Thailand Statistician
April 2025; 23(2): 407-419
http://statassoc.or.th
Contributed paper

Construction of Quality Regions Based on Consumer's and Producer's Risks for Two Sided Group Chain Sampling Plan with Binomial Distribution

Waqar Hafeez* [a], Nazrina Aziz [a, b], Javid Shabbir [c], Said Farooq Shah [d] and Akbar Ali Khan [e]
[a] School of Quantitative Sciences, University Utara Malaysia, Kedah Darul Aman, Malaysia.
[b] Institute of Strategic Industrial Decision Modelling (ISIDM), Universiti Utara Malaysia, Kedah, Malaysia.
[c] Department of Statistics, University of Wah, Punjab, Pakistan.
[d] Department of Statistics, University of Peshawar, Peshawar, Pakistan.
[e] Department of Statistics, Govt Postgraduate College Kohat, Karak Khyber Pakhtunkhwa, Pakistan.
*Corresponding author; e-mail: waqarhafeez78601@yahoo.com

Received: 24 May 2022
Revised: 20 November 2022
Accepted: 31 December 2022

Abstract

Acceptance sampling is a technique that is used to make decision about the lot under inspection. Based on representative sample the decision is made about the whole lot that is under inspection. Mostly existing plans consider consumer's risk and they do not care about producer loss. This study will consider consumer's risk as well as producer's risk and provide a criterion to satisfy both parties at the same time. A two sided group chain sampling plan (TSGChSP) is used in this paper. Based on both risks probability of lot acceptance, $L(p)$ is determined. Four different quality regions are estimated for specified values of producer's and consumer's risks. By satisfying the specified design parameters, it is assessed that as the value of design parameters increases, the proportion of defectives decreases such as g, r, i, j, β and $\alpha.$ In comparison TSGChSP is compared with existing Bayesian two sided group chain sampling plan (BTSGChSP). If both plans are applied under similar environments, then the results explain that the TSGChSP produces a lower proportion of defectives than the BTSGChSP. Hence, we suggest that TSGChSP is better substitute for lot inspection in the manufacturing industry, particularly for those working with destructive testing of high-quality products.

Keywords: Acceptance sampling, quality region, consumer's risk, producer's risk, two sided group chain.

1. Introduction

Acceptance sampling is a technique that falls somewhere between zero and full inspection. The process of inspecting a sample of items from a production lot is known as acceptance sampling. Meanwhile, in zero inspection, items are accepted without being inspected at all, whereas in 100 percent inspection, each item is inspected before being accepted. As a result, acceptance sampling was developed as a substitute for 100% inspection. This is because 100% inspection is impractical, especially when testing is destructive or costly. Acceptance sampling is used to help the manufacturer decide whether to accept or reject a batch, not to estimate or improve the batch's quality Montgomery (2009). Acceptance sampling may encourage manufacturers to improve quality, lowering the risk of batch rejection. In many industries, acceptance sampling is used to ensure the quality of raw materials and components, work-in-progress, and finished goods.

To track the evolution of acceptance sampling, several sampling plans that consider consumer's risk, producer's risk, and sample size can be developed. The producer's risk is defined as the probability of rejecting a good lot, while the consumer's risk is defined as the probability of accepting a bad lot. Sankar and Jeganathan (2019). The purpose of developing a sampling plan is to determine the smallest number of samples that will be inspected. To obtain the smallest sample size, many researchers have proposed various combinations of sampling plans with different distributions (Hafeez et al. 2022a, 2022b, Hafeez and Aziz 2019, and Dobbah et al. 2018). Hafeez (2022) developed a family of different group chain sampling plans with Bayesian and estimate quality regions by considering both risks at the same time.

Dodge and Romig (1941) proposed the single sampling plan (SSP), in which he considers only one item when making a decision about the lot under inspection. By addressing the weaknesses in SSP with zero acceptance number, Epstein (1954) extended SSP to the chain sampling plan (ChSP-1). Unlike SSP, ChSP-1 makes decisions based on cumulative sample results rather than just the current lot. ChSP-1 was developed by Ramaswamy and Jayasri (2014, 2015) using the generalized Rayleigh and Weibull distributions. If only one defective item is detected in the sample, and no further defective items are detected in subsequent lots, the current lot under inspection will be accepted. While compared to the SSP and ChSP-1 has been shown to have a higher probability of lot acceptance when the acceptance number is zero. However, the ChSP-1 only inspects one item at a time.

As a result, the group acceptance sampling plan (GASP) is proposed for simultaneously inspecting multiple items. GASP has been developed with different distributions by researchers like Aslam and Jun (2009), Aslam et al. (2011). Mughal (2018) later proposed a group chain sampling plan (GChSP) based on the GASP and ChSP-1 concepts. GChSP makes decisions based on cumulative results, and this plan can perform multiple inspections at once Teh et al. (2020). Mughal (2018) works to introduce a traditional two sided group chain sampling plan (TSGChSP). In TSGChSP the decision about the current lot is based on preceding as well as succeeding lots.

All of these plans calculate a single point at which the lot under inspection will be accepted or rejected. In this paper, first-time quality regions for TSGChSP will be estimated in order to satisfy both the consumer and the producer. Based on consumer's and producer's risks, these quality regions will provide a range of acceptable quality. Each quality region will be assigned two points: acceptable quality level (AQL) and limiting quality level (LQL). For all possible combinations of specified design parameters, four quality regions will be estimated namely, probabilistic quality region (PQR) denoted by R_1, quality decision region (QDR) denoted by R_2, limiting quality region (LQR) denoted by R_3 and indifference quality region (IQR) denoted by R_4.

1.1. Glossary of symbols

g : Number of groups

i : Number of preceding lots

j : Number of succeeding lots

r : Group size (available number of testers)

n : Sample size

d : Number of defective items

α : Producer's risk (Probability of rejecting a good lot)

β : Consumer's risk (Probability of accepting a bad lot)

$L(p)$: Probability of lot acceptance

T : Operating ratio between PQR and QDR

T_1 : Operating ratio between PQR and LQR

T_2 : Operating ratio between PQR and IQR

2. Methodology

2.1. Operating procedure

The operational procedure of TSGChSP is described below:

1. Select an ideal sample of size n and divide it into g groups. Such as each group contain r items and the required sample size $n = r * g$.

2. Start the inspection and count the number of defectives d, which is the sum of defective from current lot, preceding lot i and succeeding lot j.

3. In the current, preceding and succeeding sample if $d = 0$ in total, accept the lot.

4. In the current sample if $d > 1$, reject the lot.

5. Accept the lot, if $d = 0$ in the current sample and the preceding i and succeeding j samples have only one defective in total $d_i + d_j = 1$.

All the above steps can be summarized in the flow chart, presented in Figure 1. This procedure is illustrated through a tree diagram for $i = j = 1$ in Figure 2. The defective and non-defective products are denoted by D and $\overline{D}$, respectively. Based on Figure 2, when $i = j = 1$ the following outcomes meet the acceptance criteria $\left\{D\overline{D}\overline{D}, \overline{D}\overline{D}D, \overline{D}\overline{D}\overline{D}\right\}$.

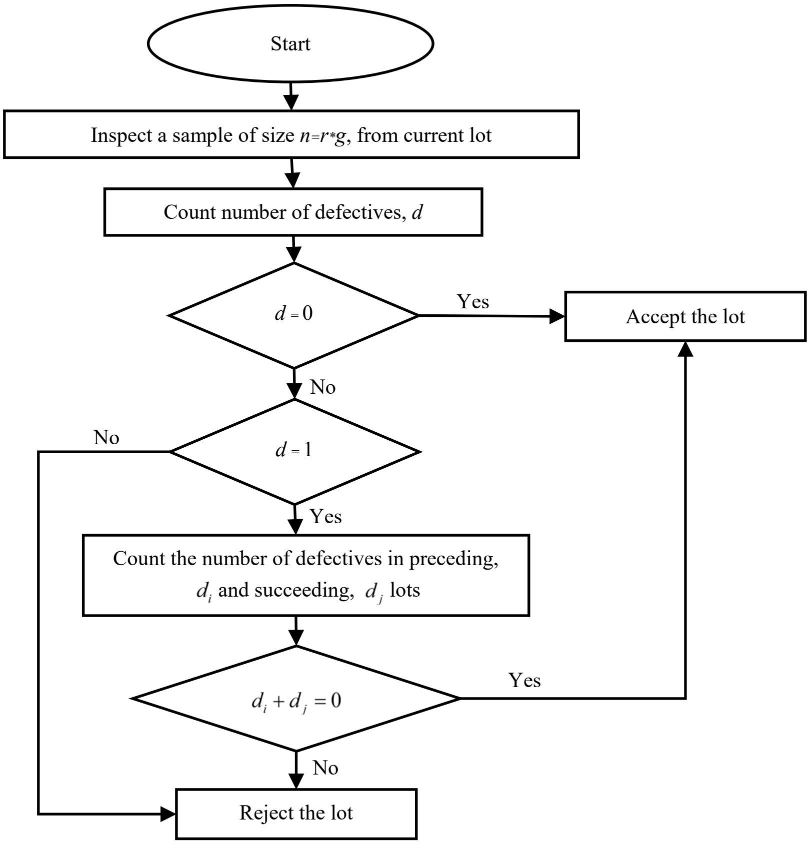

Figure 1 Operating procedure for TSGChSP

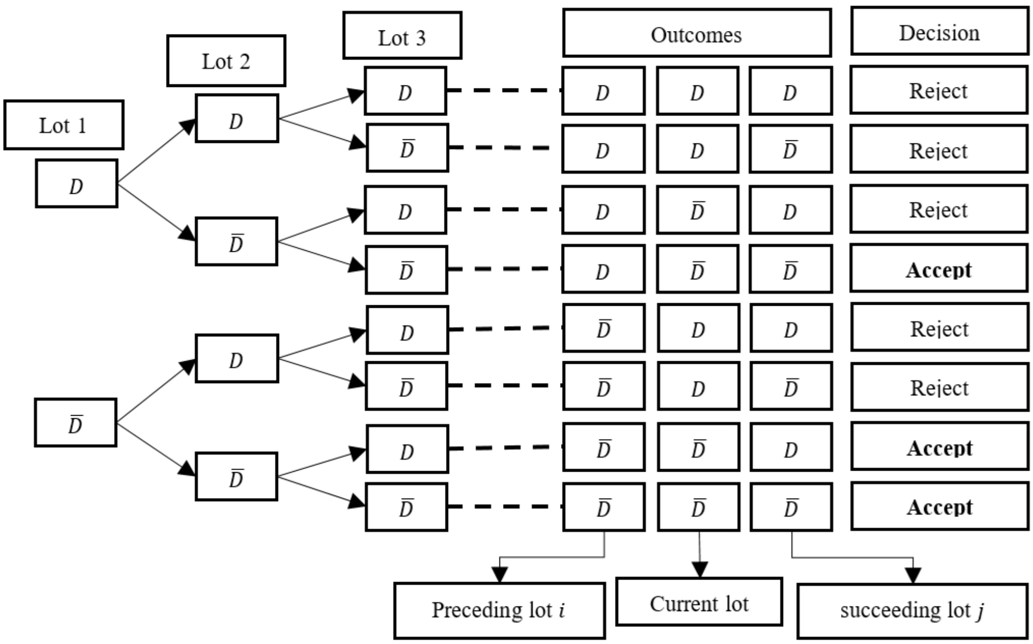

Figure 2 Tree diagram of TSGChSP for lot inspection when $i = j = 1$

2.2. Probability of defective and probability of lot acceptance

By using group chain probability of lot acceptance, the possible outcomes can be written in the form of an equation for TSGChSP. Where P_0 shows the probability of non-defective and P_1 shows the probability of defective product in Equations (1)-(3):

$$L(p)_{TSGChSP} = P_1P_0P_0 + P_0P_0P_1 + P_0P_0P_0 \tag{1}$$

$$L(p)_{TSGChSP} = P_1\left(P_0\right)^2 + P_1\left(P_0\right)^2 + \left(P_0\right)^3 \tag{2}$$

$$L(p)_{TSGChSP} = \left(P_0\right)^3 + 2P_1\left(P_0\right)^2. \tag{3}$$

Finally, the general expression is the OC function the general expression for $i = j = 1$ will be

$$L(p)_{TSGChSP} = \left(P_0\right)^{i+j+1} + (i+j)P_1\left(P_0\right)^{i+j}. \tag{4}$$

By considering binomial distribution in order to achieve the probability of lot acceptance for zero and one defective product. Here the binomial distribution is applicable because the product fulfills all four properties of the binomial experiment. This is applicable when a lot consists of identical and independent trails, the inspection outcomes are categorized into two mutually exclusive and independent outcomes. So, the probability of lot acceptance can be written as

$$L(p) = \sum_{c=0}^{1}\binom{r*g}{c}p^c\left(1-p\right)^{r*g-c}. \tag{5}$$

where p is the probability of defective. After simplifying (5) for zero and one defective product by replacing $c = 0$ and $c = 1$, we get their corresponding probabilities

$$P_0 = \left(1-p\right)^{r*g}. \tag{6}$$

$$P_1 = (r*g)p\left(1-p\right)^{r*g-1}. \tag{7}$$

For TSGChSP, the probability of lot acceptance after replacing (6) and (7) in (4) is (Mughal 2018),

$$L(p)_{TSGChSP} = \left(1-p\right)^{rg(i+j+1)} + rgp(i+j)\left(1-p\right)^{rg-1}\left(1-p\right)^{rg(i+j)}. \tag{8}$$

For TSGChSP the probability of lot acceptance based on the binomial distribution, we can rewrite (8) as the OC function of the binomial model under the condition that $i = j$ is

$$L(p)_{TSGChSP} = \left(1-p\right)^{rg(2i+1)} + 2irgp\left(1-p\right)^{rg(2i+1)-1}. \tag{9}$$

For specified design parameters, the fraction of defectives is estimated from (9) and presented in Table 1. Here Newton's approximation is used in (9), where p is used as a control point by reducing $L(p)_{TSGChSP}$.

Table 1 Certain p values in TSGChSP for specified g, r, i and $L(p)$

g	r	i	$L(p)$								
			0.99	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.01
1	2	1	0.0049	0.0237	0.0457	0.1071	0.2107	0.3419	0.4710	0.5478	0.6814
		2	0.0047	0.0202	0.0360	0.0766	0.1427	0.2293	0.3206	0.3793	0.4921
		3	0.0044	0.0170	0.0289	0.0586	0.1070	0.1715	0.2419	0.2885	0.3819
		4	0.0041	0.0144	0.0240	0.0473	0.0854	0.1369	0.1940	0.2324	0.3113
	3	1	0.0033	0.0158	0.0304	0.0715	0.1427	0.2375	0.3372	0.4007	0.5209
		2	0.0031	0.0134	0.0239	0.0510	0.0961	0.1569	0.2237	0.2681	0.3579
		3	0.0030	0.0113	0.0192	0.0391	0.0719	0.1166	0.1667	0.2007	0.2713
		4	0.0027	0.0096	0.0159	0.0315	0.0573	0.0926	0.1327	0.1602	0.2182
	4	1	0.0025	0.0118	0.0227	0.0537	0.1079	0.1818	0.2622	0.3149	0.4191
		2	0.0024	0.0100	0.0179	0.0383	0.0724	0.1192	0.1716	0.2071	0.2805
		3	0.0022	0.0084	0.0144	0.0293	0.0541	0.0883	0.1271	0.1538	0.2101
		4	0.0020	0.0072	0.0119	0.0236	0.0431	0.0700	0.1008	0.1222	0.1678
2	2	1	0.0025	0.0118	0.0227	0.0537	0.1079	0.1818	0.2622	0.3149	0.4191
		2	0.0024	0.0100	0.0179	0.0383	0.0724	0.1192	0.1716	0.2071	0.2805
		3	0.0022	0.0084	0.0144	0.0293	0.0541	0.0883	0.1271	0.1538	0.2101
		4	0.002	0.0072	0.0119	0.0236	0.0431	0.0700	0.1008	0.1222	0.1678
	3	1	0.0016	0.0079	0.0151	0.0358	0.0725	0.1238	0.1812	0.2201	0.3000
		2	0.0016	0.0067	0.0119	0.0255	0.0486	0.0805	0.1170	0.1422	0.1955
		3	0.0015	0.0056	0.0096	0.0195	0.0362	0.0594	0.0861	0.1047	0.1447
		4	0.0014	0.0048	0.0080	0.0158	0.0288	0.0470	0.0681	0.0829	0.1147
	4	1	0.0012	0.0059	0.0114	0.0269	0.0546	0.0938	0.1384	0.1690	0.2332
		2	0.0012	0.0050	0.0089	0.0191	0.0365	0.0608	0.0888	0.1082	0.1500
		3	0.0011	0.0042	0.0072	0.0146	0.0272	0.0448	0.0651	0.0794	0.1103
		4	0.0010	0.0036	0.0059	0.0118	0.0216	0.0354	0.0514	0.0627	0.0872
3	2	1	0.0016	0.0079	0.0151	0.0358	0.0725	0.1238	0.1812	0.2201	0.3000
		2	0.0016	0.0067	0.0119	0.0255	0.0486	0.0805	0.1170	0.1422	0.1955
		3	0.0015	0.0056	0.0096	0.0195	0.0362	0.0594	0.0861	0.1047	0.1447
		4	0.0014	0.0048	0.008	0.0158	0.0288	0.0470	0.0681	0.0829	0.1147
	3	1	0.0011	0.0053	0.0101	0.0239	0.0486	0.0836	0.1238	0.1514	0.2099
		2	0.0011	0.0044	0.0079	0.0170	0.0325	0.0541	0.0792	0.0967	0.1343
		3	0.0010	0.0037	0.0064	0.0130	0.0242	0.0399	0.0581	0.0708	0.0986
		4	0.0009	0.0032	0.0053	0.0105	0.0192	0.0315	0.0458	0.0559	0.0778
	4	1	0.0008	0.0039	0.0076	0.0179	0.0365	0.0631	0.0940	0.1154	0.1613
		2	0.0008	0.0033	0.0060	0.0128	0.0244	0.0408	0.0598	0.0732	0.1022
		3	0.0007	0.0028	0.0048	0.0098	0.0182	0.0300	0.0438	0.0535	0.0747
		4	0.0007	0.0024	0.0040	0.0079	0.0145	0.0237	0.0345	0.0421	0.0589
4	2	1	0.0012	0.0059	0.0114	0.0269	0.0546	0.0938	0.1384	0.1690	0.2332
		2	0.0012	0.0050	0.0089	0.0191	0.0365	0.0608	0.0888	0.1082	0.1500
		3	0.0011	0.0042	0.0072	0.0146	0.0272	0.0448	0.0651	0.0794	0.1103
		4	0.0010	0.0036	0.0059	0.0118	0.0216	0.0354	0.0514	0.0627	0.0872
	3	1	0.0008	0.0039	0.0076	0.0179	0.0365	0.0631	0.0940	0.1154	0.1613
		2	0.0008	0.0033	0.0060	0.0128	0.0244	0.0408	0.0598	0.0732	0.1022
		3	0.0007	0.0028	0.0048	0.0098	0.0182	0.0300	0.0438	0.0535	0.0747
		4	0.0007	0.0024	0.0040	0.0079	0.0145	0.0237	0.0345	0.0421	0.0589
	4	1	0.0006	0.0029	0.0057	0.0134	0.0275	0.0476	0.0711	0.0876	0.1231
		2	0.0006	0.0025	0.0045	0.0096	0.0183	0.0307	0.0452	0.0553	0.0776
		3	0.0005	0.0021	0.0036	0.0073	0.0136	0.0226	0.0330	0.0404	0.0565
		4	0.0005	0.0018	0.0030	0.0059	0.0108	0.0178	0.0259	0.0317	0.0445

2.3. Construction of quality region

For a sampling plan, a method of quality interval derived based on the range of quality instead of a point-wise description is called the quality region. This method can be adopted in the elementary production process, where the stipulated quality level is advised to be fixed at a later stage and provides a new concept for designing sampling plans through quality levels. For the product acceptance to meet the present product quality requirements, the sampling plans provide the decision rules for producer and consumer. Suppliers require high-quality products with a very low fraction of defectives due to rapid advancements in manufacturing technology. Unfortunately, in some situations, the traditional method may fail to discover minute defectives among the products. Quality interval sampling plans are introduced to overcome such problems. This idea provides a higher probability of acceptance, which depends on the quality measure of the AQL and LQL (Suresh and Divya 2009).

A quality region is based on two points AQL and LQL. Resembling the conventional OC curve, the AQL refers to the producer's risk α and LQL refers to the consumer's risk β, which needs to be minimized. The interval between these two points, AQL and LQL, is called the quality region (QR).

2.3.1 Probabilistic Quality Region (PQR)

In this interval, the product is accepted in PQR with the highest probability of 0.95 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL (β). It is clear that, PQR (R_1) is exactly the standard setting of AQL (p_1) and LQL $\left(p_2\right)$. In Figure 3, it is indicated that PQR lies between two points $p_1 \leq R_1 \leq p_2$. From the specified design parameters, this region considers $\alpha = \beta = 0.05$ and the range of PQR is based on $R_1 = p_2 - p_1$, where the values of R_1 are given in Table 2.

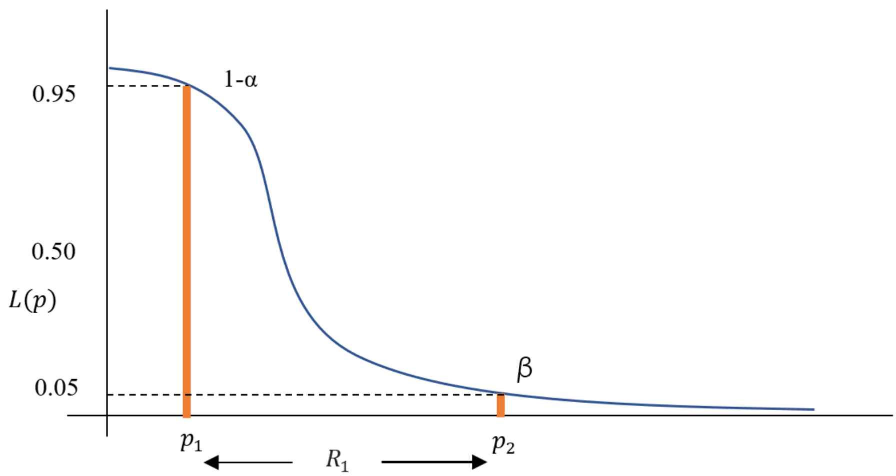

Figure 3 Probabilistic Quality Region (PQR)

2.3.2 Quality Decision Region (QDR)

In this interval, the product is accepted in QDR with the highest probability of 0.95 and the lowest probability of 0.25. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL (β). It is clear that, QDR (R_2) is exactly the standard setting of AQL (p_1) and LQL (p_β). It is also presented in Figure 4, that QDR lies between $p_1 \le R_2 \le p_\beta$. This region considers consumer's risk $\alpha = 0.05$ and producer's risk $\beta = 0.25$. In Table 4, the range of QDR is $R_2 = p_\beta - p_1$ given.

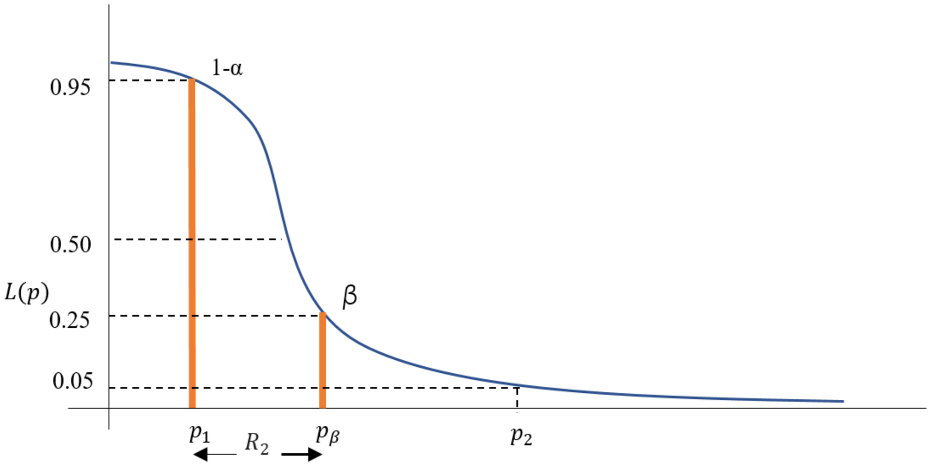

Figure 4 Quality Decision Region (QDR)

2.3.3 Limiting Quality Region (LQR)

In this interval, the product is accepted in LQR with the highest probability of 0.75 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL (β). It is clear that, LQR (R_3) is exactly the standard setting of AQL (p_α) and LQL (p_2). It is shown in Figure 5 that LQR lies between $p_\alpha \le R_3 \le p_2$ points on the x-axis. This region considers $\alpha = 0.25$ and $\beta = 0.05$. Thus, the range of LQR is $R_3 = p_2 - p_\alpha$ presented in Table 2.

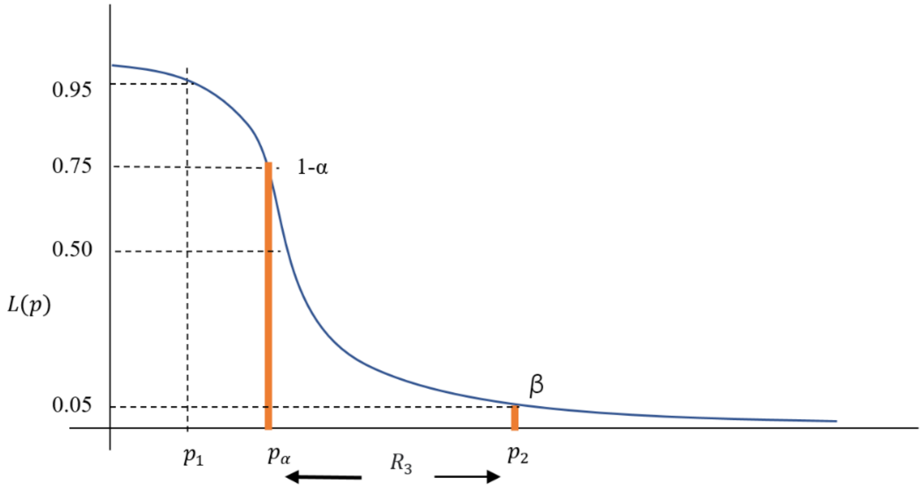

Figure 5 Limiting Quality Region (LQR)

2.3.4 Indifference Quality Region (IQR)

In this interval, the product is accepted in IQR with the highest probability of 0.5 and the lowest probability of 0.05. These probabilities are corresponding to AQL $(1-\alpha)$ and LQL (β). It is clear that, IQR (R_4) is exactly the standard setting of AQL (p_*) and LQL (p_2). IQR lies between two points $p_* \le R_4 \le p_2$, that are highlighted on x-axes in Figure 6. This region considers producer's risk $\alpha = 0.5$ and consumer's risk $\beta = 0.05$, also estimated values of the range of IQR $R_4 = p_2 - p_*$ are given in Table 2.

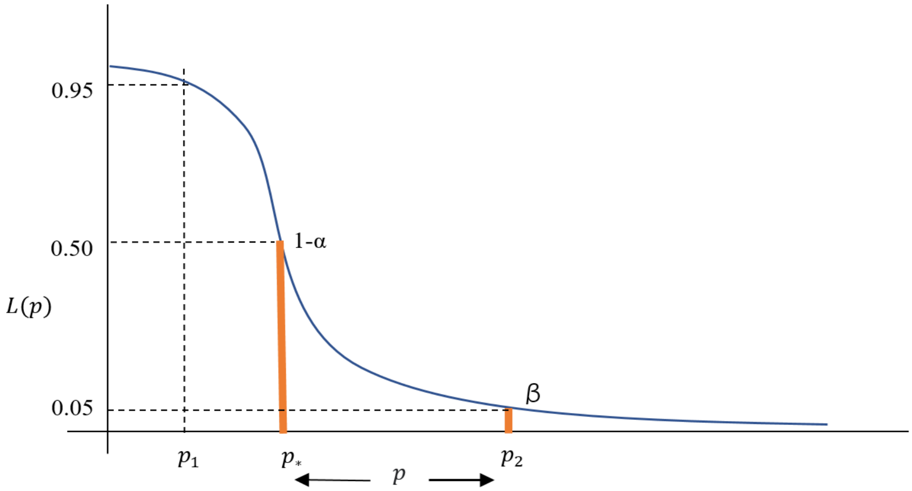

Figure 6 Indifference Quality Region (IQR)

2.4. Selection of Sampling Plan for GChSP

For any given value of PQR (R_1), QDR (R_2), LQR (R_3), IQR (R_4), we can find the operating ratio $T=\frac{R_1}{R_2}, T_1=\frac{R_1}{R_3}$ and $T_2=\frac{R_1}{R_4}$. Find the value which is approximately equal to the required ratio under the column of T, T_1 and T_2 in Table 2. From this operating ratio the corresponding design parameters g, r and i can be determined in Table 2. By using these design parameters for TSGChSP, the values of quality regions can be assessed from Table 2 and required AQL and LQL can be obtained from Table 1.

Table 2 For specified values of design parameters, the values of quality regions and operating ratios for TSGChSP

g	r	i	R_1	R_2	R_3	R_4	T	T_1	T_2
1	2	1	0.5240	0.3182	0.4407	0.3371	1.6470	1.1891	1.5547
		2	0.3591	0.2091	0.3027	0.2366	1.7174	1.1863	1.5180
		3	0.2715	0.1546	0.2299	0.1815	1.7566	1.1812	1.4961
		4	0.2180	0.1225	0.1851	0.1470	1.7801	1.1776	1.4829
	3	1	0.3849	0.2217	0.3292	0.2580	1.7360	1.1691	1.4919
		2	0.2547	0.1435	0.2171	0.1720	1.7748	1.1735	1.4808
		3	0.1894	0.1053	0.1617	0.1289	1.7988	1.1718	1.4701
		4	0.1506	0.0830	0.1287	0.1030	1.8140	1.1704	1.4629
	4	1	0.3031	0.1700	0.2613	0.2070	1.7831	1.1600	1.4639
		2	0.1971	0.1092	0.1688	0.1346	1.8044	1.1673	1.4637
		3	0.1454	0.0799	0.1245	0.0997	1.8203	1.1679	1.4585
		4	0.1150	0.0628	0.0986	0.0791	1.8317	1.1670	1.4536
2	2	1	0.3031	0.1700	0.2613	0.207	1.7831	1.1600	1.4639
		2	0.1971	0.1092	0.1688	0.1346	1.8044	1.1673	1.4637
		3	0.1454	0.0799	0.1245	0.0997	1.8203	1.1679	1.4585
		4	0.1150	0.0628	0.0986	0.0791	1.8317	1.1670	1.4536
	3	1	0.2122	0.1159	0.1843	0.1476	1.8316	1.1514	1.4378
		2	0.1355	0.0739	0.1167	0.0936	1.8348	1.1616	1.4475
		3	0.0991	0.0538	0.0852	0.0686	1.8438	1.1631	1.4458
		4	0.0781	0.0423	0.0671	0.054	1.8477	1.1641	1.4457
	4	1	0.1631	0.0878	0.1421	0.1144	1.857	1.1475	1.4255
		2	0.1032	0.0558	0.0891	0.0717	1.8511	1.1588	1.4391
		3	0.0752	0.0406	0.0648	0.0522	1.8535	1.1616	1.4409
		4	0.0591	0.0319	0.0508	0.0410	1.8552	1.1625	1.4404
3	2	1	0.2122	0.1159	0.1843	0.1476	1.8316	1.1514	1.4378
		2	0.1355	0.0739	0.1167	0.0936	1.8348	1.1616	1.4475
		3	0.0991	0.0538	0.0852	0.0686	1.8438	1.1631	1.4458
		4	0.0781	0.0423	0.0671	0.0540	1.8477	1.1641	1.4457
	3	1	0.1462	0.0784	0.1276	0.1028	1.8656	1.1460	1.4216
		2	0.0923	0.0497	0.0797	0.0642	1.8560	1.1578	1.4366
		3	0.0671	0.0361	0.0578	0.0466	1.8561	1.1609	1.4395
		4	0.0527	0.0283	0.0453	0.0366	1.8608	1.1620	1.4382
	4	1	0.1114	0.0592	0.0975	0.0788	1.8829	1.1433	1.4134
		2	0.0699	0.0374	0.0605	0.0488	1.8676	1.1560	1.4315
		3	0.0507	0.0272	0.0437	0.0353	1.8650	1.1604	1.4368
		4	0.0397	0.0213	0.0343	0.0277	1.8656	1.1604	1.4361

Table 2 (Continued)

g	r	i	R_1	R_2	R_3	R_4	T	T_1	T_2
4	2	1	0.1631	0.0878	0.1421	0.1144	1.8570	1.1475	1.4255
		2	0.1032	0.0558	0.0891	0.0717	1.8511	1.1588	1.4391
		3	0.0752	0.0406	0.0648	0.0522	1.8535	1.1616	1.4409
		4	0.0591	0.0319	0.0508	0.0410	1.8552	1.1625	1.4404
	3	1	0.1114	0.0592	0.0975	0.0788	1.8829	1.1433	1.4134
		2	0.0699	0.0374	0.0605	0.0488	1.8676	1.1560	1.4315
		3	0.0507	0.0272	0.0437	0.0353	1.8650	1.1604	1.4368
		4	0.0397	0.0213	0.0343	0.0277	1.8656	1.1604	1.4361
	4	1	0.0847	0.0447	0.0741	0.0601	1.8942	1.1417	1.4077
		2	0.0528	0.0282	0.0457	0.0370	1.8753	1.1544	1.4280
		3	0.0383	0.0205	0.0330	0.0267	1.8686	1.1583	1.4306
		4	0.0300	0.0160	0.0258	0.0209	1.8680	1.1598	1.4323

3. Results

3.1. Numerical examples

For specified PQR and QDR:

Assume a manufacturer produces defectives in the PQR and QDR regions 0.18% and QDR 0.10%, respectively, these values are based on the manufacturer's requirement he can change these values according to his need. Then, $R_1 = 0.0018, R_2 = 0.001$ and the determined operating ratio is 1.8. In Table 2, find a T value that is approximately equal to the specified ratio, which is found to be $T = 1.7988,$ with corresponding design parameters $g = 1, r = 3$ and $i = 3.$ For this operating ratio, the ranges of PQR and QDR are $R_1 = 0.1894$ and $R_2 = 0.1053$ respectively in Table 2. Hence the required design parameters for TSGChSP are found $g = 1, r = 3$ and $i = 3,$ with $p_1 = 0.0113, p_\beta = 0.1166$ and $p_2 = 0.2007$ from Table 1.

For specified PQR and LQR:

Assume a manufacturer produces defectives in the PQR and LQR regions 0.20% and LQR 0.17%, respectively. Then $R_1 = 0.0020, R_3 = 0.0017,$ and the determined operating ratio is 1.1765. In Table 2, for the specified ratio, the value is found to be $T_1 = 1.1776,$ with parallel design parameters $g = 1, r = 2$ and $i = 4.$ Therefore, for this operating ratio, the ranges of PQR and LQR are $R_1 = 0.218$ and $R_3 = 0.1851,$ respectively from Table 2. Hence for TSGChSP required design parameters are found to be $g = 1, r = 2$ and $i = 4,$ with $p_1 = 0.0144, p_\alpha = 0.0473,$ and $p_2 = 0.2324$ from Table 1.

For specified PQR and IQR:

Assume a manufacturer produces defectives in the PQR and IQR regions of 0.15% and 0.10%, respectively. Then $R_1 = 0.0015, R_4 = 0.0010$ and the determined operating ratio is 1.5. For the specified ratio, the value is found to be $T_2 = 1.518$ with parallel design parameters $g = 1, r = 2$ and $i = 2,$ in Table 2. Therefore, the ranges of PQR and IQR for this operating ratio are $R_1 = 0.3591$ and $R_4 = 0.2366$ respectively from Table 2. For TSGChSP the parameters $g = 1, r = 2$ and $i = 2$ are required, with $p_1 = 0.0202, p_* = 0.1427$ and $p_2 = 0.3793$ from Table 1.

3.2. Performances comparison

In this section for the specified values of design parameters, the probability of defective by all four quality regions is compared with BTSGChSP suggested by Hafeez and Aziz (2022).

Table 3 Comparison between quality regions for TSGChSP and BTSGChSP for $g = 2, r = 3$ and $i = 4$

Quality region	BTSGChSP			TSGChSP
	$s = 1$	$s = 2$	$s = 3$	
R_1	0.4000	0.1902	0.1424	0.0781
R_2	0.0974	0.0647	0.0562	0.0423
R_3	0.3878	0.1787	0.1310	0.0671
R_4	0.3642	0.1609	0.1149	0.0540

Table 4 Comparison between quality regions for TSGChSP and BTSGChSP for $g = 4, r = 3$ and $i = 2$

Quality region	BTSGChSP			TSGChSP
	$s = 1$	$s = 2$	$s = 3$	
R_1	0.3641	0.1687	0.1261	0.0699
R_2	0.0846	0.0564	0.0492	0.0374
R_3	0.3537	0.1589	0.1165	0.0605
R_4	0.3334	0.1434	0.1024	0.0488

From Tables 3 and 4, it can be observed that for all quality regions, TSGChSP gives the lowest probability of defective than BTSGChSP. From Table 2, it can be observed that as the value of g and r increases, the range of quality regions decreases. Also, for TSGChSP as the value of i and j increases the range of quality regions increases for PQR, QDR, LQR and IQR. From these results, we conclude that if in industry TSGChSP and BTSGChSP are used on the same product for inspection under the same conditions. Then TSGChSP will accept fewer defective products than the BTSGChSP for the same values of design parameters.

4. Conclusion

The probability of lot acceptance and quality regions for TSGChSP are calculated using binomial distributions in this paper. For both producer and consumer, these quality regions provide an acceptable range of quality. The results show that as the value of design parameters such as g, r, i and j increases, the proportion of defective product decreases. TSGChSP has a lower number of defects than BTSGChSP while still having a higher chance of lot acceptance, according to the comparison. We believe TSGChSP with these quality regions has the potential to reduce inspection, operating costs, and the risk of item damage due to mishandling. Finally, we believe that TSGChSP is a better alternative for manufacturers to use in their manufacturing processes.

Acknowledgments

All authors are grateful to the reviewers for their valuable suggestions, which helped to improve the manuscript.

References

Aslam M, Jun CH. A group acceptance sampling plan for truncated life test having Weibull distribution. J Appl Stat. 2009; 36(9): 1021-1027.

Aslam M, Jun CH, Lio YL, Ahmad M. Group acceptance sampling plans for the generalized Rayleigh distribution. Int J Intell Technol Appl Stat. 2011; 4(3): 355-365.

Dodge HF, Romig HG. Single sampling and double sampling inspection tables. Bell Syst Tech J. 1941; 20(1): 1-61.

Dobbah SA, Aslam M, Khan K. Design of a new synthetic acceptance sampling plan. Symmetry (Basel). 2018; 10(11): 653.

Epstein B. Truncated life tests in the exponential case. Ann Math Stat. 1954; 25(3): 555-564.

Hafeez W. Developing a family of Bayesian group chain sampling plans for quality regions [dissertation]. Sintok, Malaysia: Universiti Utara Malaysia; 2022.

Hafeez W, Aziz N. Bayesian group chain sampling plan based on beta binomial distribution through quality region. Int J Supply Chain Manag. 2019; 8(6): 1175-1180.

Hafeez W, Aziz N. Bayesian two-sided group chain sampling plan for beta binomial distribution under quality regions. Int J Qual Reliab Manag. 2022; 39(10): 2424-2437.

Hafeez W, Aziz N, Zain Z, Kamarudin NA. Bayesian group chain sampling plan for Poisson distribution with gamma prior. Comput Mater Contin. 2022; 70(2): 3891-3902.

Hafeez W, Aziz N, Zain Z, Kamarudin NA. Designing Bayesian new group chain sampling plan for quality regions. Comput Mater Contin. 2022; 70(2): 4185-4198.

Montgomery DC. Introduction to Statistical Quality Control. 6th ed. New York: John Wiley & Sons; 2009.

Mughal AR. A family of group chain acceptance sampling plans based on truncated life test [dissertation]. Sintok, Malaysia: Universiti Utara Malaysia; 2018.

Ramaswamy ARS, Jayasri S. Time truncated chain sampling plans for generalized Rayleigh distribution. Int Ref J Eng Sci. 2014; 3(2): 49-53.

Ramaswamy ARS, Jayasri S. Time truncated modified chain sampling plan for selected distributions. Int J Res Eng Sci. 2015; 3(3): 1-18.

Sankar SR, Jeganathan M. Comparison of double sampling plan with single sampling plan in supply chain management system - A simulation study. Int J Adv Sci Res Manag. 2019; 4(5): 34-39.

Suresh KK, Divya PR. Selection of single sampling plan through decision region. Int J Appl Math Stat. 2009; 14(9): 66-78.

Teh MAP, Aziz N, Zain Z. A new method in designing group chain acceptance sampling plans (GChSP) for generalized exponential distribution. Int J Qual Reliab Manag. 2020; 38(5): 1116-1129.