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One-Period Coupon Bond Valuation Using the Variance Gamma Model

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Abstract

Bonds issued by companies all have coupons. One-period coupons are issued once and are paid together with the principal amount of the bonds at maturity. Many studies on coupon bond valuations have been carried out using a structural model approach. The model involves the assets of the company. The asset model that is often used is the Black-Scholes-Merton model, which assumes in asset returns with a normal distribution. But in reality, the financial data is not normally distributed and indicates the presence of heavy tails and excess kurtosis. In this study, bond valuation was carried out using the variance gamma (VG) model approach. An empirical study was conducted on bond data from one of the banking companies in Indonesia, namely the Continuous Bond III of Bank CIMB Niaga Phase I 2019 Series B. The bonds were issued on December 19, 2019, with a face value of IDR 1,066,000,000,000 and a maturity date of December 19, 2022. The VG asset model in this study has a mean absolute percentage error (MAPE) value of 1.59 % which gives the conclusion that the asset model is very accurate. The bond valuation shows an expected equity value of IDR 271,071,400,000,000, an expected liability of IDR 31,359,700,000,000, and a default probability of 0. This result indicates that PT Bank CIMB Niaga Tbk is able to fulfill its obligations when the bonds mature.

Keywords: Heavy tails, excess kurtosis, ln asset returns, MAPE, default probability.

1. Introduction

One of the mathematical models that can be used to model and predict stock prices with ln returns of stock normally distributed is the geometric Brownian motion (GBM) model. The GBM model assumes that ln returns from the asset are normally distributed. The Research uses data from asset prices traded in Indonesia, showing that there is excess kurtosis and tail on the distribution of ln returns, so that the performance of the GBM model is not good enough to describe the dynamics of asset prices. Seneta and Madan (1990), and Madan and Milne (1991) had suggested the variance gamma (VG) approach, which has the advantage that there are parameters added to the distribution of ln returns of the asset to control volatility and kurtosis in the distribution of ln returns of the asset. Then, this VG model is generalized by Madan et al. (1998) by developing a three-parameter VG process, namely the addition of a parameter that controls skewness. The same thing was conveyed by Fiorani (2001). In his research, Fiorani suggests that the VG model not only controls volatility but also the slope and kurtosis of the distribution of asset returns. The results of the study were compared with the GBM model. The conclusion is that the VG model better describes the dynamics of stock prices in the market. The model that is often used to evaluate bonds is the Black Scholes model, whose asset returns follow the GBM model, which is normally distributed with constant volatility. Research on the valuation of corporate bonds in Indonesia has been carried out by Maruddani (2018) who mathematically modeled extreme data for bonds with asset data of bond issuing companies containing extreme data.

Maruddani et al. (2015) in their research used the GBM model for the valuation of one-period coupon bonds based on the default time with an empirical study of bond data in Indonesia. Abdurakhman and Maruddani (2018) had conducted research on bond valuation, namely the bond valuation of the Black-Scholes model coupled with equations related to the third and fourth moments, namely skewness and kurtosis. The valuation carried out includes the calculation of the estimated equity and probability of default of the company that issues bonds based on the standard normal distribution of the Gram-Charlier expansion model with the Hermite polynomial approach. Aniska et al. (2021) conducted a study on the valuation of one-period coupon bonds using the GBM model with a jump element. This study introduces a new method for bond valuation in Indonesia with the Variance Gamma model approach. The variance gamma process is one of the Levy process that are more widely used for data that has jumped. According to Landschoot (2004), there are three measures of corporate credit risk for company valuation, namely the capital value or equity, the debt value or liability, and probability of default. So that the bond valuation carried out in this study is to determine equity, liability and probability of default using the VG model.

2. Theoretical Results

2.1. The variance gamma distribution

According to Madan et al. (1998) the distribution of variance gamma (VG) for ln returns of stock price at time t, which follows the VG process, is

$$h(z)=\frac{2e^{\left(\frac{\theta x}{\sigma^2}\right)}}{v^{\frac{t}{v}}\sqrt{2\pi}\sigma\Gamma\left(\frac{t}{v}\right)}\left(\frac{x^2}{\frac{2\sigma^2}{v}+\theta^2}\right)^{\frac{t}{2v}-\frac{1}{4}}\cdot\kappa_{\left(\frac{t}{v}-\frac{1}{2}\right)}\cdot\frac{1}{\sigma^2}\cdot\sqrt{x^2\left(\frac{2\sigma^2}{v}+\theta^2\right)},$$

where κ is the modified Bessel function of the second kind, $x=z-\mu t-\frac{t}{v}\ln\left(1-\theta v-\frac{\sigma^2}{2}\right)$,

$z=x+\mu t+\frac{t}{v}\ln\left(1-\theta v-\frac{\sigma^2}{2}\right)$, μ is a parameter mean, σ is a parameter to control volatility, v is a parameter to control kurtosis, and θ is a parameter to control skewness.

According to Yang and Chu (2017) the modified Bessel function of the second kind $\kappa_n(x^*)$ is defined by

$$\kappa_n(x^*)=\frac{\pi\left(I_{-n}(x^*)-I_n(x^*)\right)}{2\sin(\pi n)},$$

where $I_n(x^*)$ (the modified Bessel function of the first kind x^*) is a particular solution of the second order differential equation

$$x^{*2}y''+x^*y-\left(x^{*2}+n^2\right)y=0,$$

and it can be expressed by the infinite series

$$I_n(x^*)=\sum_{n^*=0}^{\infty}\frac{1}{n^*!\Gamma\left(n+n^*+1\right)}\left(\frac{x^*}{2}\right)^{n+n^*},$$

where n is a real number (the order).

2.2. Variance gamma process

The variance gamma process is one of the Lévy process that is more widely used for data that has jumped. According to Matsuda (2005), a real valued stochastic process $(X_{t\in[0,\infty)})$ on a filtered probability space $(\Omega,\mathcal{F}_{t\in[0,\infty)},\mathrm{P})$ is said to be a Lévy process on R if it satisfies the following conditions:

1) Its increments are independent. In other words, for $0\le t_0<t_1<\ldots<t_n<\infty$,

$$P(X_{t_0}\cap\left(X_{t_1}-X_{t_o}\right)\cap\left(X_{t_2}-X_{t_1}\right)\cap\ \ldots\cap\left(X_{t_n}-X_{t_{n-1}}\right)=$$
$$P\left(X_{t_o}\right)P\left(X_{t_1}-X_{t_o}\right)P\left(X_{t_2}-X_{t_1}\right)\ldots\ P\left(X_{t_n}-X_{t_{n-1}}\right).$$

2) Its increments are stationary (time homogeneous): i.e. for $h\ge 0,\ X_{t+h}-X_t,$ has the same distribution as X_h. In other words, the distribution of increments does not depend on t.

3) Process starts from 0 almost surely (with probability 1):

$$P\left(X_0=0\right)=1.$$

4) The process is stochastically continuous:

$$\forall\varepsilon>0,\ \lim_{h\to 0}\mathrm{P}\left(\left|X_{t+h}-X_t\right|\ge\varepsilon\right)=0.$$

5) Its sample path (trajectory) is right continuous with left limit almost surely.

The variance gamma process is obtained by evaluating Brownian motion (with constant drift and volatility). Evaluation is done by substituting random time changes with the gamma process. Brownian equation of motion is written as follows

$$w\left(t;\theta,\sigma\right)=\theta t+\sigma W\left(t\right),$$

where $W(t)$ is the standard Brownian motion. The process $w\left(t;\theta,\sigma\right)$ is called Brownian motion with drift θ and volatility σ. The variance gamma process (Z_{VG}) can be stated as a function of the Brownian motion $w(t;\theta,\sigma)$ and the gamma process $g\left(t;1,v\right)$ as follows:

$$Z_{VG}\left(t;\theta,v,\sigma\right)=w\left(g\left(t;1,v\right);\theta,\sigma\right),$$

which has three parameters, namely, σ,v and θ.

2.3. Parameter estimation

According to Madan et al. (1998), one method of estimating the variance gamma parameter is the moment method. This method is easy to do and has a closed form. For example, $Z_{VG}(t)$ at time interval

t is a random variable VG with normal distribution with mean θg and $\sigma\sqrt{g}$ variance written as follows:

$$Z_{VG}(t) = \theta g + \sigma\sqrt{g}z,$$

where z is a random variable with independent standard normal distribution with random variable g having gamma distribution with mean t and variance νt. The initial step taken to estimate the VG parameter is to determine the first four moments (m) of $Z_{VG}(t)$ as follows:

1. First moment

$$m_1 = E\left(Z_{VG}(t)\right) = E\left(\theta g + \sigma\sqrt{g}z\right) = \theta t. \tag{1}$$

2. Second moment

$$m_2 = E\left[\left(Z_{VG}(t) - E\left(Z_{VG}(t)\right)\right)^2\right],$$

by assuming $\varkappa = Z_{VG}(t) - E\left(Z_{VG}(t)\right)$ that is $\varkappa = (g-t)\theta + \sigma\sqrt{g}z$, so that

$$\varkappa^2 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2,$$

then elaborate in the form $\left[(g-t)\theta + \sigma\sqrt{g}z\right]^2$, then calculate the expected value the result obtained

$$m_2 = E(\varkappa^2) = \left(\theta^2 v + \sigma^2\right)t. \tag{2}$$

3. Third moment

As in step (2) where $\varkappa^3 = \varkappa^2.\varkappa$ so that

$$\varkappa^3 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2\left[(g-t)\theta + \sigma\sqrt{g}z\right],$$

by elaborating this form and calculating the expectation obtained

$$m_3 = E(\varkappa^3) = \left(2\theta^3 v^2 + 3\sigma^2\theta v\right)t. \tag{3}$$

4. Fourth moment

The same steps are carried out, that is $\varkappa^4 = \varkappa^2\varkappa^2$, so that

$$\varkappa^4 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2\left[(g-t)\theta + \sigma\sqrt{g}z\right]^2,$$

by elaborating this form and calculating the expectation obtained

$$m_4 = E(\varkappa^4) = \left(3\sigma^4 v + 12\sigma^2\theta^2 v^2 + 6\theta^4 v^3\right)t + \left(3\sigma^4 + 6\sigma^2\theta^2 v + 3\theta^2 v^2\right)t^2. \tag{4}$$

According to Seneta (2004), the solution of Equations (2)-(4) may in practice require iterative procedures, but the following simple arguments can be used to begin such a procedure, and will sometimes suffice for estimation. From (2)-(4), so ignoring terms in $\theta^2, \theta^3, \theta^4$ we get

$$\text{Var}\left(Z_{VG}(t)\right) = \sigma^2, \tag{5}$$

$$\text{Skewness}\left(Z_{VG}(t)\right) = \frac{3\theta v}{\sigma}, \tag{6}$$

$$\text{Kurtosis}\left(Z_{VG}(t)\right) = 3(v+1). \tag{7}$$

If θ is small, then the full Equations (2)-(4) will be close to-satisfied by σ^2, υ, and θ obtained from approximate Equations (5)-(7) hence, these themselves form suitable (moment) estimates, so that the variance gamma parameters are estimated as follows:

(i) Estimated parameter σ :

$$\operatorname{Var}\left(Z_{VG}\left(t\right)\right)=\sigma^2,\text{ and } \hat{\sigma}=\sqrt{Z_{VG}\left(t\right)}.$$

(ii) Estimated parameter θ:

$$\text{Skewness}\left(Z_{VG}\left(t\right)\right)=\frac{3\theta v}{\sigma},\text{ and } \hat{\theta}=\frac{\sigma\,\text{Skewness}\left(Z_{VG}\left(t\right)\right)}{3v}.$$

(iii) Estimated parameter v:

$$\text{Kurtosis}\left(Z_{VG}\left(t\right)\right)=3\left(v+1\right),\text{ and } \hat{v}=\frac{\text{Kurtosis}\left(Z_{VG}\left(t\right)\right)}{3}-1.$$

The distribution fit test uses the chi-square test by making k sub-data intervals. The chi-square statistical value is determined as follows:

$$\chi^2=\sum_{i=1}^{k}\frac{\left(o_i-Np_i\right)^2}{Np_i},\tag{8}$$

where o_i is the observed value in the i-th sub interval, N is the sample size and p_i is the probability obtained randomly in the i-th sub interval. The value of the chi-square statistical value is compared with the value of $\chi^2_{\alpha^*;k-1-m^*}$, where α^* is the level of significance and m^* is the number of parameters in the VG model (Göncü et al. 2013).

2.4. The geometric Brownian motion (GBM)

The geometric Brownian motion (GBM) model has two parameters: the first parameter is μ the expected value of stock returns, the second parameter is σ the volatility of stock prices. Based on Brigo et al. (2007) the GBM model is determined as follows:

$$dM\left(t\right)=\mu M\left(t\right)dt+\sigma M\left(t\right)dW\left(t\right),\tag{9}$$

which is a stochastic differential equation. In Equation (9) M is the price of the asset and t is the time. W is standard Brownian motion with normal distribution with average 0 and variance equal to t_j-t_{j-1},μ is the expected value of stock returns, and σ is stock price volatility. The solution to Equation (9) to obtain the GBM asset price model is obtained through the Ito theorem. If there is an Equation:

$$dM(t)=\mu M(t)dt+\sigma M(t)dW(t),$$

then based on the Ito theorem the function $H=H\left(M,t\right)$ is as follows:

$$dH=\left(\frac{\partial H}{\partial M(t)}\mu M(t)+\frac{\partial H}{\partial t}+\frac{1}{2}\frac{\partial^2 H}{\partial M(t)^2}\sigma^2 M(t)^2\right)dt+\frac{\partial H}{\partial M(t)}\sigma M(t)dW(t).$$

For example, function $H=\ln M(t)$, with the condition $\frac{\partial H}{\partial M(t)}=\frac{1}{M(t)}$,

$$\frac{\partial^2 H}{\partial M(t)^2}=-\frac{1}{M(t)^2},\ \frac{\partial H}{\partial t}=0,$$

so that it is obtained

$$dH=\left(\mu-\frac{\sigma^2}{2}\right)dt+\sigma\,dW\left(t\right),$$

then by integrating both sides of the Equation from 0 to t, we get:

$$M(t) = M(0)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\ dW(t)\right). \tag{10}$$

According to Brian et al. (2007) to simulate this process, the discrete time continuous model $t_0 < t_1 < \ldots < t_n$ is solved as follows:

$$M(t_{i+1}) = M(t_i)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_{i+1} - t_i) + \ \sigma\sqrt{t_{i+1} - t_i}\,N_{i+1}\right),$$

where $N_1, N_2, \ldots, N_n$ is generated randomly independent of the standard normal distribution.

According to Avramidis and L'Ecuyer (2006) if the process follows the VG process, the Equation (10) becomes as follows:

$$M(t) = M(0).\exp\left[\left(\omega^* + r - q\right)t + Z_{VG}(t)\right],$$

where q is the dividend. So if there is no dividend, the model becomes:

$$M(t) = M(0)\cdot\exp\left[\left(\omega^* + r\right)t + Z_{VG}(t)\right], \tag{11}$$

where $\omega^* = \frac{1}{v}\ln\left(1 - \theta v - \frac{1}{2}\sigma^2 v\right)$, r is risk-free interest rate. So, according to Equation (11) to simulate the VG model as follow:

$$\begin{aligned} M(t_{i+1}) &= M(t_i)\cdot\exp\left(\left(\omega^* + r\right)\Delta t_i + Z_{VG}(t_i)\right), \\ &= M(t_i)\exp\left(\left(\omega^* + r\right)\Delta t_i + \ \theta\Delta g_i + \sigma\sqrt{\Delta g_i}\ z_i\right), \end{aligned} \tag{12}$$

where $\Delta g_i \sim \Gamma\left(\frac{\Delta t_i}{v}, v\right)$, $z_i \sim \text{Normal}(0,1)$. In Bond valuation, Equation (12) is used to forecast asset.

According to Goodwin and Lawton (1999) the forecast error at time $t = e_t = A_t^* - F_t^*$ where A_t^* the actual observation at time t and F_t^* the forecast made for period t. The MAPE for periods 1 to N^* of a single series is defined as follows:

$$\text{The percentage error } = \frac{A_t^* - F_t^*}{A_t^*} \times 100,$$

so that the absolute percentage error for period t.

$$\text{APE}_\text{t} = \frac{A_t^* - F_t^*}{A_t^*} \times 100,$$

and the mean absolute percentage error (MAPE) is

$$\text{MAPE} = \sum_{t=1}^{N^*} \frac{\text{APE}_\text{t}}{N^*}. \tag{13}$$

2.5. One-period coupon bonds

Based on Maruddani and Abdurakhman (2018), one-period coupon (c^*) bonds are bonds that provide coupon payments to investors once during the bond period. Coupons are given at maturity. In addition to the obligation to pay a coupon of K, the bond issuer has an obligation to pay the principal debt (face value) to bondholders. For example, suppose a company with a total asset value of M_{T1}, with a face value of *K*, the movement of the total asset value follows a geometric Brownian motion, the risk-free interest rate is denoted by *r*, the bond coupon amount is c^* and the maturity date is bond is T_1, then at maturity there are two possible circumstances, namely: If the asset value is more than or equal to the principal plus the coupon $(K + c^* = K_1)$ i.e. $M_{T_1} \geq K_1$, the bond issuer will pay the investor K_1 and the bond issuer has a capital or equity of $M_{T_1} - K_1$. If the asset value is less than the principal debt plus a coupon, i.e. $M_{T_1} < K_1$, then the bond issuer has 0 capital or equity, means that the bond issuer is default.

Based on Maruddani et al. (2015), a description of the situation for one coupon payment period is shown in Figure 1.

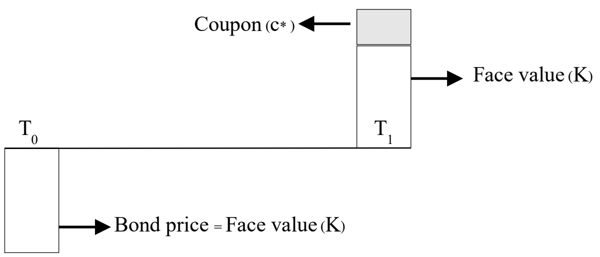

Figure 1 One-time cash flow coupon payment

While the payoff at maturity can be given in Table 1.

Table 1 Payoff or equity at maturity

Status	Asset	Liability	Equity
Non Default	$M_{T_1} \geq K_1$	K_1	$M_{T_1} - K_1$
Default	$M_{T_1} < K_1$	M_{T_1}	0

A new method for bond valuation in Indonesia with the variance gamma model is as follows : if a bond with principal K, maturity T_1, one coupon payment of c^*, initial asset value $M(0)$ and risk free interest rate r, then

1) The equity of the bond issuer company based on default at maturity is

$$\varphi_{T_0}^{T_1} = M(0)\Psi\left(d\sqrt{\frac{1-c_1}{v}}, (\alpha+s)\sqrt{\frac{v}{1-c_1}}, \frac{T_1}{v}\right) - K\exp(-rT_1)\Psi\left(d\sqrt{\frac{1-c_2}{v}}, \alpha\sqrt{\frac{v}{1-c_1}}, \frac{T_1}{v}\right). \quad (14)$$

2) The probability of default of the bond issuer based on default at maturity is:

$$P(\eta = T_1) = P\left(M_{T_1} < K_1\right),$$

$$= \int_{-\infty}^{Ln\left(\frac{K_1}{M(0)}\right)-\left(r+\omega^*\right)T_1} \frac{2e^{\left(\frac{\theta x}{\sigma^2}\right)}}{v^{\frac{t}{v}}\sqrt{2\pi}\sigma\Gamma\left(\frac{t}{v}\right)} \left(\frac{x^2}{\frac{2\sigma^2}{v} + \theta^2}\right)^{\frac{t}{2v}-\frac{1}{4}} \times$$

$$\kappa_{\frac{t}{v}-\frac{1}{2}} \frac{1}{\sigma^2}\sqrt{x^2(\frac{2\sigma^2}{v}+\theta^2)}\ dx. \quad (15)$$

To calculate liability (ℓ) using the basic accounting equation i.e. Asset = Equity + Liability, or in symbols the equation is written as follows

$$M_{T_1} = \varphi_{T_0}^{T_1} + \ell. \quad (16)$$

3. Methods

The data used in this study is bond data on banking companies in Indonesia, namely Bank CIMB Niaga Continuous Bonds III Phase I 2019 Series B. The PT Bank CIMB Niaga Tbk bonds were issued on December 19, 2019 and matured on December 19, 2022. Data The bonds were obtained from The Indonesia Central Securities Depository (KSEI) (2022). Meanwhile, the company asset in sample data used for modeling is obtained from the financial statements of PT Bank CIMB Niaga Tbk for the period January, 2012–December, 2019 and the out sample data used to see the accuracy of the model is the asset data of PT Bank CIMB Niaga Tbk for the period January–August, 2020. The asset data can be accessed at The Financial Services Authority (OJK) (2022).

The stages carried out in this research are:

(i) Exploring the data by making an opportunity plot to see the accuracy of the distribution and the fit test of the VG distribution using Equation (8),
(ii) Estimating the VG model parameters using maximum likelihood,
(iii) Doing the VG modeling for the company asset forecast using Equation (12),
(iv) Calculating asset prediction error using MAPE using Equation (13),
(v) Calculating the bond valuation, which consists of the expected value of equity (using Equation (14)), expected liabilities (using Equation (16)), and the probability of default (using Equation (15)).

The data is processed using the R4.0.2 software (R Core Team 2020). Some of the R packages used are: Variance Gamma (Scott et al., 2018), BAS (Clyde et al. 2011), and Bessel (Martin and Maechler 2019).

4. Results and Discussion

There have been many prediction models for assets or stocks. Aniska et al. (2020) compared the prediction accuracy of asset models using GBM (MAPE = 6.11%) and GBM with jump diffusion (MAPE = 3.87%). These results indicate that the GBM model with jump diffusion is better than the GBM model. We have observed the same data using the VG model (MAPE = 2.59%). These results indicate that the VG model is better. Hoyyi et al (2021) have conducted modeling to predict daily stock prices. The models used were the GBM (MAPE = 9.71%), the GBM with jump diffusion (MAPE = 11.77%), and the VG model (MAPE = 5.75%). These results show that the VG model is the most accurate. Therefore, the VG model is more appropriate to use as a reason for bond valuation.

The first stage in this research is data exploration, namely making quantile plots to see the distribution fitting.

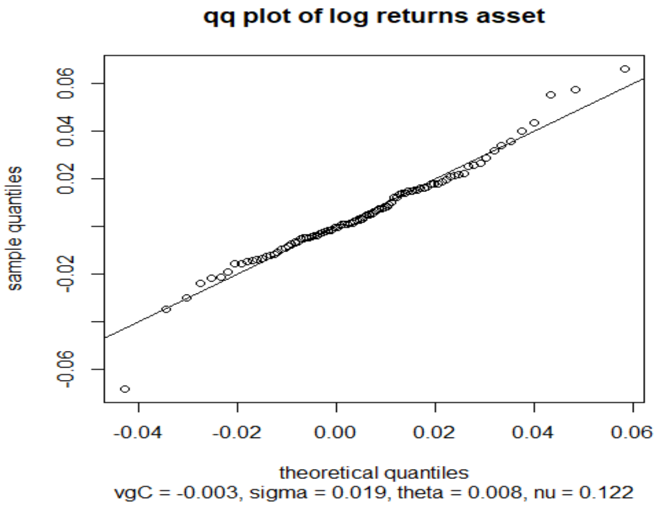

Figure 2 Plot of the quantiles of the variance gamma distribution

Figure 2 is a plot of the quantiles of the variance gamma distribution, which shows the pattern forming a straight line, so that there is a match between the data and the distribution used, namely the variance gamma distribution.

The fit test of the VG distribution model is as follows:

H_0: The ln returns of asset PT Bank CIMB Niaga Tbk follow VG distribution,

H_1: The ln returns of asset PT Bank CIMB Niaga Tbk are not following VG distribution.

The test was carried out using the chi-square test with $\alpha^* = 5\%$ the statistical value obtained was $\chi^2 = 3.89$ which was smaller than $\chi^2_{0.05;2} = 5.99$. Based on these result, gave a decision that the null hypothesis is accepted, which the distribution of the ln returns of asset PT Bank CIMB Niaga Tbk is VG distribution.

The estimation of the variance gamma model parameters is obtained by using the maximum likelihood method. This method has been discussed in the paper Fragiadakis et al. (2013). We use the software R to estimate these parameters with the 'VarianceGamma' package. The syntax used is 'vgFit(data)'. The results are as follows:

Table 2 Parameter estimation results of the variance gamma model

Company Name	$\hat{\mu}$	$\hat{\sigma}$	$\hat{v}$	$\hat{\theta}$
PT Bank CIMB Niaga Tbk.	0.013818	0.018697	0.122136	0.008455

Table 2 shows the results of the estimation of the parameter ln return of asset on the monthly data of PT Bank CIMB Niaga Tbk. The value $\hat{v}$ = 0.122136 is used to determine the kurtosis. From Equation (7) the monthly kurtosis is 3(1+(0.122136)(12)), or 7.40. These results indicate excess kurtosis. A non-zero value of $\hat{\theta}$ indicates that the distribution is asymmetric.

The results of calculating the MAPE value to determine the accuracy of the model obtained a MAPE value of 1.59%. These results provide a very accurate model conclusion.

The next step is to predict asset prices. In this study, the value of $T=0$ is in 2019 with an asset value of IDR 272,172,690,000,000. The value of the asset is the first M. The results of asset value predictions for the next three years are as follows:

Table 3 Predicted results of asset value

T	Year	Asset Prediction Results
1	2020	IDR 276,963,800,000,000
2	2021	IDR 296,319,300,000,000
3	2022	IDR 302,431,100,000,000

Table 3 shows the results of asset predictions for $T=1$ (2020), $T=2$ (2021), and $T=3$ (2022). The predicted value of asset at $T=3$ (2022) is used to determine liabilities.

Before calculating the expected value of capital (equity), it is necessary to know the bond data variables. In addition to the VG parameter, the required bond data variables in Equation (14) are: Face Value (*K*), Term Bond (*T*), Coupon (c^*), risk-free interest rate (*r*).

Table 4 Bond data variables

Data	Value
Bond Name	Continuous Bonds III Bank CIMB Niaga Phase I 2019 Series B
Bond Code	BNGA03BCN1
Publisher	PT Bank CIMB Niaga Tbk.
Face Value	IDR 1,066,000,000,000
Publication Date	December 19, 2019
Maturity Date	December 19, 2022
Term Bonds	3 Years
Rating	AAA
Coupon	7.55% p.a

Value of $K_1 = K(1+3c^*) = \text{IDR}\,1{,}307{,}449{,}000{,}000.$ The risk-free interest rate (*r*) used refers to the Bank Indonesia interest rate of 4.95%. The company's equity at maturity T_1, can be written as:

$$\varphi_{T_1} = \begin{cases} M_{T_1} - K_1 & \text{for } M_{T_1} \geq K_1 \\ 0 & \text{for } M_{T_1} < K_1, \end{cases} = \left(M_{T_1} - K_1\right)^+ = \text{Max}\left(M_{T_1} - K_1, 0\right).$$

According to Maruddani et al. (2016), the present value of company equity at $t = T_0$

$$\varphi_{T_0}^{T_1} = \exp\left(-rT_1\right)E\left[(M_{T_1} - K_1)^+\right] = \exp\left(-rT_1\right)\left[\text{Max}\left(M_{T_1} - K_1, 0\right)\right]. \tag{17}$$

Equation (17) represents the price of the European VG call option $\left(c_o\right)$ as stated by Madan et al. (1998). The solution to Equation (17) is obtained by integrating $W(g)$ to the gamma density which has a mean T_1 and a variance vT_1.

$$\text{c}_\text{o}\left(M(0);K,T_1\right) = \varphi_{T_0}^{T_1} = \exp\left(-rT_1\right)E\left[(M_{T_1} - K)^+\right] = \int_0^\infty W(g)f(g)dg\ ,$$

where $W(g)$ is the option price that has been stated by Madan and Milne (1991) namely

$$W(g)=M(0)\left(1-\frac{v(\alpha+\sigma)^2}{2}\right)^{\frac{T_1}{v}}\exp\left(\frac{(\sigma+\alpha)^2 g}{2}\right)\Phi\left(\frac{d}{\sqrt{g}}+(\sigma+\alpha)\sqrt{g}\right)$$

$$-K\exp(-rT_1)\left(1-\frac{v\alpha^2}{2}\right)^{\frac{T_1}{n}}\exp\left(\frac{\alpha^2 g}{2}\right)\Phi\left(\frac{d}{\sqrt{g}}+\alpha\sqrt{g}\right).$$

While $f(g)$ is the gamma density which has a mean T_1 and a variance vT_1, namely

$$f(g) = \frac{g^{\frac{T_1}{v}-1}e^{-g/v}}{v^{\frac{T_1}{v}}\Gamma\left(\frac{T_1}{v}\right)},$$

$$c_o(M(0);K,T_1) = \int_0^\infty W(g)\frac{g^{\frac{T_1}{v}-1}e^{-g/v}}{v^{\frac{T_1}{v}}\Gamma\left(\frac{T_1}{v}\right)}dg, \tag{18}$$

where g is a random variable with gamma distribution. Madan and Milne (1991) solved Equation (18) numerically. However, the closed form has been found by Madan et al. (1998). By substituting $y=\frac{g}{v}$ and defining $\gamma=\frac{T_1}{v}$, $c_1=\frac{v(\alpha+s)^2}{2}$, and $c_2=\frac{v\alpha^2}{2}$, Equation (18) becomes:

$$c_o(M(0);K,T_1)=\int_0^\infty\left\{M(0)(1-c_1)^\gamma\exp(c_1 y)N\left(\frac{d/\sqrt{v}}{\sqrt{y}}+(\alpha+s)\sqrt{v}\sqrt{y}\right)-\right.$$

$$\left.K\exp(-rT_1)(1-c_2)^\gamma\exp(c_2 y)N\left(\frac{d/\sqrt{v}}{\sqrt{y}}+\alpha\sqrt{v}\sqrt{y}\right)\right\}\frac{y^{\gamma-1}\exp(-y)}{\Gamma(\gamma)}dy. \tag{19}$$

By involving the modified Bessel function of the second kind, and the degenerate hypergeometric function of two variables, the solution to Equation (19) is as follows:

$$c_o(M(0);K,T_1)=M(0)\Psi\left(d\sqrt{\frac{1-c_1}{v}},(\alpha+s)\sqrt{\frac{v}{1-c_1}},\frac{T_1}{v}\right)$$

$$-K\exp(-rT_1)\Psi\left(d\sqrt{\frac{1-c_2}{v}},\alpha\sqrt{\frac{v}{1-c_1}},\frac{T_1}{v}\right), \tag{20}$$

where the function $\Psi(a,b,\gamma)$ is the solution of the following integral:

$\Psi(a,b,\gamma)=\int_0^\infty N\left(\frac{a}{\sqrt{u}},b\sqrt{u}\right)\frac{\exp(-u)u^{\gamma-1}}{\Gamma(\gamma)}du$, which resulting:

$$\psi(a,b,\gamma) = \frac{c^{\gamma+\frac{1}{2}}\exp\left[sign(a)c\right](1+u)^{\gamma}}{\sqrt{2\pi}\Gamma(\gamma)\gamma}\times$$

$$\kappa_{\gamma+\frac{1}{2}}(c)\Phi\left[\gamma,1-\gamma,1+\gamma;\frac{1+u}{2},-sign(a)c(1+u)\right]$$

$$-sign(a)\frac{c^{\gamma+\frac{1}{2}}\exp\left[sign(a)c\right](1+u)^{1+\gamma}}{\sqrt{2\pi}\Gamma(\gamma)(1+\gamma)}\times$$

$$\kappa_{\gamma-\frac{1}{2}}(c)\Phi\left[1+\gamma,1-\gamma,2+\gamma;\frac{1+u}{2},-sign(a)c(1+u)\right]$$

$$+\,sign(a)\frac{c^{\gamma+\frac{1}{2}}\exp\left[sign(a)c\right](1+u)^{\gamma}}{\sqrt{2\pi}\Gamma(\gamma)\gamma}\times$$

$$\kappa_{\gamma-\frac{1}{2}}(c)\Phi\left[\gamma,1-\gamma,1+\gamma;\frac{1+u}{2},-sign(a)c(1+u)\right],$$

where $d=\frac{1}{s}\left[\ln\left(\frac{M(0)}{K}\right)+rT_1+\frac{T_1}{v}\ln\left(\frac{1-c_1}{1-c_2}\right)\right]$, $s=\frac{\sigma}{\sqrt{1+\left(\frac{\theta}{\sigma}\right)^2\frac{v}{2}}}$, $\alpha=-\frac{\theta}{\sigma^2}s,$

$c=|a|\sqrt{2+b^2}$, $u=\frac{b}{\sqrt{2+b^2}}$, κ_δ is a modified Bessel function of the second order type δ,Φ is a degenerated hypergeometric function of two variables with the integral form of Humbert as follows:

$$\Phi(\alpha_1,\beta_1,\gamma_1;x_1,y_1) = \frac{\Gamma(\gamma_1)}{\Gamma(\alpha_1)\Gamma(\gamma_1-\alpha_1)}\times\int_0^1 u_1^{\alpha_1-1}(1-u_1)^{\gamma_1-\alpha_1-1}(1-u_1x_1)^{-\beta_1}\exp(u_1y_1)du_1.$$

Company Default Probability at Time T_1 (Maturity Time):

Consider Figure 1, based on the default at maturity the company is assumed to be default only at the maturity of the bonds T_1. So the default time (η) is a discrete random variable with

$$\eta=\begin{cases}\infty & Jika\ \ M_{T_1}\geq K_1\\ T_{\mathring{a}T} & Jika\ \ M_{\ 1}<K_1.\end{cases}$$

Probability of default at maturity, namely

$$P(\eta=T_1)=P\left(M_{T_1}<K_1\right)=P\left(M(0)\exp\left\{\left(r+\omega^*\right)T_1+Z_{VG}\right\}<K_1\right),$$

$$=P\left(Z_{VG}<\left(Ln\left(\frac{K_1}{M(0)}\right)-\left(r+\omega^*\right)T_1\right)\right),$$

$$=\int_{-\infty}^{Ln\left(\frac{K_1}{M(0)}\right)-\left(r+\omega^*\right)T_1} h(z)dx\ ,$$

$$= \int_{-\infty}^{Ln\left(\frac{K_1}{M(0)}\right)-\left(r+\omega^*\right)T_1} \frac{2e^{\left(\frac{\theta x}{\sigma^2}\right)}}{v^{\frac{t}{v}}\sqrt{2\pi}\sigma\Gamma\left(\frac{t}{v}\right)} \left(\frac{x^2}{\frac{2\sigma^2}{v}+\theta^2}\right)^{\frac{t}{2v}-\frac{1}{4}}$$

$$\times \kappa_{\frac{t}{v}-\frac{1}{2}} \frac{1}{\sigma^2}\sqrt{x^2(\frac{2\sigma^2}{v}+\theta^2)}dx,$$

where $h(z)$ is the density function of the variance gamma distribution. Then calculate the company's equity expectations in 2022 using Equation (14). The results obtained are IDR 271,071,400,000,000. Because the value of asset and expected equity in 2020 has been determined, according to the Equation (16) the value of liability (ℓ) can be calculated from the value of asset minus equity which results in IDR 31,359,700,000,000.

Probability of default at maturity is calculated using Equation (15)

$$P\left(M_{T_1} < K_1\right) = P\left(Z_{VG} < Ln\left(\frac{K_1}{M(0)}\right) - \left(r+\omega^*\right)T_1\right) = P\left(Z_{VG} < -5.460956\right) = 0.$$

These results provide information that PT Bank CIMB Niaga Tbk can certainly be able to pay its debts at the maturity date of the bonds. This is also supported by the expected value of equity which is much larger than the value of its liabilities.

5. Conclusions

The application of a one-period coupon bond valuation using the VG model on Bank CIMB Niaga Phase I 2019 Series B Continuous Bonds III data ensured that the company does not default. These results are indicated by the default probability value equal to zero. Other supporting factors are the high asset value and the good performance of PT Bank CIMB Niaga Tbk based on the rating. This research can be developed because bond trading is very complex, such as coupon payments more than once in the bond period, types of default time patterns, and the use of floating interest rates.

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