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Inventory Model for Non-Instantaneous Deteriorating Items with Stock-Dependent Demand and Preservation Technology Investment

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Abstract

The paper considers an inventory system for non-instantaneous deteriorating items when demand is dependent on the stock level. Time dependent deterioration and partial waiting time dependent backlogging rates are assumed to model the inventory system. Deterioration, although non-instantaneous, is not desirable. Hence some technique that can reduce deterioration is very necessary. However, that will involve a certain amount of cost. This preservation technology investment and its effect on the inventory decision are considered in this paper. The optimum cost function is formulated, and its convexity is proven. The study is supported by hypothetical illustrations that suggest that the minimum cost decreases owing to the preservation technology adopted. Managerial aspects, future scope and the limitations of the study are also discussed alongside the findings and conclusions. It is concluded that although preservation technology involves certain amount of investment, the total cost is reduced by its application.

Keywords: Non-instantaneous and time dependent deterioration, waiting time dependent backlogging, preservation cost, cost optimization**.**

1. Introduction

Deterioration is a phenomenon observed in many stored items. like fruits, vegetables, medicines, volatile liquids, etc. Ghare and Schrader (1963) first developed an economic order quantity model for deteriorating items assuming exponential decay and constant demand rate. However, in real life, the rate of deterioration, especially for perishable food items, is seldom constant. Subsequently, Covert and Philip (1973), Philip (1974) and Tadikamalla (1978) modified the model of Ghare and Schrader (1963) to include variable rate of deterioration. Further, research with focus on time varying deterioration rate has been carried out by researchers like Wee (1997), Benkherouf and Balkhi (1997), Chakrabarty et al. (1998), Skouri and Papachristos (2003), Dye (2013), Tsao and Sheen (2007), Sarkar and Sarkar (2013), to name a few.

Most of the existing literature on inventory for deteriorating items assumes that deterioration occurs as soon as the commodities arrive in inventory. In actual practice, however, goods like

medicines, food stuffs, vegetables and fruits retain their original quality for a span of time before deteriorating. Ouyang et al. (2006) and Wu et al. (2006) first introduced this phenomenon in inventory model and termed it as "non-instantaneous deterioration". They also showed that if the retailer can effectively reduce the deteriorating rate of the item by improving the storage facility, the total annual inventory cost will be lowered. Geetha and Uthayakumar (2016) extended the model of Wu et al. (2006) by considering reciprocal time-dependent partial backlogging rate. Maihami and Abadi (2012a) and Maihami and Abadi (2012b) further extended the model of Ouyang et al. (2006) by considering a price and time dependent demand. To characterize the more practical situation, recently Shah et al. (2013) integrated time varying deterioration and holding cost rates in the inventory model where shortages are not prohibited. The main objective in their model is to find the retailer's replenishment, selling price and advertisement strategies which maximize the retailer's unit time profit.

In the above-mentioned papers, it is assumed that the deterioration rate of items in stock is not controllable. However, in the real market, the retailer can reduce the deterioration rate by investing in preservation technology. A simple example is cold storage that preserves freshness of food items for longer period. Hsu et al. (2010) studied an inventory model for exponentially decaying items when demand occurs at a uniform rate, assuming that the preservation technology cost is fixed per inventory cycle. The assumption is rather unrealistic when new preservation equipment is necessary, as that will involve capital costs. Dye and Hsieh (2012) modified the model by assuming the preservation technology cost to be a function of the length of a replenishment cycle. Dye (2013) investigated an inventory model for non-instantaneous deteriorating items with constant demand rate and time-varying deterioration rate, under the effect of preservation technology investment. Lee et al. (2012) studied a model with preservation technology with stock dependent demand where deterioration was uniform and instantaneous. He and Huang (2013) incorporated the idea of pricing strategies of deteriorating seasonal products along with the preservation technology investment. Zhang et al. (2014) optimised a price policy model with the investment done to preserve items in inventory. A model on instantaneous constant deterioration and stock dependent demand with permissible delay in payments was developed by Singh et al. (2016). Giri et al. (2017) formulated an integrated vendor-buyer inventory model for deteriorating item under preservation technology investment. Mishra et al. (2018) developed a model on permissible delay in payments. Bardhan et al. (2019) studied a model with constant non-instantaneous deterioration and stock dependent demand while preservation technology is applied. Khanna et al. (2020) considered the deterioration rate to be a decreasing function of investment in preservation technology. Price discount facility was incorporated with preservation by De (2020). Mahata et al. (2022) developed a model with preservation technology investment with price dependent demand and constant deterioration rate.

As per the literature review the construction of inventory models with deterioration dates back to 1960s but the inclusion of preservation technology has happened in the last decade. Many assumptions have been considered along with this factor in different papers which are not applicable in real-life. The aim of this paper is to minimize those assumptions like instantaneous deterioration, no shortages, constant demand, constant deterioration rate etc. In this paper, we consider a periodic review inventory model for non-instantaneous deteriorating items, where the demand is stock dependent as long as there is stock on hand, and becomes constant during the shortage period. The deterioration rate when the items start to deteriorate is time dependent and the reduction in deterioration rate caused by preservation technology increases with increase in the preservation technology cost per unit time. During shortage, demand may be lost or may be backlogged with the backlogging proportion being a decreasing function of the waiting time.

Table 1 shows various works done on preservation technology to slow down the rate of deterioration. Although there is a vast literature that encompasses various studies pertinent to non-instantaneous deterioration with deterioration rate being a function of time and a few papers on preservation technology investment with shortages, but the combination of the two has not been researched so far. This paper attempts to bridge this gap, assuming stock dependent demand.

The paper is organized as follows. In Section 2, the notations and assumptions are given. Section 3 deals with formulation of the model. In Section 4, the cost function is studied, and the optimal values of the decision variables, namely the length of a reorder interval and the preservation technology cost, are discussed. Examples are cited in Section 5, and a sensitivity analysis of the model to change in parameters is addressed in Section 6. In Sections 7 and 8, managerial aspects and the limitations of the paper are discussed, respectively. Finally, in Section 9, a discussion on the model, the findings and the future scope are given.

Table 1 Comparison of some existing literature on preservation technology

Source and Year	Non-instantaneous deterioration	Time Dependent Deterioration rate	Stock dependent demand	Shortage
Dye (2013)	Yes	Yes	No	Yes
Lee et al. (2012)	No	No	Yes	Yes
He and Huang (2013)	No	No	No	No
Zhang et al. (2014)	No	No	No	No
Sing et al. (2016)	No	No	Yes	Yes
Giri et al. (2017)	No	Yes	No	No
Mishra et al. (2018)	No	No	No	No
Bardhan et al. (2019)	Yes	No	Yes	No
Khanna et al. (2020)	No	No	Yes	No
De (2020)	No	No	Yes	No
Mahata et al. (2022)	No	No	Yes	No
Present paper	Yes	Yes	Yes	Yes

2. Model Notations and Assumptions

The inventory model is developed based on the following notations and assumptions.

Notations

$\theta(t)$: Deterioration rate.

$[0,T]$: Time horizon.

W : the maximum inventory level for each ordering cycle.

S : the maximum amount of demand backlogged for each ordering cycle.

Q : Order quantity for each ordering cycle.

μ : Time point when deterioration starts. $0 < \mu \leq T$, μ is fixed.

t_1 : Time point at which shortage starts. .

C_S : Ordering cost per unit.

C_1 : Purchase cost per unit.

C_2 : Deterioration cost per unit.

C_3 : Backlogging cost per unit per unit time.

C_4 : Opportunity cost due to lost sales per unit lost.

p : Carrying cost per unit; p is a fraction of the value of one unit in inventory.

$I(t)$: Inventory level at time *t*.

ξ : the preservation technology cost per unit time for reducing deterioration rate in order to preserve the products, where $0 \le \xi \le w$.

w : Maximum capital constraint.

Assumptions

1) There is no lead time.
2) Deterioration rate is a function of time.
3) The replenishment rate is infinite.
4) Non-instantaneous deterioration is considered i.e. the commodities have a span to maintain the freshness and after some time they start to decay.
5) There is no repair or replacement of deteriorated units. The items will be withdrawn from warehouse immediately as they become deteriorated.
6) The reduction in deterioration rate, $m(\xi,t)$, is an increasing function of the preservation technology cost ξ and is also dependent on *t*, where $\lim_{\xi\to\infty} m(\xi,t) = \theta(t)$.
7) Shortages are allowed. During the shortage period, the backlogging rate is a variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time. the smaller would be the backlogging rate. Hence, the proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T-t)$ for the next replenishment. To take care of this situation we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$, where δ, the backlogging parameter, is a positive constant for $t_1 \le t \le T$.
8) The demand rate is a function of the inventory level and is given by

$$\begin{aligned} D(t) &= \alpha + \beta I(t), \quad 0 \le t \le t_1 \\ &= \alpha, \quad t_1 \le t \le T, \end{aligned}$$

where $\alpha > 0$ and $0 < \beta < 1$ are termed as scale and shape parameters, respectively.

3. Model Formulation

Let $I(t)$ denote the inventory level at time t in the reorder interval $[0,T]$. When the stock on hand is non-zero, depletion from stock occurs due to both demand and deterioration. However, during stock-out, shortage accumulates owing to partial demand that is backlogged. Hence, $I(t)$ satisfies the following differential equations:

$$\begin{aligned} &\frac{dI(t)}{dt} = -(\alpha + \beta I(t), \quad 0 \le t \le \mu, \\ &\frac{dI(t)}{dt} + \{\theta(t) - m(\xi,t)\} I(t) = -(\alpha + \beta I(t), \quad \mu \le t \le t_1, \\ &\frac{dI(t)}{dt} = -\frac{\alpha}{1+\delta(T-t)}, \quad t_1 \le t \le T, \end{aligned}$$

where t_1 denotes the time point in $[0,T]$ at which the stock on hand is completely exhausted.

Applying the boundary conditions $I(0) = W$ and $I(t_1) = 0$, we get

$$I(t) = W\frac{\alpha}{\beta}(e^{\beta i} - 1), \quad \text{if } 0 \le t \le \mu,$$

$$= \alpha e^{-\beta i}\int_{t}^{t_1} e^{\beta y + \int_{t}^{y}\{\theta(x) - m(\xi,x)\}dx} dy, \quad \text{if } \mu \le t \le t_1,$$

$$= \frac{\alpha}{\delta}[\ln(1+\delta(T-t)) - \ln(1+\delta(T-t_1))], \text{ if } t_1 \le t \le T.$$

Hence,

$$W = \frac{\alpha}{\beta}(e^{\beta\mu} - 1) + \alpha\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x) - m(\xi,x)\}dx} dy. \tag{1}$$

Further, $I(t) = -S$ gives

$$S = \ln(1+\delta(T-t_1)). \tag{2}$$

This establishes the relationship between S and T. From (1) and (2), the order quantity is

$$Q = W + S = \frac{\alpha}{\beta}(e^{\beta\mu} - 1) + \alpha\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x) - m(\xi,x)\}dx} dy + \frac{\alpha}{\delta}\ln(1+\delta(T-t_1)).$$

The decision variables are W, T and ξ, or equivalently, t_1, T and ξ.

4. Cost Function

The different costs incurred during a reorder interval $[0, T]$ are as follows:

1) Ordering cost $= C_S$.

2) Purchase cost (P) =

$$C_1 Q = C_1\frac{\alpha}{\beta}(e^{\beta\mu} - 1) + C_1\alpha\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x) - m(\xi,x)\}dx} dy + C_1\frac{\alpha}{\delta}\ln(1+\delta(t-t_1)).$$

3) Deterioration cost $(D^*) = C_2\int_{\mu}^{t_1}\{\theta(t) - m(\xi,t)\}I(t)dt,$

$$= C_2\alpha\int_{\mu}^{t_1}\{\theta(t) - m(\xi,t)\}\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(t) - m(\xi,x)\}dx} dydt.$$

4) Holding cost $(H) = C_1 p\int_{0}^{t_1} I(t)dt,$

$$= C_1 p[\frac{\alpha}{\beta}(e^{\beta\mu} - 1)(\mu - \frac{1}{\beta}) + \frac{\alpha\mu}{\beta} + \alpha\mu e^{-\beta\mu}\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x) - m(\xi,x)\}dx} dy$$

$$\times\int_{\mu}^{t_1}\alpha\int_{t}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x) - m(\xi,x)\}dx} dydt].$$

5) Backorder cost $(SC) = C_3[-\int_{t_1}^{T} I(t)dt = \frac{\alpha C_3}{\delta}[(T-t_1) - \frac{1}{\delta}\ln(1+\delta(T-t_1))]$.

6) Opportunity cost due to lost sales $(L) = C_4\alpha\int_{t_1}^{T}(1-\frac{1}{1+\delta(T-t)})dt,$

$$= C_4\alpha[(T-t_1) - \frac{1}{\delta}\ln(1+\delta(T-t_1))].$$

7) Preservation technology cost $(TC) = \xi\ t_1$.

Since the length of a reorder cycle is itself a decision variable, we consider the cost per unit length of a cycle. Let it be denoted by $C(T,t_1,\xi)$. Then, after some algebraic manipulation, we have

$$\begin{aligned} C(T,\xi,t_1) &= \frac{1}{T}[C_S + P + D^* + H + SC + LS + TC], \\ &= \frac{1}{T}[A_1 + A_2(T-t_1) + A_3\ln(1+\delta(T-t_1)) + \xi\ t_1 \\ &\quad + C_1\alpha(1+p\mu)\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x)-m(\xi,x)\}dx} dy \\ &\quad + C_2\alpha\int_{\mu}^{t_1}\{\theta(t)-m(\xi,t)\}\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} dydt \\ &\quad + C_1 p\int_{\mu}^{t_1}\alpha\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} dydt], \end{aligned}$$

where $A_1 = C_S + C_1\frac{\alpha}{\beta}(e^{\beta\mu}-1) + C_1 p\frac{\alpha}{\beta}(e^{\beta\mu}-1)(\mu-\frac{1}{\beta}) + C_1 p\frac{\alpha\mu}{\beta}$, $A_2 = C_4\alpha + C_3\frac{\alpha}{\delta}$,

$A_3 = C_1\frac{\alpha}{\delta} - \frac{1}{\delta}(C_4\alpha + C_3\frac{\alpha}{\delta}) = \frac{1}{\delta}(C_1\alpha - A_2)$.

Solution Procedure*:*

The optimal values of (T,t_1,ξ) that minimize $C(T,\xi,t_1)$ must satisfy

$$\frac{\partial C(T,\xi,t_1)}{\partial T} = 0,\ \ \frac{\partial C(T,\xi,t_1)}{\partial \xi} = 0 \text{ and } \frac{\partial C(T,\xi,t_1)}{\partial t_1} = 0,$$

which give the following equations:

$$A_2 = C_4\alpha + C_3\frac{\alpha}{\delta},$$

$$A_2 + A_3\frac{\delta}{1+\delta(T-t_1)} = C(T,\xi,t_1),$$

$$t_1 = C_1\alpha(1+p\mu)\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x)-m(\xi,x)\}dx}\left(\int_{\mu}^{y} m'(\xi,x)dx\right)dy$$

$$+ C_1 p\int_{\mu}^{t_1}\alpha\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx}\left(\int_{t}^{y} m'(\xi,x)dx\right)dydt$$

$$+ C_2\alpha\int_{\mu}^{t_1}\{\theta(x)-m(\xi,x)\}\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx}\left(\int_{t}^{y} m'(\xi,x)dx\right)dydt$$

$$+ C_2\alpha\int_{\mu}^{t_1} m'(\xi,x)\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx}dydt,$$

$$A_2 + A_3\frac{\delta}{1+\delta(T-t_1)} - \xi = g(t,\xi),$$

where

$$g(t_1,\xi) = C_1\alpha(1+p\mu)e^{\beta(t_1-\mu)+\int_{\mu}^{t_1}\{\theta(x)-m(\xi,x)\}dx} + C_1 p\alpha\int_{\mu}^{t_1} e^{\beta(t_1-t)+\int_{t}^{t_1}\{\theta(x)-m(\xi,x)\}dx}dt$$

$$+ C_2\alpha\int_{\mu}^{t_1}\{\theta(x)-m(\xi,x)\}e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} + C_2\alpha\int_{\mu}^{t_1} m'(\xi,x)\int_{t}^{t_1} e^{\beta(t_1-t)+\int_{t}^{t_1}\{\theta(x)-m(\xi,x)\}dx}dt.$$

The following theorems help to find the optimal values of (T,ξ, t_1).

Theorem 1. *For given* T *and* ξ, *the cost function is convex in* t_1 *when* $C_1 < \frac{C_3}{\delta} + C_4$.

Proof: We have $C_1 < \frac{C_3}{\delta} + C_4$, i.e., $C_1 - \frac{C_3}{\delta} - C_4 < 0$, and $C_1\frac{\alpha}{\delta} - \frac{1}{\delta}(C_4\alpha + C_3\frac{\alpha}{\delta}) < 0$.

Hence, $A_3 < 0$. Then,

$$\frac{\partial^2 C(T,\xi,t_1)}{\partial t_1^2} = \frac{1}{T}[g(t_1,\xi)\{\beta+\theta(t_1)-m(\xi,t_1)\} + C_1 p + C_2\{\theta(t_1)-m(\xi,t_1)\} - \frac{A_3\delta^2}{1+\delta(T-t_1)} > 0,$$

since $\theta(t_1)-m(\xi,t_1) > 0$ and $A_3 < 0$. Hence, the cost function is convex in t_1 for given T and ξ.

Theorem 2. *For given* T *and* t_1, *the cost function is convex in* ξ *if* $m(\xi,t_1)$ *is strictly concave in* ξ *for all* t.

Proof: We have

$$\frac{\partial^2 C(T,\xi,t_1)}{\partial \xi^2} = \frac{\alpha}{T}[C_1\alpha(1+p\mu)\int_{\mu}^{t_1} e^{\beta(y-\mu)+\int_{\mu}^{y}\{\theta(x)-m(\xi,x)\}dx} (\int_{\mu}^{y}(m''(\xi,x)-m'^2(\xi,x))dx)dy$$

$$+C_1 p\int_{\mu}^{t_1}\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} (\int_{t}^{y}(m''(\xi,x)-m'^2(\xi,x))dx)dydt$$

$$+C_2\int_{\mu}^{t_1}\{\theta(x)-m(\xi,x)\}\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} (\int_{t}^{y}(m''(\xi,x)-m'^2(\xi,x))dx)dydt$$

$$+C_2\alpha\int_{\mu}^{t_1} m''(\xi,x)\int_{t}^{t_1} e^{\beta(y-t)+\int_{t}^{y}\{\theta(x)-m(\xi,x)\}dx} dydt] > 0,$$

when $m''(\xi,t) = \frac{\partial^2 m(\xi,t)}{\partial \xi^2} < 0$, that is, $m(\xi,t)$ is strictly concave in ξ.

Theorem 3. *For given* ξ *and* t_1, *the cost function* $C(T,\xi,t_1)$ *is strictly convex at* $T = T_0$ $(> t_1)$ *satisfying* $\frac{\partial C(T,\xi,t_1)}{\partial T}|_{T=T_0} = 0$, *provided* $C_1 < \frac{C_3}{\delta} + C_4$.

Proof: Suppose. $C_1 < \frac{C_3}{\delta} + C_4$. For given (ξ,t_1), let us write $C(T,\xi,t_1) = \frac{N(T)}{T}$.

Then,

$$\frac{\partial C(T,\xi,t_1)}{\partial T} = \frac{N'(T)}{T} - \frac{N(T)}{T^2}, \text{ and } \frac{\partial^2 C(T,\xi,t_1)}{\partial T^2} = \frac{N''(T)}{T} - \frac{2N'(T)}{T^2} + \frac{2N(T)}{T^3}.$$

Therefore, $\frac{\partial C(T,\xi,t_1)}{\partial T} = 0$ gives $C(T,\xi,t_1) = N'(T)$. Let $T_0 (> t_1)$ be a solution to the above equation. Then,

$$\frac{\partial^2 C(T,\xi,t_1)}{\partial T^2}\Big|_{T=T_0} = \left[\frac{N''(T)}{T} - \frac{2N'(T)}{T^2} + \frac{2N(T)}{T^3}\right]_{T=T_0} = \left[\frac{N''(T)}{T} - \frac{2}{T}\left(\frac{N'(T)}{T} - \frac{N(T)}{T^2}\right)\right]_{T=T_0},$$

$$= \frac{N''(T)}{T}\Big|_{T=T_0} > 0,$$

since $\frac{N''(T)}{T} = -\frac{A_3\delta^2}{T\{1+\delta(T-t_1)\}^2}$, which is positive for all $T > t_1$ as $A_3 < 0$ when $C_1 < \frac{C_3}{\delta} + C_4$.

5. Numerical Examples

Example 1 Consider an item that can maintain its freshness for 10 months and then starts deteriorating with a deteriorating rate $\theta(t-\mu)$, where θ = 0.08. The costs are: C_S = Rs. 1500, C_1 = Rs. 1000 per unit, p = 0.10, C_2 = Rs.250 per unit deteriorating, C_3 = Rs. 400 per unit short per unit time and C_4 = Rs. 250 per unit demand lost. The demand rate is $\alpha + \beta I(t)$, where α = 20 and β = 10. Let $m(\xi,t) = \theta(1-e^{-k\xi})(t-\mu)$, $k > 0$. Suppose δ = 0.4 and k = 1.

The optimum reorder interval length and time to exhaust the stock on hand in a reorder interval come out to be T = 1.048 years and t_1 = 1 year, and the optimum preservation technology cost per unit time is ξ = Rs.453. The resulting cost per unit length of the inventory cycle is $C(T,\xi,t_1)$ = Rs. 2593.50.

Example 2 In this example, the values of all the parameters are kept exactly the same as in Example 1, but the preservation technology is not applied. The optimum reorder interval length and time to exhaust the stock on hand in a reorder interval come out to be T = 0.96 years and t_1 = 0.88 year. The resulting cost per unit length of the inventory cycle is $C(T,t_1)$ = Rs. 3715.35.

Comparing the above two examples, it is observed that if the preservation technology is applied, then the cost is diminished by Rs. 3715.35 - Rs. 2593.5 = Rs. 1121.85, when the other parameters are the same.

6. Sensitivity Analysis

We next examine the sensitivity of the model to a change in the model (cost) parameters. We also check the effect of the preservation technology on the minimum cost. Table 2 shows the change in the optimal values of T,ξ,t_1 and $C(T,\xi,t_1)$ when preservation technology is applied. Table 3 shows the effect of preservation technology on the minimum cost for different values of the cost parameters.

The sensitivity analysis carried out in Table 2 shows that there is considerably high change in the minimum cost with change in the purchase cost per unit (C_1). In fact, the minimum cost increases with increase in C_1. Further, the re-order interval length T and the preservation technology cost ξ per unit time diminish with increase in C_1. However, change in any other parameter does not affect the minimum cost considerably. So, from managerial point of view, the model is fairly robust with respect to all cost parameters except C_1. This means, apart from C_1, changes in the cost parameters will not considerably affect the cost per unit length of an inventory cycle. Hence, the manager must pay close attention while buying the items, and negotiation at this stage is highly recommended. Further, though an added expenditure is faced with the introduction of preservation technology, this helps not only to increase the longevity of the items in inventory but also helps to considerably reduce the cost per unit length of the inventory cycle. This is evident from comparison of the minimum cost in Example 1, which includes preservation cost, and that in Example 2, which does not involve preservation cost. In fact, a reduction in cost by almost 43% is noted by using preservation technology. This is motivational to the inventory manager to use preservation methods for the deteriorating items in inventory.

Table 2 Change in the optimal values of T, ξ and t_1 and percentage change in minimum cost with change in the model parameters

C_1	t_1	T	ξ	% cost change	C_3	t_1	T	ξ	% cost change
600	1.23	2.32	453.30	-39.96	300	0.97	1.022	450.96	-0.3920
800	1.15	1.652	453.09	-20.04	350	0.99	1.032	451.50	-0.2166
1000	1.00	1.048	453.00	0.000	400	1.00	1.048	453.00	0.0000
1200	0.93	0.985	452.60	20.00	500	1.00	1.069	453.95	0.1612
1400	0.85	0.91	452.10	39.90	600	1.17	1.098	434.09	0.2786
C_2	t_1	T	ξ	% cost change	C_4	t_1	T	ξ	% cost change
50	1	1.048	453	-0.004	50	0.88	0.977	452.55	-0.1555
150	1	1.048	453	-0.002	150	0.95	1.001	452.89	-0.0791
250	1	1.048	453	0.000	250	1.00	1.048	453.00	0.0000
350	1	1.048	453	0.0435	350	1.06	1.126	453.34	0.0262
450	1	1.048	453	0.0592	450	1.09	2.022	453.89	0.0651
p	t_1	T	ξ	% cost change	C_S	t_1	T	ξ	% cost change
0.06	2.55	2.99	454.87	-0.181	500	0.83	0.89	450.98	-0.8700
0.08	1.60	2.06	453.65	-0.089	1000	0.93	0.92	452.12	-0.5220
0.10	1.00	1.048	453.00	0.000	1500	1.00	1.048	453.00	0.0000
0.12	0.93	1.002	452.45	0.1002	2000	1.36	1.41	143.80	0.4111
0.14	0.82	0.99	451.90	0.1966	2500	1.99	2.11	144.30	0.7584

Table 3 Decrease in the minimum cost owing to preservation technology with change in the model parameters

C_1	Decrease in cost	C_3	Decrease in cost
600	604.45	200	1204.84
800	863.59	300	1163.69
1000	1121.85	400	1121.85
1200	1381.19	500	1082.45
1400	1645.95	600	1045.15
C_2	Decrease in cost	C_4	Decrease in cost
50	1121.80	50	1154.75
150	1121.82	150	1138.28
250	1121.85	250	1121.85
350	1120.81	350	1119.75
450	1120.78	450	1117.36
p	Decrease in cost	C_S	Decrease in cost
0.06	1143.56	500	1143.89
0.08	1132.59	1000	1135.14
0.10	1121.85	1500	1121.85
0.12	1110.89	2000	1111.46
0.14	1100.22	2500	1102.68

7. Limitation of the Study

- The numerical example is done based on hypothetical figures.
- This model is applicable only if there is no lead time which means that the order arrives as soon as it is placed.
- The model is applicable to those businesses that deal with only deteriorating items.

8. Conclusion and Future Scope of the Study

The paper obtains an optimum periodic review inventory model for non-instantaneous deteriorating items with time dependent deterioration and stock dependent demand. During shortage, there is partial backlogging, with the backlogging rate dependent on the waiting time. The system also invests on preservation technology, whereby the deterioration of items is effectively reduced. The decision variables are the reorder interval and the preservation technology cost per unit time. To sum up, the major findings are

- Reduction of total cost due to application of preservation technology. This implies that reduction of deterioration brings reduction in cost, hence profit will be more.
- The model is highly sensitive to change in the procurement cost per unit.
- Reduction of total cost increases with procurement cost per unit.

The proposed model can further be extended for future research in many aspects like introducing lead time, considering other types of demand such as time dependent demand, random demand etc., using fuzzy cost parameters.

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