MSE of the mentioned estimators in section 2, respectively.

From equation (1), (2), (3), (4), (5) and (17), we find the efficiency comparisons of the proposed family of estimators as

i) $MSE_{min}(\tau_{1,id})<MSE(T_{Rd})$,

$$\left(\frac{A}{1+A}\right)<\left(\lambda C_y^2+\lambda^*(C_x^2-2C_{yx})\right).$$

ii) $MSE_{min}(\tau_{1,id})<MSE(T_{BTd})$,

$$\left(\frac{A}{1+A}\right)<\left(\lambda C_y^2+\lambda^*\left(\frac{C_x^2}{4}-C_{yx}\right)\right).$$

iii) $MSE_{min}(\tau_{1,id})<MSE_{min}(T_{YKd})$,

$$\left(\frac{A}{1+A}\right)<\left(1-\frac{\left(\lambda^*(2\xi^2C_x^2-\xi C_{yx})+1\right)^2}{(\lambda C_y^2+\lambda'(5\xi^2C_x^2-4\xi C_{yx})+1)}\right).$$

iv) $MSE_{min}(\tau_{1,id}) < MSE(T_{SPd})$,

$$\left(\frac{A}{1+A}\right) < \left(\lambda C_y^2 + \frac{3\lambda^*\theta C_x^2}{4}\left(3\theta - 4\rho_{yx}\frac{C_y}{C_x}\right)\right).$$

v) $MSE_{min}(\tau_{1,id}) < MSE(T_{regd})$,

$$\left(\frac{A}{1+A}\right) < \left(\lambda + \lambda^*\rho_{yx}^2\right).$$

For the Situation 1, when the above conditions are satisfied, we infer that the proposed family of estimators $\tau_{1,id};\ i = 1,2,...,11$ are more efficient than the compared estimators T_{Rd}, T_{BTd}, T_{YKd}, T_{SPd}, T_{regd}.

Situation 2: In this section, we compare the MSE equation of the proposed family of estimators $\tau_{2,id};\ i = 1,2,...,11$ with the MSE of the mentioned estimators in section 2, respectively.

From equation (6), (7), (8), (9), (10), (11), (12), (13) and (20), we find the efficiency comparisons of the proposed family of estimators as:

i) $MSE_{min}(\tau_{2,id}) < MSE(T_{Rd}^*)$,

$$\left(\frac{B}{1+B}\right) < \left(\lambda C_y^2 + \lambda^*\left(C_x^2 - 2C_{yx}\right) + \frac{W_2(h-1)}{n}C_{y(2)}^2\right).$$

ii) $MSE_{min}(\tau_{2,id}) < MSE(T_{regd}^*)$,

$$\left(\frac{B}{1+B}\right) < \left(\left(\lambda + \lambda^*\rho_{yx}^2\right) + \frac{W_2(h-1)}{n}C_{y(2)}^2\right).$$

iii) $MSE_{min}(\tau_{2,id}) < MSE(T_{BTd}^*)$,

$$\left(\frac{B}{1+B}\right) < \left(\lambda C_y^2 + \lambda^*\left(\frac{C_x^2}{4} - C_{yx}\right) + \frac{W_2(h-1)}{n}C_{y(2)}^2\right).$$

iv) $MSE_{min}(\tau_{2,id}) < MSE(T_{Rd}^{**})$,

$$\left(\frac{B}{1+B}\right) < \left(\lambda C_y^2 + \lambda^*\left(C_x^2 - 2C_{yx}\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}\right)\right).$$

v) $MSE_{min}(\tau_{2,id}) < MSE(T_{BTd}^{**})$,

$$\left(\frac{B}{1+B}\right) < \left(\lambda C_y^2 + \lambda^*\left(\frac{C_x^2}{4} - C_{yx}\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} - \rho_{yx(2)}C_{y(2)}C_{x(2)}\right)\right).$$

vi) $MSE_{min}(\tau_{2,id}) < MSE(T_{regd}^{**})$,

$$\left(\frac{B}{1+B}\right) < \left(\left(\lambda + \lambda^*\rho_{yx}^2\right) + \frac{W_2(h-1)}{n}\left(C_{y(2)}^2 + \rho_{yx}^2\frac{C_y^2}{C_x^2}C_{x(2)}^2 - 2\rho_{yx}\frac{C_y}{C_x}C_{yx(2)}\right)\right).$$

vii) $MSE_{min}(\tau_{2,id}) < MSE_{min}(T_{1,id})$,

$$\left(\frac{B}{1+B}\right) < \left(1 - \frac{A_1^2}{2A_2}\right).$$

viii) $MSE_{min}(\tau_{2,id}) < MSE_{min}(T_{2,id})$,

$$\left(\frac{B}{1+B}\right) < \left(1 - \frac{A_3^2}{2A_4}\right).$$

For the Situation 2, when the above conditions are satisfied, we conclude that the proposed family of estimators $\tau_{2,id};\ i = 1,2,...,11$ are more efficient than the compared estimators T_{Rd}^*,T_{BTd}^*,T_{Rd}^{**}, T_{BTd}^{**}, T_{regd}^{**}, $T_{1,id}$, $T_{2,id}$.

5. Simulation Study

To illustrate numerically, we compared efficiency of proposed class of estimators with considered estimators defined in section 2. For empirical analysis, we used R software. For this we generated two populations-

Population I is generated with $N = 89$ units from normal distribution, where $X \sim N(0,1)$ and $Y = X + N(0,1)$. We then took a random sample of 45 units for first phase and then another random sample of 20 from first phase units. This population is used for the numerical illustration of Situation 1 (complete response on study and auxiliary variable).

Population II is generated with $N = 100$ units from normal distribution, where $X \sim N(0,1)$ and $Y = X + N(0,1)$. We then took a random sample of 50 units for first phase and then another random sample of 30 from first phase units. There is 25% of non-response in second phase both for variables Y and X. This population is used for the numerical illustration of Situation 2 (non-response on study and auxiliary variable).

The percent relative efficiency (PREs) of proposed estimators for Situation 1 is calculated by using the following formulae

$$PRE(T,\bar{y}) = \frac{Var(\bar{y})}{MSE(T)} \times 100,$$

where $T = T_{Rd}, T_{BTd}, T_{regd}, T_{YKd}, T_{SPd}$ and $\tau_{(1,i)}$; $i = 1,2,\ldots,11$.

Table 2 and Table 3 shows the PRE values of the estimators (T) with respect to $\bar{y}$.

Table 2 PRE of existing estimators with respect to $\bar{y}$ for Situation 1

	Population I
T_{Rd}	137.1301
T_{BTd}	120.2381
T_{regd}	145.4506

Table 3 PRE of estimators with respect to $\bar{y}$ for different values of c_i and d_i; $i = 1,2,...,11$ for Situation 1

	Population I				
S.No.	c_i	d_i	T_{YKd}	T_{SPd}	$\tau_{1,id}$
1	1	1	516.7556	83.3394	546.7819
2	1	-4.3416	519.9926	102.9881	526.7632
3	1	0.6603	510.4757	71.9236	545.8451
4	1	0	500.9595	145.3568	543.8567
5	-4.3416	0.6603	544.0398	134.4114	412.3036
6	0.6603	-4.3416	493.8301	102.0036	546.5760
7	0.6603	1	512.7114	89.7487	522.7470
8	1	3.3704	516.7556	83.3394	546.7819
9	3.3704	1	517.4011	17.1267	440.0743
10	3.3704	-4.3416	87.6477	109.0793	482.1222
11	-4.3416	3.3704	82.8837	113.7385	411.3565

According to the results in Table 2 and Table 3, we would like to remark that for complete response on study and auxiliary variables (Situation 1), our proposed class of estimators $\tau_{1,id}$ performs better than $T_{Rd}, T_{BTd}, T_{YKd}, T_{SPd}$ and T_{regd} for populations I in terms of PRE with respect to $\bar{y}$.

For population I, the estimator $\tau_{1,5}, c_i = C_x$ and $d_i = \rho_{yx}$ and $\tau_{1,9}, c_i = \beta_2(x)$ and $d_i = 1$, performs less efficient than T_{YKd} and T_{SP}.

The percent relative efficiency (PREs) of proposed estimators for Situation 2 is calculated as

$$PRE(T^*) = \frac{Var(\bar{y}^*)}{MSE(T^*)} \times 100,$$

where $T^* = T^*_{Rd}, T^*_{BTd}, T^*_{regd}, T^{**}_{Rd}, T^{**}_{BTd}, T^{**}_{regd}, T_{1,id}, T_{2,id}$ and $\tau_{2,id}$; $i = 1, 2, ..., 11$ gives the different estimators obtained from generalized estimator for situation 2.

The values of PRE's are shown in Table 4 and Table 5.

Table 4 PRE of existing estimators with respect to $\bar{y}^*$ for Situation 2

Population II				
Estimators	h=2	h=3	h=4	h=5
T^*_{Rd}	131.0453	128.4719	126.2925	124.4231
T^*_{regd}	93.7367	94.1170	94.4538	94.7541
T^*_{BTd}	43.6857	45.3329	46.8864	48.3540
T^{**}_{Rd}	130.6286	127.7246	125.2771	123.1864
T^{**}_{BTd}	43.7032	45.3680	46.9394	48.4250
T^{**}_{regd}	93.1394	92.9960	92.8717	92.7613

From Tables 4 and 5, we can see that our proposed class of estimators for Situation 2 are more efficient than already existing estimators. Also the estimator $\tau_{2,8}$; $i = 8$ is more efficient among other class of estimators. Also, the estimators $T_{1,id}$ and $T_{2,id}$; $i = 1, 2, ..., 11$ are not efficient for the consider population II with respect to $\bar{y}^*$.

Table 5 PRE of estimators with respect to $\bar{y}^*$ for different values of c_i and d_i; $i = 1, 2, ..., 11$ for Situation 2

Population II													
c_i	d_i	$\tau_{2,id}$				$T_{1,id}$				$T_{2,id}$			
		h=2	h=3	h=4	h=5	h=2	h=3	h=4	h=5	h=2	h=3	h=4	h=5
1	1	916.4484	851.8463	820.3213	806.5823	6.6103	6.1681	5.7828	5.4440	3.9909	3.8446	3.7075	3.5792
1	-6.4072	531.2065	397.5454	577.2513	600.9683	5.0581	2.6896	4.5759	4.3675	5.4017	9.4838	4.8551	4.6215
1	0.6879	861.0944	544.9505	793.0547	785.3454	7.6927	5.7016	6.3786	6.1379	3.4394	4.1936	3.2423	3.1499
1	0	772.6096	751.7723	745.0419	746.8984	1.3597	1.3664	1.3701	1.3710	14.3362	12.5701	11.1745	10.0467
-6.4072	0.6879	371.8272	397.2816	422.7020	448.0859	2.4010	2.3654	2.3281	2.2898	11.6404	10.4084	9.4054	8.5742
0.6879	-6.4072	481.3007	507.4146	533.5373	559.6673	5.1048	4.8467	4.6133	4.4013	5.3436	5.0622	4.8090	4.5800
0.6879	1	521.3634	544.9505	568.9301	593.2194	6.0731	5.7016	5.3738	5.0823	4.3752	4.1936	4.0258	3.8705
1	3.0379	2694.3547	1539.9731	1215.1596	1072.8626	5.5808	5.2692	4.9908	4.7407	4.8159	4.5907	4.3855	4.1976
3.0379	1	360.7829	385.6930	410.6065	435.5233	1.8443	1.8257	1.8055	1.7842	1.8574	1.8506	1.8408	1.8289
3.0379	-6.4072	339.7385	362.8485	385.9118	408.9273	5.7745	5.4430	5.1477	4.8830	5.7754	5.4447	5.1501	4.8860
-6.4072	3.0379	373.3726	398.9942	424.5937	450.1696	7.5192	6.1692	6.4944	6.0801	7.5116	6.9563	6.4777	6.0610

6. Conclusions

In this paper, a modified class of exponential estimator using double sampling is suggested for the estimation of population mean of the study variable by using the information of the auxiliary variable under two situations i.e. Situation 1: when there is full response on study and auxiliary variable with known population mean of auxiliary variable; Situation 2: when there is non-response on study and auxiliary variables with known population mean of auxiliary variable. The bias and minimum MSE equations of the proposed family of estimators ($\tau_{1,id}$; $i = 1,2,...,11$ and $\tau_{2,id}$; $i = 1,2,...,11$) are obtained to the first degree of approximation. For empirical results, we studied the model based simulation and obtained the percent relative efficiency of both the situations. The results are shown in Tables (2-5) which indicate that the performance of the proposed class of estimators is better than the already existing estimators. From the above discussion, we recommend our proposed class of estimator will perform efficient in complete response case and also when there is a non-response present on both study as well as auxiliary variables.

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