Thailand Statistician
April 2025; 23(2): 258-268
http://statassoc.or.th
Contributed paper

Mean Estimation in Presence of Measurement Errors Using Log Type Estimators

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Received: 3 March 2022
Revised: 23 September 2022
Accepted: 30 September 2022

Abstract

This article introduces some log type class of mean estimators in the case of measurement errors (ME) using simple random sampling (SRS). The mean square error of the proposed estimators is obtained when data on both the study and auxiliary variables are commingled with ME. The performance of the proposed estimators is compared with the existing estimators and the efficiency conditions are derived. Further, the performance of the proposed estimators is illustrated through numerical and simulation studies using some real and artificially generated populations. The results of numerical and simulation studies show that the proposed estimators dominate the usual mean estimator, classical ratio and product estimators.

Keywords: Mean square error, efficiency, simulation study.

1. Introduction

In survey sampling, it is well-known that the consideration of auxiliary information helps in improving the efficiency of estimation procedures. These estimation procedures include ratio, product, regression, exponential and logarithmic methods that consider information on an auxiliary variable. The ratio method of estimation provides a better estimate when the study and auxiliary variables are positively correlated, see Cochran (1940) and Bhushan and Kumar (2022a). The product method of estimation is best suited when study and auxiliary variables are negatively correlated, see Murthy (1964). The regression method of estimation is the most efficient procedure provided that the regression line passes through the origin, see Cochran (1977). The exponential estimators perform better when the exponential functions model a relationship in which a constant change in the independent variable gives the same proportional change in the dependent variable. Zaman (2021) considered an efficient exponential estimator of the mean under stratified random sampling. Zaman and Kadilar (2021a) suggested exponential ratio and product type estimators of population mean in stratified two-phase sampling, whereas Zaman and Kadilar (2021b) developed a new class of exponential estimators for finite population mean in two-phase sampling. The logarithmic estimators would work in situations when the study variable is logarithmically related to the auxiliary variable. Cekim and Kadilar (2020a, b) suggested ln-type variance estimators under SRS and stratified random sampling.

Bhushan et al. (2021) and Bhushan and Kumar (2022b) suggested some classes of log type estimators of population mean under ranked set sampling. Bhushan et al. (2022a) considered logarithmic type predictive estimators under SRS, whereas Bhushan et al. (2022b) developed some efficient logarithmic type imputation methods in the presence of missing data.

In sampling surveys, it is assumed that the data collected on the study variable y and the auxiliary variable x are the actual recorded values of observation. However, in practicality, the observation under study may be recorded with some errors known as ME. The ME is defined as the discrepancy between the observed and the actual values of the parameters. The impact of ME in survey sampling was studied by Cochran (1968) and Murthy (1967). Fuller (1987) also examined the effects of ME models in his text book. Cheng and Van Ness (1994) suggested the estimation of linear relationships when the study and auxiliary variables are recorded with ME. Carroll et al. (2006) studied the impact of ME in non-linear models. Various estimation procedures such as ratio, product and regression for estimating different parameters have been proposed by many prominent authors including Shalabh (1997), Manisha and Singh (2001), Allen et al. (2003), Sahoo et al. (2006), Kumar et al. (2011) to deal with the issue of ME. This article aims to propose some log type estimators of the population mean of study variable in the presence of ME. The impacts of ME on the performance of the proposed and existing estimators have been studied.

Consider a finite population of size N from which a sample of size n is drawn using simple random sampling without replacement. We consider the situation where data values may be recorded with ME. Let the observed values be denoted by (y_i, z_i); $i = 1, 2, ..., n$ and the true values be denoted by (Y_i, Z_i). Let the observed values be expressible in additive forms as $y_i = Y_i + U_i$ and $z_i = Z_i + V_i$ such that $U \sim N(0, \sigma_U^2)$ and $V \sim N(0, \sigma_V^2)$. It is assumed that the error variables U and V are uncorrelated to each other as well as uncorrelated to other combinations of X and Y, respectively. Let μ_Y, μ_Z be the population means and σ_Y^2, σ_Z^2 be the population variance of study and auxiliary variables, respectively. In the presence of ME $s_z^2 = (n-1)^{-1}\sum_{i=1}^{n}(z_i - \bar{z})^2$ and $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variance σ_Z^2 and σ_Y^2, respectively. Therefore, the expected values of s_z^2 and s_y^2 in the presence of ME are given by $E(s_z^2) = \sigma_Z^2 + \sigma_V^2$ and $E(s_y^2) = \sigma_Y^2 + \sigma_U^2$, respectively.

To find the properties of the proposed estimators in the presence of ME, we assume that $\bar{y} = \mu_Y(1 + e_0)$ and $\bar{z} = \mu_Z(1 + e_1)$ such that $E(e_0) = 0,\ E(e_1) = 0,\ E(e_0^2) = C_Y^2/n\phi_y,\ E(e_1^2) = C_Z^2/n\phi_z$ and $E(e_1 e_0) = \rho C_Y C_Z/n$.
where $C_Y = S_y/\mu_Y$ and $C_Z = S_Z/\mu_Z$ are the population coefficient of variations for study and auxiliary variables, respectively, ρ is the population correlation coefficient between study and auxiliary variables. Also, $\phi_y = \sigma_Y^2/(\sigma_Y^2 + \sigma_U^2)$ and $\phi_z = \sigma_Z^2/(\sigma_Z^2 + \sigma_V^2)$ are the reliability ratio of the study and auxiliary variables that lies between 0 and 1.

The variance of usual mean estimator $\bar{y}$ in the presence of ME is given by

$$Var(\bar{y}) = \frac{\sigma_Y^2}{n} + \frac{\sigma_U^2}{n} \tag{1}$$

Shalabh (1997) developed the conventional ratio and product estimators in the case of ME using SRS as

$$t_r = \bar{y}\left(\frac{\mu_Z}{\bar{z}}\right) \tag{2}$$

$$t_p = \bar{y}\left(\frac{\bar{z}}{\mu_Z}\right) \tag{3}$$

The MSE of the estimators t_r and t_p is given by

$$MSE(t_r) = \mu_Y^2\left[\frac{C_Y^2}{n} + \frac{C_Z^2}{n} - \frac{2\rho C_Z C_Y}{n}\right] + \mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_Z^2}{n}\frac{\sigma_V^2}{\sigma_Z^2}\right] \tag{4}$$

$$MSE(t_p) = \mu_Y^2\left[\frac{C_Y^2}{n} + \frac{C_Z^2}{n} + \frac{2\rho C_Z C_Y}{n}\right] + \mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_Z^2}{n}\frac{\sigma_V^2}{\sigma_Z^2}\right] \tag{5}$$

where the first term in the expressions of the $MSE(t_r)$ and $MSE(t_p)$ represent the MSE of t_r and t_p estimator without ME whereas the last terms of the expressions of $MSE(t_r)$ and $MSE(t_p)$ represent the contribution of ME.

The article is designed in the following sections. Section 2 considers the proposed log type estimators of population mean along with their properties in the presence of ME. A comparative study is performed between the proposed and existing estimators in Section 3. The numerical and simulation studies are carried out in Section 4 and Section 5, respectively. The discussion of the results and concluding remarks are given Section 6.

2. Proposed Estimators

Motivated by the works of Bhushan and Kumar (2020, 2022c), we propose some log type estimators in the case of ME using SRS as

$$t_1 = \bar{y}\left[1 + \log\left(\frac{\bar{z}}{\mu_Z}\right)\right]^{\delta_1} \tag{6}$$

$$t_2 = \bar{y}\left[1 + \delta_2 \log\left(\frac{\bar{z}}{\mu_Z}\right)\right] \tag{7}$$

where $\delta_i,\ i = 1, 2$ are suitably chosen scalars to optimize the MSE.

Theorem 1 *The minimum MSE of the proposed estimators* $t_i,\ i = 1, 2$ *is given by*

$$minMSE(t_i) = \frac{\mu_Y^2}{n}\frac{C_Y^2}{\phi_y}\left[1 - \rho^2\phi_z\phi_y\right]. \tag{8}$$

Proof: Using the notations defined in the preceding section, we express the proposed estimators $t_i,\ i = 1, 2$ as

$$t_1 - \mu_Y = \mu_Y\left\{e_0 + \delta_1 e_1 + \left(\frac{\delta_1^2}{2} - \delta_1\right)e_1^2 + \delta_1 e_0 e_1\right\} \tag{9}$$

$$t_2 - \mu_Y = \mu_Y\left(e_0 + \delta_1 e_1 - \frac{\delta_1}{2}e_1^2 + \delta_1 e_0 e_1\right) \tag{10}$$

Squaring and taking expectations on both sides of (9) and (10), we get the $MSE(t_i),\ i = 1, 2$ to the first order of approximation as

$$MSE(t_i) = \frac{\mu_Y^2}{n}\left[\frac{C_Y^2}{\phi_y} + \delta_i^2\frac{C_Z^2}{\phi_z} + 2\delta_i\rho C_Y C_Z\right] \tag{11}$$

Differentiating (11) with respect to $\delta_i,\ i = 1, 2$ and equating to zero, we get the optimum values of δ_i as

$$\delta_{i(opt)} = -\rho\phi_z\frac{C_Y}{C_Z} \tag{12}$$

Now, putting the value of $\delta_{i(opt)}$ in (11), we get the minimum MSE of the proposed estimators $t_i,\ i = 1, 2$ as

$$minMSE(t_i) = \frac{\mu_Y^2}{n}\frac{C_Y^2}{\phi_y}\left[1 - \rho^2\phi_z\phi_y\right] \tag{13}$$

3. Comparative Study

This section presents the comparative study of the proposed estimators regarding the existing estimators. We compare the minimum MSE of the proposed estimators from (13) with the:

(i) usual mean estimator $\bar{y}$, we get

$$\rho^2\phi_y\phi_z > 1 - \frac{1}{n\mu_Y^2} \tag{14}$$

(ii) classical ratio estimator t_r, we get

$$\rho^2 > \frac{2\rho C_Z}{\phi_z C_Y} - \frac{C_Z^2}{\phi_z^2 C_Y^2} \tag{15}$$

(iii) classical product estimator t_p, we get

$$\rho^2 > -\frac{2\rho C_Z}{\phi_z C_Y} - \frac{C_Z^2}{\phi_z^2 C_Y^2} \tag{16}$$

Under the above conditions, the proposed estimators will dominate the existing estimators. These conditions are further assessed in next sections through numerical and simulation studies.

4. Numerical Study

This section exemplifies the performance of the proposed class of estimators using two real populations. Population 1 is taken from Gujarati and Sangeetha (2007) where consumption expenditure is denoted by the study variable and income is denoted by the auxiliary variable. Population 2 is taken from the book of U.S. Census Bureau 1986, where the product sold is denoted by the study variable and the size of the farms is denoted by the auxiliary variable. The descriptive statistics of these populations are given in Table 1 for ready reference.

Table 1 Descriptive statistics of real populations

Descriptive statistics	N	n	μ_Z	μ_Y	σ_Z^2	σ_Y^2	ρ	σ_U^2	σ_V^2
Population 1	10	4	170	127	3300	1278	0.964	36	36
Population 2	56	15	75.79	61.59	155.5	577.44	-0.508	16	16

Based on the above populations, we have calculated percent relative efficiency (PRE) of the classical ratio estimator t_r, the product estimator t_p and the proposed estimators $t_i,\ i = 1, 2$ regarding the usual mean estimator $\bar{y}$ using the following formula.

$$PRE = \frac{MSE(\bar{y})}{MSE(T)} \times 100 \tag{17}$$

where $T = t_r, t_p$ and $t_i,\ i = 1, 2$. The results of the numerical study for these populations are reported in Table 2.

Table 2 PRE of different estimators based on real populations

Estimators	PRE without ME	PRE with ME
Population 1		
t_r	789.79	664.25
t_p	21.02	21.49
$t_i,\ i=1,2$	**1414.34**	**944.12**
Population 2		
t_r	62.25	62.89
t_p	133.44	129.18
$t_i,\ i=1,2$	**134.78**	**129.48**

5. Simulation Study

To generalize the numerical exemplification carried out in the preceding section, we accomplished a simulation study over a normal population of size N=800. The population is generated artificially with R software by using a multivariate normal distribution based on mean vector $(\mu_Y, \mu_Z, 0, 0)$ and covariance matrix

$$\begin{pmatrix} \sigma_Y^2 & \rho\sigma_Z\sigma_Y & 0 & 0 \\ \rho\sigma_Z\sigma_Y & \sigma_Z^2 & 0 & 0 \\ 0 & 0 & \sigma_U^2 & 0 \\ 0 & 0 & 0 & \sigma_V^2 \end{pmatrix}$$

such that $\mu_Y = 40$, $\mu_Z = 30$, $\sigma_Z^2 = (20, 25)$, $\sigma_Y^2 = (20, 25)$, $\sigma_U^2 = (2, 4)$, $\sigma_V^2 = (2, 4)$, $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$. Based on 15000 iterations, we have computed the PRE of the classical ratio estimator, the classical product estimator and the proposed log type class of estimators regarding the unbiased estimator for the above population by using the following formula.

$$PRE = \frac{\frac{1}{15{,}000}\sum_{i=1}^{15{,}000}(\bar{y} - \bar{Y})^2}{\frac{1}{15{,}000}\sum_{i=1}^{15{,}000}(T - \bar{Y})^2} \times 100 \tag{18}$$

The simulation study is conducted in the following steps.

(i) Normal population of size N=800 is generated using multivariate normal distribution with R software.

(ii) Random samples of sizes $n = 50$ and $n = 100$ are drawn from the generated population.

(iii) The required descriptive statistics are computed for both samples.

(iv) The PRE of different estimators for the parameters σ_Z^2, σ_Y^2, σ_U^2, σ_V^2 and ρ is calculated by using (18) and the results are reported from Table 3 to Table 6.

(v) The PRE is also calculated for different amounts of ME such as $10\%, 20\%, 30\%, 40\%$ by using (18) and the results are given from Table 7 to Table 10.

Table 3 PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_Y^2 = 20$

			n=50			n=100		
σ_U^2	σ_V^2	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
2	2	-0.9	20.832	189.486	322.18	20.835	189.837	320.535
		-0.5	24.363	63.432	129.748	24.381	63.572	128.768
		-0.1	32.189	39.244	102.764	32.208	39.304	101.9
		0.1	40.928	33.509	102.764	40.977	33.518	101.9
		0.5	63.738	24.451	129.885	63.811	24.45	128.851
		0.9	171.183	19.788	322.671	171.488	19.803	320.782
2	4	-0.9	20.199	147.439	272.735	20.203	147.726	271.366
		-0.5	23.441	57.543	126.737	23.459	57.664	125.863
		-0.1	30.601	36.91	102.532	30.62	36.965	101.743
		0.1	38.543	31.893	102.532	38.587	31.902	101.743
		0.5	57.889	23.539	126.899	57.953	23.539	125.973
		0.9	134.401	19.181	274.205	134.645	19.196	272.595
4	2	-0.9	22.312	177.501	273.167	22.312	177.769	272.116
		-0.5	26.031	65.659	126.756	26.049	65.795	125.888
		-0.1	34.179	41.431	102.54	34.198	41.49	101.747
		0.1	43.143	35.539	102.54	43.191	35.546	101.747
		0.5	65.99	26.145	126.832	66.055	26.142	125.922
		0.9	162.402	21.253	272.267	162.639	21.265	271.14
4	4	-0.9	21.558	142.111	239.972	21.563	142.355	238.99
		-0.5	24.912	59.491	124.177	24.931	59.61	123.402
		-0.1	32.338	38.804	102.332	32.357	38.86	101.606
		0.1	40.467	33.667	102.332	40.513	33.677	101.606
		0.5	59.854	25.031	124.292	59.919	25.032	123.471
		0.9	130.752	20.527	240.255	130.967	20.542	239.139

Table 4 PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_Y^2 = 25$

			n=50			n=100		
σ_U^2	σ_V^2	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
2	2	-0.9	21.909	213.776	329.245	21.927	214.68	327.555
		-0.5	27.439	74.041	130.235	27.462	74.202	129.215
		-0.1	36.924	44.512	102.698	36.968	44.573	101.79
		0.1	46.054	38.156	102.698	46.095	38.189	101.79
		0.5	74.593	27.476	130.664	74.69	27.476	129.615
		0.9	228.428	22.661	329.245	228.643	22.665	327.555
2	4	-0.9	21.235	166.277	279.458	21.246	166.752	277.917
		-0.5	26.482	67.467	127.161	26.506	67.607	126.251
		-0.1	35.216	42.054	102.472	35.258	42.111	101.642
		0.1	43.564	36.431	102.472	43.603	36.462	101.642
		0.5	68.031	26.533	127.575	68.117	26.534	126.638
		0.9	176.675	21.929	279.458	176.771	21.927	277.917
4	2	-0.9	23.205	201.51	287.38	23.212	202.05	286.109
		-0.5	28.912	75.648	127.731	28.935	75.803	126.806
		-0.1	38.66	46.37	102.518	38.702	46.429	101.672
		0.1	47.92	39.912	102.518	47.96	39.944	101.672
		0.5	76.206	28.967	128.083	76.294	28.966	127.135
		0.9	212.489	23.933	287.38	212.524	23.927	286.109
4	4	-0.9	22.037	157.021	252.542	22.063	157.562	251.251
		-0.5	27.781	68.891	125.003	27.805	69.029	124.178
		-0.1	36.728	43.661	102.312	36.771	43.719	101.536
		0.1	45.184	37.964	102.312	45.223	37.996	101.536
		0.5	69.457	27.847	125.355	69.541	27.849	124.506
		0.9	165.512	22.709	252.542	165.741	22.725	251.251

Table 5 PRE of different estimators when $\sigma_Z^2 = 25$ and $\sigma_Y^2 = 20$

			n=50			n=100		
σ_U^2	σ_V^2	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
2	2	-0.9	17.208	129.534	330.042	17.225	130.107	328.417
		-0.5	21.309	53.949	130.567	21.326	54.083	129.57
		-0.1	29.026	34.628	102.707	29.022	34.641	101.798
		0.1	35.461	29.105	102.815	35.489	29.098	101.953
		0.5	57.281	22.287	130.567	57.357	22.291	129.57
		0.9	141.953	17.916	330.042	142.104	17.92	328.417
2	4	-0.9	16.681	106.583	287.998	16.692	106.897	286.436
		-0.5	20.599	49.618	127.979	20.616	49.736	127.078
		-0.1	27.795	32.89	102.524	27.792	32.902	101.678
		0.1	33.657	27.878	102.618	33.683	27.873	101.819
		0.5	52.727	21.562	127.979	52.794	21.566	127.078
		0.9	115.705	17.341	287.998	115.76	17.341	286.436
4	2	-0.9	18.52	127.928	279.096	18.528	128.278	277.919
		-0.5	22.828	56.272	127.445	22.845	56.404	126.565
		-0.1	22.845	56.404	126.565	30.936	36.75	101.647
		0.1	37.572	31.001	102.582	37.599	30.993	101.792
		0.5	59.578	23.854	127.445	59.651	23.857	126.565
		0.9	138.172	19.228	279.096	138.181	19.224	277.919
4	4	-0.9	17.562	102.943	251.315	17.586	103.36	250.19
		-0.5	21.944	51.625	125.273	21.962	51.744	124.475
		-0.1	29.475	34.73	102.32	29.474	34.745	101.543
		0.1	35.498	29.543	102.409	35.525	29.539	101.675
		0.5	54.724	22.952	125.273	54.791	22.957	124.475
		0.9	110.65	18.217	251.315	110.868	18.232	250.19

Table 6 PRE of different estimators when $\sigma_Z^2 = 25$ and $\sigma_Y^2 = 25$

			n=50			n=100		
σ_U^2	σ_V^2	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
2	2	-0.9	19.369	174.015	343.258	19.388	174.882	341.465
		-0.5	25.061	67.093	131.257	25.072	67.246	130.168
		-0.1	32.97	40.368	102.891	32.976	40.424	101.981
		0.1	41.106	33.533	102.866	41.16	33.545	101.969
		0.5	67.763	25.236	131.116	67.857	25.242	130.082
		0.9	189.969	20.142	343.258	190.174	20.146	341.465
2	4	-0.9	18.828	141.105	297.355	18.84	141.579	295.693
		-0.5	24.312	61.987	128.636	24.325	62.126	127.645
		-0.1	31.672	38.44	102.692	31.679	38.493	101.847
		0.1	39.133	32.208	102.666	39.183	32.22	101.834
		0.5	62.558	24.477	128.469	62.64	24.484	127.534
		0.9	152.645	19.546	297.355	152.724	19.545	295.693
4	2	-0.9	20.543	167.956	296.931	20.553	168.494	295.568
		-0.5	26.466	68.881	128.582	26.477	69.023	127.602
		-0.1	34.642	42.203	102.693	34.647	42.257	101.847
		0.1	42.923	35.193	102.673	42.975	35.204	101.837
		0.5	69.508	26.629	128.495	69.598	26.635	127.56
		0.9	180.606	21.3	296.931	180.631	21.295	295.568
4	4	-0.9	19.549	134.182	265.307	19.576	134.753	263.989
		-0.5	25.558	63.542	126.307	25.571	63.679	125.412
		-0.1	33.138	40.035	102.513	33.147	40.091	101.725
		0.1	40.716	33.666	102.491	40.767	33.678	101.715
		0.5	64.084	25.713	126.183	64.166	25.72	125.337
		0.9	143.766	20.25	265.307	144.021	20.265	263.989

Table 7 PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_Y^2 = 20$

		n=50			n=100		
% of ME	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
10	-0.9	20.832	189.486	322.18	20.835	189.837	320.535
	-0.5	24.363	63.432	129.748	24.381	63.572	128.768
	-0.1	32.189	39.244	102.764	32.208	39.304	101.9
	0.1	40.928	33.509	102.764	40.977	33.518	101.9
	0.5	63.738	24.451	129.885	63.811	24.45	128.851
	0.9	171.183	19.788	322.671	171.488	19.803	320.782
20	-0.9	21.558	142.111	239.972	21.563	142.355	238.99
	-0.5	24.912	59.491	124.177	24.931	59.61	123.402
	-0.1	32.338	38.804	102.332	32.357	38.86	101.606
	0.1	40.467	33.667	102.332	40.513	33.677	101.606
	0.5	59.854	25.031	124.292	59.919	25.032	123.471
	0.9	130.752	20.527	240.255	130.967	20.542	239.139
30	-0.9	22.22	117.118	200.05	22.226	117.313	199.29
	-0.5	25.402	56.483	120.122	25.422	56.588	119.481
	-0.1	32.466	38.435	101.995	32.486	38.488	101.376
	0.1	40.081	33.803	101.995	40.124	33.814	101.376
	0.5	56.889	25.55	120.218	56.948	25.552	119.538
	0.9	108.893	21.2	200.233	109.061	21.215	199.388
40	-0.9	22.824	101.678	176.608	22.832	101.843	175.96
	-0.5	25.843	54.113	117.058	25.863	54.208	116.511
	-0.1	32.578	38.121	101.728	32.599	38.172	101.192
	0.1	39.752	33.923	101.728	39.793	33.934	101.192
	0.5	54.552	26.016	117.138	54.606	26.019	116.559
	0.9	95.199	21.815	176.735	95.339	21.83	176.029

Table 8 PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_Y^2 = 25$

		n=50			n=100		
% of ME	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
10	-0.9	22.293	213.023	319.229	22.301	213.65	317.551
	-0.5	27.882	74.703	129.598	27.905	74.863	128.596
	-0.1	37.467	45.114	102.653	37.509	45.173	101.76
	0.1	46.658	38.704	102.653	46.699	38.737	101.76
	0.5	75.266	27.925	129.999	75.356	27.924	128.973
	0.9	226.027	23.002	319.229	226.057	22.996	317.551
20	-0.9	22.715	155.348	240.489	22.738	155.818	239.441
	-0.5	28.615	70.136	124.066	28.639	70.272	123.269
	-0.1	37.736	44.771	102.24	37.777	44.827	101.489
	0.1	46.3	38.983	102.24	46.338	39.014	101.489
	0.5	70.721	28.693	124.382	70.794	28.692	123.566
	0.9	163.242	23.401	240.489	163.418	23.413	239.441
30	-0.9	23.51	127.877	200.371	23.532	128.203	199.589
	-0.5	29.272	66.662	120.034	29.295	66.781	119.373
	-0.1	37.97	44.484	101.918	38.008	44.536	101.276
	0.1	45.999	39.224	101.918	46.034	39.253	101.276
	0.5	67.265	29.381	120.289	67.325	29.38	119.614
	0.9	133.784	24.221	200.371	133.913	24.232	199.589
40	-0.9	24.238	111.03	176.828	24.258	111.277	176.178
	-0.5	29.863	63.93	116.985	29.886	64.037	116.421
	-0.1	38.173	44.239	101.662	38.21	44.288	101.106
	0.1	45.742	39.435	101.662	45.775	39.462	101.106
	0.5	64.547	30.001	117.195	64.598	30	116.621
	0.9	115.846	24.972	176.828	115.946	24.982	176.178

Table 9 PRE of different estimators when $\sigma_Z^2 = 25$ and $\sigma_Y^2 = 20$

		n=50			n=100		
% of ME	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
10	-0.9	17.036	122.982	319.643	17.047	123.36	317.851
	-0.5	21.127	52.797	129.873	21.144	52.926	128.903
	-0.1	28.708	34.177	102.659	28.705	34.189	101.766
	0.1	34.992	28.788	102.763	35.02	28.782	101.918
	0.5	56.07	22.101	129.873	56.144	22.105	128.903
	0.9	133.914	17.702	319.643	133.962	17.701	317.851
20	-0.9	17.325	95.299	239.822	17.349	95.676	238.718
	-0.5	21.59	49.71	124.274	21.608	49.821	123.508
	-0.1	28.877	33.902	102.245	28.876	33.917	101.494
	0.1	34.641	28.947	102.328	34.668	28.943	101.619
	0.5	52.717	22.591	124.274	52.78	22.596	123.508
	0.9	102.337	17.977	239.822	102.54	17.992	238.718
30	-0.9	17.878	81.581	199.933	17.903	81.869	199.096
	-0.5	22.003	47.338	120.199	22.022	47.438	119.566
	-0.1	29.022	33.671	101.923	29.024	33.688	101.281
	0.1	34.346	29.085	101.99	34.372	29.083	101.386
	0.5	50.147	23.029	120.199	50.203	23.034	119.566
	0.9	87.063	18.555	199.933	87.224	18.57	199.096
40	-0.9	18.383	72.594	176.508	18.408	72.829	175.808
	-0.5	22.374	45.46	117.121	22.393	45.55	116.582
	-0.1	29.148	33.475	101.667	29.153	33.493	101.11
	0.1	34.095	29.205	101.722	34.121	29.205	101.201
	0.5	48.116	23.421	117.121	48.166	23.428	116.582
	0.9	77.157	19.083	176.508	77.293	19.098	175.808

Table 10 PRE of different estimators when $\sigma_Z^2 = 25$ and $\sigma_Y^2 = 25$

		n=50			n=100		
% of ME	ρ	t_r	t_p	$t_i,\ i=1,2$	t_r	t_p	$t_i,\ i=1,2$
10	-0.9	19.115	158.73	322.671	19.142	159.496	320.782
	-0.5	25.189	66.123	129.885	25.201	66.271	128.851
	-0.1	33.014	40.28	102.788	33.021	40.336	101.912
	0.1	41.003	33.568	102.764	41.056	33.58	101.9
	0.5	66.757	25.359	129.748	66.848	25.366	128.768
	0.9	171.349	19.798	322.671	171.67	19.813	320.782
20	-0.9	19.825	122.57	240.255	19.852	123.058	239.139
	-0.5	25.79	62.053	124.292	25.804	62.185	123.471
	-0.1	33.225	39.941	101.616	33.225	39.941	101.616
	0.1	40.541	33.726	102.332	40.591	33.739	101.606
	0.5	62.544	25.935	124.177	62.62	25.942	123.402
	0.9	130.862	20.537	240.255	131.087	20.553	239.139
30	-0.9	20.472	102.687	200.233	20.499	103.047	199.388
	-0.5	26.327	58.95	120.218	26.343	59.07	119.538
	-0.1	33.388	39.556	102.012	33.401	39.61	101.384
	0.1	40.154	33.863	101.995	40.201	33.877	101.376
	0.5	59.335	26.45	120.122	59.402	26.458	119.481
	0.9	108.976	21.21	200.233	109.152	21.226	199.388
40	-0.9	21.063	90.11	176.735	21.09	90.397	176.029
	-0.5	26.81	56.506	117.138	26.828	56.616	116.559
	-0.1	33.54	39.274	101.743	33.554	39.327	101.199
	0.1	39.825	33.983	101.728	39.869	33.997	101.192
	0.5	56.81	26.913	117.058	56.869	26.921	116.511
	0.9	95.267	21.826	176.735	95.414	21.841	176.029

6. Discussions and Concluding Remarks

The following concluding remarks can be read out.

(i) From the results of the numerical study revealed in Table 2, it is observed that the superiority of the proposed estimators dominates the usual mean estimator, classical ratio and product estimators by PRE.

(ii) From the findings of the simulation study disclosed in Table 3 for $\sigma_Z^2 = 20$, $\sigma_Y^2 = 20$, n=50 & 100, it is observed that when $\sigma_U^2 = 2$ and $\sigma_V^2 = 2$:

  (a) the PRE of the classical ratio estimator t_r increases as the correlation coefficient ρ varies from -0.9 to +0.9.

  (b) the PRE of the classical product estimator t_p decreases as the correlation coefficient ρ varies from -0.9 to +0.9.

  (c) the PRE of the proposed estimators $t_i,\ i = 1, 2$ decreases as the correlation coefficient ρ varies from -0.9 to -0.1 and increases as the correlation coefficient ρ varies from +0.1 to +0.9 and dominates the existing estimators.

  (d) The similar conclusion can be observed when $\sigma_U^2 = 2$, $\sigma_V^2 = 4$; $\sigma_U^2 = 4$, $\sigma_V^2 = 2$ and $\sigma_U^2 = 4$, $\sigma_V^2 = 4$.

(iii) The conclusion like point (ii) can also be observed from Table 4 based on $\sigma_Z^2 = 20$, $\sigma_Y^2 = 25$, Table 5 based on $\sigma_Z^2 = 25$, $\sigma_Y^2 = 20$ and Table 6 based on $\sigma_Z^2 = 25$, $\sigma_Y^2 = 25$.

(iv) From the outcomes summarized in Table 7 for $\sigma_Z^2 = 20$, $\sigma_Y^2 = 20$, n=50 & 100, it is seen that when the level of ME is 10% then:

  (a) the PRE of the classical ratio estimator t_r varies as the correlation coefficient ρ varies from -0.9 to +0.9.

  (b) the PRE of the classical product estimator t_p decreases as the correlation coefficient ρ varies from -0.9 to +0.9.

  (c) the PRE of the proposed estimators $t_i,\ i = 1, 2$ decreases as the correlation coefficient ρ varies from -0.9 to -0.1 and increases as the correlation coefficient ρ varies from +0.1 to +0.9 and dominates the existing estimators.

  (d) The same tendency can be seen when the levels of ME are 20%, 30% and 40%.

  (e) Moreover, the PRE of the proposed estimators decreases as the level of ME increases.

(v) The interpretation like point (iv) can also be drawn from Table 8 based on $\sigma_Z^2 = 20$, $\sigma_Y^2 = 25$, Table 9 based on $\sigma_Z^2 = 25$, $\sigma_Y^2 = 20$ and Table 10 based on $\sigma_Z^2 = 25$, $\sigma_Y^2 = 25$.

Based on the above discussions drawn after the perusal of the findings of numerical and simulation studies, it is clear that the performance of the proposed class of estimators is highly justifiable over the conventional estimators. Therefore, the proposed class of estimators can be recommended to the survey practitioners whenever ME occurs in the survey.
Furthermore, the proposed estimators can be developed under stratified random sampling in case of measurement errors.

Acknowledgement

The authors are thankful to the editor-in-chief Wararit Panichkitkosolkul and anonymous reviewers for their encouraging suggestions which improved the quality of contents and presentation of the original manuscript.

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