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# Acceptance Sampling Plans Based on Truncated Life Tests for the Nadarajah-Haghighi Distribution

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## Abstract

In this paper, we develop a time truncated single acceptance sampling plan when the lifetime follows the Nadarajah-Haghighi distribution. We obtain the minimum sample size necessary to ensure a certain mean lifetime for various selected acceptance numbers, consumer's confidence levels and values of the ratio of the experimental time to the specified mean lifetime. The operating characteristic function and the associated producers risks are also studied. A numerical example is provided to illustrate some results of the paper.

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**Keywords:** Acceptance sampling plan, consumer and producer's risk, Nadarajah-Haghighi distribution, operating characteristic function value

## 1. Introduction

In reliability sampling plans, a time truncated sampling plan is used to check whether a product can be accepted or not. In such a plan, the number of failures leads to the final decision. Due to several conatraints like time and cost, checking all products is hardly possible or it is not possible. Thus, one can think of a test based on a random sample drawn from the lot. The test is ended and the decision is made by a pre-specified time $t$. Let $P^\star$ be the consumers confidence level, then the goal is to find a confidence limit on the mean life of products, $\mu$ and to set a specified mean life, $\mu_0$, with a probability of at least $P^\star$.

Several types of acceptance sampling plans are developed in the literature like single, two stage and group acceptance sampling plans. Here, we discuss the single sampling plan, that includes the following parameters: (i) the number of items $n$ on test, (ii) the acceptance number $c$ and (iii) the ratio $t/\mu_0$. In such a plan, the lot is accepted if and only if the observed number of failures does not become bigger than $c$. One can end the test whenever the number of failures gets more than $c$ before the time $t$ and decide to reject the lot. The single sampling plan has been proposed for many lifetime distributions, see for example Sobel and Tischendrof (1959) for the exponential model, Goode and Kao (1961) for the Weibull model, Gupta and Gupta (1961) for the gamma model, Gupta (1962) for the log-normal model, Rao et al. (2008) for the Marshall-Olkin extended Lomax model, Aslam et al. (2010) for the generalized exponential model, Lu et al. (2013) for the half normal model and Al-Nasser and Al-Omari (2013) for the exponentiated Fréchet model. Recently, Gui and Aslam (2017), Al-Omari and Al-Hadhrami (2018), Hamurkaroglu et al. (2020), Sahaa et al. (2021), Mahmood et al. (2021) and Al-Nasser and Ahsan ul Haq (2021) discussed the single sampling plan for the weighted exponential, extended exponential, compound Weibull-exponential, transmuted Rayleigh, Topp-Leone Gompertz and power Lomax models, respectively.

In this paper, we provide the single acceptance sampling plan when the lifetime distribution follows a Nadarajah-Haghighi (NH) distribution with the following probability density function (PDF)

$$f(t,\alpha,\lambda) = \alpha\lambda(1+\lambda t)^{\alpha-1}\exp\{1-(1+\lambda t)^{\alpha}\}, \quad t>0, \tag{1}$$

whose corresponding cummulative density function (CDF) is given by

$$F(t,\alpha,\lambda) = 1-\exp\{1-(1+\lambda t)^{\alpha}\}, \quad t>0, \tag{2}$$

where $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters, respectively. We write $X \sim NH(\alpha,\lambda)$ if $X$ has the pdf (1). The NH model reduces to the exponential distribution when $\alpha = 1$.

The NH distribution was introduced by Nadarajah and Haaghighi (2011) and it is a special case of a formerly introduced distribution by Dimitrakopoulou et al. (2007). The hazard rate function of the NH model can be increasing or decreasing depending on the value of parameter $\alpha$. The NH distribution enjoys a closed form for its CDF that makes it preferable in comparison with some other two parameter models like the gamma distribution. These advantages as well as the others tempted the researchers to focus on modelling life data using the NH model and many authors worked on the inferential aspects of the NH model, see for instance MirMostafaee et al. (2016), Minić (2020) and Ashour et al. (2020).

The $k$th moment of the NH distribution is given by (see Nadarajah and Haaghighi (2011))

$$E(T^k) = \alpha\,\lambda\,\mathrm{e}\,I(k,0,1),$$

where

$$I(k,0,1) = \frac{1}{\alpha\lambda^{k+1}}\sum_{i=0}^{k}\binom{k}{i}(-1)^{k-i}\Gamma(\frac{i}{\alpha}+1,1),$$

in which $\Gamma(\cdot,\cdot)$ is the incomplete gamma function defined as $\Gamma(a,x) = \int_x^{\infty} t^{a-1}\mathrm{e}^{-t}\mathrm{d}t$. Consequently, the mean of the NH distribution becomes

$$\mu = \frac{1}{\lambda}[\mathrm{e}\,\Gamma(1+\frac{1}{\alpha},1)-1]. \tag{3}$$

The paper is organized as follows. In Section 2, the proposed acceptance sampling plan is presented and the minimum sample sizes to be tested for a preassigned time necessary to assert the mean life to exceed a given value is calculated. Section 3 is devoted to a discussion regarding the operating characteristic values of the plan. We then intend to find the minimum ratio of $\mu/\mu_0$ for the acceptability of a lot with a certain producer's risk in Section 4. An illustrative example in order to discuss the applicability of some theoretical results of the paper is given in Section 5. Finally we end the paper with some conclusions.

## 2. Acceptance Sampling Plans

In this section, we assume that the lifetime follows an $NH(\alpha,\lambda)$, where $\alpha$ is fixed. Let $\mu$ denote the true average lifetime of a product, then a lot is declared to be good if $\mu \geq \mu_0$, otherwise it is called a bad lot. Thus, the consumers' risk is defined as the probability of accepting a bad lot,whereas the producers risk is the probability of rejecting a good lot. In this study, we fixed the consumers risk not to become greater than $1-P^{\star}$, where $0 < P^{\star} < 1$. We suppose that size of the lot is so large that we can use the binomial distribution in our work. The acceptance or rejection of the lot corresponds to the acceptance or rejection of the hypothesis $\mu \geq \mu_0$. From (3), we can say that the hypothesis $\mu \geq \mu_0$ corresponds to $\lambda \leq \lambda_0$, where $\lambda_0 = m/\mu_0$ and $m = \mathrm{e}\,\Gamma(1+\frac{1}{\alpha},1)-1$.

The plan is to set a specified time, $t$ and is characterized by the triplet $(n, c, t/\mu_0)$ including
1- the number of items $n$ to be drawn from the lot,
2- the acceptance number $c$,
3- the ratio $t/\mu_0$.

We intend to find the minimum sample size $(n)$ satisfying the consumer's risk, the probability of accepting a bad lot, is fixed not to exceed $1 - P^{\star}$, when $\mu = \mu_0$. The probability of accepting a lot is given by $\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$ where $p = F(t, \alpha, \lambda)$ =probability of a failure before time $t$ that is given by

$$\begin{aligned} p &= 1 - \exp\{1 - (1 + \lambda t)^{\alpha}\} \\ &= 1 - \exp\{1 - (1 + \frac{mt}{\mu})^{\alpha}\}. \end{aligned}$$

For $\mu = \mu_0$, we have $p_0 = 1 - \exp\{1 - (1 + mt/\mu_0)^{\alpha}\}$. Therefore, the required $n$ is the smallest positive integer that satisfies the following inequality

$$\sum_{i=0}^{c} \binom{n}{i} p_0^i (1 - p_0)^{n-i} \leq 1 - P^{\star}. \tag{4}$$

If the number of observed failures is at most $c$, then from Equation (4) we can make the confidence statement that $F(t, \alpha, \lambda) \leq F(t, \alpha, \lambda_0)$ with probability $P^{\star}$. This indicates that $\lambda \leq \lambda_0$ or equivalently, $\mu \geq \mu_0$.

The minimum values of $n$ satisfying (4) are computed and summarized in Tables 1-2 for $\alpha = 1, 2$, $P^{\star} = 0.75, 0.90, 0.95, 0.99$, and $t/\mu_0 = 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0$. Suppose that the lifetime distribution of the test items follows the NH distribution with shape parameter $\alpha = 2$ and we wish to establish that the mean lifetime is at least 1000 hours with probability $P^{\star} = 0.95$. The life test is terminated at $t = 1500$ hours. Therefore, from Table 2 for $t/\mu_0 = 1.5$ and $c = 3$, the minimum sample size is 8, i.e. 8 items should be put on test. if no more than 3 items fail during 1500 hours, then the experimenter can assert that the true mean lifetime $\mu$ of the items is at least 1000 hours with a confidence level of 0.95.

Moreover, we plotted the shape of the required minimum sample size versus $t/\mu_0$ for $\alpha = 2,\ c = 4$ and selected values of the confidence level $P^{\star}$ in Figure 1. From Figure 1, we see that the required minimum sample size decreases as the ratio $t/\mu_0$ increases.

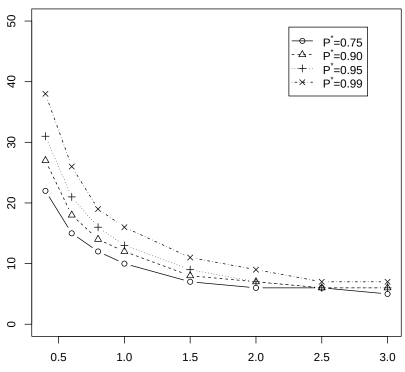


**Figure 1** The minimum sample size versus $t/\mu_0$ for $\alpha = 2$ and $c = 4$

**Table 1** Minimum sample size $n$ to be tested for a time $t$ in order to assert with probability $P^{\star}$ and acceptance number $c$ that $\mu \geq \mu_0$ ($\alpha = 1$)

| | | $t/\mu_0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $P^{\star}$ | $c$ | 0.4 | 0.6 | 0.8 | 1 | 1.5 | 2 | 2.5 | 3 |
| 0.75 | 0 | 4 | 3 | 2 | 2 | 1 | 1 | 1 | 1 |
| | 1 | 8 | 6 | 4 | 4 | 3 | 3 | 2 | 2 |
| | 2 | 11 | 8 | 7 | 6 | 4 | 4 | 3 | 3 |
| | 3 | 15 | 11 | 9 | 7 | 6 | 5 | 5 | 4 |
| | 4 | 18 | 13 | 11 | 9 | 7 | 6 | 6 | 5 |
| | 5 | 22 | 16 | 13 | 11 | 9 | 7 | 7 | 7 |
| | 6 | 25 | 18 | 15 | 13 | 10 | 9 | 8 | 8 |
| | 7 | 28 | 20 | 17 | 14 | 11 | 10 | 9 | 9 |
| | 8 | 32 | 23 | 19 | 16 | 13 | 11 | 10 | 10 |
| 0.9 | 0 | 6 | 4 | 3 | 3 | 2 | 2 | 1 | 1 |
| | 1 | 11 | 8 | 6 | 5 | 4 | 3 | 3 | 2 |
| | 2 | 15 | 10 | 8 | 7 | 5 | 4 | 4 | 4 |
| | 3 | 19 | 13 | 11 | 9 | 7 | 6 | 5 | 5 |
| | 4 | 23 | 16 | 13 | 11 | 8 | 7 | 6 | 6 |
| | 5 | 26 | 19 | 15 | 13 | 10 | 8 | 8 | 7 |
| | 6 | 30 | 21 | 17 | 14 | 11 | 10 | 9 | 8 |
| | 7 | 34 | 24 | 19 | 16 | 13 | 11 | 10 | 9 |
| | 8 | 37 | 27 | 21 | 18 | 14 | 12 | 11 | 10 |
| 0.95 | 0 | 8 | 5 | 4 | 3 | 2 | 2 | 2 | 1 |
| | 1 | 13 | 9 | 7 | 6 | 4 | 3 | 3 | 3 |
| | 2 | 17 | 12 | 9 | 8 | 6 | 5 | 4 | 4 |
| | 3 | 21 | 15 | 12 | 10 | 7 | 6 | 6 | 5 |
| | 4 | 25 | 18 | 14 | 12 | 9 | 8 | 7 | 6 |
| | 5 | 29 | 21 | 16 | 14 | 11 | 9 | 8 | 7 |
| | 6 | 33 | 23 | 19 | 16 | 12 | 10 | 9 | 9 |
| | 7 | 37 | 26 | 21 | 18 | 13 | 12 | 10 | 10 |
| | 8 | 41 | 29 | 23 | 19 | 15 | 13 | 12 | 11 |
| 0.99 | 0 | 12 | 8 | 6 | 5 | 4 | 3 | 2 | 2 |
| | 1 | 18 | 12 | 9 | 8 | 6 | 4 | 4 | 3 |
| | 2 | 23 | 16 | 12 | 10 | 7 | 6 | 5 | 5 |
| | 3 | 27 | 19 | 15 | 12 | 9 | 7 | 6 | 6 |
| | 4 | 32 | 22 | 17 | 14 | 11 | 9 | 8 | 7 |
| | 5 | 36 | 25 | 20 | 16 | 12 | 10 | 9 | 8 |
| | 6 | 40 | 28 | 22 | 18 | 14 | 12 | 10 | 9 |
| | 7 | 44 | 31 | 24 | 20 | 15 | 13 | 11 | 11 |
| | 8 | 48 | 34 | 27 | 22 | 17 | 14 | 13 | 12 |

**Table 2** Minimum sample size $n$ to be tested for a time $t$ in order to assert with probability $P^\star$ and acceptance number $c$ that $\mu \geq \mu_0$ ($\alpha = 2$)

| | | $t/\mu_0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $P^\star$ | $c$ | 0.4 | 0.6 | 0.8 | 1 | 1.5 | 2 | 2.5 | 3 |
| 0.75 | 0 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 |
| | 1 | 9 | 6 | 5 | 4 | 3 | 2 | 2 | 2 |
| | 2 | 14 | 9 | 7 | 6 | 4 | 4 | 3 | 3 |
| | 3 | 18 | 12 | 9 | 8 | 6 | 5 | 4 | 4 |
| | 4 | 22 | 15 | 12 | 10 | 7 | 6 | 6 | 5 |
| | 5 | 26 | 18 | 14 | 12 | 9 | 7 | 7 | 6 |
| | 6 | 30 | 21 | 16 | 13 | 10 | 9 | 8 | 7 |
| | 7 | 34 | 23 | 18 | 15 | 11 | 10 | 9 | 8 |
| | 8 | 38 | 26 | 20 | 17 | 13 | 11 | 10 | 9 |
| 0.9 | 0 | 8 | 5 | 4 | 3 | 2 | 2 | 1 | 1 |
| | 1 | 13 | 9 | 7 | 5 | 4 | 3 | 3 | 2 |
| | 2 | 18 | 12 | 9 | 8 | 5 | 4 | 4 | 3 |
| | 3 | 23 | 15 | 12 | 10 | 7 | 6 | 5 | 5 |
| | 4 | 27 | 18 | 14 | 12 | 8 | 7 | 6 | 6 |
| | 5 | 32 | 21 | 17 | 14 | 10 | 8 | 7 | 7 |
| | 6 | 36 | 25 | 19 | 16 | 11 | 9 | 8 | 8 |
| | 7 | 40 | 28 | 21 | 18 | 13 | 11 | 10 | 9 |
| | 8 | 45 | 30 | 24 | 19 | 14 | 12 | 11 | 10 |
| 0.95 | 0 | 10 | 6 | 5 | 4 | 3 | 2 | 2 | 1 |
| | 1 | 16 | 10 | 8 | 6 | 4 | 3 | 3 | 3 |
| | 2 | 21 | 14 | 11 | 9 | 6 | 5 | 4 | 4 |
| | 3 | 26 | 17 | 13 | 11 | 8 | 6 | 5 | 5 |
| | 4 | 31 | 21 | 16 | 13 | 9 | 7 | 6 | 6 |
| | 5 | 35 | 24 | 18 | 15 | 11 | 9 | 8 | 7 |
| | 6 | 40 | 27 | 21 | 17 | 12 | 10 | 9 | 8 |
| | 7 | 44 | 30 | 23 | 19 | 14 | 11 | 10 | 9 |
| | 8 | 49 | 33 | 26 | 21 | 15 | 13 | 11 | 10 |
| 0.99 | 0 | 15 | 10 | 7 | 6 | 4 | 3 | 2 | 2 |
| | 1 | 21 | 14 | 11 | 8 | 6 | 4 | 4 | 3 |
| | 2 | 27 | 18 | 14 | 11 | 7 | 6 | 5 | 4 |
| | 3 | 33 | 22 | 17 | 13 | 9 | 7 | 6 | 5 |
| | 4 | 38 | 26 | 19 | 16 | 11 | 9 | 7 | 7 |
| | 5 | 43 | 29 | 22 | 18 | 12 | 10 | 9 | 8 |
| | 6 | 48 | 32 | 25 | 20 | 14 | 11 | 10 | 9 |
| | 7 | 53 | 36 | 27 | 22 | 16 | 13 | 11 | 10 |
| | 8 | 58 | 39 | 30 | 24 | 17 | 14 | 12 | 11 |

## 3. Operating Characteristic Function

The operating characteristic function is the probability of accepting the lot. A sampling plan is more preferable if its operating characteristics approach more rapidly to one. For the proposed sampling plan $(n, c, t/\mu_0)$, the OC is defined as

$$OC = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}. \tag{5}$$

Note that increasing the ratio $\mu/\mu_0$ will lead to increasing the acceptance probability.

**Table 3** Operating characteristic values of the sampling plan for $c = 3,\ \alpha = 1$ and selected values of $P^\star$

| | | | $\mu/\mu_0$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| $P^\star$ | $n$ | $t/\mu_0$ | 2 | 4 | 6 | 8 | 10 | 12 |
| 0.75 | 15 | 0.4 | 0.7172 | 0.9523 | 0.9867 | 0.9949 | 0.9977 | 0.9988 |
| | 11 | 0.6 | 0.6877 | 0.9448 | 0.9843 | 0.9940 | 0.9972 | 0.9985 |
| | 9 | 0.8 | 0.6592 | 0.9370 | 0.9818 | 0.9930 | 0.9967 | 0.9983 |
| | 7 | 1.0 | 0.7227 | 0.9530 | 0.9868 | 0.9950 | 0.9977 | 0.9988 |
| | 6 | 1.5 | 0.6031 | 0.9189 | 0.9756 | 0.9904 | 0.9955 | 0.9976 |
| | 5 | 2.0 | 0.6053 | 0.9178 | 0.9750 | 0.9901 | 0.9953 | 0.9975 |
| | 5 | 2.5 | 0.4438 | 0.8534 | 0.9509 | 0.9796 | 0.9901 | 0.9946 |
| | 4 | 3.0 | 0.6357 | 0.9224 | 0.9760 | 0.9904 | 0.9954 | 0.9976 |
| 0.9 | 19 | 0.4 | 0.5392 | 0.8999 | 0.9692 | 0.9878 | 0.9942 | 0.9969 |
| | 13 | 0.6 | 0.5534 | 0.9047 | 0.9709 | 0.9885 | 0.9946 | 0.9971 |
| | 11 | 0.8 | 0.4830 | 0.8778 | 0.9612 | 0.9843 | 0.9925 | 0.9960 |
| | 9 | 1.0 | 0.4990 | 0.8837 | 0.9633 | 0.9852 | 0.9930 | 0.9962 |
| | 7 | 1.5 | 0.4397 | 0.8568 | 0.9530 | 0.9807 | 0.9907 | 0.9950 |
| | 6 | 2.0 | 0.3893 | 0.8297 | 0.9418 | 0.9756 | 0.9881 | 0.9935 |
| | 5 | 2.5 | 0.4438 | 0.8534 | 0.9509 | 0.9796 | 0.9901 | 0.9946 |
| | 5 | 3.0 | 0.3106 | 0.7760 | 0.9178 | 0.9641 | 0.9821 | 0.9901 |
| 0.95 | 21 | 0.4 | 0.4555 | 0.8668 | 0.9571 | 0.9826 | 0.9917 | 0.9955 |
| | 15 | 0.6 | 0.4286 | 0.8543 | 0.9523 | 0.9804 | 0.9906 | 0.9949 |
| | 12 | 0.8 | 0.4035 | 0.8418 | 0.9474 | 0.9782 | 0.9895 | 0.9943 |
| | 10 | 1.0 | 0.3987 | 0.8389 | 0.9461 | 0.9777 | 0.9892 | 0.9942 |
| | 7 | 1.5 | 0.4397 | 0.8568 | 0.9530 | 0.9807 | 0.9907 | 0.9950 |
| | 6 | 2.0 | 0.3893 | 0.8297 | 0.9418 | 0.9756 | 0.9881 | 0.9935 |
| | 6 | 2.5 | 0.2311 | 0.7198 | 0.8923 | 0.9518 | 0.9756 | 0.9864 |
| | 5 | 3.0 | 0.3106 | 0.7760 | 0.9178 | 0.9641 | 0.9821 | 0.9901 |
| 0.99 | 27 | 0.4 | 0.2526 | 0.7476 | 0.9071 | 0.9595 | 0.9798 | 0.9889 |
| | 19 | 0.6 | 0.2344 | 0.7323 | 0.8999 | 0.9559 | 0.9779 | 0.9878 |
| | 15 | 0.8 | 0.2178 | 0.7172 | 0.8926 | 0.9523 | 0.9760 | 0.9867 |
| | 12 | 1.0 | 0.2395 | 0.7355 | 0.9012 | 0.9565 | 0.9782 | 0.9880 |
| | 9 | 1.5 | 0.2026 | 0.7000 | 0.8837 | 0.9478 | 0.9735 | 0.9852 |
| | 7 | 2.0 | 0.2303 | 0.7227 | 0.8943 | 0.9530 | 0.9762 | 0.9868 |
| | 6 | 2.5 | 0.2311 | 0.7198 | 0.8923 | 0.9518 | 0.9756 | 0.9864 |
| | 6 | 3.0 | 0.1293 | 0.6031 | 0.8297 | 0.9189 | 0.9573 | 0.9756 |

Tables 3-4 provide the OC function values for the NH distribution adopted from Tables 1-2 for $c = 3$, various values of $P^\star$, $\mu/\mu_0$ and $\alpha = 1, 2$, respectively. The values of OC versus $\mu/\mu_0$ are plotted in Figure 2 for $\alpha = 1$ and $c = 3$. From Tables 3-4 and Figure 2, we observe that the OC function values increase as the ratio $\mu/\mu_0$ increases. If we consider $P^\star = 0.95$, $t/\mu_0 = 0.6,\ c = 3$ and $\mu/\mu_0 = 8$, then from Table 3, the sample size is $15$, and the OC value is obtained as $0.9804$.

**Table 4** Operating characteristic values of the sampling plan for $c = 3,\ \alpha = 2$ and selected values of $P^\star$

| | | | $\mu/\mu_0$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| $P^\star$ | $n$ | $t/\mu_0$ | 2 | 4 | 6 | 8 | 10 | 12 |
| 0.75 | 18 | 0.4 | 0.5831 | 0.9149 | 0.9745 | 0.9900 | 0.9954 | 0.9976 |
| | 12 | 0.6 | 0.6202 | 0.9262 | 0.9783 | 0.9916 | 0.9961 | 0.9980 |
| | 9 | 0.8 | 0.6593 | 0.9371 | 0.9818 | 0.9930 | 0.9968 | 0.9983 |
| | 8 | 1.0 | 0.6092 | 0.9222 | 0.9769 | 0.9910 | 0.9958 | 0.9978 |
| | 6 | 1.5 | 0.6031 | 0.9190 | 0.9756 | 0.9904 | 0.9955 | 0.9977 |
| | 5 | 2.0 | 0.6054 | 0.9179 | 0.9750 | 0.9901 | 0.9954 | 0.9976 |
| | 4 | 2.5 | 0.7408 | 0.9534 | 0.9865 | 0.9948 | 0.9976 | 0.9987 |
| | 4 | 3.0 | 0.6358 | 0.9225 | 0.9760 | 0.9904 | 0.9955 | 0.9976 |
| 0.9 | 23 | 0.4 | 0.3791 | 0.8299 | 0.9427 | 0.9762 | 0.9885 | 0.9938 |
| | 15 | 0.6 | 0.4286 | 0.8544 | 0.9524 | 0.9805 | 0.9907 | 0.9950 |
| | 12 | 0.8 | 0.4036 | 0.8418 | 0.9474 | 0.9783 | 0.9895 | 0.9944 |
| | 10 | 1.0 | 0.3988 | 0.8389 | 0.9462 | 0.9777 | 0.9892 | 0.9942 |
| | 7 | 1.5 | 0.4397 | 0.8569 | 0.9530 | 0.9807 | 0.9907 | 0.9950 |
| | 6 | 2.0 | 0.3893 | 0.8297 | 0.9419 | 0.9756 | 0.9881 | 0.9936 |
| | 5 | 2.5 | 0.4438 | 0.8535 | 0.9510 | 0.9796 | 0.9901 | 0.9947 |
| | 5 | 3.0 | 0.3107 | 0.7761 | 0.9179 | 0.9641 | 0.9821 | 0.9901 |
| 0.95 | 26 | 0.4 | 0.2808 | 0.7690 | 0.9168 | 0.9642 | 0.9823 | 0.9903 |
| | 17 | 0.6 | 0.3214 | 0.7961 | 0.9286 | 0.9697 | 0.9852 | 0.9919 |
| | 13 | 0.8 | 0.3326 | 0.8026 | 0.9313 | 0.9709 | 0.9858 | 0.9923 |
| | 11 | 1.0 | 0.3119 | 0.7890 | 0.9254 | 0.9682 | 0.9844 | 0.9915 |
| | 8 | 1.5 | 0.3047 | 0.7822 | 0.9222 | 0.9666 | 0.9835 | 0.9910 |
| | 6 | 2.0 | 0.3893 | 0.8297 | 0.9419 | 0.9756 | 0.9881 | 0.9936 |
| | 5 | 2.5 | 0.4438 | 0.8535 | 0.9510 | 0.9796 | 0.9901 | 0.9947 |
| | 5 | 3.0 | 0.3107 | 0.7761 | 0.9179 | 0.9641 | 0.9821 | 0.9901 |
| 0.99 | 33 | 0.4 | 0.1271 | 0.6144 | 0.8390 | 0.9248 | 0.9609 | 0.9778 |
| | 22 | 0.6 | 0.1400 | 0.6321 | 0.8488 | 0.9300 | 0.9638 | 0.9796 |
| | 17 | 0.8 | 0.1372 | 0.6278 | 0.8463 | 0.9286 | 0.9630 | 0.9791 |
| | 13 | 1.0 | 0.1809 | 0.6800 | 0.8740 | 0.9430 | 0.9709 | 0.9838 |
| | 9 | 1.5 | 0.2026 | 0.7001 | 0.8838 | 0.9478 | 0.9735 | 0.9853 |
| | 7 | 2.0 | 0.2303 | 0.7227 | 0.8944 | 0.9530 | 0.9763 | 0.9868 |
| | 6 | 2.5 | 0.2311 | 0.7198 | 0.8924 | 0.9519 | 0.9756 | 0.9864 |
| | 5 | 3.0 | 0.3107 | 0.7761 | 0.9179 | 0.9641 | 0.9821 | 0.9901 |

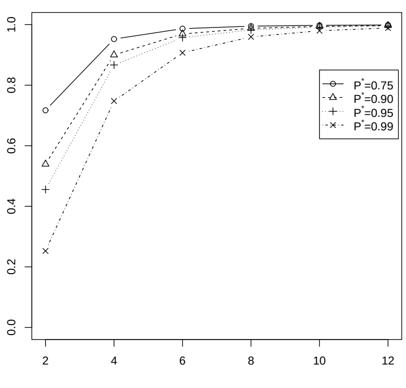


**Figure 2** OC values versus $\mu/\mu_0$ for $\alpha = 1$ and $c = 3$

This reveals that the lot is accepted if less than or equal to 3 items out of 15 items fail before time point $t = 600$ hours and $\mu \geq 8\mu_0 = 8t/0.6 = 8000$ hours, then the lot will be accepted with a probability at least 0.9804.

## 4. Minimum Ratio of $\mu/\mu_0$ for the Acceptability of a Lot

For a specified producer's risk and a sampling plan $(n, c, t/\mu_0)$, one may seek the value of quality level $\mu/\mu_0$ that will ensure the producer's risk not to become greater than $\gamma$. Recall that the producer's risk is the probability of rejection of the lot when it is good i.e. $\mu > \mu_0$, or equivalently $\lambda < \lambda_0$. Then, we seek the minimum values of $\mu/\mu_0$ that satisfy

$$\sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \leq \gamma, \tag{6}$$

or equivalently $OC \geq 1 - \gamma$.

**Table 5** Minimum ratio of $\mu/\mu_0$ for the acceptability of a lot with producer's risk of $\gamma = 0.05$ $(\alpha = 1)$

| | | $t/\mu_0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $P^\star$ | $c$ | 0.4 | 0.6 | 0.8 | 1 | 1.5 | 2 | 2.5 | 3 |
| 0.75 | 0 | 31.20 | 35.10 | 31.20 | 39.00 | 29.25 | 39.00 | 48.74 | 58.49 |
| | 1 | 8.43 | 9.25 | 7.79 | 9.74 | 10.32 | 13.76 | 9.88 | 11.86 |
| | 2 | 4.88 | 5.10 | 5.81 | 6.02 | 5.25 | 7.00 | 5.45 | 6.53 |
| | 3 | 3.94 | 4.14 | 4.33 | 3.92 | 4.74 | 4.77 | 5.97 | 4.69 |
| | 4 | 3.24 | 3.32 | 3.60 | 3.46 | 3.60 | 3.70 | 4.62 | 3.77 |
| | 5 | 2.97 | 3.07 | 3.16 | 3.17 | 3.55 | 3.07 | 3.84 | 4.60 |
| | 6 | 2.67 | 2.71 | 2.87 | 2.96 | 3.01 | 3.35 | 3.32 | 3.99 |
| | 7 | 2.45 | 2.46 | 2.66 | 2.55 | 2.63 | 2.95 | 2.96 | 3.55 |
| | 8 | 2.38 | 2.40 | 2.50 | 2.47 | 2.70 | 2.65 | 2.69 | 3.23 |
| 0.9 | 0 | 46.79 | 46.79 | 46.79 | 58.49 | 58.49 | 77.99 | 48.74 | 58.49 |
| | 1 | 11.81 | 12.64 | 12.33 | 12.58 | 14.61 | 13.76 | 17.20 | 11.86 |
| | 2 | 6.84 | 6.58 | 6.80 | 7.26 | 7.15 | 7.00 | 8.75 | 10.50 |
| | 3 | 5.11 | 5.02 | 5.52 | 5.42 | 5.88 | 6.32 | 5.97 | 7.16 |
| | 4 | 4.26 | 4.24 | 4.42 | 4.50 | 4.40 | 4.80 | 4.62 | 5.54 |
| | 5 | 3.59 | 3.77 | 3.78 | 3.95 | 4.15 | 3.92 | 4.89 | 4.60 |
| | 6 | 3.28 | 3.26 | 3.36 | 3.27 | 3.49 | 4.01 | 4.18 | 3.99 |
| | 7 | 3.06 | 3.07 | 3.07 | 3.07 | 3.43 | 3.50 | 3.68 | 3.55 |
| | 8 | 2.80 | 2.92 | 2.85 | 2.91 | 3.04 | 3.13 | 3.32 | 3.23 |
| 0.95 | 0 | 62.39 | 58.49 | 62.39 | 58.49 | 58.49 | 77.99 | 97.48 | 58.49 |
| | 1 | 14.06 | 14.33 | 14.59 | 15.41 | 14.61 | 13.76 | 17.20 | 20.63 |
| | 2 | 7.82 | 8.05 | 7.78 | 8.50 | 9.03 | 9.54 | 8.75 | 10.50 |
| | 3 | 5.70 | 5.91 | 6.11 | 6.16 | 5.88 | 6.32 | 7.90 | 7.16 |
| | 4 | 4.66 | 4.85 | 4.83 | 5.01 | 5.19 | 5.86 | 5.99 | 5.54 |
| | 5 | 4.05 | 4.23 | 4.09 | 4.34 | 4.75 | 4.73 | 4.89 | 4.60 |
| | 6 | 3.65 | 3.63 | 3.86 | 3.89 | 3.97 | 4.01 | 4.18 | 5.02 |
| | 7 | 3.36 | 3.37 | 3.48 | 3.58 | 3.43 | 4.04 | 3.68 | 4.42 |
| | 8 | 3.15 | 3.17 | 3.20 | 3.13 | 3.37 | 3.59 | 3.91 | 3.98 |
| 0.99 | 0 | 93.58 | 93.58 | 93.58 | 97.48 | 116.98 | 116.98 | 97.48 | 116.98 |
| | 1 | 19.69 | 19.40 | 19.10 | 21.06 | 23.11 | 19.48 | 24.35 | 20.63 |
| | 2 | 10.76 | 10.99 | 10.73 | 10.96 | 10.89 | 12.04 | 11.92 | 14.30 |
| | 3 | 7.46 | 7.67 | 7.87 | 7.63 | 8.12 | 7.84 | 7.90 | 9.48 |
| | 4 | 6.09 | 6.08 | 6.06 | 6.04 | 6.74 | 6.91 | 7.33 | 7.19 |
| | 5 | 5.12 | 5.15 | 5.33 | 5.11 | 5.33 | 5.53 | 5.91 | 5.87 |
| | 6 | 4.50 | 4.55 | 4.60 | 4.51 | 4.91 | 5.29 | 5.01 | 5.02 |
| | 7 | 4.07 | 4.13 | 4.09 | 4.09 | 4.21 | 4.57 | 4.38 | 5.25 |
| | 8 | 3.74 | 3.82 | 3.89 | 3.78 | 4.03 | 4.05 | 4.49 | 4.69 |

**Table 6** Minimum ratio of $\mu/\mu_0$ for the acceptability of a lot with producer's risk of $\gamma = 0.05$ ($\alpha = 2$)

| | | $t/\mu_0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $P^\star$ | $c$ | 0.4 | 0.6 | 0.8 | 1 | 1.5 | 2 | 2.5 | 3 |
| 0.75 | 0 | 29.63 | 26.71 | 23.80 | 29.74 | 22.45 | 29.93 | 37.41 | 44.89 |
| | 1 | 7.32 | 7.12 | 7.78 | 7.57 | 8.10 | 6.35 | 7.94 | 9.52 |
| | 2 | 4.89 | 4.54 | 4.55 | 4.75 | 4.25 | 5.66 | 4.56 | 5.47 |
| | 3 | 3.73 | 3.58 | 3.43 | 3.72 | 3.86 | 3.96 | 3.38 | 4.05 |
| | 4 | 3.15 | 3.09 | 3.19 | 3.20 | 2.99 | 3.14 | 3.93 | 3.34 |
| | 5 | 2.80 | 2.79 | 2.78 | 2.88 | 2.95 | 2.66 | 3.32 | 2.92 |
| | 6 | 2.56 | 2.58 | 2.51 | 2.42 | 2.53 | 2.87 | 2.93 | 2.63 |
| | 7 | 2.39 | 2.32 | 2.32 | 2.30 | 2.24 | 2.57 | 2.65 | 2.42 |
| | 8 | 2.26 | 2.22 | 2.17 | 2.22 | 2.30 | 2.34 | 2.44 | 2.26 |
| 0.9 | 0 | 47.36 | 44.44 | 47.44 | 44.52 | 44.61 | 59.48 | 37.41 | 44.89 |
| | 1 | 10.73 | 10.97 | 11.21 | 9.72 | 11.35 | 10.79 | 13.49 | 9.52 |
| | 2 | 6.37 | 6.21 | 6.05 | 6.62 | 5.69 | 5.66 | 7.08 | 5.47 |
| | 3 | 4.84 | 4.59 | 4.78 | 4.85 | 4.73 | 5.15 | 4.95 | 5.94 |
| | 4 | 3.92 | 3.79 | 3.81 | 3.98 | 3.60 | 3.98 | 3.93 | 4.71 |
| | 5 | 3.49 | 3.32 | 3.48 | 3.47 | 3.41 | 3.31 | 3.32 | 3.99 |
| | 6 | 3.12 | 3.14 | 3.07 | 3.13 | 2.90 | 2.87 | 2.93 | 3.51 |
| | 7 | 2.85 | 2.90 | 2.78 | 2.89 | 2.86 | 2.99 | 3.21 | 3.17 |
| | 8 | 2.72 | 2.61 | 2.70 | 2.55 | 2.56 | 2.71 | 2.92 | 2.92 |
| 0.95 | 0 | 59.18 | 53.31 | 59.26 | 59.29 | 66.78 | 59.48 | 74.35 | 44.89 |
| | 1 | 13.30 | 12.26 | 12.92 | 11.87 | 11.35 | 10.79 | 13.49 | 16.19 |
| | 2 | 7.49 | 7.33 | 7.54 | 7.56 | 7.12 | 7.59 | 7.08 | 8.49 |
| | 3 | 5.51 | 5.26 | 5.22 | 5.41 | 5.58 | 5.15 | 4.95 | 5.94 |
| | 4 | 4.53 | 4.48 | 4.43 | 4.37 | 4.20 | 3.98 | 3.93 | 4.71 |
| | 5 | 3.84 | 3.84 | 3.72 | 3.76 | 3.86 | 3.93 | 4.14 | 3.99 |
| | 6 | 3.49 | 3.42 | 3.44 | 3.37 | 3.27 | 3.38 | 3.59 | 3.51 |
| | 7 | 3.16 | 3.13 | 3.09 | 3.09 | 3.16 | 2.99 | 3.21 | 3.17 |
| | 8 | 2.98 | 2.91 | 2.96 | 2.88 | 2.81 | 3.06 | 2.92 | 2.92 |
| 0.99 | 0 | 88.73 | 88.77 | 82.90 | 88.85 | 88.94 | 89.03 | 74.35 | 89.22 |
| | 1 | 17.56 | 17.38 | 18.05 | 16.15 | 17.80 | 15.13 | 18.91 | 16.19 |
| | 2 | 9.71 | 9.56 | 9.77 | 9.42 | 8.53 | 9.49 | 9.49 | 8.49 |
| | 3 | 7.06 | 6.93 | 7.01 | 6.53 | 6.43 | 6.30 | 6.43 | 5.94 |
| | 4 | 5.61 | 5.64 | 5.36 | 5.54 | 5.38 | 5.60 | 4.98 | 5.97 |
| | 5 | 4.77 | 4.71 | 4.65 | 4.64 | 4.31 | 4.54 | 4.92 | 4.96 |
| | 6 | 4.23 | 4.12 | 4.19 | 4.07 | 3.99 | 3.87 | 4.22 | 4.31 |
| | 7 | 3.84 | 3.81 | 3.71 | 3.67 | 3.75 | 3.81 | 3.74 | 3.85 |
| | 8 | 3.56 | 3.49 | 3.48 | 3.37 | 3.32 | 3.41 | 3.38 | 3.51 |

The minimum values of $\mu/\mu_0$ satisfying (6) are presented in Tables 5-6 with producer's risk of $\gamma = 0.05$ and $\alpha = 1, 2$. Consider a situation where one intends to establish that the mean life $\mu_0$ is at least 1000 hours with probability $P^\star = 0.95$ and desires to end the life test at $t = 600$ hours. If the actual mean life $\mu$ is about 6000 hours (i.e. the value of $\mu/\mu_0$ is about 6), then from Table 5 with the producer's risk 0.05, we find $c = 3$. We also perceive that the required $n$ in Table 1 corresponding to the values of $P^\star = 0.95$, $c = 3$, and $t/\mu_0 = 0.6$ is 15. Hence, a sampling plan $(n, c, t/\mu_0) = (15, 3, 0.6)$ may be taken.

## 5. A Real Data Application

In this section, we consider real data regarding the lifetimes of 20 small electric carts measured in months. These electric carts were used by the manufacturing company for internal transportation and delivery services. The data are as follows (see Zimmer et al. (1998) and Al-Nasser and Ahsan ul Haq (2021)).

$$0.9,\ 1.5,\ 2.3,\ 3.2,\ 3.9,\ 5.0,\ 6.2,\ 7.5,\ 8.3,\ 10.4,\ 11.1,\ 12.6,\ 15.0,\ 16.3,\ 19.3,\ 22.6,\ 24.8,\ 31.5,\ 38.1,\ 53.0.$$

We perform a formal Kolmogorov-Smirnov (K-S) test to check if the NH distribution fits the above data well. The K-S statistic is computed as $D = 0.052344$ with the corresponding $p$-value that equals 1. The maximum likelihood (ML) estimates of the scale and shape parameters are calculated as $\hat{\lambda} = 0.04518172$ and $\hat{\alpha} = 1.31386385$, respectively. Thus, we conclude that the two parameter NH distribution can model the above data perfectly.

Now, consider the null hypothesis $H_0 : \alpha = 2$ versus $H_1 : \alpha \neq 2$. We use the likelihood ratio test to see if we can accept $H_0$. The ML estimate of $\lambda$ under the null hypothesis is obtained as $\lambda_0 = 0.02512025$. Besides, the asymptotic likelihood ratio test statistic is computed as $-2\lambda^*(\boldsymbol{x}) = 0.2657774$ and its corresponding $p$-value is calculated as $0.6061785$, so we can accept $H_0$.

Now, once again we check if the NH distribution with $\alpha = 2$ and $\lambda = \lambda_0$ fits the data. We apply the K-S test, Anderson-Darling (A-D) test and Cramér-von Mises (C-M) test to detecting the suitability of $NH(2, \lambda_0)$ for the data. We also calculate the Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC) and the numerical results are given in Table 7. From Table 7, we see that $NH(2, \lambda_0)$ fits the data very well.

**Table 7** Goodness of fit statistics and the information criteria for the electric carts data

| | K-S | A-D | C-M | AIC | BIC | HQIC |
|---|---|---|---|---|---|---|
| Stataistic | 0.08099 | 0.17115 | 0.02649 | 149.5264 | 150.5221 | 149.7208 |
| $p$-value | 0.9980 | 0.9965 | 0.9882 | | | |

The mean lifetime can be estimated as $\widehat{\mu} = \frac{1}{\lambda_0}[\mathrm{e}\,\Gamma(1.5, 1) - 1] = 15.08488$. So if we assume that the actual mean life is about $15 \times 30 = 450$ days and emphasize that mean life $\mu_0$ is not less than 103 days with probability $P^\star = 0.95$ and we like to end the test at $t = 103$ days, then we note that $\mu/\mu_0$ is about 4.37 and from Table 6 with the producer's risk 0.05, we find out that $c$ equals 4. Next, from Table 2, the minimum sample size is observed to be 13. Summing up, we take a plan $(n, c, t/\mu_0) = (13, 4, 1)$. Therefore, if we put on 13 electric carts on test and no more than 4 items fail within 103 days, then we can declare that the true mean of the electric carts is not less than 103 days with a confidence level 95 percent.

## 6. Conclusions

The Nadarajah-Haghighi model is a well-established rather new model introduced by Nadarajah and Haaghighi (2011) as a generalization of the exponential distribution and it is actually a special case of a three parameter model that had been introduced by Dimitrakopoulou et al. (2007). This model enjoys increasing and decreasing hazard rate functions depending on the value of its shape parameter. In this paper, we developed a single acceptance sampling plan based on the truncated life test for the Nadarajah-Haghighi distribution. The minimum sample size required to guarantee a certain mean lifetime of the test items is provided. The operating characteristic function values are obtained and discussed. Also, some tables are provided for minimum ratio of true mean life to specified life for the acceptability of a lot with certain procedure's risk. Moreover, a real data application is provided for the sake of illustrative purposes. The computations of the paper were done using R (R Core Team, 2022) and some packages therein such as AdequacyModel (Marinho et al., 2016), nleqslv (Hasselman, 2018), goftest (Faraway et al., 2021) and ADGofTest (Bellosta, 2015).

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