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The Zero-Truncated Poisson Generalized Lindley Distribution: Regression Model and Applications

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Abstract

This article introduces a new discrete distribution for nonzero count data. Some distributional properties, such as shape, probability generating function, moment generating function, and raw moments, are studied. The maximum likelihood method is applied to estimate the distribution parameters. A nonzero count regression model based on the proposed distribution is developed. The performance of the proposed distribution and its regression model are shown through real nonzero count data, which shows that they outperform other competitive models. Therefore, the proposed model can be considered for count data excluding zero.

Keywords: Count data analysis, regression model, maximum likelihood estimation, truncated count, mixed poisson distribution.

1. Introduction

Count data refers to data that represent the number of occurrences of a particular event or outcome within a specified time period or area. While nonzero count data refers to a type of data where the focus is on the number of occurrences or events, and the values are counts that are greater than zero. Nonzero count data is commonly encountered in various fields, including biology, epidemiology, finance, and social sciences. There are many examples of nonzero count data in real life such as the length of hospital stay, the number of occupants in passenger cars, or the number of items purchased by a customer making a purchase. Using a standard count distributions like the Poisson distribution without accounting for zero-truncation may bias estimates. For nonzero count data, zero-truncated distributions often provide a significantly better fit than their non-truncated counterparts. Furthermore, the zero-truncated distribution can correctly identify the likelihood function when nonzero observations.

Let Y be a nonnegative integer-valued random variable, the left truncated distribution of Y at k is given by

$$f(y|y > k) = \frac{f(y)}{1 - F(k)}, \tag{1}$$

for $y = k + 1, k + 2, \ldots$, $f(y)$ is the probability mass function (pmf), and $F(y)$ is the cumulative distribution function of Y (see Johnson (2005)).

Left truncation at zero is the most standard form of truncation in count data. The pmf of zero-truncated distribution can be written as

$$f(y|y>0)=\frac{f(y)}{1-f(0)}. \tag{2}$$

The zero-truncated distribution was originated by David and Johnson (1952). The distribution is modified from the Poisson distribution, called the zero-truncated Poisson distribution, for excluding zero data. Recently, various zero-truncated distributions have been proposed to apply for nonzero count data (Ghitany et al., 2008; Shanker, 2016; Shanker and Shukla, 2017a,b; Shukla et al., 2020; Hussain, 2020; Atikankul et al., 2021). Although these nonzero count distributions are more flexible than the zero-truncated Poisson distribution, they have not been developed for modeling nonzero count data with covariates.

The Poisson generalized Lindley (PGL) distribution was proposed by Atikankul (2023). The distribution is a mixture of the Poisson distribution and the generalized Lindley distribution (Abouammoh et al., 2015). The pmf of the PGL distribution is

$$f(y)=\frac{\theta^{\alpha}\Gamma\left(\alpha+y-1\right)\left(\alpha(\theta+2)-\theta+y-2\right)}{\Gamma(\alpha)\Gamma(y+1)(\theta+1)^{\alpha+y+1}}, \tag{3}$$

for $y=0,1,2,...$, with the shape parameter $\alpha>1$, and the scale parameter $\theta>0$, where the gamma function $\Gamma(y)=\int_0^{\infty}u^{y-1}e^{-u}du$ and $\Gamma(y)=(y-1)!$ for $y\in$ N.

The PGL distribution is log-concave and unimodal. The distribution was compared with other well-known count distributions that the distribution outperformed compared distributions.

The main goal of this paper is to introduce a new nonzero count distribution for analyzing count data excluding zero, called the zero-truncated Poisson generalized Lindley distribution (ZTPGL). The distribution is developed from the PGL distribution. Some statistical properties of the proposed distribution are studied. Maximum likelihood estimators of unknown parameters are derived. A nonzero count regression model based on the proposed distribution is also constructed. Some count data sets excluding zero are applied to show the potential of the proposed distribution and its associate regression model. The proposed model provides better fits than other models.

The rest of the paper is structured as follows. The ZTPGL distribution and its statistical properties are introduced in Section 2. Unknown parameters of the proposed distribution are derived by the maximum likelihood estimation in Section 3. A new regression model based on the proposed distribution is developed in Section 4. In Section 5, the potentiality of the ZTPGL distribution and its regression model are shown by nonzero real data sets. Finally, some concluding remarks are presented in Section 6.

2. The Zero-Truncated Poisson Generalized Lindley Distribution

In this section, we present a new nonzero count distribution, called the ZTPGL distribution. Furthermore, we also study its shape and some statistical properties.

Proposition 1 *Let Y be the a random variable follows the ZTPGL distribution with parameter $\alpha>1$ and $\theta>0$ if its pmf is*

$$f(y|y>0)=\frac{\theta^{\alpha}\left(\alpha(\theta+2)-\theta+y-2\right)\Gamma(\alpha+y-1)}{(\theta+1)^{y}\Gamma(y+1)\left((\theta+1)^{\alpha+1}\Gamma(\alpha)-\theta^{\alpha}(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right)}. \tag{4}$$

The pmf of the ZTPGL distribution in Equation (4) can be given by substituting Equation (3) into Equation (2). In addition, we can obtain the pmf of the ZTPGL distribution in another way, which is a mixture of the zero-truncated Poisson (ZTP) and partly-gamma distributions, it can be proof as follows:

Proof: Suppose a random variable $Y|\Lambda$ follows the ZTP distribution with pmf

$$f(y|\lambda) = \frac{e^{-\lambda}\lambda^y}{\Gamma(y+1)(1-e^{-\lambda})},$$

for $y = 1, 2, \ldots$. We assume that Λ follows the partly-gamma distribution with probability density function

$$h(\lambda) = \frac{\lambda^{\alpha-2}\theta^\alpha(\theta+1)^\alpha(\lambda+\alpha-1)e^{-\theta\lambda}(1-e^{-\lambda})}{(\theta+1)^{\alpha+1}\Gamma(\alpha) - \theta^\alpha\left(\alpha(\theta+2)-\theta-2\right)\Gamma(\alpha-1)},$$

for $\lambda > 0$.

Hence, the ZTPGL distribution can be obtained by

$$\begin{aligned}
f(y|y>0) &= \int_0^\infty \Big[\frac{e^{-\lambda}\lambda^y}{\Gamma(y+1)(1-e^{-\lambda})} \\
&\quad \times \left(\frac{\lambda^{\alpha-2}\theta^\alpha(\theta+1)^\alpha(\lambda+\alpha-1)e^{-\theta\lambda}(1-e^{-\lambda})}{(\theta+1)^{\alpha+1}\Gamma(\alpha) - \theta^\alpha(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)}\right)\Big] d\lambda \\
&= \frac{\theta^\alpha(\theta+1)^\alpha}{\Gamma(y+1)\left((\theta+1)^{\alpha+1}\Gamma(\alpha) - \theta^\alpha(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right)} \\
&\quad \times \int_0^\infty \left[e^{-(\theta+1)\lambda}\lambda^{\alpha+y-1} + \alpha e^{-(\theta+1)\lambda}\lambda^{\alpha+y-2} - e^{-(\theta+1)\lambda}\lambda^{\alpha+y-2}\right] d\lambda \\
&= \frac{\theta^\alpha(\theta+1)^\alpha}{\Gamma(y+1)\left((\theta+1)^{\alpha+1}\Gamma(\alpha) - \theta^\alpha(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right)} \\
&\quad \times \left[\frac{\Gamma(\alpha+y)}{(\theta+1)^{\alpha+y}} + \frac{\alpha\Gamma(\alpha+y-1)}{(\theta+1)^{\alpha+y-1}} - \frac{\Gamma(\alpha+y-1)}{(\theta+1)^{\alpha+y-1}}\right] \\
&= \frac{\theta^\alpha\left(\alpha(\theta+2)-\theta+y-2\right)\Gamma(\alpha+y-1)}{(\theta+1)^y\Gamma(y+1)\left((\theta+1)^{\alpha+1}\Gamma(\alpha) - \theta^\alpha(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right)}.
\end{aligned}$$

The ZTPGL distribution is unimodal since

$$\frac{f(y+1|y>0)}{f(y|y>0)} = \frac{\alpha+y-1}{(y+1)(\theta+1)}\left(1 + \frac{1}{\alpha(\theta+2)-\theta+y-2}\right), \tag{5}$$

is a decreasing function in y.

Furthermore, the $f(y)$ is log-concave and the distribution has an increasing failure rate (Johnson, 2005) because

$$\frac{f(y+2|y>0)f(y|y>0)}{f^2(y+1|y>0)} = \frac{(y+1)(\alpha+y)}{(y+2)(\alpha+y-1)}\left(\frac{1+\frac{1}{\alpha(\theta+2)-\theta+y-1}}{1+\frac{1}{\alpha(\theta+2)-\theta+y-2}}\right) < 1. \tag{6}$$

Figure 1 shows some pmf plots of the ZTPGL distribution for some parameter values. The figure confirms that the distribution is decreasing function and unimodal.

Proposition 2 *The probability generating function of the ZTPGL distribution is given by*

$$G(z) = \frac{\theta^\alpha(\theta-z+1)^{-\alpha}\left((\theta+1)^\alpha(-\theta+z-2)+(\theta+2)(\theta-z+1)^\alpha\right)}{(\theta+2)\theta^\alpha - (\theta+1)^{\alpha+1}}, \tag{7}$$

where $|\ z\ | < \theta + 1$.

Proposition 3 *The moment generating function of the ZTPGL distribution is given by*

$$M(t) = \frac{\theta^\alpha(\theta-e^t+1)^{-\alpha}\left((\theta+1)^\alpha(-\theta+e^t-2)+(\theta+2)(\theta-e^t+1)^\alpha\right)}{(\theta+2)\theta^\alpha - (\theta+1)^{\alpha+1}}, \tag{8}$$

where $t < \log(\theta+1)$.

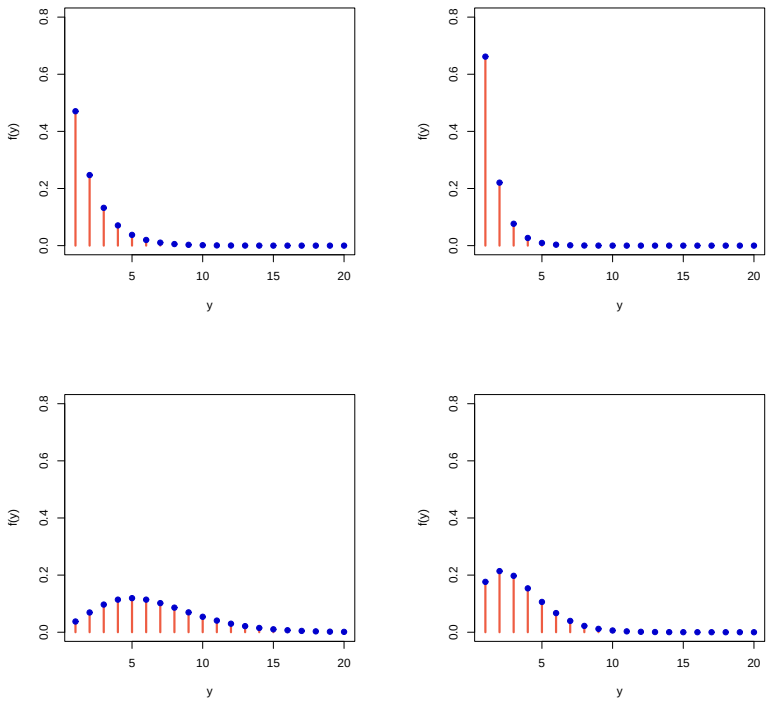


Figure 1 Pmf plots of the ZTPGL distribution with some parameter values

The first four raw moments about the origin of the ZTPGL are, respectively

$$\begin{aligned}
\mathrm{E}(Y|Y>0) &= \delta\left[\frac{(\alpha-1)\,\theta+\alpha}{\theta}\right],\\
\mathrm{E}(Y^2|Y>0) &= \delta\left[\frac{(\alpha-1)\theta(\alpha+\theta)+\alpha(\alpha+\theta+1)}{\theta^2}\right],\\
\mathrm{E}(Y^3|Y>0) &= \delta\left[\frac{(\alpha-1)\theta^3+\alpha(3\alpha-2)\theta^2+\alpha(\alpha+1)(\alpha+2)\theta+\alpha(\alpha+1)(\alpha+2)}{\theta^3}\right],\\
\mathrm{E}(Y^4|Y>0) &= \delta\Big[\frac{(\alpha-1)\theta^4+\alpha(7\alpha-6)\theta^3+\alpha(\alpha+1)(6\alpha+1)\theta^2}{\theta^4}\\
&\quad+\frac{\alpha(\alpha+1)(\alpha+2)(\alpha+5)\theta+\alpha(\alpha+1)(\alpha+2)(\alpha+3)}{\theta^4}\Big],
\end{aligned}$$

where $\delta=\dfrac{(\alpha-1)\,(\theta+1)^{\alpha}}{(\alpha-1)\,(\theta+1)^{\alpha+1}-\theta^{\alpha}\,(\alpha\,(\theta+2)-\theta-2)}$.

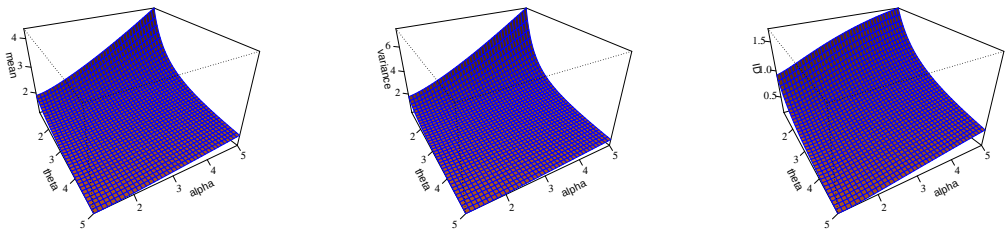
The variance of the ZTPGL distribution is given by

$$\mathrm{V}(Y|Y>0) = \frac{\alpha(\theta+1)^3-\theta^2(\theta+2)}{\theta^2(\theta+1)\left(-(\theta+2)\left(\frac{\theta}{\theta+1}\right)^{\alpha}+\theta+1\right)}-\frac{(\theta+2)\theta^{\alpha-2}(\theta+1)^{\alpha-1}\left((\alpha-1)\theta+\alpha\right)^2}{\left((\theta+1)^{\alpha+1}-(\theta+2)\theta^{\alpha}\right)^2}.$$

The index of dispersion (ID) of the ZTPGL distribution can be written by

$$\mathrm{ID}(Y|Y>0) = \frac{\sigma^2}{\mu}. \tag{9}$$

Figure 2 displays the mean and variance plots of the ZTPGL distribution. We can see that the mean and variance are increasing as α increases, but the mean and variance are increasing as θ decreases. From the ID plot, the ZTPGL distribution the ZTPGL distribution can be underdispersion, equidispersion, or overdispersion.



(a) Mean **(b)** Variance **(c)** ID

Figure 2 Mean plot (a), variance plot (b) and ID plot (c) of the ZTPGL distribution

3. Parameter Estimation

In this section, the method of maximum likelihood is employed to estimate the parameters of the ZTPGL distribution

Suppose $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed (iid) random sample from the PGL distribution with the parameters α and θ; and $y_1, y_2, \ldots, y_n$ be the observations. The likelihood function of the ZTPGL distribution is given by

$$L(\alpha,\theta) = \prod_{i=1}^{n} \frac{\theta^{\alpha}(\theta+1)^{-y_i}\left(\alpha(\theta+2)-\theta+y_i-2\right)\Gamma(\alpha+y_i-1)}{\Gamma(y_i+1)\left((\theta+1)^{\alpha+1}\Gamma(\alpha)-\theta^{\alpha}(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right)}.$$

The corresponding log-likelihood function is

$$\begin{aligned}\log L(\alpha,\theta) &= n\alpha\log(\theta)+\sum_{i=1}^{n}\log\left(\Gamma(\alpha+y_i-1)\right)+\sum_{i=1}^{n}\log\left(\alpha(\theta+2)-\theta+y_i-2\right)\\ &\quad -\sum_{i=1}^{n} y_i\log(\theta+1)-\sum_{i=1}^{n}\log\Gamma(y_i+1)\\ &\quad -n\log\left((\theta+1)^{\alpha+1}\Gamma(\alpha)-\theta^{\alpha}(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1)\right).\end{aligned}$$

The partial derivatives of the log-likelihood for the parameter α and θ are

$$\begin{aligned}\frac{\partial\log L(\alpha,\theta)}{\partial\alpha} &= n\log(\theta)+\sum_{i=1}^{n}\left(\Psi(\alpha+y_i-1)\right)+\sum_{i=1}^{n}\frac{\theta+2}{(\theta+2)\,\alpha+y_i-\theta-2}\\ &\quad -\frac{n}{(\theta+1)^{\alpha+1}\Gamma(\alpha)-\theta^{\alpha}(\theta+2)\Gamma(\alpha-1)(\alpha-1)}\\ &\quad \times\Big[(\theta+1)^{\alpha+1}\Psi(\alpha)\Gamma(\alpha)+(\theta+1)^{\alpha+1}\log(\theta+1)\Gamma(\alpha)\\ &\quad -\theta^{\alpha}(\theta+2)\Psi(\alpha-1)\Gamma(\alpha-1)(\alpha-1)\\ &\quad -\theta^{\alpha}(\theta+2)\log(\theta)\Gamma(\alpha-1)(\alpha-1)-\theta^{\alpha}(\theta+2)\Gamma(\alpha-1)\Big], \quad (10)\end{aligned}$$

$$\begin{aligned}\frac{\partial \log L(\alpha,\theta)}{\partial \theta} &= \frac{n\alpha}{\theta}+\sum_{i=1}^{n}\frac{\alpha-1}{\alpha(\theta+2)-\theta+y_i-2}-\sum_{i=1}^{n}\frac{y_i}{\theta+1}\\ &\quad -\frac{n}{\Gamma(\alpha)(\theta+1)^{\alpha+1}-\Gamma(\alpha-1)\,\theta^{\alpha}(\alpha(\theta+2)-\theta-2)}\\ &\quad \times\Big[(\alpha+1)\Gamma(\alpha)(\theta+1)^{\alpha}-\Gamma(\alpha-1)(\alpha-1)\theta^{\alpha}\\ &\quad -\Gamma(\alpha-1)\alpha\theta^{\alpha-1}(\alpha(\theta+2)-\theta-2)\Big]. \end{aligned} \tag{11}$$

The maximum likelihood estimates of the parameters α and θ can be obtained by setting Equations (10) and (11) to zeros and solving them. In this paper, we apply the numerical methods of the optimx function in the optimx package (Nash and Varadhan, 2011) available in the R programming language (R Core Team, 2023) (see Appendix).

For small sample sizes, maximum likelihood estimators are biased estimators. Moreover, the likelihood surface can be flat or irregular, making it hard for optimization algorithms to locate the maximum likelihood estimate.

Proposition 4 *Let Θ be the vector of parameters, the maximum likelihood estimator is consistency and asymptotic normality:*

$$\sqrt{n}(\hat{\Theta}_{MLE}-\Theta)\xrightarrow{d} N(0,I^{-1}(\Theta)), \tag{12}$$

where I is the Fisher information about Θ. The distribution of the maximum likelihood estimates becomes approximately normal, centered at the true parameter, with variance equal to the inverse of the Fisher Information.

4. The Zero-Truncated Poisson Generalized Lindley Regression Model

Let $Y_1, Y_2, \ldots, Y_n$ be iid from the PGL distribution, denoted by $Y_i \sim$PGL(α,θ). The pmf of Y is given by

$$f(y)=\frac{\theta^{\alpha}\Gamma(\alpha+y-1)\,(\alpha(\theta+2)-\theta+y-2)}{\Gamma(\alpha)\Gamma(y+1)(\theta+1)^{\alpha+y+1}}.$$

The mean of Y is given by

$$\mu=\frac{(\alpha-1)\theta+\alpha}{\theta(\theta+1)}.$$

The PGL regression model can be derived by parameterization $\alpha=\frac{\theta(\theta\mu+\mu+1)}{\theta+1}$; thus, the pmf of PGL distribution with parameter θ and μ can be defined by

$$f(y)=\frac{\theta^{\frac{\theta(\theta\mu+\mu+1)}{\theta+1}}\Gamma\left(\frac{\theta(\theta\mu+\mu+1)}{\theta+1}+y-1\right)\left(\frac{\theta(\theta\mu+\mu+1)}{\theta+1}(\theta+2)-\theta+y-2\right)}{\Gamma\left(\frac{\theta(\theta\mu+\mu+1)}{\theta+1}\right)\Gamma(y+1)\,(\theta+1)^{\frac{\theta(\theta\mu+\mu+1)}{\theta+1}+y+1}}.$$

The ZTPGL regression model with parameter μ and θ can be defined by

$$f(y|y>0)=\frac{\theta^{\alpha}(\theta+1)^{-y}\,(\alpha(\theta+2)-\theta+y-2)\,\Gamma(\alpha+y-1)}{\Gamma(y+1)\,((\theta+1)^{\alpha+1}\Gamma(\alpha)-\theta^{\alpha}(\alpha(\theta+2)-\theta-2)\Gamma(\alpha-1))},$$

where $\alpha=\frac{\theta(\theta\mu+\mu+1)}{\theta+1}$.

The ZTPGL regression model can be defined by

$$g(\boldsymbol{\mu}_i)=\boldsymbol{x}_i^T\boldsymbol{\beta},\quad i=1,\ldots,n,$$

where $g(\boldsymbol{\mu}_i)$ is the link function, $\boldsymbol{x}_i^T = (1, x_{i1}, x_{i2}, \ldots, x_{ip})$ is a vector of explanatory variables and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_p)$ is a vector of unknown regression coefficients.

In this paper, the logarithmic link function is considered for constructing the ZTPGL regression model to confirm that the mean is positive. Hence

$$\log(\mu_i) = \boldsymbol{x}_i^T \boldsymbol{\beta}.$$

The log-likelihood function of the ZTPGL regression model with the parameter vector $\Theta = (\boldsymbol{\beta}^T, \theta)^T$ is given by

$$\begin{aligned}
\log L(\Theta) \quad = \quad & n\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1}\log(\theta) + \sum_{i=1}^{n}\log\left(\Gamma\left(\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1} + y_i - 1\right)\right) \\
& + \sum_{i=1}^{n}\log\left(\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1}(\theta + 2) - \theta + y_i - 2\right) \\
& - \sum_{i=1}^{n} y_i \log(\theta + 1) - \sum_{i=1}^{n}\log\Gamma(y_i + 1) \\
& - n\log\left[(\theta + 1)^{\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1} + 1}\Gamma\left(\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1}\right)\right. \\
& \left. - \theta^{\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1}}\left(\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1}(\theta + 2) - \theta - 2\right)\Gamma\left(\frac{\theta(\theta\mu_i + \mu_i + 1)}{\theta + 1} - 1\right)\right].
\end{aligned}$$

The maximum likelihood estimates can be obtained by the numerical methods. Here, we apply the optimx package (Nash and Varadhan, 2011) in the R programming language (R Core Team, 2023).

5. Applications

In this paper, nonzero count applications are applied to show the potential of the proposed model. We compare the ZTPGL model with the Poisson, ZTP, and PGL models in nonzero count data sets. We determine the log-likelihood value, the Akaike information criterion (AIC) (Akaike, 1974), the Bayesian information criterion (BIC) (Schwarz, 1978) and the discrete Anderson-Darling (AD) (Choulakian et al., 1994) goodness of fit test for selecting models. The model with the smallest the AIC value, the smallest BIC value, the largest log-likelihood, and the largest p-value based on the discrete AD test indicate the best model.

5.1. Application 1

For the first application, we consider the number of estate areas where at least one live birth and one neonatal death. It appears in Borah and Saikia (2017).

Table 1 shows the maximum likelihood estimates, the log-likelihood values, the AIC values, the BIC values, the AD statistics, and p-values based on the discrete AD test. The table indicates that the ZTPGL distribution provides a better fit than the Poisson, ZTP, and PGL distributions because it provides the largest log-likelihood value, the largest p-value, the smallest AIC value, the smallest BIC value, and the smallest AD statistic. In addition, the PGL distribution cannot be used to fit this data set because the p-value of the discrete AD test is less than 0.05.

Table 1 Comparison of count distributions for nonzero data

Number of neonatal death	Number of mother	Expected frequency			
		Poisson	ZTP	PGL	ZTPGL
1	176	83.799	164.408	83.400	173.380
2	44	59.412	61.139	58.869	49.056
3	16	28.082	15.157	28.013	14.890
4	6	9.955	2.818	10.108	4.600
5	2	2.823	0.419	2.950	1.429
Estimated parameter		$\hat{\lambda} = 1.418$	$\hat{\lambda} = 0.744$	$\hat{\alpha} = 89.221$ $\hat{\theta} = 62.211$	$\hat{\alpha} = 1.235$ $\hat{\theta} = 2.272$
Log-likelihood		-312.962	-214.343	-314.068	-209.572
AIC		627.925	430.685	632.135	423.144
BIC		631.422	434.183	639.130	430.139
Discrete AD statistic		71.681	1.723	72.393	0.113
p-value		< 0.001	0.080	< 0.001	0.846

5.2. Application 2

The second data set consists of 1959 observations of patients in Arizona Medicare, 1991 (Hilbe, 2014). The Poisson, ZTP, PGL and ZTPGL regression models are considered with variables associated with length of stay in hospital. Variables of azcabgptca data are shown in Table 2.

Table 2 Summary statistics for the azcabgptca data

Variable	Measurement	Min	Max
Los	Length of stay in hospital	1	51
Procedure	(1 = CABG; 0 = PTCA)	0	1
Gender	Gender (1 Male; 0 = Female)	0	1
Type	(1 = Emerg/Urgent; 0 = Elective)	0	1

Table 3 displays the maximum likelihood estimates of the parameters, their standard errors (S.E.), and the p-values for azcabgptca data set. The variable age cannot be considered because it is not significant at the level of 5%. The values of the log-likelihood, AIC, and BIC criterion are shown in Table 4. Since the ZTPGL regression model provides the largest log-likelihood value, the lowest AIC value, and the lowest BIC value, the proposed model is the best regression model among the competing regression models.

Table 3 Parameter estimates and the p-values of the fitted regression models for the azcabgptca data set

	Poisson		ZTP		PGL		ZTPGL	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	1.284	< 0.001	1.254	< 0.001	1.286	< 0.001	1.159	< 0.001
Procedure	1.123	< 0.001	1.152	< 0.001	1.110	< 0.001	1.241	< 0.001
Type	0.196	< 0.001	0.202	< 0.001	0.186	< 0.001	0.198	< 0.001
Gender	-0.105	< 0.001	-0.109	< 0.001	-0.088	< 0.001	-0.095	< 0.001
θ	-	-	-	-	1.286	< 0.001	1.159	< 0.001

Table 4 Measures for azcabgptca data

Criterion	Poisson	ZTP	PGL	ZTPGL
Log-likelihood	−5235.500	−5206.756	−4956.587	−4867.469
AIC	10479.000	10421.510	9923.174	9744.938
BIC	10501.320	10443.830	9951.075	9772.839

5.3. Application 3

The third data set contains 3589 observations of Arizona cardiovascular patients in 1991 (Hilbe, 2011). Table 5 displays summary statistics of the response variable and explanatory variables of the azpro data set. We consider the length of hospital stay as the response variable.

Table 5 Summary statistics for the azpro data set

Variables	Measurement	Min	Max
Los	Length of hospital stay	1	83
Procedure	Standard cardiovascular procedures (1= CABG; 0 = PTCA)	0	1
Sex	Sex (1 = Male; 0 = Female)	0	1
Admit	Type of admission (1 = Urgent/Emerg; 0 = Elective)	0	1
Age75	Age (1 = Age $>$ 75; 0 = $\leq$ 75)	0	1

Table 6 presents parameter estimates, and p-values of the Poisson, ZTP, PGL, and ZTPGL regression models for azpro data set. From Table 6, we can see that p-values for all parameter estimates are less than 0.05; thus, the estimates of all parameters are significant at the level of 5%.

Table 6 Parameter estimates, S.E. and the p-values of the fitted regression models for the azpro data set

	Poisson		ZTP		PGL		ZTPGL	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	1.456	< 0.001	1.446	< 0.001	1.482	< 0.001	1.396	< 0.001
Procedure	0.960	< 0.001	0.968	< 0.001	0.942	< 0.001	1.023	< 0.001
Sex	−0.123	< 0.001	−0.125	< 0.001	−0.114	< 0.001	−0.123	< 0.001
Admit	0.326	< 0.001	0.331	< 0.001	0.300	< 0.001	0.320	< 0.001
Age75	0.122	< 0.001	0.123	< 0.001	0.115	< 0.001	0.124	< 0.001
θ	-	-	-	-	0.7663	< 0.001	0.6600	< 0.001

Table 7 reports the measures for azpro data set. The ZTPGL regression also provides the best model among the three.

Table 7 Measures for azpro data set

Criterion	Poisson	ZTP	PGL	ZTPGL
Log-likelihood	−11190.000	−11173.100	−9970.605	−9863.058
AIC	22390.000	22356.200	19953.210	19738.120
BIC	22420.930	22387.130	19990.320	19775.230

Based on the lowest of the negative log-likelihood value, the lowest of the AIC value, the lowest of BIC value, and the largest the p-value of the discrete AD test, the proposed models provide better results than the other compared models considered for the following reasons: First, the Poisson and PGL models are inappropriate for all three applications because these data sets are nonzero count data. Second, the ZTPGL distribution has more the number of parameters than the ZTP distribution; thus, the ZTPGL distribution is more flexible than the ZTP distribution.

Furthermore, all three applications are nonzero health data, the ZTPGL models can be applied to health data excluding zero.

6. Conclusions and Discussions

In this paper, we propose the ZTPGL distribution for nonzero count data. the distribution can be underdispersion, equidispersion, or overdispersion. It is developed from the PGL distribution. Some statistical properties of the proposed distribution are investigated. The method of maximum likelihood is used to estimate its parameters. Furthermore, we also proposed a new nonzero count regression model based on the ZTPGL distribution for the analysis of data excluding zero counts. The usefulness of the proposed distribution and its regression model are shown in real-world data sets. All three data sets are health data, in which the response variable is nonzero. Based on the model selection criteria, including log-likelihood, AIC, BIC, and the discrete AD goodness of fit test, the proposed model provides better fit than other competitive models. Therefore, the proposed model can be considered as an alternative nonzero count data, especially health data.

The ZTPGL distribution is undimodal distribution, thus the model is not suitable for bimodal data. Furthermore, the maximum likelihood estimation can be biased for small sample size and it is sensitive to outliers. For the future study, other parameter estimation methods such as the Bayesian approach will be applied.

Appendix

```
library(optimx)
ll = function (par)
{
theta=par[1]
alpha=par[2]
f = log(ztpgwl(data,theta,alpha))
return(-sum(f))
}
fit <- optimx(start,ll,method="BFGS",hessian=TRUE)
```

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