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A Synthetic Variable Parameter $\bar{X}$ Control Charts to Detect Small and Moderate Shifts

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Abstract

Due to the technological advances, the use of control charts is more important than ever. The variable parameters (VP) control charts are based on the varying all $\bar{X}$ chart parameters, sample size n, sampling interval h and the k used to determine the action limits. In this paper, we introduce the synthetic-VP $\bar{X}$ chart for monitoring changes in the process mean especially moderate and small shifts. The optimal charting parameters of the proposed chart are obtained using the Markov chain approach. An example is also presented to show the performance of the proposed chart against other charts.

Keywords: Control charts, markov chain, synthetic control chart, VP control chart.

1. Introduction

The control chart is primarily a tool used to analyze data which is generated over a period of time. Control charts simply tell us if the process is in-control or out of control, that is, whether a search for special causes is warranted or the process should be left alone. Shewhart (1931) was the first to introduce the concept of a control chart. An Xbar-chart is a type of control chart used to monitor the process mean when measuring subgroups at regular intervals from a process. The Shewhart $\bar{X}$ control chart is quite effective in detecting large scale shifts in the mean. The design and operation of Shewhart control charts require the determination of three design parameters: the sample size (n), the sampling interval (h), and the width coefficient of control limits (k).

Variable Parameters (VP)$\bar{X}$ chart Costa (1999a), which is the $\bar{X}$ chart with all the design parameters variable: the sample size, (n); the sampling interval, (h); and the factor used in determining the width of the control limits, (k). These three parameters vary between minimum and maximum values as a function of the last sample point position. The effect of varying all $\bar{X}$ chart parameters has been studied by Costa (1999a), Costa (1999c) and Yeong et al. (2018). Zhou et al. (2020) introduced a synthetic control chart for monitoring small size shifts in the process mean. Costa (1999a) indicated that the $\bar{X}$ chart with all parameters variable can detect moderate shifts in the process mean. Against the Shewhart $\bar{X}$ control chart, the VP $\bar{X}$ chart is the sensitive scheme in detecting small shifts in the process mean.

Wu and Spedding (2000) introduced the combined Shewhart $\bar{X}$ and conforming run length (CRL) charts, which is called the synthetic $\bar{X}$ control chart.

A synthetic control chart for the process mean which integrates the $\bar{X}$ chart and the conforming run length (CRL) chart provides significant improvement in terms of detection power over the basic $\bar{X}$ chart for all levels of mean shifts. There are Numerous studies on synthetic control charts. Wu et al. (2010) presented the combined synthetic $\bar{X}$ and $\bar{X}$ chart and Khoo et al. (2011) proposed the synthetic double sampling $\bar{X}$ chart. Khoo et al. (2012) provided an optimal design of the synthetic $\bar{X}$ chart using the median run length. Zhang et al. (2011) studied the run-length performance of the synthetic $\bar{X}$ chart. wan (2020) proposed a variable sampling interval $np_x - \bar{X}$ control chart by combining attribute-variable inspection.

In this paper, we propose the synthetic-VP $\bar{X}$ chart. The VP synthetic $\bar{X}$ chart is a type of synthetic chart, it also comprises two sub-charts, namely the VP $\bar{X}$ sub-chart and the CRL sub-chart.

In the rest of this paper, Section 2 reviews some literature on the VP $\bar{X}$ and the synthetic $\bar{X}$ charts. The synthetic-VP $\bar{X}$ chart is proposed in Section 3. An example is illustrating the use of the synthetic-VP $\bar{X}$ chart is given in the Section 4.

2. Control Charts

A control chart is one of the primary techniques of statistical process control (SPC). The control charts are used to detect and eliminate unwanted special causes of variation occurred during a period of time. Here we review the VP $\bar{X}$ and synthetic $\bar{X}$ charts, then we introduce a synthetic - VP $\bar{X}$ chart.

2.1. The VP $\bar{X}$ chart

Control Charting techniques put great emphasis on the monitoring of shifts in the process mean. The $\bar{X}$ chart is a quality control chart monitoring the mean of a process. To monitor a process using $\bar{X}$ chart, a sample size n_0 is randomly chosen every h_0 hours and the $\bar{X}$ values from the samples are plotted on the control chart with upper and lower control limites. The upper and lower control limits for $\bar{X}$ chart are

$$\mu_0 + k_0\sigma/\sqrt{n_0},$$
$$\mu_0 - k_0\sigma/\sqrt{n_0},$$

where k_0 is the control limit coefficient.

The VP chart is divided into central region, warning region and action region. As seen in Figure 1, the central region is bounded by the lower and upper warning limits. The region between the lower warning limit and lower control limit or upper warning limit and upper control limit is named as the warning region and the action region that is bounded by the values that are lower than lower control limit or exceed the upper control limit.

The VP $\bar{X}$ is the control chart with design parameters n, h and k, which vary between two values. These values are a function of the most recent process information. In fact in the VP $\bar{X}$, the position of each sample point on the chart establishes the size of the next sample and the time of its sampling. Let n_1 and n_2 be sample size values with the sampling interval values h_1 and h_2. Also consider k_1 and k_2 are the coefcient values used in determining the width of the control limits. If the sample point falls in the central region, the next sample should be small, that is $n_1 < n_0$, and it should be sampled after a long time interval, i.e. $h_1 > h_0$. If the sample point falls in the warning region, the next sample should be large, that is $n_2 > n_0$, and it should be taken after a short time interval, that is $h_2 < h_0$.

For the VP $\bar{X}$ chart, in addition to the control limits, the lower and upper warning limits are also defined. Thus there are two regions: warning region and central region. The region between the lower warning limit and the lower central limit or the upper warning limit and the upper control limit is the warning limit. The warning and control limits respectively are

$$\mu_0 \pm w_i\sigma/\sqrt{n_i},$$
$$\mu_0 \pm k_i\sigma/\sqrt{n_i},$$

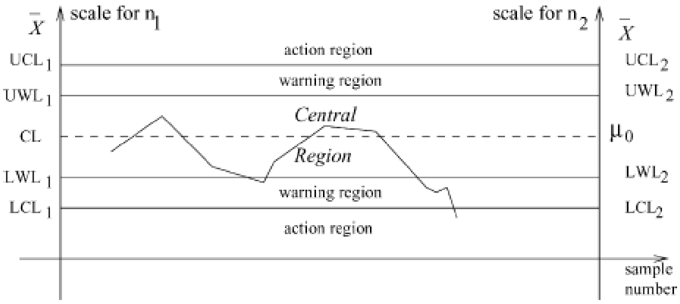

Figure 1 The VP $\bar{X}$ chart

where $w_i, i = 1, 2$ denote the width coefficients of the warning limits. Note that $k_1 > k_0 > k_2$ and $w_1 > w_2$. Since $\bar{X}$ values come from small and large samples, two charts should be plotted. To avoid the use of two control charts, one can construct the VP $\bar{X}$ chart with two scales, one on the left side for samples of size n_1 and the other on the right side for samples of size n_2.

To use the VP $\bar{X}$ chart, the design parameters should be chosen. Note that if the sample point falls in the central region, then the next sample should be small and sampled after a long time interval. The control limit also should be wide for this sample. On the other hand, if the sample point falls in the warning region, then the next sample should be large and sampled after a short time interval. The warning limit also should be narrow.

If a sample point falls in the action region, the VP $\bar{X}$ chart produces a signal. So the process is out of control or has occurred just a false alarm.

The size of the first sample, that is taken from the process when it is just starting or after a false alarm, is chosen at random. If the sample was chosen to be large (small) it should be sampled after a short (long) time interval.

During the in-control period all samples have probability p_0 of being small and $1 - p_0$ of being large.

2.2. The synthetic $\bar{X}$ chart

One possible enhancement of the Shewhart control chart in increasing its speed in detecting shifts in the process mean is to construct a synthetic control chart.The synthetic$\bar{X}$ control chart improve upon the performance of the Shewhart$\bar{X}$control chart for detecting small and moderate shifts in the process mean.The synthetic $\bar{X}$ chart integrates the standard $\bar{X}$ chart and the conforming run length (CRL) chart. Let X be a random variable distributed as $N(\mu_0, \sigma_0)$, where μ_0 is the in-control mean and σ_0 is the in-control standard deviation. The synthetic $\bar{X}$ chart consists of two sub-charts, a $\bar{X}/S$ sub-chart and a CRL/S sub-chart. The CRL is defined as the number of inspected units between two consecutive non-conforming units, inclusive of the non-conforming unit at the end. Figure 2 shows a process starting at $t = 0$ and the open and closed dots represent conforming and non-conforming units, respectively, where $CRL_1 = 5$, $CRL_2 = 2$ and $CRL_3 = 4$. The operations of the synthetic $\bar{X}$ chart are outlined as follows.

1. Determine the control limits:
   Determine the lower control limit $LCL_{\bar{X}/S}$ and the upper control limit $UCL_{\bar{X}/S}$ for the $\bar{X}/S$ sub-chart and the lower control limit L for the CRL sub-chart. The $LCL_{\bar{X}/S}$ and $UCL_{\bar{X}/S}$ are as follows

$$LCL_{\bar{X}/S} = \mu_0 - K\sigma_{\bar{x}},$$
$$UCL_{\bar{X}/S} = \mu_0 + K\sigma_{\bar{x}},$$

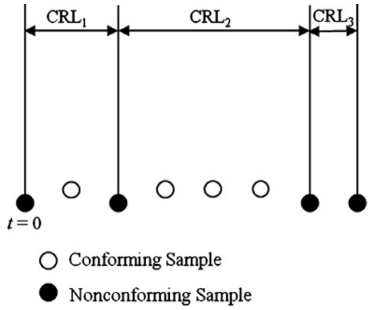

Figure 2 Conforming run length

where the standard deviation $\sigma_{\bar{x}} = \dfrac{\sigma_0}{\sqrt{n}}$. Note that $K = \dfrac{K'}{\sqrt{n}}$, i.e. K includes the sample size n.

2. Take a sample:
   Take a random sample of size n at each inspection point i and compute the sample mean $\bar{X}$.

3. Classify the sample:
   If the value of $\bar{X}$ falls between the lower control limit $LCL_{\bar{X}/S}$ and upper control limit $UCL_{\bar{X}/S}$, the sample is classified as a conforming sample and it is accepted that the process is in-control, then the control flow returns to Step 2.

4. Determine the CRL values:
   Count the number of $\bar{X}$ samples between the current and the previous non-conforming samples. This is the CRL value of the CRL/S sub-chart in the synthetic chart.

5. Declare the process:
   If the value of $CRL > L$, the process is declared to be in-control and the control flow returns to Step 2. Otherwise, the process is deemed to be out-of-control and the control flow goes to the next step.

6. Signal an out-of-control status:
   This step indicates a process mean shift by an out-of-control signal.

7. Find and remove assignable cause(s). Then return to Step 2.

3. The Synthetic - VP $\bar{X}$ Chart

The Shewhart $\bar{X}$ chart is the popular control chart that can detect large shifts in the mean of a process. This chart can be a good choice for the large shifts but responds slowly to small and moderate shifts. The Synthetic $\bar{X}$ chart is the combination of the Shewhart $\bar{X}$ and conforming run length (CRL) charts. This chart can detect all levels of the mean shift of processes better that Shewhart $\bar{X}$ chart. The VP $\bar{X}$ chart is a modification of the Shewhart $\bar{X}$ chart with all design parameters. These parameters vary between two values as a function of the last sample point position. This control chart is effective in detecting the small mean shifts.

Here we introduce the synthetic VP $\bar{X}$ chart that integrates the VP $\bar{X}$ chart and the conforming run length (CRL) chart. The synthetic - VP $\bar{X}$ chart is a conservative approach to signal false alarm. In this section, we describe it in detail.

Let X be a Normal random variable with distribution $N(\mu, \sigma)$, where μ and σ are the in-control mean and standard deviation respectively. The Synthetic - VP $\bar{X}$ chart consists of two sub-charts, a

VP $\bar{X}$ chart and a CRL sub-chart.We are interest in detecting the small and moderate process mean shifts.The synthetic vp $\bar{X}$chart is a type of synthetic chart,it also comprises two sub-charts. The steps for constructing the synthetic-VP $\bar{X}$ chart are as follows:

1. Determine the control limits:
   Determine the lower and upper control limites $LCL_{VP\bar{X}}$ and $UCL_{VP\bar{X}}$ and warning limites $LWL_{VP\bar{X}}$ and $LWL_{VP\bar{X}}$ for the $VP\bar{X}$ sub-chart and the lower control limit L for the CRL sub-chart. The $LCL_{VP\bar{X}}$ and $UCL_{VP\bar{X}}$ are

$$LCL = \mu - k_i\sigma/\sqrt{n_i},$$
$$UCL = \mu + k_i\sigma/\sqrt{n_i},$$

   The lower and upper warning limites are as follows:

$$LWL = \mu - w_i\sigma/\sqrt{n_i},$$
$$UWL = \mu + w_i\sigma/\sqrt{n_i},$$

2. Take a sample:
   Take a random sample of size n_i at each inspection point i and compute the sample mean $\bar{X}$. n_1 references to the small sample and n_2 denotes large sample.

3. Classify the sample:
   3.1. If the value of $\bar{X}$ falls in the central region, the next sample should be small and sampled after a long time interval. So the process is in-control, then the control flow goes back to Step 2.
   3.2. If the $\bar{X}$ value fall in the warning region, the next sample should be large and taken after a short time interval. So the process is in-control, then the control flow return to Step 2.
   3.3. If the value of $\bar{X}$ falls in the action region, the sample is non-conforming and the control flow goes to the next step.

4. Determine the CRL values:
   Count the number of $\bar{X}$ samples between the current and the previous non-conforming samples as the CRL value.

5. Declare the process:
   5.1 If the value of CRL is larger than L,the next sample should be large and sampled after a short time interval, thus the process is declared to be in-control and the control flow returns to Step 2. Otherwise, the process is out-of-control and the control flow goes to the next step.

6. Signal an out-of-control status:
   This step indicates a process mean shift by an out-of-control signal.

7. Take actions to investigate and eliminate the assignable cause(s). Then return to Step 2.

3.1. Properties of the synthetic-VP $\bar{X}$ chart

When the process is in-control, depending on the size of the sample point, $\bar{X} \sim N(\mu, \sigma/\sqrt{n_1})$ or $\bar{X} \sim N(\mu, \sigma/\sqrt{n_2})$.

We are interested in computing type I and type II errors. Since the probability of choosing samples depends on the size of them, we compute errors in two parts. For $i = 1, 2$, α is obtained by the following:

$$\alpha = p_0\alpha_1 + (1 - p_0)\alpha_2, \tag{1}$$

where α_1 and α_2 is the type I errors when the sample size is small and large respectively. Also p_0 is the probability of choosing the small sample and obtained as follows:

$$\begin{aligned} p_0 &= P(\mid Z \mid < w_1 \mid \mid Z \mid < k_1) \\ &= P(-w_1 < Z < w_1 \mid -k_1 < Z < k_1). \end{aligned}$$

Since $w_1 < k_1$, then $(-w_1, w_1) \subseteq (-k_1, k_1)$. It means that $(-w_1, w_1) \cap (-k_1, k_1) = (-w_1, w_1)$ and p_0 is computed as:

$$\begin{aligned} p_0 &= \frac{P(-w_1 < Z < w_1)}{P(-k_1 < Z < k_1)} \\ &= \frac{\Phi(w_1) - \Phi(-w_1)}{\Phi(k_1) - \Phi(-k_1)}. \end{aligned}$$

To compute type I error, depending on the sample size we need to obtain α_1 for the small samples and α_2 for the large ones, so:

$$\begin{aligned} \alpha =& p_0[P_{H_0}(\bar{X} > UCL_1) + P_{H_0}(\bar{X} < LCL_1)] \\ &+ (1 - p_0)[P_{H_0}(\bar{X} > UCL_2) + P_{H_0}(\bar{X} < LCL_2)] \\ =& p_0[P(\bar{X} > \mu + k_1\sigma/\sqrt{n_1}) + P(\bar{X} < \mu - k_1\sigma/\sqrt{n_1})] \\ &+ (1 - p_0)[P(\bar{X} > \mu + k_2\sigma/\sqrt{n_2}) + P(\bar{X} < \mu - k_2\sigma/\sqrt{n_2})] \\ =& p_0[P(Z > k_1) + P(Z < -k_1)] \\ &+ (1 - p_0)[P(Z > k_2) + P(Z < -k_2)] \\ &= p_0[2\Phi(-k_1)] + (1 - p_0)[2\Phi(-k_2)]. \end{aligned}$$

To compute type II error, note that $X \sim N(\mu + \delta\sigma, \sigma)$ when the process is out of control. The β is divided into two parts, β_1 for the small samples with probability p_0 and β_2 for the large samples with probability $(1 - p_0)$, thus:

$$\begin{aligned} \beta =& p_0[P(LCL_1 < \bar{X} < UCL_1 | H_1)] \\ &+ (1 - p_0)[P(LCL_2 < \bar{X} < UCL_2 | H_1)] \\ =& p_0[P(\mu - k_1\sigma/\sqrt{n_1} < \bar{X} < \mu + k_1\sigma/\sqrt{n_1})] \\ &+ (1 - p_0)[P(\mu - k_2\sigma/\sqrt{n_2} < \bar{X} < \mu + k_2\sigma/\sqrt{n_2})] \\ =& p_0[\Phi(k_1 - \delta\sqrt{n_1}) - \Phi(-k_1 - \delta\sqrt{n_1})] \\ &+ (1 - p_0)[\Phi(k_2 - \delta\sqrt{n_2}) - \Phi(-k_2 - \delta\sqrt{n_2})]. \end{aligned}$$

Now we are going to compute the ARL and AATS for the proposed chart. The Markov chain approach suggested by Davis and Woodall Davis and Woodall (2002) is used to compute the in-control and out-of-control AATS and ARL. Assume that the process starts with the mean μ_o and a single assignable cause occurs at a random time, thus the process mean shifts to $\mu_1 = \mu_0 \pm \delta\sigma_0$, $(\delta > 0)$. The time until an assignable cause occurs described by an exponential distribution with parameter λ.

AATS is adjusted average time to signal, i.e. AATS is the average time from the process mean shift until the chart produces a signal.To compute AATS, we need to obtain ATC. In other words we have:

$$AATS = ATC + \frac{1}{\lambda} \tag{2}$$

The memoryless property of the exponential distribution allows the computation of the ATC, using the Markov chain approach. Depending on the status of the process, the fowllowing transient states is reached.

1. the process is in-control and the sample is small;
2. the process is in-control and the sample is large;
3. the process is out of control and the sample is small; and
4. the process is out of control and the sample is large.

The transition probability matrix is given by following

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 \\ p_{21} & p_{22} & p_{23} & p_{241} & 0 \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & p_{43} & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & p_{55} \end{pmatrix} \quad (3)$$

where p_{ij} denotes the transition probability and obtains as follow:

$$\begin{aligned}
p_{11} &= P(\mid Z \mid < w_1 \mid \mid Z \mid < k_1) exp(-\lambda h_1), \\
p_{12} &= P(w_1 < \mid Z \mid < k_1 \mid \mid Z \mid < k_1) exp(-\lambda h_2), \\
p_{21} &= P(\mid Z \mid < w_2 \mid \mid Z \mid < k_2) exp(-\lambda h_1), \\
p_{22} &= P(w_2 < \mid Z \mid < k_2 \mid \mid Z \mid < k_2) exp(-\lambda h_2), \\
p_{13} &= P(\mid Z \mid < w_1 \mid \mid Z \mid < k_1)(1 - exp(-\lambda h_1)), \\
p_{23} &= P(\mid Z \mid < w_2 \mid \mid Z \mid < k_2)(1 - exp(-\lambda h_1)), \\
p_{14} &= P(w_1 < \mid Z \mid < k_1 \mid \mid Z \mid < k_1)(1 - exp(-\lambda h_2)), \\
p_{24} &= P(w_2 < \mid Z \mid < k_2 \mid \mid Z \mid < k_2)(1 - exp(-\lambda h_2)), \\
p_{33} &= P(\mid Y \mid < w_1 \mid \mid Y \mid < k_1), \text{ where } Y \sim N(\delta\sqrt{n_1}, 1), \\
p_{34} &= P(w_1 < \mid Z \mid \mid \mid Y \mid < k_1), \text{ where } Y \sim N(\delta\sqrt{n_1}, 1), \\
p_{43} &= P(\mid Y \mid < w_2 \mid \mid Y \mid < k_2), \text{ where } Y \sim N(\delta\sqrt{n_2}, 1), \\
p_{44} &= P(w_2 < \mid Z \mid \mid \mid Y \mid < k_2), \text{ where } Y \sim N(\delta\sqrt{n_2}, 1), \\
p_{35} &= (P \mid Y \mid > k_1 | Y \sim N(\delta\sqrt{n_1}, 1)), \\
p_{45} &= (P \mid Y \mid > k_2 | Y \sim N(\delta\sqrt{n_2}, 1)).
\end{aligned}$$

Let $b^{'} = (p_{11}, p_{12}, p_{13}, p_{14})$ be the vector starting probabilities and Q is the transition probability matrix where the elements associated with the absorbing state have been deleted, ATC is obtained by

$$ATC = b^{'}(I - Q)I. \quad (4)$$

Thus we can write as follows

$$(I-Q)^{-1}I = \begin{pmatrix} \frac{1-p_{22}}{A} & \frac{p_{12}}{A} & \frac{C(1-p_{44}) + Dp_{43}}{AB} & \frac{Cp_{34} + D(1-p_{33})}{AB} \\ \frac{p_{21}}{A} & \frac{1-p_{11}}{A} & \frac{E(1-p_{44}) + Fp_{43}}{AB} & \frac{Ep_{34} + F(1-p_{33})}{AB} \\ 0 & 0 & \frac{1-p_{44}}{B} & \frac{p_{34}}{B} \\ 0 & 0 & \frac{p_{43}}{B} & \frac{1-p_{33}}{B}. \end{pmatrix} \quad (5)$$

$$\begin{aligned}
A &= (1-p_{11})(1-p_{22}) - p_{12}p_{21}, \\
B &= (1-p_{44})(1-p_{33}) - p_{34}p_{43}, \\
C &= p_{12}p_{21} + p_{13}(1-p_{22}), \\
D &= p_{24}p_{12} + p_{14}(1-p_{22}), \\
E &= p_{13}p_{21} + p_{14}(1-p_{11}), \\
F &= p_{14}p_{21} + p_{24}(1-p_{11}).
\end{aligned}$$

Average run length (ARL) is an important performance metric of control charts. ARL represents the average number of sample points required by the chart to signal a shift. The ARL of the synthetic-VP $\bar{X}$ chart corresponds to the specific values of K, L, n and δ. So the ARL formula for the synthetic-VP $\bar{X}$ chart is givin by

$$ARL(\delta) = \frac{1}{P_i(1-(1-P_i)^L)}, \tag{6}$$

and

$$P_i = 1 - \Phi(k_i - \delta\sqrt{n_i}) + \Phi(-k_i - \delta\sqrt{n_i}). \tag{7}$$

The ARL formula of the synthetic-VP $\bar{X}$ chart also can be considered as

$$ARL = 1 + [ARL_{VP\bar{X}} - 1](ARL_{CRL}) + ARL_{CRL} - 1, \tag{8}$$

where the first part is the initial sampling interval, $[ARL_{VP\bar{X}} - 1](ARL_{CRL})$ is the expected number of sampling intervals, where the length is determined by the $\bar{X}$ samples and the last part is the expected number of sampling intervals, where the length is dependent on the CRL samples.

3.2. Optimal design of the synthetic $\bar{X}$ chart

In the synthetic $\bar{X}$ chart, we aim to minimize AATS. To obtain the optimal values (L, k_1, w_1, k_2, w_2), the following procedure is used:

1. Set $ARL(0) = 370.4$ and intial values.
2. Set the sample size, $n_i, i = 1, 2$ and $AATS_0$.
3. Choose the magnitude of the standardized mean shift, δ_{opt}, where a quick detection is needed.
4. Determine the optimal values of L and k_1 and k_2 from $ARL(0)$.
5. Determine the values w_i from the AATS.
6. Obtain the optimal values of parameters when the smallest $AATS_1(\delta_{opt})$ is recorded.

The optimal parameters of the synthetic-VP $\bar{X}$ Chart are presented in 1. In Table 1, when the mean shift δ increases, the AATS value decreases towards unity. Also as the sample size increases AATS value decreases. This indicates that the synthetic-VP $\bar{X}$ chart provides a quicker detection of shifts in the mean when the sample size increases. Note that the $AATS_0$ is set as 370. This means that the performance of the synthetic-VP $\bar{X}$ chart matches that of a typical 3 sigma Shewhart $\bar{X}$ chart when the process is in-control.

Table 1 Values of AATS for the synthetic VP $\bar{X}$ Chart

δ	n_1	n_2	h_1	h_2	k_1	k_2	w_1	w_2	L	$AATS$
2.00	11	15	2.15	0.1	1.15	1.08	2.62	1.97	1	1.14
1.75	7	9	1.70	0.1	1.23	1.18	3.03	2.22	1	1.62
1.50	5	6	1.43	0.1	1.30	1.26	3.26	2.38	2	2.03
1.25	3	5	1.25	0.1	1.43	1.39	3.44	2.47	2	2.05
1.00	3	5	1.11	0.1	1.71	1.6	3.62	2.44	3	2.37
0.75	2	5	1	0.1	1.9	1.8	3.65	2.42	5	3.19
0.50	2	5	1	0.1	2.06	1.9	3.56	2.42	6	3.31

Table 2 AATS(δ_{opt}) for the $\bar{X}$,VP$\bar{X}$ and synthetic VP$\bar{X}$ (n_1, n_2) = (3,5)

δ_{opt}	$\bar{X}$	$VP\bar{X}$	$syntheticvp\bar{X}$
0.2	124.21	117.11	99.82
0.4	93.99	91.11	73.15
0.6	21.19	18.17	8.3
0.8	7.31	4.17	3.14
1	5.70	4.91	2.36
1.2	4.31	3.72	1.93
1.4	3.54	2.49	1.65
1.6	2.40	1.50	1.37
1.8	1.70	1.32	1.16
2	1.47	1.21	1.11

Table 3 AATS(δ_{opt}) for the $\bar{X}$,VP$\bar{X}$ and synthetic VP$\bar{X}$ (n_1, n_2) = (11,15)

δ_{opt}	$\bar{X}$	$VP\bar{X}$	$syntheticvp\bar{X}$
0.2	119.56	101.21	73.52
0.4	48.03	39.59	23.38
0.6	16.81	15.73	2.73
0.8	5.91	5.42	1.89
1	3.62	3.04	1.47
1.2	2.47	2.19	1.26
1.4	2.21	1.87	1.21
1.6	1.91	1.23	1.14
1.8	1.52	1.04	1
2	1.13	1	1

4. Example

Here we want to show the performance of the synthetic-VP $\bar{X}$ chart using Example Wu and Spedding (2000).

Suppose that the mean value of a quality characteristic, x, is to be monitored by $\mu = 200$, and known standard deviation, $\sigma = 0.02$. Presently, one sample of size four is taken from the process at 20 minute intervals. The $\bar{X}$ values are plotted on the chart with control limits placed at $\mu_0 \pm 3\sigma_0 = 200 \pm 0.06$.

Figure 3 demonstrates Curves of the ARL for the synthetic $\bar{X}$, VP $\bar{X}$ and synthetic-VP $\bar{X}$ control charts. As seen in Figure 3, the performance of the synthetic-VP $\bar{X}$ and VP $\bar{X}$ charts is better than the synthetic $\bar{X}$ chart for all value of mean shift. The synthetic-VP $\bar{X}$ is the most effective when δ is larger than 0.80. It is found that the synthetic-VP $\bar{X}$ chart is more sensitive than both the synthetic $\bar{X}$ and VP $\bar{X}$ charts. In other word, It performs better than other two charts in detecting all sizes of shifts.

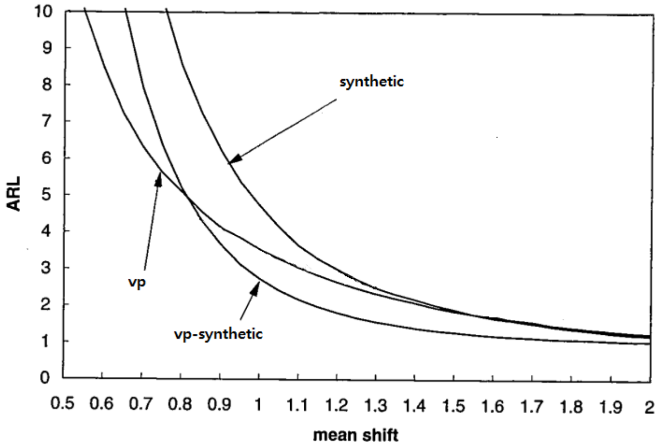

Figure 3 Curves of the ARL for Various Control Charts

5. Conclusions

In this paper, we proposed the synthetic-VP $\bar{X}$ chart. The properties of the proposed chart is obtained. The optimal design parameters are also computed. The results show that the synthetic-VP $\bar{X}$ chart perform better than both synthetic $\bar{X}$ chart and VP $\bar{X}$ chart. This chart is effective in detecting small and moderate shifts. In other word to find all level of shifts we can use the synthetic-VP $\bar{X}$ chart. The suggested chart is conservative approach which rarely signal false alarm.

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