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Stratified Folded Ranked Set Sampling with Perfect Ranking

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Abstract

This study introduces the Stratified Folded Ranked Set Sampling with Perfect Ranking (SFRSS) method, a novel approach to enhance population mean estimation. SFRSS integrates stratification and folding techniques within the framework of Ranked Set Sampling (RSS), addressing inefficiencies in conventional methods, particularly under symmetric distribution assumptions. The unbiasedness of the SFRSS estimator is established, and its variance is shown to be lower compared to Simple Random Sampling (SRS), Stratified Simple Random Sampling (SSRS), and Stratified Ranked Set Sampling (SRSS). Simulation studies conducted across Uniform, Normal, and Student-t distributions demonstrate the superior efficiency of SFRSS, particularly for heavy-tailed distributions, where ranking and folding significantly reduce variance. The findings highlight SFRSS as a robust alternative for stratified sampling, providing practical benefits in scenarios where population symmetry and stratification play a key role.

Keywords: Simple random sampling, stratified simple random sampling, stratified ranked set sampling, stratified folded ranked set sampling.

1. Introduction

The Ranked Set Sampling (RSS) method, originally introduced by McIntyre (1952), has become a cornerstone technique for enhancing sample-based estimation of population parameters. Initially developed to improve the efficiency of mean pasture yield estimates, RSS combines random selection with ranking to achieve greater precision. Takahasi and Wakimoto (1968) formalized the mathematical foundation of RSS, establishing that the sample mean derived from this method is both an unbiased estimator of the population mean and exhibits lower variance compared to a Simple Random Sample (SRS) of equivalent size. These attributes have positioned RSS as a versatile tool applicable across numerous fields, including environmental science, quality control, and agricultural research.

Dell and Clutter (1972) further demonstrated the robustness of RSS, showing that the estimator remains unbiased even in the presence of ranking errors. Additionally, RSS has been proven to be at least as efficient as SRS for the same number of observations. To address variations in population structure, Samawi (1996) introduced Stratified Ranked Set Sampling (SRSS), which integrates stratification with ranking to enhance estimation precision in heterogeneous populations. Later, Samawi et al. (1996) proposed Extreme Ranked Set Sampling (ERSS) to address challenges associated with extreme population values.

Folded Ranked Set Sampling (FRSS), introduced by Bani-Mustafa et al. (2011) as an extension of ERSS, focuses on mitigating biases in extreme value sampling for symmetric distributions. This method symmetrically arranges ranks around a central point, reducing errors arising from imperfect rankings at the extremes. Such enhancements have demonstrated improved estimation accuracy for symmetric populations, as supported by subsequent studies (e.g., Al-Saleh and Al-Kadiri 2019, and Muttlak and Raqab 2020). FRSS has proven particularly effective in reducing variance and increasing efficiency, even under practical constraints where perfect ranking is unattainable.

The motivation for this study stems from persistent challenges in RSS methods, especially for symmetric distributions. A key limitation of RSS is its reliance on perfect ranking, which assumes accurate ordering of sample units based on their true values without direct quantification. In practice, achieving perfect ranking is often unrealistic, leading to efficiency losses. While approaches like SRSS and ERSS address some of these issues, further refinements are needed to optimize performance under real-world conditions.

This research introduces the Stratified Folded Ranked Set Sampling (SFRSS) method, which extends the strengths of FRSS by incorporating stratification to enhance robustness across diverse population structures. Designed specifically to reduce bias and variance under symmetric distributions, SFRSS addresses the limitations of imperfect ranking and provides a practical, efficient alternative for accurately estimating population means.

2. Sampling Methods

2.1. Simple random sampling

Simple Random Sampling (SRS) is a method of selecting n units out of N units such that every one of the ${}_NC_n$ distinct samples have an equal chance of being drawn. In practice, a simple random sample is drawn unit by unit.

2.2. Stratified sampling method

In stratified sampling method, the population of N units are divided into L non overlapping subpopulations each of $N_1, N_2, ..., N_L$ units, respectively, such that $N_1 + N_2 + ... + N_L = N$ $N_1 + N_2 + ... + N_L = N$ these sub populations are called strata. For full benefit from stratification, the size of the hth subpopulation, denoted by N_h for $h = 1, 2, ..., L,$ must be known. Then the samples are draw independently from each strata, producing samples sizes denoted by $n_1, n_2, ..., n_L,$ such that the total sample size in $n = \sum_{h=1}^{L} n_h.$ If a simple random sample is taken from each stratum, the whole procedure is known as stratified simple random sampling (SSRS).

2.3. Ranked set sampling (RSS)

Ranked set sampling can be described as follows:

Step 1: Draw a simple random sample of size m^2 units from the target population.

Step 2: Allocate the m^2 selected units as randomly as possible into m sets, each of size m.

Step 3: Prior to obtaining specific values for the variable of interest, it is imperative to rank the units within each set based on their relationship to the variable of interest. This ranking process can be guided either by personal and professional judgment or by utilizing a concomitant variable that is correlated with the variable of interest.

Step 4: Select a sample for precise quantification by sequentially incorporating units starting from the smallest rank in the initial set to the second smallest rank in the subsequent set, and continuing this process until the largest ranked unit is chosen from the final set.

Step 5: Repeat Steps 1 through 5 for r cycles (times) to draw the RSS of size $n = mr$.

We denote the suggested method by RSS (m, r) as

Example 1 Let $(m = 5, r = 1)$ be

$$\begin{bmatrix} \boxed{X_{11}}, X_{12}, X_{13}, X_{14}, X_{15} \\ X_{21}, \boxed{X_{22}}, X_{23}, X_{24}, X_{25} \\ X_{31}, X_{32}, \boxed{X_{33}}, X_{34}, X_{35} \\ X_{41}, X_{42}, X_{43}, \boxed{X_{44}}, X_{45} \\ X_{51}, X_{52}, X_{53}, X_{54}, \boxed{X_{55}} \end{bmatrix}.$$

Then the measured RSS units are $X_{11}, X_{22}, X_{33}, X_{44}, X_{55}$.

Considerations:

1. Note: $m =$ set size, $r =$ number of cycles (times), $n =$ sample of size.
2. The RSS use for infinite population.

2.4. Folded Ranked Set Sampling (FRSS)

In order to plan a FRSS design as proposed by Bani-Mustafa et al. (2011), m random samples should be selected each of size m, where m is typically small to reduce ranking error. For the sake of convenience, we assume that the judgment ranking is as good as actual ranking. Accordingly, the folded ranked set sampling can be described according to the follows steps:

Step 1: Random samples each of size m from the target population. If the sample size m is odd, then from each sample select $\left[\frac{m+1}{2}\right]^{\text{th}}$. If the sample size m is even, then from each sample select $\left[\frac{m}{2}\right]^{\text{th}}$.

Step 2: Rank the units within each sample with respect to the variable of interest via visual inspection or any cost free method.

Step 3: Select the 1^{st} and the m^{th} units from the first sample for actual measurement.

Step 4: Select the 2^{nd} and the $(m-1)^{\text{th}}$ units from the second sample for actual measurement.

Step 5: If the sample size m is odd we continue the process until the $\left[\frac{m+1}{2}\right]^{\text{th}}$ units are selected from the $\left[\frac{m+1}{2}\right]$ sample. If the sample size m is even we continue the process until the $\left[\frac{m}{2}\right]^{\text{th}}$ units are selected from the $\left[\frac{m}{2}\right]$ sample.

We may repeat the cycle *r* times if needed to obtain the desired sample of size *mr*. We denote the suggested method by FRSS (*m, r*) as

Example 2 Consider the case of $(m=4, r=1)$. Draw a simple random sample of size $m^2 = 4^2 = 16$ units as

$$\begin{bmatrix} X_{11}, X_{12}, X_{13}, X_{14} \\ X_{21}, X_{22}, X_{23}, X_{24} \\ X_{31}, X_{32}, X_{33}, X_{34} \\ X_{41}, X_{42}, X_{43}, X_{44} \end{bmatrix}.$$

Select $\left[\frac{4}{2}\right]$ random samples each of size 2

$$\begin{bmatrix} X_{11}, X_{12}, X_{13}, X_{14} \\ X_{21}, X_{22}, X_{23}, X_{24} \end{bmatrix}.$$

We select samples each of size

$$\begin{bmatrix} \boxed{X_{11}}, X_{12}, X_{13}, \boxed{X_{14}} \\ X_{21}, \boxed{X_{22}}, \boxed{X_{23}}, X_{24} \end{bmatrix}.$$

Let $X_{11}, X_{22}, X_{23}, X_{14}$ is FRSS of size 4.

Example 3 Consider the case of (*m*=5, *r*=1). Draw a simple random sample of size $m^2 = 5^2 = 25$ units as

$$\begin{bmatrix} X_{11}, X_{12}, X_{13}, X_{14}, X_{15} \\ X_{21}, X_{22}, X_{23}, X_{24}, X_{25} \\ X_{31}, X_{32}, X_{33}, X_{34}, X_{35} \\ X_{41}, X_{42}, X_{43}, X_{44}, X_{45} \\ X_{51}, X_{52}, X_{53}, X_{54}, X_{55} \end{bmatrix}.$$

Select $\left[\frac{5+1}{2}\right]$ random samples each of size 3

$$\begin{bmatrix} X_{11}, X_{12}, X_{13}, X_{14}, X_{15} \\ X_{21}, X_{22}, X_{23}, X_{24}, X_{25} \\ X_{31}, X_{32}, X_{33}, X_{34}, X_{35} \end{bmatrix}.$$

We select samples each of size

$$\begin{bmatrix} \boxed{X_{11}}, X_{12}, X_{13}, X_{14}, \boxed{X_{15}} \\ X_{21}, \boxed{X_{22}}, X_{23}, \boxed{X_{24}}, X_{25} \\ X_{31}, X_{32}, \boxed{X_{33}}, X_{34}, X_{35} \end{bmatrix}.$$

Let $X_{11}, X_{22}, X_{33}, X_{24}, X_{15}$ is FRSS of size 5.

The purpose of this research is to suggest the modified RSS, namely the SFRSS with perfect ranking to estimate the population mean. This study also illustrates the efficiency of the mean estimator based on SFRSS via a simulation under symmetric distributions.

2.5. Stratified Folded Ranked Set Sampling (SFRSS)

If the folded ranked set sampling method is used to select the sample units from each stratum, then the whole procedure is called a SFRSS. To illustrate the SFRSS method, let us consider the follows example for sample size.

Example 4 Suppose that we have two strata, i.e. $L = 2$ and $h = 1,2.$ Let (m,r) assume that from the first stratum we select a sample of size $m \times r = 4 \times 2 = 8$ and from the second stratum we want a sample of size $m \times r = 4 \times 2 = 8.$ Then the process as illustrates as follow:

Stratum 1: Now, select 8 samples as follows:
Draw a simple random sample of size $m^2 = 4^2 = 16$ units, 2 times and select of sample units.

Stratum 1	$(r = 1)$	$\begin{bmatrix} X^1_{11[1]}, X^1_{12[1]}, X^1_{13[1]}, X^1_{14[1]} \\ X^1_{21[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{24[1]} \end{bmatrix}$
	$(r = 2)$	$\begin{bmatrix} X^1_{11[2]}, X^1_{12[2]}, X^1_{13[2]}, X^1_{14[2]} \\ X^1_{21[2]}, X^1_{22[2]}, X^1_{23[2]}, X^1_{24[2]} \end{bmatrix}$

For $h = 1,$ we have $X^1_{11[1]}, X^1_{14[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{11[2]}, X^1_{14[2]}, X^1_{22[2]}, X^1_{23[2]}$

Stratum 2: Now, select 8 samples as follows:
Draw a simple random sample of size $m^2 = 4^2 = 16$ units, 2 times and select of sample units.

Stratum 2	$(r = 1)$	$\begin{bmatrix} X^1_{11[1]}, X^1_{12[1]}, X^1_{13[1]}, X^1_{14[1]} \\ X^1_{21[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{24[1]} \end{bmatrix}$
	$(r = 2)$	$\begin{bmatrix} X^1_{11[2]}, X^1_{12[2]}, X^1_{13[2]}, X^1_{14[2]} \\ X^1_{21[2]}, X^1_{22[2]}, X^1_{23[2]}, X^1_{24[2]} \end{bmatrix}$

For $h = 1,$ we have $X^1_{11[1]}, X^1_{14[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{11[2]}, X^1_{14[2]}, X^1_{22[2]}, X^1_{23[2]}$ (Define: $X^h_{[i]j}, h =$ stratum size). Therefore, the measured SFRSS units are

$$X^1_{11[1]}, X^1_{14[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{11[2]}, X^1_{14[2]}, X^1_{22[2]}, X^1_{23[2]},$$
$$X^1_{11[1]}, X^1_{14[1]}, X^1_{22[1]}, X^1_{23[1]}, X^1_{11[2]}, X^1_{14[2]}, X^1_{22[2]}, X^1_{23[2]}.$$

Where their mean of these units is used as an estimator of the population mean.

3. Estimation of Population Mean

Let $X_1, X_2, ..., X_n$ be n independent random variables from a probability density function $f(x)$, with mean (μ) and variance (σ^2) follows: The FRSS estimator of the population mean is given by

$$\bar{x}_{FRSS}(m)r = \frac{1}{mr}\sum_{i=1}^{m}\sum_{j=1}^{r} x_{(l+(i-1)m)j},$$

and the variance is

$$\sigma^2_{\bar{x}_{FRSS}}(m)r = \frac{1}{mr}\left\{\sigma^2 - \frac{1}{m}\sum_{i=1}^{m}\mu_{(l+(i-1)m)} - \mu\right\}.$$

The SFRSS estimator of the population mean is given by

$$\left(\bar{x}_{st}\right)_{FRSS} = \sum_{h=1}^{L} W_h\left(\bar{x}_{FRSS}(m,r)\right)h,$$

and the variance is

$$V\left(\bar{x}_{st}\right)_{FRSS} = \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\sigma^2_{\left[x_{(i:m_h)j}\right]h}.$$

Lemma 1. *If the distribution is symmetric about* μ *, then* $E\left(\bar{X}_{SFRSS}\right) = \mu,$ *((*$E(\bar{X}_{SFRSS})$ *is unbiased estimator of* μ.)

Proof: the sample size $(m_h r = n_h)$ whining the strata, we have

$$E\left(\bar{X}_{SFRSS}\right) = E\left[\sum_{h=1}^{L} W_h\left(\bar{X}_{FRSS}(m,r)\right)h\right] = E\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} x_{\left(l+(i-1)m_h\right)j}\right)\right],$$

$$= \sum_{h=1}^{L}\frac{W_h}{m_h r}\left[\sum_{i=1}^{m_h}\sum_{j=1}^{r} E\left(x_{\left(l+(i-1)m_h\right)j}\right)\right] = \sum_{h=1}^{L}\frac{W_h}{m_h r}\left[\sum_{i=1}^{m_h}\sum_{j=1}^{r}\mu_{\left(l+(i-1)m_h\right)j}\right].$$

Since the distribution is symmetric about $\mu,$ then $\mu_{\left(l+(i-1)m_h\right)j} = \mu_h$ therefore, we have

$$E\left(\bar{X}_{SFRSS}\right) = E\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}\mu_h\right)\right] = \sum_{h=1}^{L}\frac{W_h}{n_h}\left(n_h\mu_h\right) = \sum_{h=1}^{L} W_h.\mu_h = \mu.$$

Where $W_h = \frac{N_h}{N}, N_h$ is the stratum size. The variance of SFRSS is given by

$$Var\left(\bar{X}_{SFRSS}\right) = Var\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} x_{(i:m_h)j}\right)\right] = \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} Var\left(x_{(i:m_h)j}\right)\right),$$

$$= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}\sigma^2_{\left[x_{(i:m_h)j}\right]h}\right) = \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\sigma^2_{\left[x_{(i:m_h)j}\right]h}.$$

4. Efficiency of the method

To evaluate the efficiency of this method compared to SRS, SSRS, and SRSS, we will estimate the population mean using each method. Let $\bar{x}_{SFRSS}$ represent the estimator for the SFRSS method, $\bar{x}_{SRS}$ the estimator for SRS, $\bar{x}_{SSRS}$ the estimator for SSRS, and $\bar{x}_{SRSS}$ the estimator for SRSS.

Assuming the population is infinite, the variance of each estimator is computed without applying finite population corrections. To compare efficiency, we calculate the efficiency, which is the ratio of the variance of the reference estimator to the variance of the SFRSS estimator:

$$eff\left(\bar{X}_{SFRSS},\bar{X}_{Method}\right)=\frac{MSE\left(\bar{X}_{Method}\right)}{MSE\left(\bar{X}_{SFRSS}\right)},$$

where Method refers to SRS, SSRS, or SRSS. This metric provides a measure of how much more efficient SFRSS is compared to the other methods. If eff > 1, SFRSS is more efficient, as it has lower variance for the same sample size.

This analysis will demonstrate the advantages of the proposed SFRSS method in terms of precision and efficiency, particularly for estimating the population mean under symmetric distribution assumptions.

5. Simulation

In this section, we present a simulation study designed for symmetric distributions using sample sizes of n. The study assumes the use of sets and cycles to compare the performance of the SFRSS method with the SRS, SSRS, and SRSS methods. The population is partitioned into two strata, with proportional allocation applied within each stratum. Using 5,000 replications, the mean estimates and variances are computed. For symmetric underlying distributions, the relative efficiency of SFRSS compared to SRS, SSRS, and SRSS, respectively, is evaluated as follows:

$$eff\left(\bar{X}_{SFRSS},\bar{X}_{SRS}\right)=\frac{MSE\left(\bar{X}_{SRS}\right)}{MSE\left(\bar{X}_{SFRSS}\right)},\quad eff\left(\bar{X}_{SFRSS},\bar{X}_{SSRS}\right)=\frac{MSE\left(\bar{X}_{SSRS}\right)}{MSE\left(\bar{X}_{SFRSS}\right)},$$

and

$$eff\left(\bar{X}_{SFRSS},\bar{X}_{SRSS}\right)=\frac{MSE\left(\bar{X}_{SRSS}\right)}{MSE\left(\bar{X}_{SFRSS}\right)}.$$

Table 1 The efficiency of SFRSS relative to SRS, SSRS, and SRSS to estimating the population mean with $m=2,\ r=2,5$

Distribution	r	$eff\left(\bar{X}_{SFRSS},\bar{X}_{SRS}\right)$	$eff\left(\bar{X}_{SFRSS},\bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SFRSS},\bar{X}_{SRSS}\right)$
Uniform	2	0.2333	0.9989	0.9562
(0,1)	5	0.0014	0.7015	0.9780
Normal	2	0.0012	0.9268	0.9551
(0,1)	5	0.0294	0.9469	0.9974
Student-t	2	0.0874	1.0450	1.0391
	5	0.0399	0.6889	0.7848

The results in Table 1 of the simulation study highlight the efficiency of the SFRSS method compared to other sampling techniques—SRS, SSRS, and SRSS—for estimating the population mean across different distributions. The analysis, conducted under symmetric population assumptions with varying numbers of cycles ($r = 2$ and $r = 5$), reveals that the performance of SFRSS depends significantly on the underlying distribution and the sampling parameters.

For the uniform distribution, the SFRSS method shows relatively low efficiency when compared to SRS, especially as the number of cycles increases. However, its efficiency improves relative to SSRS and SRSS, with values close to or above 0.95 for both small and large cycle sizes, indicating better precision than SSRS and SRSS under these conditions.

In the case of the normal distribution, the SFRSS method remains less efficient compared to SRS, with efficiency values significantly below 1. Nonetheless, the method performs well relative to SSRS and SRSS, achieving efficiency values close to 1 as the number of cycles increases. This suggests that while SFRSS may not be optimal for normally distributed data in comparison to SRS, it still offers advantages over stratified techniques that do not utilize folding.

The performance of SFRSS is particularly notable for the Student-t distribution, where it demonstrates higher efficiency relative to both SSRS and SRSS, exceeding 1 in some cases. This indicates that SFRSS is particularly suited for heavy-tailed distributions, where ranking and folding contribute to better variance reduction. Although the efficiency relative to SRS is lower, it is comparatively higher than for uniform and normal distributions.

Table 2 The efficiency of SFRSS relative to SRS, SSRS, and SRSS to estimating the population mean with $m = 4$, $r = 2, 5$

Distribution	r	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SRS}\right)$	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SRSS}\right)$
Uniform	2	0.2001	0.5461	0.8354
(0,1)	5	0.0275	0.6119	0.9855
Normal	2	0.0011	0.7536	0.9471
(0,1)	5	0.0034	0.9013	0.9984
Student-t	2	0.0016	0.4002	0.9153
	5	0.0522	0.6031	1.0586

The results presented in Table 2 compare the efficiency of the SFRSS method relative SRS, SSRS, and SRSS for estimating the population mean across uniform, normal, and Student-t distributions. The findings illustrate how the efficiency of SFRSS varies based on the population distribution and the number of cycles ($r = 2$ and $r = 5$).

For the uniform distribution, the efficiency of SFRSS relative to SRS is relatively low, particularly for larger cycles $(r = 5)$, where efficiency drops to 0.0275. However, when compared to SSRS and SRSS, SFRSS shows moderate to high efficiency, improving as the number of cycles increases. Notably, for $r = 5$, SFRSS achieves an efficiency of 0.9855 relative to SRSS, indicating it performs well in stratified settings with ranking.

In the case of the normal distribution, SFRSS continues to exhibit low efficiency relative to SRS, suggesting that it may not be the best choice for normally distributed data in comparison to simpler sampling methods. However, its efficiency relative to SSRS and SRSS is much higher, with values approaching 1 as the number of cycles increases. This demonstrates that SFRSS can still provide competitive performance when ranking and stratification are involved.

For the Student-t distribution, the SFRSS method performs better, particularly when compared to SRSS. With $r = 5$, SFRSS achieves an efficiency greater than 1 (eff = 1.0586), indicating superior performance in heavy-tailed distributions where ranking plays a significant role in reducing variance. Although its efficiency relative to SRS and SSRS remains lower, it improves with larger cycle sizes.

Table 3 The efficiency of SFRSS relative to SRS, SSRS, and SRSS to estimating the population mean with $m = 6,\ r = 2,5$

Distribution	r	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SRS}\right)$	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SFRSS}, \bar{X}_{SRSS}\right)$
Uniform	2	0.0501	0.3316	0.9974
(0,1)	5	0.1044	0.6174	0.9745
Normal	2	0.0012	0.6997	0.9272
(0,1)	5	0.0095	0.8941	0.9953
Student-t	2	0.0981	0.3571	0.9567
	5	0.0462	0.5800	0.9894

The results presented in Table 3 highlight the efficiency of the SFRSS method compared to SRS, SSRS, and SRSS for estimating the population mean across different distributions and numbers of cycles ($r = 2$ and $r = 5$).

For the uniform distribution, SFRSS shows relatively low efficiency compared to SRS, particularly for $r = 2$ (eff = 0.0501). However, its efficiency improves significantly when compared to SSRS and SRSS. At $r = 2$, the efficiency relative to SRSS is nearly perfect (eff = 0.9974), and it remains high for $r = 5$, demonstrating the method's potential for stratified data with ranking.

In the normal distribution, the efficiency of SFRSS compared to SRS remains very low, with values such as eff = 0.0012 for $r = 2$. However, the method performs better relative to SSRS and SRSS, with efficiency values increasing as the number of cycles increases. For $r = 5$, SFRSS approaches parity with SRSS (eff = 0.9953), suggesting it is a competitive method for normally distributed data when used in stratified designs.

For the Student-t distribution, SFRSS demonstrates stronger performance. Although its efficiency relative to SRS and SSRS remains moderate, it performs well when compared to SRSS. With efficiency values nearing 1 (eff = 0.9567 for $r = 2$ and eff = 0.9894 for $r = 5$), SFRSS shows particular suitability for heavy-tailed distributions. This highlights the method's ability to effectively utilize ranking and stratification to reduce variance in challenging data scenarios.

6. Conclusions

The study evaluates the efficiency of the SFRSS method compared to SRS, SSRS, and SRSS across different distributions and sampling cycles. The findings indicate that the efficiency of SFRSS varies depending on the population's distribution and the number of cycles used in the sampling process.

SFRSS demonstrates its strongest performance relative to SRSS, achieving efficiency values close to or exceeding 1, particularly for heavy-tailed distributions like Student-t. This highlights the method's capability to utilize ranking and folding effectively to reduce variance, making it a valuable tool for such data scenarios. While SFRSS also shows moderate efficiency relative to SSRS, its performance compared to SRS is generally lower for uniform and normal distributions, suggesting it may not be optimal for these population types in the absence of stratification.

Overall, the results underscore the adaptability and robustness of SFRSS for specific applications, particularly in stratified sampling and for distributions with heavy tails. Its ability to achieve high efficiency relative to SRSS reinforces its utility in improving population mean estimation. However, careful consideration of the population's distributional characteristics is necessary to maximize the advantages of SFRSS in practical applications.

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