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The Exponentiated Rayleigh-Rayleigh Model on Peak over Threshold Method and Application to Danish Fire Claims

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Abstract

This work introduces a new proposed model, the exponentiated Rayleigh-Rayleigh distribution created by Rayleigh-Rayleigh and exponentiated distributions. It better fits the data sets, whereas other distributions could not be implemented. The peak over threshold method is considered for model fitting in the tailed distribution. The parameters estimation is the maximum likelihood estimate and measurements of model fitting are the Kolmogorov-Smirnov test, Anderson-Darling test, Akaike Information Criterion and Bayesian information criterion. The exponentiated Rayleigh-Rayleigh distribution is compared to the current distributions, which are lognormal, gamma, Weibull and generalized Pareto, exponential, exponential-exponential, gamma-exponential and Rayleigh-Rayleigh. The data are considered based on simulation and Danish Fire data. We have found that the Exponentiated Rayleigh-Rayleigh distribution is the better fit for the small size of the data.

Keywords: Exponentiated distribution, extreme value theorem, infinite mixture distribution, Rayleigh distribution, tailed distribution.

1. Introduction

One method of constructing a new model is an exponentiated distribution. This was originally introduced by Gupta et al. (1999) and it was proposed for the failure time data model. Many papers and publications have been modified and built on exponentiated distribution. They are mostly applied to the data of lifetime models and medical sciences. The model of exponentiated distribution, G, is in the form of

$$G_\alpha(x) = [F(x)]^\alpha,$$

where a cumulative distribution function (cdf) has a baseline distribution function $F(x)$ and α is a positive real number. Many researchers have created various baseline distributions. For example, Gupta and Kundu (2001) proposed and studied some mathematical and statistical properties of the exponentiated exponential (EE) distribution. They found that the EE might work better than Weibull or Gamma. AL-Jammal (2008) studied the reliability function and failure rate by using the EE distribution. Hamedani (2013) presented the various characterizations of the class of the exponentiated distributions. Masoom et al. (2007) studied a number of new exponentiated distributions, consisting of exponential inverse Weibull, exponentiated logistic, exponentiated Pareto, exponentiated generalized uniform, exponentiated general exponential, exponentiated double exponential and exponentiated double Weibull.

In 2014, Cordeiro et al. (2014) proposed the two parameters exponentiated half-logistic (EHL) distributions, which represent a competitive alternative for lifetime data, while Elbatal et al. (2014) introduced the transmuted exponentiated Fechet (TEF) distribution with some study of the transmuted probability distribution. The new lifetime distribution of exponentiated Gumbel (EG) type-2 distribution was presented by Okorie et al. (2016). They recommended applying the model to complex data and the exponentiated moment exponential power series (EMEPS) family was proposed by Iqbal et al. (2017). It is suitable for several data types. In 2018, Handique et al. (2018) proposed the exponentiated generalized MarshallOlkin family of distributions, a better fit for three data sets that are increasing hazard rates. Fatima et al. (2018) proposed the new exponentiated inverse Kumaraswamy distribution. Al-Omari et al. (2019) created the exponentiated new Weibull-Pareto (ENWP) distribution and derived several properties. In 2020, Al-Sulami (2020) proposed the exponentiated Exponential Weibull distribution (EEWD), in which the mathematical and statistical properties are described. Abbas et al. (2020) presented exponentiated exponential-exponentiated Weibull distribution, which deals with linear mixing of exponentiated exponential and exponentiated Weibull.

Many papers have discussed and considered the Danish fire data set which is globally recognized data of fire insurance losses, such as Boonradsamee et al. (2022) and Wasinrat and Choopradit (2022). It is not easy to model a whole data set for a model fitting by using a single parametric distribution. The peaks over threshold (POT) method is a model of the extreme value theorem (EVT), which is a model of tail distribution. Thus, the data exceeds the threshold u, which would be fitted by some models. The generalized Pareto distribution (GPD) is most popularly used for modelling extreme events, especially in insurance, based on the POT method, which benefits tail distribution modelling and reinsurance work.

We present a new model called the exponentiated Rayliegh-Rayliegh (ERR) distribution derived from an exponentiated distribution with the baseline distribution function of the Rayliegh-Rayliegh (RR) distribution. The RR is created by the infinite distribution based on the Rayleigh distribution proposed by Jaroengeratikun et al. (2022). The heavy-tailed properties of ERR distribution are described and applied to the model, which simulates the insurance claims for model fitting. The comparisons of the models are based on individual data and the POT method in which the data exceed a threshold u are analyzed. The ERR distribution is applied to the Danish fire data set and also compared to some current distributions.

2. System Descriptions

In each distribution procedure, we consider the sample sizes n that are independent identically distributed (i.i.d.). The distributions take into account the current distributions and a new distribution that we propose in this paper.

We employ the current distributions for comparisons of model fitting. The current models are composed of the traditional distributions and the new distributions, which have been proposed by recent authors. They are sometimes called a single parametric distribution, which is different from a finite mixture distribution. The current distributions are lognormal (LN), gamma (Gam), Weibull (Wei), generalized Pareto distribution (GP), exponential (Exp), exponential-exponential (EE), gamma-exponential (GE) and Rayliegh-Rayliegh (RR). The cumulative distribution function (cdf) and the probability density function (pdf) are described as follows.

The RR distribution was constructed by Jaroengeratikun et al. (2022). It was developed from the infinite mixture Rayleigh model. The cdf and pdf are in the following form:

$$F(x) = \frac{x^2}{x^2 + t^2}; \quad t > 0, x \geq 0,$$

$$f(x) = \frac{2xt^2}{(x^2+t^2)^2}; \quad t > 0, x \geq 0.$$

2.1. A new distribution

We created a new model called the exponentiated Rayleigh-Rayleigh (ERR) distribution. The distribution is derived from the infinite mixture distribution of the Rayleigh distribution, which was modified to be the exponentiated Rayleigh-Rayleigh distribution under the exponentiated distribution.

Exponentiated Rayleigh-Rayleigh distribution: Suppose a random variable X follows a Rayleigh-Rayleigh distribution. Denote its cdf by $F(x|t)$ where

$$F(x|t) = \frac{x^2}{x^2+t^2}; \quad t > 0, x \geq 0.$$

The cdf of the exponentiated Rayleigh-Rayleigh distribution is

$$G_\alpha(x) = \left(\frac{x^2}{x^2+t^2}\right)^\alpha; \quad t > 0, \alpha > 0, x \geq 0.$$

The pdf is given by

$$g_\alpha(x) = \frac{2\alpha t^2 x^{2\alpha-1}}{(x^2+t^2)^{\alpha+1}}; \quad t > 0, \alpha > 0, x \geq 0.$$

Theorem 1 *Let a random variable X follow an exponentiated Rayleigh-Rayleigh distribution. Then the distribution function $G_\alpha(x)$ is:*

$$G_\alpha(x) = \left(\frac{x^2}{x^2+t^2}\right)^\alpha; \quad t > 0, \alpha > 0, x \geq 0. \tag{1}$$

Proof: It is easy to see that $G_\alpha(x)$ is an increasing function and a right continuous function and that it also has $\lim_{x\to 0} G_\alpha(x) = 0$ and $\lim_{x\to\infty} G_\alpha(x) = 1$.

2.1.1 Properties of ERR distribution

This section presents the properties of the distributions of exponentiated Rayleigh-Rayleigh (ERR), such as survival, hazard functions, and value-at-risk (VaR).

Survival function:

$$S(x) = 1 - \left(\frac{x^2}{x^2+t^2}\right)^\alpha. \tag{2}$$

Hazard function:

$$h(x) = \frac{2\alpha t^2 x^{(2\alpha-1)}}{\left(1 - \left(\frac{x^2}{x^2+t^2}\right)^\alpha\right)(x^2+t^2)^{(\alpha+1)}}. \tag{3}$$

Value-at-risk:

$$\pi_p = \sqrt{\frac{t^2}{p^{-1/\alpha}-1}}. \tag{4}$$

The theorem below shows that the ERR distribution has a heavier tail based on exponential distribution.

Theorem 2 *The ERR distribution has a heavier tail based on exponential distribution.*

Proof: Consider the ratio

$$\lim_{x\to\infty} \frac{S_{\text{ERR}}(x)}{S_{\text{Exp}}(x)} = \lim_{x\to\infty} \frac{f_{\text{ERR}}(x)}{f_{\text{Exp}}(x)} = \lim_{x\to\infty} \frac{t^2 \alpha x^{\alpha-1} \exp(\lambda x)}{\lambda (x^2+t^2)^{\alpha+1}}.$$

The limit is infinity since the exponential goes to infinity faster than polynomials. So, the ERR has a heavier tail than the exponential. Therefore, the ERR is a heavy-tailed distribution.

2.1.2 Parameters estimation

Let $X_i, i = 1, 2, \ldots, n$ be an i.i.d. random sample of size n from the ERR distribution. The likelihood function can be written as follows:

$$L(t, \alpha) = \prod_{i=1}^{n} \frac{2\alpha t^2 x_i^{2\alpha-1}}{(x_i^2 + t^2)^{\alpha+1}}$$

and the natural log-likelihood function is in the form

$$\ln L(t, \alpha) = n \ln 2 + n \ln \alpha + 2n \ln t + (2\alpha - 1) \sum_{i=1}^{n} \ln x_i - (\alpha + 1) \sum_{i=1}^{n} \ln (x_i^2 + t^2).$$

Taking the partial derivatives of the log-likelihood function with respect to the parameters are as follows:

$$\frac{\partial}{\partial t} \ln L(t, \alpha) = \frac{2n}{t} - 2t(\alpha + 1) \sum_{i=1}^{n} \frac{1}{(x_i^2 + t^2)},$$

$$\frac{\partial}{\partial \alpha} \ln L(t, \alpha) = \frac{n}{\alpha} + 2 \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln (x_i^2 + t^2).$$

We estimate $\hat{t}$ and $\hat{\alpha}$ for parameter 1, t and parameter 2, α by setting $\frac{\partial}{\partial t} \ln L(t, \alpha) = 0$ and $\frac{\partial}{\partial \alpha} \ln L(t, \alpha) = 0$. Thus,

$$\hat{t} = \frac{n}{t(\alpha + 1) \sum_{i=1}^{n} \frac{1}{(x_i^2 + t^2)}},$$

and

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \ln (x_i^2 + t^2) - 2 \sum_{i=1}^{n} \ln x_i}.$$

The equations can be solved numerically by using a fixed-point iteration method.

2.2. Peaks Over Threshold (POT) Method

The Peak Over Threshold (POT) method is one method in the extreme value theorem (EVT) [see in Embrechts et al. (1997)]. The data considered exceed a given threshold u, and the GPD is always represented for modelling in the tail distribution based on extreme event data. Let basic losses data $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with distribution functions F. The order data are denoted by $X_{1,n} \leq X_{2,n} \leq \cdots \leq X_{n,n}$. Right endpoint $x_F = \sup\{x; F(x) < 1\}$. For all $u < x_F$, the function

$$F_u(x) = P\{X - u \leq x | X > u\}, \quad x \geq u \tag{5}$$

is called the distribution function of exceedances above threshold u. By the conditional probabilities, F_u can also be defined as

$$F_u(x) = \begin{cases} \dfrac{F(u + x) - F(u)}{1 - F(u)}, & \text{if } x \geq 0, \\ 0 & \text{else.} \end{cases}$$

Let $Y = X - u$ for $X > u$ and for n observed variables $X_1, X_2, X_3, \ldots, X_n$, we can write $Y_j = X_i - u$ such that i is the index of the j^{th} exceedance, $j = 1, 2, \ldots, n_u$. The exceedances data $Y_1, Y_2, \ldots, Y_{n_u}$ are independent.

We consider the various thresholds u for model fitting in tailed distribution for simulation data and the Danish fire claims. We vary threshold $u = 1, 2, \ldots, m$ in millions of Thai Baht depending on the distributed sample and the sample size. Table 2 shows the suitable threshold u (or m) regarding the distributions and the sample size n.

2.3. Measurements of Model Fitting

There are two goodness of fit tests (GOF) (see in Klugman et al. (2008)) for model fitting, which are the Kolmogorov-Smirnov Test (K-S test) and the Anderson-Darling Test (AD test), and two paradoxes in model selections for Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

The K-S test decides if a sample comes from a hypothesized continuous distribution. It is based on the empirical cumulative distribution function (ECDF) and denoted by

$$F_X^n(x) = \frac{1}{n}[\text{Number of observations } x_i \leq x].$$

The K-S test statistic is defined by

$$D = \sup_x |F_X^n(x) - F_X(x)|.$$

The AD test statistic is defined as

$$\text{AD} = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left(\ln F(x_i) + \ln\left(1 - F(x_{n-i+1})\right)\right)$$

where F_X is the theoretical cumulative distribution of the distribution being tested and n is the number of data points, the number of observations, or, equivalently, the sample size.

The Akaike information criterion (AIC)

$$\text{AIC} = 2k - 2\text{ln}L(\theta),$$

where k is the number of parameters estimated and $\ln L(\theta)$ is the log-likelihood function.

The Bayesian information criterion (BIC)

$$\text{BIC} = -2\ln L(\theta) + k\ln(n),$$

where $\ln L(\theta)$ is the log-likelihood function and n is the number of observations.

3. Simulation

The simulation data are generated by the loss distributions that are composed of loglogistic, lognormal, Burr, Pareto and Weibull distributions and the mixture of loss distributions for 2 and 3 components. The simulated data include 200 repetitions of individual data, which are based on imposed parameters according to Table 1. The original sample sizes n were 50, 100, 500, 1,000, 3,000 and 5,000. We consider the exceedance data over threshold u. To accurately model the tailed loss distributions, a high percentile of the distributed sample data is necessary to determine a suitable threshold u as specification on the Table 2. This ensures a sufficiently large sample size of data points exceeding the threshold for model fitting.

Table 1 The parameters of distribution for simulation data

Distribution	Parameters
Loglogistic (LL)	a = 4.6189, b = 20.5570
Lognormal (LN)	μ = 3.0693, σ = 0.3905
Burr	b = 1.0296, g = 1.1342, s =1.2320
Pareto	a = 5.3693, s = 13.8426
Weibull (Wei)	a = 0.9586, b = 3.2920

Table 2 The values of a suitable thresholds u (or m)

	m (million baht)					
Distributed samples	$n = 50$	$n = 100$	$n = 500$	$n = 1000$	$n = 3000$	$n = 5000$
LL	28	34	50	59	74	84
LN	31	36	49	55	65	69
Burr	3	5	18	30	77	110
Pareto	19	22	31	34	41	43
Wei	6	7	14	17	20	23
LL and LN	30	34	47	57	66	75
LL and Burr	24	30	47	56	72	92
LL and Pareto	23	29	42	48	60	72
LL and Wei	23	30	41	53	63	64
LN and Burr	26	31	46	51	67	74
LN and Pareto	26	31	43	51	54	62
LN and Wei	26	31	43	51	54	61
Burr and Pareto	4	6	16	27	43	62
Burr and Wei	5	7	14	24	43	62
Pareto and Wei	5	8	15	18	25	27
LL, LN and Burr	29	34	46	53	68	81
LL, LN and Pareto	30	31	46	52	64	72
LL, LN and Wei	28	33	48	53	64	69
LL, Burr and Pareto	25	27	41	47	64	69
LL, Burr and Wei	21	27	41	47	63	75
LL, Pareto and Wei	23	27	39	48	57	64
LN, Burr and Pareto	25	29	43	48	64	73
LN, Burr and Wei	21	27	41	49	60	67
LN, Pareto and Wei	24	30	43	45	55	58
Burr, pareto and Wei	17	18	25	30	48	49

We fit the distributions of loglogistics (LN), gamma (Gam), Weibull (Wei), generalized Pareto (GP), exponential (Exp), exponential-exponential (EE), gamma-exponential (GE), Rayleigh-Rayleigh (RR) and exponentiated Rayleigh-Rayleigh (ERR) to the simulation data. At a significant level $\alpha = 0.05$, none of the data can be fitted to any of the models with regard to the individual data. For the data under the POT method, the ERR provides a higher p-value than the other comparable distributions, except for the Burr-distributed sample. In particular, for sample size $n \geq$ 1,000, the highest p-value of ERR is obviously based on the exceedance threshold u of 20 to 40 million Baht. The p-value of RR was mostly higher than Exp for the sample sizes $n =$ 50 and 100. We can use AIC and BIC to show that ERR is generally suitable for the data that is generated by the components of lognormal and Burr. The RR is generally less suitable for the data. The ERR provides a better fit for the data when the exceedance data are decreased. Some results are presented in Table 3 - 16.

Table 3 Distribution fitting to mixed components of lognormal and Burr distributed samples with $n =$ 3,000 and threshold $u = 23$ (Number of exceedance is 667.2450 or 77.7585 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		5.9215	10.6055	3.0677	5.5975	0.1025	1.6469	0.9342	0.9210	8.1785
parameter 2			0.5018	1.6203			1.2554	0.0969	9.1874	0.1246
K-S test	D-value	0.0925	0.0324	0.1187	0.1326	0.0561	0.0820	0.0508	0.0469	0.0455
	p-value	0.0003	0.5056	0.0000	0.0000	0.2763	0.0017	0.3300	0.3018	0.2221
AD test	AD test	19.8600	1.1865	15.1804	22.4811	1.6970	8.7762	1.5386	2.3286	2.3416
	p-value	0.0000	0.3343	0.0000	0.0000	0.2090	0.0004	0.2790	0.2226	0.1786
AIC		4,449.4061	4,300.5837	4,447.4839	4,479.5968	4,390.2737	4,397.8387	4,375.8562	4,351.1642	4,306.7014
BIC		4,453.9087	4,309.5891	4,456.4893	4,484.0995	4,394.7764	4,406.8306	4,384.8616	4,360.1696	4,315.7068

Table 4 Distribution fitting to mixed components of lognormal and Burr distributed samples with $n =$ 5,000 and threshold $u = 24$ (Number of exceedance is 1,005.1900 or 79.8962 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		5.7847	10.4711	3.0829	5.4677	0.1038	1.6218	0.9196	0.9094	7.9618
parameter 2			0.4958	1.5741			1.2692	0.0961	8.9951	0.1352
K-S test	D-value	0.0916	0.0296	0.1164	0.1269	0.0541	0.0812	0.0497	0.0457	0.0431
	p-value	0.0000	0.3759	0.0000	0.0000	0.1729	0.0001	0.2070	0.1805	0.1216
AD test	AD test	30.3700	1.5288	22.5054	32.9465	1.9233	13.2044	1.9080	3.1247	3.2151
	p-value	0.0000	0.2252	0.0000	0.0000	0.1194	0.0000	0.1554	0.1192	0.0892
AIC		6,674.9884	6,441.4648	6,661.4144	6,706.3084	6,577.9067	6,577.1918	6,560.5441	6,521.4191	6,451.9172
BIC		6,679.9009	6,451.2899	6,671.2395	6,711.2209	6,582.8192	6,605.0169	6,570.3692	6,531.2442	6,461.7423

Table 5 Distribution fitting to mixed components of lognormal, Burr and Pareto distributed samples with $n = 50$ and threshold $u = 8$ (Number of exceedance is 33.7950 or 32.4100 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		10.2189	3.7520	0.5264	10.2657	0.0820	2.3191	3.0742	1.7264	17.1635
parameter 2			148.9300	397.7646			0.6206	0.2553	13.8458	-0.4147
K-S test	D-value	0.1887	0.1437	0.2643	0.3047	0.3202	0.1508	0.1597	0.1667	0.2532
	p-value	0.2023	0.4859	0.0282	0.0050	0.0045	0.4583	0.3841	0.3249	0.0424
AD test	AD test	1.5336	0.8879	3.8489	5.0287	3.6049	0.8980	1.0573	1.2574	2.3650
	p-value	0.2226	0.4962	0.0167	0.0038	0.0229	0.5191	0.4200	0.3265	0.0834
AIC		226.9776	222.3547	242.5481	258.5933	239.1742	223.0464	224.0610	226.9483	234.4117
BIC		228.4975	225.3946	245.5879	260.1132	240.6941	226.0863	227.1009	229.9882	237.4515

Table 6 Distribution fitting to mixed components of lognormal, Burr and Pareto distributed samples with $n = 50$ and threshold $u = 10$ (Number of exceedance is 33.3600 or 33.2800 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		8.1008	4.4432	0.6886	8.1483	0.0970	2.0828	2.2433	1.4830	13.5352
parameter 2			57.9183	277.3878			0.7409	0.2204	11.5772	-0.3033
K-S test	D-value	0.1436	0.1281	0.2229	0.2739	0.2629	0.1425	0.1530	0.1566	0.2150
	p-value	0.4853	0.6157	0.0950	0.0167	0.0340	0.5251	0.4367	0.3983	0.1209
AD test	AD test	0.8733	0.6701	2.6471	3.9239	2.3929	0.8052	0.9533	1.0568	1.6602
	p-value	0.4911	0.6451	0.0627	0.0131	0.0903	0.5779	0.4748	0.4102	0.1910
AIC		215.9660	214.1474	228.7232	241.9198	225.1156	216.3651	217.1330	218.9968	223.1204
BIC		217.4729	217.1612	231.7371	243.4267	226.6226	219.3790	220.1469	222.0107	226.1342

Table 7 Distribution fitting to mixed components of lognormal, Burr and Pareto distributed samples with $n = 100$ and threshold $u = 9$ (Number of exceedance is 68.1200 or 31.8800 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		9.1482	4.9842	0.6695	9.1914	0.0883	2.2092	2.5646	1.5664	14.0689
parameter 2			53.8358	248.0067			0.6789	0.2295	12.6997	-0.2617
K-S test	D-value	0.1606	0.1133	0.2289	0.2856	0.2876	0.1253	0.1332	0.1491	0.2449
	p-value	0.0780	0.3745	0.0040	0.0001	0.0002	0.3129	0.2175	0.1358	0.0020
AD test	AD test	2.0618	1.0952	6.2962	8.9922	5.7370	1.2050	1.5791	2.0237	4.0651
	p-value	0.1245	0.3758	0.0016	0.0001	0.0030	0.3737	0.2423	0.1464	0.0157
AIC		447.1559	441.2038	476.3205	506.3106	469.5734	443.8729	445.5243	450.7330	463.8524
BIC		449.3769	445.6460	480.7627	508.5317	471.7945	448.3151	449.9664	455.1752	468.2946

Table 8 Distribution fitting to mixed components of lognormal, Burr and Pareto distributed samples with $n = 500$ and threshold $u = 11$ (Number of exceedance is 331.2150 or 33.7570 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		6.9685	5.4933	0.7702	7.0438	0.1036	1.9476	1.7460	1.2638	10.2100
parameter 2			1.4498	34.5602			0.8078	0.1824	10.4217	-0.0694
K-S test	D-value	0.0963	0.0697	0.1554	0.2407	0.2118	0.0860	0.1053	0.1301	0.1988
	p-value	0.0084	0.1136	0.0000	0.0000	0.0000	0.0354	0.0044	0.0003	0.0000
AD test	AD test	2.8586	1.9687	17.0326	31.1480	13.8614	2.4971	5.4348	6.9865	12.2913
	p-value	0.0425	0.1248	0.0000	0.0000	0.0000	0.0747	0.0041	0.0010	0.0000
AIC		2,082.6367	2,076.2958	2,189.6945	2,307.6087	2,172.1180	2,091.4690	2,117.4690	2,133.5565	2,157.7795
BIC		2,086.4394	2,083.9012	2,197.2999	2,311.4114	2,175.9207	2,099.0744	2,124.9089	2,141.1620	2,165.3849

Table 9 Distribution fitting to mixed components of loglogistic, Burr and Weibull distributed samples with $n = 500$ and threshold $u = 16$ (Number of exceedance is 131.8450 or 73.6310 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		6.4702	9.9211	2.6635	6.1750	0.1048	1.7556	1.1598	1.0702	9.4320
parameter 2			0.5959	2.1670			1.1436	0.1227	9.7520	0.0029
K-S test	D-value	6.4702	9.9211	2.6635	6.1750	0.1048	1.7556	1.1598	1.0702	9.4320
	p-value	0.0853	0.0594	0.1403	0.1582	0.0923	0.1025	0.0694	0.0670	0.0801
AD test	AD test	2.4610	0.5948	4.0841	6.4114	1.4247	2.0570	0.7392	0.8418	1.1464
	p-value	0.1367	0.6930	0.0175	0.0023	0.3233	0.1791	0.6573	0.6305	0.4154
AIC		872.7893	857.2610	891.4892	902.4554	862.6280	875.3923	860.7250	859.7545	858.6417
BIC		875.6699	863.0223	897.2505	905.3361	865.5087	881.1536	866.4863	865.5158	864.4030

Table 10 Distribution fitting to mixed components of loglogistic, Burr and Weibull distributed samples with $n = 1{,}000$ and threshold $u = 14$ (Number of exceedance is 298.6950 or 70.1305 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		7.2639	10.1821	2.6979	6.9547	0.0956	1.8820	1.2406	1.1001	10.2043
parameter 2			0.6512	2.2847			1.0826	0.1203	10.7324	0.0051
K-S test	D-value	0.0624	0.0511	0.1482	0.1656	0.0977	0.0941	0.0606	0.0605	0.0879
	p-value	0.2577	0.4520	0.0000	0.0000	0.0523	0.0301	0.4265	0.3728	0.0538
AD test	AD test	3.1775	1.2649	10.7686	17.1538	3.9753	4.4840	1.1436	1.7221	3.8638
	p-value	0.0636	0.3267	0.0000	0.0000	0.0293	0.01091	0.4153	0.3176	0.0364
AIC		2,001.5785	1,978.4945	2,067.4510	2,100.8268	2,008.8331	2,021.9393	1,996.0436	1,992.8487	1,990.3176
BIC		2,005.2776	1,985.8926	2,074.8491	2,104.5258	2,012.5322	2,029.3374	2,003.4417	2,000.2468	1,997.7158

Table 11 Distribution fitting to mixed components of loglogistic, Burr and Weibull distributed samples with $n = 1{,}000$ and threshold $u = 16$ (Number of exceedance is 236.3100 or 73.6690 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		6.3613	9.9573	2.7840	6.0666	0.1031	1.7367	1.0934	1.0234	8.9870
parameter 2			0.5769	1.9521			1.1669	0.1146	9.6656	0.0520
K-S test	D-value	0.0773	0.0459	0.1332	0.1459	0.0807	0.0932	0.0617	0.0588	0.0715
	p-value	0.1383	0.6374	0.0008	0.0001	0.2330	0.0588	0.4770	0.4521	0.2110
AD test	AD test	4.5693	0.7939	7.7118	12.0030	1.9246	3.8203	1.0585	1.4627	2.0250
	p-value	0.0234	0.5578	0.0004	0.0000	0.1841	0.0353	0.4704	0.3990	0.1797
AIC		1,742.3105	1,707.0403	1,774.4234	1,794.7737	1,732.9384	1,745.8634	1,725 .8940	1,720.4858	1,712.6398
BIC		1,745.8832	1,714.1858	1,781.5688	1,798.3464	1,736.5111	1,753.0088	1,733.0394	1,727.6312	1,719.7852

Table 12 Distribution fitting to mixed components of loglogistic, Burr and Weibull distributed samples with $n = 3{,}000$ and threshold $u = 16$ (Number of exceedance is 790.7500 or 73.64167 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		6.4040	10.0388	2.8419	6.1099	0.1019	1.7453	1.0736	1.0002	8.8511
parameter 2			0.5728	1.8740			1.1695	0.1101	9.7208	0.0783
K-S test	D-value	0.0701	0.0345	0.1286	0.1445	0.0614	0.0845	0.0498	0.0511	0.0657
	p-value	0.0041	0.3453	0.0000	0.0000	0.0375	0.0003	0.1837	0.1273	0.0115
AD test	AD test	12.2895	1.6343	22.6506	35.4377	4.2476	10.5504	2.1039	3.3584	5.8775
	p-value	0.0000	0.2071	0.0000	0.0000	0.0184	0.0001	0.1462	0.0817	0.0083
AIC		5,240.3516	5,133.0991	5,337.1641	5,398.7337	5,201.4859	5,254.5152	5,194.1692	5,185.1255	5,155.8372
BIC		5,245.0244	5,142.4448	5,346.5098	5,403.4065	5,206.1588	5,263.8609	5,203.5149	5,194.4712	5,165.1828

Table 13 Distribution fitting to mixed components of lognormal, Burr and Weibull distributed samples with $n = 3{,}000$ and threshold $u = 23$ (Number of exceedance is 445.8100 or 85.13967 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		5.8801	10.4428	2.9835	5.5663	0.1048	1.6425	0.9629	00.9470	8.2938
parameter 2			0.5075	1.6588			1.2475	0.1025	9.1159	0.0980
K-S test	D-value	0.0933	0.0355	0.1198	0.1343	0.0592	0.0832	0.0517	0.0462	0.0459
	p-value	0.0034	0.6309	0.0000	0.0000	0.3796	0.0133	0.4802	0.4770	0.3998
AD test	AD test	12.9886	0.8607	10.1989	15.3207	1.2196	5.7584	1.0541	1.5065	1.5209
	p-value	0.0000	0.4883	0.0000	0.0000	0.3288	0.0042	0.4637	0.4199	0.3507
AIC		2,960.6777	2,865.2209	2,963.5861	2,986.0422	2,916.0575	2,928.4901	2,905.8275	2,890.4733	2,866.6575
BIC		2,964.7770	2,873.4194	2,971.7846	2,990.1414	2,920.1568	2,936.6886	2,914.0260	2,898.6719	2,874.8560

Table 14 Distribution fitting to mixed components of loglogistic, Burr and Weibull distributed samples with $n = 5{,}000$ and threshold $u = 16$ (Number of exceedance is 1,321.5450 or 73.5691 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		6.4014	9.9918	2.8005	6.1059	0.1015	1.7462	1.0697	0.9924	8.7761
parameter 2			0.5747	1.8853			1.1632	0.1094	9.7049	0.0861
K-S test	D-value	0.0676	0.0313	00.1273	0.1435	0.0582	0.0815	0.0486	0.0496	0.0642
	p-value	0.0002	0.1853	0.0000	0.0000	0.0074	0.0000	0.0643	0.0361	0.0006
AD test	AD test	19.9164	2.5492	37.7341	59.3852	6.6539	17.1463	2.9576	4.9234	10.0981
	p-value	0.0000	0.0724	0.0000	0.0000	0.0014	0.0000	0.0405	0.0127	0.0002
AIC		8,747.2297	8,571.5721	8,912.6188	9,018.6866	8,701.8667	8,768.0334	8,687.0063	8,669.4477	8,613.5592
BIC		8,752.4161	8,581.9450	8,922.9917	9,023.8730	8,707.0532	8,778.4063	8,697.3792	8,679.8206	8,623.9321

Table 15 Distribution fitting to mixed components of lognormal, Burr and Pareto distributed samples with $n = 5{,}000$ and threshold $u = 23$ (Number of exceedance is 853.0800 or 82.9384 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		5.4881	10.3187	3.0450	5.1802	0.1091	1.5647	0.9083	0.9121	7.6745
parameter 2			0.4791	1.5201			1.2913	0.1001	8.5903	0.1296
K-S test	D-value	0.0983	0.0287	0.1120	0.1245	0.0497	0.0802	0.0439	0.0394	0.0356
	p-value	0.0000	0.5012	0.0000	0.0000	0.3032	0.0003	0.3596	0.3517	0.3195
AD test	AD test	29.8572	1.1737	17.6394	25.4275	1.4558	10.8147	1.3393	1.9774	1.7360
	p-value	0.0000	0.3291	0.0000	0.0000	0.2476	0.0000	0.3053	0.2633	0.2730
AIC		5,626.2435	5,404.4491	5,581.8597	5,614.2811	5,500.0793	5,529.7241	5,482.8671	5,452.5965	5,405.2747
BIC		5,630.9919	5,413.9460	5,591.3566	5,619.0296	5,504.8278	5,539.2210	5,492.3640	5,462.0934	5,414.7715

Table 16 Distribution fitting to mixed components of lognormal, Burr and Weibull distributed samples with $n = $ 5,000 and threshold $u = $ 23 (Number of exceedance is 743.7600 or 85.1248 percentile)

Measurements of model fitting		Distributions								
		RR	ERR	GE	EE	Exp	LN	Gam	Wei	GP
parameter 1		5.9041	10.5626	3.0530	5.5815	0.1034	1.6439	0.9434	0.9297	8.2082
parameter 2			0.5020	1.6201			1.2548	0.0987	9.1418	0.1149
K-S test	D-value	0.0922	0.0321	0.1184	0.1323	0.0537	0.0820	0.0483	0.0445	0.0441
	p-value	0.0001	0.4541	0.0000	0.0000	0.2734	0.0010	0.3326	0.3001	0.2034
AD test	AD test	21.9665	1.2877	16.8856	25.0702	1.4144	9.6875	1.2529	2.2270	2.4064
	p-value	0.0000	0.2927	0.0000	0.0000	0.2193	0.0002	0.2917	0.2387	0.1724
AIC		4,951.7365	4,786.4164	4,950.9305	4,986.9741	4,878.5667	4,896.4832	4,863.6753	4,836.4781	4,790.9772
BIC		4,956.3479	4,795.6391	4,960.1533	4,991.5855	4,883.1780	4,905.7059	4,872.8981	4,845.7009	4,800.1999

4. Application

The Danish fire claims data are applied for model fitting. They were collected at Copenhagen Reinsurance and comprised 2,167 fire losses from 1980 to 1990. They have been adjusted for inflation to reflect 1985 values and are expressed in millions of Danish Krone (DK). We consider the individual data to exceed the threshold u of 1 to 40 million DK for model fitting under the POT method.

For the individual data, at a significant level $\alpha = 0.05$, the data cannot be fitted to any of the distributions. We can use AIC and BIC to show that the distribution of ERR is the best fit for the data, followed by the distributions of RR, GE, EE, Exp, LN, Gam, Wei and GP. Table 17 shows the estimated parameter values and model fitting test with individual danish fire claims.

Table 17 Distribution fitting to Danish fire data (million DK) $n = $ 2,167

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 2.0259$	$D = 0.19592$	AD $= 89.435$	8,089.178	8,094.859
		p-value < 0.01	p-value < 0.01		
ERR	$t = 3.0021 \times 10^{-2}$	$D = 0.083063$	AD $= 28.554$	7,200.597	7,211.960
	$\alpha = 3.0389 \times 10^{3}$	p-value < 0.01	p-value < 0.01		
GE	$b = 6.8877 \times 10^{-3}$	$D = 0.21414$	AD $= 232.91$	8,541.412	8,552.775
	$\alpha = 2.6661 \times 10^{2}$	p-value < 0.01	p-value < 0.01		
EE	$b = 2.1208$	$D = 0.32043$	AD $= 294.31$	9,919.146	9,924.827
		p-value < 0.01	p-value < 0.01		
Exp	$\lambda = 0.2954$	$D = 0.25578$	AD > 300	9,620.793	9,626.474
		p-value < 0.01	p-value < 0.01		
LN	$\mu = 0.7870$	$D = 0.13746$	AD $= 87.193$	8,119.795	8,131.157
	$\sigma = 0.7166$	p-value < 0.01	p-value < 0.01		
Gam	$a = 1.2977$	$D = 0.20188$	AD > 300	9,538.191	9,549.554
	$r = 0.3834$	p-value < 0.01	p-value < 0.01		
Wei	$a = 0.9586$	$D = 0.2732$	AD > 300	9,611.243	9,622.605
	$b = 3.2920$	p-value < 0.01	p-value < 0.01		
GP	$b = 2.5778$	$D = 0.3124$	AD $= 208.34$	9,249.666	9,261.029
	$s = 0.1864$	p-value < 0.01	p-value < 0.01		

For the individual data under the POT method, at a significant level $\alpha = 0.05$, the data are mostly fitted to all the distributions, except RR and Exp, which do not fit the exceedance data for thresholds $u = 6, 7, 8, 9, 10$ million DK, and the exceedance data on threshold $u = 8, 9$ million Krone cannot be fitted by the Gam. We can use AIC and BIC to show that the distribution of ERR is the best fit for the exceedance data on threshold $u = 6$ million DK, followed by the distributions of GP, EE, LN, GE, Wei, Gam, Exp and RR. Tables 18 - 22 show the estimated parameters' values and model fitting test with the POT method based on $u = 6, 7, 8, 9$ and 10 million DK, respectively.

Table 18 Distribution fitting to Danish fire data over threshold $u = 6$ million DK (Number of exceedance data is 186 or 91.4300 percentile)

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 4.7382$	$D = 0.15599$ p-value < 0.01	AD $= 16.595$ p-value < 0.01	1,308.247	1,311.473
ERR	$t = 12.6114$ $\alpha = 0.3638$	$D = 0.039924$ p-value $= 0.9282$	AD $= 0.27409$ p-value $= 0.9562$	1,207.803	1,214.254
GE	$b = 3.8477$ $\alpha = 1.1118$	$D = 0.07714$ p-value $= 0.2183$	AD $= 1.5233$ p-value $= 0.1695$	1,221.153	1,227.605
EE	$b = 4.4626$	$D = 0.081633$ p-value $= 0.1676$	AD $= 1.7224$ p-value $= 0.1313$	1,219.708	1,222.934
Exp	$\lambda = 0.0893$	$D = 0.14471$ p-value < 0.01	AD $= 10.843$ p-value < 0.01	1,272.741	1,275.966
LN	$\mu = 1.4130$ $\sigma = 1.5543$	$D = 0.079892$ p-value $= 0.1860$	AD $= 1.4595$ p-value $= 0.1864$	1,221.513	1,227.965
Gam	$a = 0.6138$ $r = 0.0548$	$D = 0.089372$ p-value $= 0.1025$	AD $= 2.3554$ p-value $= 0.05912$	1,238.782	1,245.234
Wei	$a = 0.7122$ $b = 8.5449$	$D = 0.058513$ p-value $= 0.5474$	AD $= 1.0903$ p-value $= 0.3131$	1,221.986	1,228.437
GP	$b = 5.8444$ $s = 0.4704$	$D = 0.035597$ p-value $= 0.9725$	AD $= 0.32851$ p-value $= 0.9152$	1,207.654	1,214.106

Table 19 Distribution fitting to Danish fire data over threshold $u = 7$ million DK (Number of exceedance data is 157 or 92.7750 percentile)

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 5.2880$	$D = 0.14398$ p-value < 0.01	AD $= 12.731$ p-value < 0.01	1,124.127	1,127.183
ERR	$t = 13.1619$ $\alpha = 0.3791$	$D = 0.029544$ p-value $= 0.9992$	AD $= 0.21483$ p-value $= 0.9858$	1,047.130	1,053.242
GE	$b = 4.0120$ $\alpha = 1.1625$	$D = 0.081164$ p-value $= 0.2522$	AD $= 1.5951$ p-value $= 0.1554$	1,059.616	1,065.729
EE	$b = 4.9459$	$D = 0.087754$ p-value $= 0.1781$	AD $= 1.8055$ p-value $= 0.1179$	1,058.530	1,061.586
Exp	$\lambda = 0.0820$	$D = 0.15032$ p-value < 0.01	AD $= 8.7519$ p-value < 0.01	1,101.218	1,104.274
LN	$\mu = 1.5208$ $\sigma = 1.5073$	$D = 0.084921$ p-value $= 0.2075$	AD $= 1.4758$ p-value $= 0.1824$	1,055.918	1,062.030
Gam	$a = 0.6268$ $r = 0.0514$	$D = 0.099789$ p-value $= 0.08771$	AD $= 2.228$ p-value $= 0.06914$	1,075.620	1,081.733
Wei	$a = 0.7199$ $b = 9.3968$	$D = 0.066928$ p-value $= 0.4828$	AD $= 1.1371$ p-value $= 0.2926$	1,061.371	1,067.484
GP	$b = 6.4819$ $s = 0.4545$	$D = 0.038747$ p-value $= 0.9725$	AD $= 0.37073$ p-value $= 0.877$	1,047.481	1,053.594

Table 20 Distribution fitting to Danish fire data over threshold $u = 8$ million DK (Number of exceedance data is 131 or 93.95478 percentile)

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 6.0937$	$D = 0.10715$	AD $= 5.5418$	938.3103	941.1855
		p-value $= 0.09878$	p-value < 0.01		
ERR	$t = 12.3312$	$D = 0.0351$	AD $= 0.1898$	904.2095	909.9598
	$\alpha = 0.4577$	p-value $= 0.9970$	p-value $= 0.9930$		
GE	$b = 3.3413$	$D = 0.087815$	AD $= 1.4594$	914.7029	920.4533
	$\alpha = 1.5335$	p-value $= 0.2646$	p-value $= 0.1865$		
EE	$b = 5.8840$	$D = 0.10406$	AD $= 2.4577$	917.8125	920.6877
		p-value $= 0.1172$	p-value $= 0.05223$		
Exp	$\lambda = 0.0740$	$D = 0.15312$	AD $= 6.0576$	946.2302	949.1054
		p-value < 0.01	p-value < 0.01		
LN	$\mu = 1.7289$	$D = 0.065542$	AD $= 0.8466$	909.8917	915.6421
	$\sigma = 0.6269$	p-value $= 0.6269$	p-value $= 0.4484$		
Gam	$a = 0.6917$	$D = 0.11811$	AD $= 2.7103$	934.5987	940.3491
	$r = 0.0512$	p-value $= 0.05172$	p-value $= 0.03858$		
Wei	$a = 0.7572$	$D = 0.083142$	AD $= 1.7163$	922.8568	928.6072
	$b = 10.9075$	p-value $= 0.3255$	p-value $= 0.1324$		
GP	$b = 7.6241$	$D = 0.053655$	AD $= 0.42064$	905.3603	911.1107
	$s = 0.4088$	p-value $= 0.8451$	p-value $= 0.8278$		

Table 21 Distribution fitting to Danish fire data over threshold $u = 9$ million DK (Number of exceedance data is 117 or 94.6008 percentile)

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 6.1737$	$D = 0.099108$	AD $= 4.7515$	804.9825	843.7447
		p-value $= 0.2006$	p-value < 0.01		
ERR	$t = 12.1574$	$D = 0.045445$	AD $= 0.29302$	815.4841	821.0084
	$\alpha = 0.4746$	p-value $= 0.9691$	p-value $= 0.9432$		
GE	$b = 2.9581$	$D = 0.082254$	AD $= 1.1242$	821.6893	827.2137
	$\alpha = 1.7161$	p-value $= 0.4071$	p-value $= 0.2981$		
EE	$b = 6.0210$	$D = 0.10286$	AD $= 2.2483$	826.2059	828.9681
		p-value $= 0.1681$	p-value $= 0.06746$		
Exp	$\lambda = 0.0710$	$D = 0.16749$	AD $= 6.1338$	854.7936	857.5558
		p-value < 0.01	p-value < 0.01		
LN	$\mu = 1.7681$	$D = 0.049741$	AD $= 0.5667$	816.8050	822.3294
	$\sigma = 1.3317$	p-value $= 0.9343$	p-value $= 0.6797$		
Gam	$a = 0.6908$	$D = 0.12518$	AD $= 2.8248$	844.5309	850.0552
	$r = 0.0491$	p-value $= 0.05111$	p-value $= 0.0337$		
Wei	$a = 0.7543$	$D = 0.086363$	AD $= 1.7786$	833.2919	838.8163
	$b = 11.2761$	p-value $= 0.3474$	p-value $= 0.1221$		
GP	$b = 7.7166$	$D = 0.051331$	AD $= 0.47032$	816.0017	821.5261
	$s = 0.4267$	p-value $= 0.9175$	p-value $= 0.7769$		

Table 22 Distribution fitting to Danish fire data over threshold $u = 10$ million DK (Number of exceedance data is 109 or 94.9700 percentile)

		Measurements of model fitting			
Distributions	Estimates	K-S test	AD test	AIC	BIC
RR	$t = 5.7689$	$D = 0.1255$ p-value $= 0.06453$	AD $= 7.7578$ p-value < 0.01	807.0222	809.7135
ERR	$t = 14.4421$ $\alpha = 0.3767$	$D = 0.036582$ p-value $= 0.9986$	AD $= 0.19696$ p-value $= 0.9912$	753.4739	758.8566
GE	$b = 4.8875$ $\alpha = 1.0883$	$D = 0.082735$ p-value $= 0.4446$	AD $= 1.1452$ p-value $= 0.2892$	761.5034	766.8861
EE	$b = 5.4898$	$D = 0.086271$ p-value $= 0.3918$	AD $= 1.2665$ p-value $= 0.2435$	759.7266	762.4179
Exp	$\lambda = 0.0710$	$D = 0.18005$ p-value < 0.01	AD $= 7.6352$ p-value < 0.01	796.5842	799.2755
LN	$\mu = 1.6136$ $\sigma = 1.5797$	$D = 0.078153$ p-value $= 0.5184$	AD $= 1.2608$ p-value $= 0.2454$	764.7827	770.1654
Gam	$a = 0.5995$ $r = 0.0426$	$D = 0.11469$ p-value $= 0.1137$	AD $= 2.1328$ p-value $= 0.07785$	775.0911	780.4738
Wei	$a = 0.7014$ $b = 10.5252$	$D = 0.082115$ p-value $= 0.4543$	AD $= 1.1545$ p-value $= 0.2854$	764.2895	769.6722
GP	$b = 6.9758$ $s = 0.4968$	$D = 0.043277$ p-value $= 0.9868$	AD $= 0.2662$ p-value $= 0.9611$	753.7860	759.1687

5. Conclusion and Discussion

5.1. Conclusion

At the significant level $\alpha = 0.05$, not all simulation data can be fitted by any of the models. For the data under the POT method, the ERR provides a higher p-value than the others, except for the Burr distributed sample only. The p-value of ERR is higher than the others for sample size $n \geq 1{,}000$ based on exceedance threshold u of 20 to 40 million Baht. By using AIC and BIC, the ERR is mostly an appropriate model for the data that is generated by the components of Lognormal and Burr. The ERR is a better fit for the data when the exceedance data is less.

At a significant level $\alpha = 0.05$, the Danish Fire claims data cannot be fitted to any of the distributions. For the individual data under the POT Danish fire claims data, the data are mostly fitted by all the distributions. Using AIC and BIC, the ERR distribution is the best fit for the data, following the distributions of GP, EE, LN, GE, Wei, Gam, Exp and RR.

5.2. Discussion

The ERR is suitable for the remaining data because of the small sizes of the simulated data and the Danish fire data. For further research, claim frequency should be studied, and a new compound distribution should be constructed. This would benefit insurance pricing and provide a reasonable price and credibility for some analysis, such as the insurer surplus.

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