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An Efficiency of DMEWMA Control Chart to Monitor Changes in Process Mean on First Order Moving Average Model with Application to Daily Natural Gas Data

Julalak Neammai [a], Yupaporn Areepong*[a], and Saowanit Sukparungsee [a]
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand.
*Corresponding author; e-mail: yupaporn.a@sci.kmutnb.ac.th

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Abstract

The double modified exponentially weighted moving average (DMEWMA) control chart is a mixture of two modified exponentially weighted moving average (MEWMA) control chart for monitoring shifts in the process mean. The average run length (ARL) is important for evaluating the performance of control charts. The purpose of this research is to derive the exact formula for ARL on the DMEWMA Control Chart for moving average process order one in one-sided. The absolute percentage relative error (APRE) was used to compare the accuracy of the exact formula to four quadrature distinct numerical integral equation approaches (NIE). Following that, Banach's fixed point theorem is applied to ensure its existence and uniqueness. Furthermore, the effectiveness of the DMEWMA control chart is compared to that of the EWMA and MEWMA control charts. The ARL, the Standard Deviation Run Length (SDRL), and the relative mean index (RMI) are measurement tools used to assess a control chart's capacity to detect process mean. The results demonstrate that the ARL generated by the exact formula and the NIE approach are almost the same, but the exact formula can assist reduce computational (CPU) time. This is further expanded to provide performance comparisons with the EWMA and MEWMA control charts. For almost all scenarios, the DMEWMA control chart outperforms the EWMA control chart and the MEWMA control chart. Finally, the ARL analytical solution gets utilized on real-world data.

Keywords: Double MEWMA control chart, moving average process, average run length, numerical integer equations, exact formula

1. Introduction

Many statistical methods have evolved and been utilized in a wide range of activities, including those often employed in industry, business, health care, the environment, engineering, and other sectors. Control charts are a critical tool in Statistical Process Control (SPC) and are widely used to control, monitor, and optimize a manufacturing system.

Control charts are used to detect changes in the process. Shewhart's initial control chart (1931), which was used to detect shifts in process means but is not powerful for detecting small changes.

Following that, Page (1954) proposed a cumulative sum, and Roberts (1959) proposed the exponentially weighted moving average (EWMA), which is an alternative to a Shewhart individuals. Many studies have since enhanced the performance of EWMA charts for minor process changes, for example Shamma and Shamma (1992) introduced Double exponentially weighted moving average (DEWMA), Capizzi and Masarotto (2003) studied adaptive exponentially weighted moving average (AEWMA), extended exponentially weighted moving average (EEWMA) (Naveed et al. 2018), and the modified exponentially weighted moving average (MEWMA) was proposed by Patel and Divecha (2011). Khan et al. (2017), another modification to the EWMA chart is the modified EWMA control chart, published by examining MEWMA charts in greater depth and provided a broad pattern of it. Alevizakos et al. (2021) had presented a novel control chart that is a hybrid of the two MEWMA control charts for monitoring shifts in the process mean under the assumption of a normal distribution. It's referred to as the double modified EWMA (DMEWMA).

Process correlation can be captured by time series models. The stationary process is a type of time series model in which it is expected that the process will remain stable around a constant mean. The type of model can give a foundation for implementing statistical control while monitoring autocorrelated activities. The moving average order one $(MA(1))$ is a fundamental time series model used extensively in statistics and econometrics, in which the present value of a series is expressed as a function of the previous errors and a white noise term. Naturally, autocorrelation can be a beneficial tool for traders to use, especially for technical analysts who can apply it to real-world data such as financial forecasting, environmental monitoring, and compliance and regulatory oversight. Moving average order one models are widely utilized in financial forecasting to discern patterns in stock prices and other financial instruments. Traditional statistical methods, such as ARMA-GARCH, have long been pivotal in analyzing time series data for volatility and trends in financial markets. For the selected chart that use to monitor changes in process mean on first order moving average model, we used a previously analyzed control chart and applied it to the correlated data.

The average run length (ARL) is the most commonly used metric for assessing the performance of a control chart. The power of a control chart is defined as the likelihood of detecting an out-of-control signal, whereas the ARL represents the average number of samples required to signal an out-of-control situation in the process (or the predicted value of the run length) and stands for the in-control and out-of-control ARLs, respectively. When the process changes unfavorably, should be as large as possible and should be as little as possible. The ARL can be calculated using a variety of methods, including numerical integration equations (IEs) based on various quadrature rules, Markov chains, Martingale, and Monte Carlo simulation. The exact form of the ARL requires the solution of second order Fredholm IEs.

Based on previous literature research, several researchers have calculated the ARL using explicit formulas based on the moving average model. Petcharat et al. (2013) used the explicit formula on the EWMA chart for a moving average model of order q with exponential white noise to assess the ARL. Petcharat et al. (2014) used the Fredhom type integral equations approach to get the explicit formula of the ARL for a CUSUM chart running the $MA(1)$ process, while Petcharat et al. (2015) used the $MA(q)$ process in the CUSUM chart to derive the explicit formula of the ARL. Supharakonsakun et al. (2020) looked at explicit formulations for both the one-sided and two-sided ARL on a MEWMA control chart for a first order moving-average process. Supharakonsakun (2021) has published an exact formula for the ARL for the $MA(q)$ model on the MEWMA chart. Many academics continue to calculate ARL using exact formulas based on various models and control charts.

However, no previous research on the precise formulations of the ARL on the double modified

EWMA chart for the $MA(1)$ model has been published. Furthermore, the purpose of this study is to derive precise formulas for the ARL on the DMEWMA chart with $MA(1)$ model with exponential white noise. Furthermore, the explicit formula efficacy for computing the ARL on the DMEWMA control chart was compared to the EWMA and MEWMA control charts for both simulated and real-world datasets.

2. Materials and Methods

2.1. A first order moving average model

A moving average model is a model that has a similarity between past errors and current values. This is because unpredictable events occur and those events need to be taken into account. A first order moving average process, written as $MA(1)$, has the general equation

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}, \tag{1}$$

where μ is the mean of the process, θ_1 is a numeric process coefficient for the value associated with the 1^{st} lag, ε_t and ε_{t-1} are exponentially distributed white noise process which represent the residuals for the current and the previous period, respectively with the starting value of ε_0 is given.

2.2. Control charts

2.2.1. Exponentially weighted moving average (EWMA)

Roberts (1959) proposed the exponentially weighted moving average (EWMA). The EWMA is a statistical inquiry procedure that averages data so that it becomes less weighted as it moves further away from the current measurement in time. The EWMA statistic is computed recursively from individual data points at time t, i.e.,

$$E_t = \lambda X_t + (1-\lambda)E_{t-1}, \tag{2}$$

where E_t is t^{th} statistic, λ is an exponential smoothing parameter such that $0 \le \lambda \le 1$, and X_t is t^{th} observation from $MA(1)$ model. The center line (CL) for the control chart is the average of historical data. The upper (UCL) and lower (LCL) control limit of the EWMA control charts are determined by

$$UCL / LCL = \mu_0 \pm L_1 \sigma \sqrt{\frac{\lambda}{2-\lambda}} \tag{3}$$

where μ_0 and σ are the estimates of the long-term process mean and standard deviation generated during control-chart setup, respectively, and L_1 is the width of the appropriate control limit.

2.2.2. Modified EWMA (MEWMA)

Khan et al. (2017) created and presented the modified exponentially weighted moving average (MEWMA) control chart. The MEWMA control chart is an attempt to improve on the conventional EWMA chart's performance. A new MEWMA control chart that considers the constant case. The MEWMA statistic control chart is described as follows:

$$M_t = \lambda X_t + (1-\lambda)M_{t-1} + k(X_t - X_{t-1}), \tag{4}$$

where λ is an exponential smoothing parameter such that $0 \le \lambda \le 1$, M_t is t^{th} statistic with M_0 is initial values and X_t is t^{th} observation from $MA(1)$ process, k is an additional parameter, in case $k = 0$ it reverts to EWMA control chart. The upper and lower control limit of the MEWMA control charts are given by

$$UCL/LCL = \mu_0 \pm L_2\sigma\sqrt{\frac{\lambda}{2-\lambda}+\frac{2k(\lambda+k)}{2-\lambda}}, \tag{5}$$

where μ_0 and σ represent mean and standard deviation of the process, respectively, and L_2 is the width of the appropriate control limit.

2.2.3 Double MEWMA (DMEWMA)

Alevizakos et al. (2021) presented the double MEWMA control chart, which is a hybrid of the two MEWMA control charts, for monitoring shifts in the process mean under the assumption of a normal distribution. The DMEWMA statistics are defined by the following equation system:

$$\begin{cases} M_t = \lambda_1 X_t + (1-\lambda_1)M_{t-1} + k_1(X_t - X_{t-1}), \\ DM_t = \lambda_2 M_t + (1-\lambda_2)DM_{t-1} + k_2(M_t - M_{t-1}), \end{cases} \tag{6}$$

where $0 \le \lambda_1, \lambda_2 \le 1$ are the smoothing parameters, k_1, k_2 are the additional parameters and $X_0 = M_0 = DM_0 = \mu_0$ are the starting values. The upper and lower control limit of the DMEWMA control charts are given as follow

$$UCL/LCL = \mu_0 \pm L_3\sigma\sqrt{Var(DM_t)}, \tag{7}$$

where $Var(DM_t)$ is the exact value of the variance, L_3 is the width of the appropriate control limit.

3. Determining of Average Run Length on DMEWMA Control Chart for MA(1) Process

3.1. Exact formula of ARL

From the recursion of DMEWMA statistics,

$$DM_t = (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)X_t - (k_1\lambda_2 + k_1k_2)X_{t-1} + (\lambda_2 - \lambda_1\lambda_2 - k_2\lambda_1)M_{t-1} + (1-\lambda_2)DM_{t-1}, \tag{8}$$

the statistic DM_t for $MA(1)$ process can be written as,

$$\begin{aligned} DM_t = {} & (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)(\mu + \varepsilon_t - \theta_1\varepsilon_{t-1}) \\ & - (k_1\lambda_2 + k_1k_2)X_{t-1} + (\lambda_2 - \lambda_1\lambda_2 - k_2\lambda_1)M_{t-1} + (1-\lambda_2)DM_{t-1}. \end{aligned} \tag{9}$$

Consider one-sided case when $0 \le DM_t \le b$ where b is upper control chart limit which $\tau_b = \{t > 0; DM_t > b\}$ is the corresponding stopping time. Let $L_E(u)$ be the analytical ARL for initial value u, as calculated from: $L_E(u) = ARL = E_\infty(\tau_b)$. In the present study, we set $DM_0 = \mu$ and also set the change-point time to $t = 1$ and X_0, X_{-1}, M_0 are initials. Consequently, the DM_t statistic in-control process is given by

$$\begin{aligned} 0 \le {} & (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)(\mu + \varepsilon_1 - \theta_1\varepsilon_0) - (k_1\lambda_2 + k_1k_2)X_0 \\ & + (\lambda_2 - \lambda_1\lambda_2 - k_2\lambda_1)\left[\lambda_1 X_0 + (1-\lambda_1)M_{-1} + k_1(X_0 - X_{-1})\right] + (1-\lambda_2)u \le b \end{aligned}$$

Arranged in the form of ε_t

$$\frac{-(1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2} \le \varepsilon_1 \le \frac{b - (1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2},$$

where $DM_0 = \mu$ and

$$\begin{aligned} G = {} & (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)(\mu - \theta_1\varepsilon_0) - (k_1\lambda_2 + k_1k_2)X_0 \\ & + (\lambda_2 - \lambda_1\lambda_2 - k_2\lambda_1)\left[\lambda_1 X_0 + (1-\lambda_1)M_{-1} + k_1(X_0 - X_{-1})\right]. \end{aligned}$$

Then, the function $L_E(u)$ can be expressed by Fredholm integral equation of the second kind as follows:

$$L_E(u) = 1 + \int_0^b L\{(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)y + G + (1-\lambda_2)u\} f(y)dy, \tag{10}$$

Let $w = (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)y + G + (1-\lambda_2)u$ thus

$$y = \frac{w-(1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2} \quad \text{and} \quad dy = \frac{1}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}dw.$$

From Equation (10), the function $L_E(u)$ is obtained as follows

$$L_E(u) = 1 + \frac{1}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}\int_0^b L(w) f\left[\frac{w-(1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}\right]dw. \tag{11}$$

Since $\varepsilon_t \sim Exp(\beta)$ then $f(x) = \frac{1}{\beta}e^{-\frac{x}{\beta}}; x \ge 0,$ i.e,

$$f\left(\frac{w-(1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}\right) = \frac{1}{\beta}e^{\frac{-w}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}}e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}}.$$

Therefore, Equation (11) can be rewritten as

$$L_E(u) = 1 + \frac{e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}}}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}\int_0^b L(w)e^{\frac{-w}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}}dw. \tag{12}$$

If we set $\gamma = \beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)$, $c(u) = \frac{1}{\gamma}e^{\frac{(1-\lambda_2)u+G}{\gamma}}$, and $D = \int_0^b L(w)e^{\frac{-w}{\gamma}}dw.$

Subsequently, this equation can be rewritten as

$$L_E(u) = 1 + c(u)D. \tag{13}$$

Now, it is to be calculate D that

$$D = \int_0^b L(w)e^{\frac{-w}{\gamma}}dw = \int_0^b (1 + c(w)D)e^{\frac{-w}{\gamma}}dw = \int_0^b e^{\frac{-w}{\gamma}}dw + D\int_0^b c(w)e^{\frac{-w}{\gamma}}dw = \int_0^b e^{\frac{-w}{\gamma}}dw + \frac{D}{\gamma}\int_0^b e^{\frac{(1-\lambda_2)w+G}{\gamma}}e^{\frac{-w}{\gamma}}dw$$

$$= \int_0^b e^{\frac{-w}{\gamma}}dw + \frac{D}{\gamma}e^{\frac{G}{\gamma}}\int_0^b e^{\frac{-\lambda_2 w}{\gamma}}dw = -\gamma\left[e^{\frac{-b}{\gamma}} - 1\right] - \frac{D}{\lambda_2}e^{\frac{G}{\gamma}}\left[e^{\frac{-\lambda_2 b}{\gamma}} - 1\right].$$

Then $D = \dfrac{-\gamma\left[e^{\frac{-b}{\gamma}} - 1\right]}{1 + \dfrac{e^{\frac{G}{\gamma}}}{\lambda_2}\left[e^{\frac{-\lambda_2 b}{\gamma}} - 1\right]}.$ (14)

Finally, substituting D in Equation (13), we obtain the exact formula for the ARL of $MA(1)$ process on the DMEWMA control chart as

$$L_E(u) = 1 + \frac{1}{\gamma}e^{\frac{(1-\lambda_2)u+G}{\gamma}}\left[\frac{-\gamma\left[e^{\frac{-b}{\gamma}} - 1\right]}{1 + \frac{e^{\frac{G}{\gamma}}}{\lambda_2}\left[e^{\frac{-\lambda_2 b}{\gamma}} - 1\right]}\right] = 1 - \frac{e^{\frac{(1-\lambda_2)u+G}{\gamma}}\left[e^{\frac{-b}{\gamma}} - 1\right]}{1 + \frac{e^{\frac{G}{\gamma}}}{\lambda_2}\left[e^{\frac{-\lambda_2 b}{\gamma}} - 1\right]}, \tag{15}$$

where $\gamma = \beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)$.

For the ARL solution of an in-control process, we set $ARL_0 = 370$ when the exponential parameter is defined to $\beta = \beta_0$ and for an out-of-control we defined ARL$_1$ which is $\beta_1 = (1+\delta)\beta_0$ where δ is the shift size.

3.2. Existence and uniqueness of ARL

The ARL solution proves that the integral equation for exact formulas only occurs once by applying the Banach's Fixed-point theorem. Let T denotes a function operation on the class of all continuous functions defined by

$$T(L_E(u)) = 1 + \frac{1}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}\int_0^b L(w) f\left[\frac{w-(1-\lambda_2)u - G}{\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2}\right]dw. \quad (16)$$

The fixed-point equation $T(L_E(u)) = L_E(u)$ has a singular solution if operator T is a contraction, according to Banach's Fixed-point theorem. The following theorem can be used to demonstrate that Equation (16) exists and has a unique solution.

Theorem 1 Banach's fixed-point Theorem

Let X defined on a complete metric space and $T : X \to X$ satisfies the conditions of a contraction mapping with contraction constant $0 \le r < 1$ such that $\|T(L_1) - T(L_2)\| \le r\|L_1 - L_2\|$, $\forall L_1, L_2 \in X$. Then there exists a unique $L(\bullet) \in X$ such that $T(L_E(u)) = L_E(u)$ i.e., a unique fixed-point in X.

Proof: Let T defined in Equation (12) is a contraction mapping for $L_1, L_2 \in F[0,b]$, such that $\|T(L_1) - T(L_2)\| \le r\|L_1 - L_2\|$, $\forall L_1, L_2 \in F[0,b]$ with $0 \le r < 1$ under the norm $\|L\|_\infty = \sup_{u\in[0,b]} |L(u)|$, so

$$\|T(L_1) - T(L_2)\|_\infty = \sup_{u\in[0,b]}\left|\frac{e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}\int_0^h (L_1(w) - L_2(w))e^{\frac{-w}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}dw\right|$$

$$\le \sup_{u\in[0,b]}\left|(L_1 - L_2)\frac{e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}}{\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)}\cdot\left(-\beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)\right)\left(e^{\frac{-b}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}} - 1\right)\right|$$

$$= \|L_1 - L_2\|_\infty \sup_{u\in[0,b]}\left|e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}\left(1 - e^{\frac{-b}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}\right)\right| \le r\|L_1 - L_2\|_\infty$$

where $r = \sup_{u\in[0,b]}\left|e^{\frac{(1-\lambda_2)u+G}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}\left(1 - e^{\frac{-b}{\beta(\lambda_1\lambda_2+k_1\lambda_2+k_2\lambda_1+k_1k_2)}}\right)\right|; 0 \le r < 1.$

As a result of Banach's contraction mapping principle, we infer that T possesses a fixed point that is the unique solution. The proof is done.

3.3. Numerical integral equation (NIE) of ARL

One of the most widely utilized methods for approximating the ARL on control charts is the NIE method. Using the quadrature rules, let $\tilde{L}_N(u)$ represent the approximation of ARL with the m linear equation systems. A form that represents the solution to an integral equation is

$$L(u) = 1 + \frac{1}{\gamma}\int_0^b L(w) f\left[\frac{w-(1-\lambda_2)u-G}{\gamma}\right] dw \tag{17}$$

where $\gamma = \beta(\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)$, and

$$G = (\lambda_1\lambda_2 + k_1\lambda_2 + k_2\lambda_1 + k_1k_2)(\mu - \theta_1\varepsilon_0) - (k_1\lambda_2 + k_1k_2)X_0 + (\lambda_2 - \lambda_1\lambda_2 - k_2\lambda_1)\left[\lambda_1 X_0 + (1-\lambda_1)M_{-1} + k_1(X_0 - X_{-1})\right].$$

The numerical method for solving the integral problem is to use the quadrature rule to approximate the integral by a finite sum. The set of points or nodes defines any quadrature rule. The approximate value for a one-sided interval integral is

$$\int_0^b f(y)dy \approx \sum_{j=1}^{n} w_j f(a_j), \tag{18}$$

where a_j is a point w_j is a weight that is determined by the quadrature rules. The system of m linear equations is represented as follows:

$$L_{m\times 1} = 1_{m\times 1} + R_{m\times m}L_{m\times 1} \text{ or } L_{m\times m} = (I_m - R_{m\times m})^{-1} 1_{m\times 1}$$

where $L_{m\times 1} = [\tilde{L}_N(a_1), \tilde{L}_N(a_2), ..., \tilde{L}_N(a_m)]^T, I_m = diag(1,1,...,1)$ is the order m unit matrix, $1_{m\times 1} = [1,1,...,1]^T$ is a column vector of $\tilde{L}_N(a_j)$ and $R_{m\times m}$ is a matrix of elements:

$$\left[R_{ij}\right] \approx \frac{1}{\gamma} w_j f\left[\frac{a_j-(1-\lambda_2)a_i-G}{\gamma}\right].$$

A numerical approximation $\tilde{L}_N(u)$ for (17) can be determined using (18), as follows:

$$\tilde{L}_N(u) = 1 + \frac{1}{\gamma}\sum_{j=1}^{m} w_j L(a_j) f\left[\frac{a_j-(1-\lambda_2)u-G}{\gamma}\right], \tag{19}$$

where a_j is a set of the division point on the close interval $[o,b]$ as a_j is the quadrature rules. This study, we choose NIE methods based on midpoint rule, trapezoidal rule, Simpson's rule, and Gauss-Legendre rule.

3.3.1. Midpoint formula

The ARL that the midpoint rule resolves are

$$\tilde{L}_M(u) = 1 + \frac{1}{\gamma}\sum_{j=1}^{m} w_j L(a_j) f\left[\frac{a_j-(1-\lambda_2)u-G}{\gamma}\right] \tag{20}$$

where $a_j = \left(j - \frac{1}{2}\right) w_j$ and $w_j = \frac{b}{m}$; $j = 1, 2, ..., m.$

3.3.2. Trapezoidal formula

The ARL that the Trapezoidal rule resolves is

$$\tilde{L}_T(u) = 1 + \frac{1}{\gamma}\sum_{j=1}^{m+1} w_j L(a_j) f\left[\frac{a_j-(1-\lambda_2)u-G}{\gamma}\right], \tag{21}$$

where $a_j = jw_j$ and $w_j = \frac{b}{m}$; $j = 1, 2, ..., m$ in other case, $w_j = \frac{b}{2m}$.

3.3.3. Simpson's formula

The ARL that Simpson's rule resolves is

$$\tilde{L}_S(u) = 1 + \frac{1}{\gamma}\sum_{j=1}^{2m+1} w_j L(a_j) f\left[\frac{a_j - (1-\lambda_2)u - G}{\gamma}\right], \tag{22}$$

where $a_j = jw_j$ and $w_j = \frac{4}{3}\left(\frac{b}{2m}\right)$; $j = 1,3,...,2m-1$ and $w_j = \frac{2}{3}\left(\frac{b}{2m}\right)$; $j = 2,4,...,2m-2$.

3.3.4. Gauss-Legendre formula

The ARL that the Gauss-Legendre rule resolves is

$$\tilde{L}_{GL}(u) = 1 + \frac{1}{\gamma}\sum_{j=1}^{m} w_j L(a_j) f\left[\frac{a_j - (1-\lambda_2)u - G}{\gamma}\right] \tag{23}$$

where $a_j = \frac{w_j}{(j-0.5)}$ and $w_j = \frac{b}{m}$; $j = 1,2,...,m.$

4. Performance of the Control Charts

4.1. Efficiency of exact formula and NIE methods

The absolute percentage relative error (APRE) (Nguyen et al. 2019) is generated to compare the ARL solution between the exact formulas and the NIE approach for the ARL results:

$$APRE(\%) = \frac{\left|L_E(u) - \tilde{L}_N(u)\right|}{L_E(u)} \times 100, \tag{24}$$

where $L_E(u)$ and $\tilde{L}_N(u)$ denote the ARL values obtained by using the exact formula and NIE approach, respectively. The very small difference in values means that the proposed formula's ARL_1 value is close to that for the NIE method, indicating an agreement that the values are not different between them. Additionally, the CPU time in seconds was used to determine the speed test results.

4.2. Efficiency of the control charts

A variety of performance metrics are widely employed to analyze their overall effectiveness. These metrics can be used to assess the chart's capacity to detect process deviations. The DMEWMA control chart is compared to the EWMA and MEWMA control charts in terms of efficiency. First, the control chart's effectiveness in identifying various sorts of process deviations may be examined and informed decisions about its performance can be made by examining the SDRL and ARL combined. A lower SDRL and ARL indicates higher performance in detecting shifts in the process mean. If the process is running in a controlled, ARL_0 is chosen to be large enough to remove the effect of the false alarm rate. On the other hand, ARL_1 should be small enough to detect changes quickly. The exact formula was used to calculate the ARL. Furthermore, the SDRL was recommend by Fonseca et al., in 2021 is a performance evaluation used to assess the effectiveness of control charts in detecting out-of-control conditions. The ARL and SDRL for an in-control process is determined as follows.

$$ARL_0 = \frac{1}{\alpha}, \text{ and } SDRL_0 = \sqrt{\frac{1-\alpha}{\alpha}}, \tag{25}$$

where α represents type I error. ARL_0 was set to 370 in this investigation. Form an ARL_0 value at around 370 that may be computed as $SDRL_0$ using Equation (25). ARL and SDRL on the other hand, are calculated in out-of-control situations by

$$ARL_1 = \frac{1}{1-\beta}, \text{ and } SDRL_1 = \sqrt{\frac{\beta}{(1-\beta)^2}} \quad (26)$$

where β represents type II error.

Moreover, the relative mean index (RMI) (Tang et al. 2018), is used to evaluate a control chart's ability to identify shifts in the process mean. The control chart outperforms the others based on the lowest RMI values. The RMI is calculated using the following formula:

$$RMI = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{ARL_i(c) - ARL_i(sm)}{ARL_i(sm)}\right], \quad (27)$$

where $ARL_i(c)$ is the ARL of each control chart for the determined shift sizes of the row i, $ARL_i(sm)$ and is the lowest ARL of the row i from all control charts.

5. Numerical Results

The ARL for detecting shifts in the process mean as a measure of its sensitivity and practicability is a popular criterion for measuring the performance of a control chart. Hence, the analytical and numerical IE approaches were compared to evaluate the ARL for monitoring shifts in the process mean on the $MA(1)$ process on a DMEWMA control chart. For this section, a simulation of the in-control process is normally provided ARL_0 = 370, and the initial parameter value at $\beta_0 = 1$ was analyzed. Using the NIE approach, we computed the number of division points $m = 1000$ for the approximated ARL. The in-control condition consists of exponential white noise with mean $\beta_0 = 1$ but the out-of-control situation is indicated by a shift in the process mean from β_0 to β_1, where $\beta_1 = (1+\delta)\beta_0$. Thus, δ = 0.001, 0.003, 0.005, 0.007, 0.01, 0.03, 0.05, 0.07, 0.1, 0.3, 0.5, 0.70, and 1, where $\delta = 0$ indicates that the process is under control and $\delta > 0$ indicates that the process is out of control, designated as ARL_0 and ARL_1, respectively.

Tables 1 and 2 show the ARL of the DMEWMA control chart at $\lambda_1 = 0.2$ and $\lambda_2 = 0.5$, respectively, and is calculated using two techniques: the exact formula and the NIE method, which is based on four quadrature rules for $MA(1)$ models. The results demonstrate that the ARL values generated using explicit equations provide results similar to those of NIE. The computing (CPU) times for the midpoint and trapezoidal are approximately 8-12 seconds, while Simpson and Gauss took more than 40 seconds. The comparison calculation for APRE% was performed using both the exact approach and the midpoint method (shortest time). Because the APRE% is less than 0.000000013%, there is no difference between the ARL values obtained for exact and NIE method. The proposed exact DMEWMA is recommended because of its guaranteed accuracy and computation time.

Following that, we compare the performance of the DMEWMA $(k_1 = 1)$ control chart to the MEWMA $(k_1 = 1)$ and EWMA $(k_1 = 0)$ control charts in Tables 3 and 4. We calculated the ARL and SDRL values acquired from the exact method for all control charts running the processes. In all cases, the control chart performance with the lowest ARL and SDRL offered the best performance, and SDRL values were smaller than ARL values. The bold letters represent the smallest ARL_1 values for a certain shift and each λ value. For $\lambda = 0.05$, we can see that the value of ARL on the DMEWMA control chart was less than on the EWMA and MEWMA control charts when the shift parameter was less than 0.5, whereas ARL on the EWMA control chart was smaller when the shift parameter was greater than 0.5. With $\lambda = 0.1$, we can see that the value of ARL on the DMEWMA control chart was lower than on the EWMA and MEWMA control charts for all shift parameter values. Thus, the DMEWMA control chart detects small-to-moderate shifts in the process mean better than the EWMA control chart and DMEWMA.

Furthermore, when the ARL values got from each control chart showed in Tables 3 and 4 were utilized to calculate the RMI values to evaluate the performance of each chart, it was discovered that the DMEWMA control chart performed the best because it provided the lowest RMI. As a result, ARL, SDRL, and RMI can all agree that the DMEWMA control chart outperforms the EWMA and MEWMA control charts.

Table 1 The ARL values for $MA(1)$ processes running on the DMEWMA control chart using the exact formula and NIE approach when $b = 0.8354, \lambda_1 = 0.2, \lambda_2 = 0.5, k_1 = 1, k_2 = 2, \mu = 2, \theta = 0.1$

δ		Exact	NIE approach				APRE(%)
			Midpoint	Trapezoidal	Simpson	Gauss	
0.000	ARL_0	370.153264	370.153260	370.153273	370.153264	370.153264	1.292×10^{-8}
	Time	<0.001	8.969	9.453	42.234	43.329	
0.001	ARL_1	240.525759	240.525757	240.525764	240.525759	240.525759	9.646×10^{-9}
	Time	<0.001	10.453	11.625	43.812	52.453	
0.003	ARL_1	141.621509	141.621508	141.6215116	141.621509	141.621509	8.332×10^{-9}
	Time	<0.001	9.766	10.516	45.156	45.141	
0.005	ARL_1	100.468544	100.468543	100.468545	100.468544	100.468544	7.764×10^{-9}
	Time	<0.001	9.562	10.828	48.469	47.953	
0.007	ARL_1	77.915275	77.915274	77.915276	77.915275	77.915275	7.444×10^{-9}
	Time	<0.001	9.266	11.016	47.046	52.719	
0.01	ARL_1	58.359586	58.359586	58.35958765	58.359586	58.35956	7.025×10^{-9}
	Time	<0.001	10.281	11.985	39.531	47.281	
0.03	ARL_1	22.138212	22.138212	22.13821245	22.138212	22.138212	5.872×10^{-9}
	Time	<0.001	6.922	7.39	32.609	38.516	
0.05	ARL_1	13.865367	13.865367	13.86536779	13.865367	13.865367	5.049×10^{-9}
	Time	<0.001	9.531	10.719	47.531	47.687	
0.07	ARL_1	10.201864	10.201864	10.20186427	10.2018641	10.201864	0
	Time	<0.001	10.406	11.391	46.469	52.328	
0.1	ARL_1	7.408913	7.408913	7.40891340	7.408913	7.408913	4.049×10^{-9}
	Time	<0.001	10.5	11.781	46.578	49.141	
0.3	ARL_1	3.014093	3.014093	3.01409368	3.014093	3.014093	0
	Time	<0.001	6.734	7.844	34.328	39.844	
0.5	ARL_1	2.148778	2.148778	2.14877877	2.148778	2.148778	0
	Time	<0.001	10.25	11.329	47.625	48.312	
0.7	ARL_1	1.788287	1.788287	1.78828779	1.788287	1.788287	0
	Time	<0.001	11.734	12.672	49.61	56.937	
1	ARL_1	1.526924	1.526924	1.52692407	1.526924	1.526924	0
	Time	<0.001	10.766	11.093	44.86	48.938	

6. Utilization of real data

We use an actual dataset on the daily natural gas price data, provided by https://th.investing.com/equities/ptt-historical-data, to show the practical use of the suggested DMEWMA control chart. There are 61 samples in the dataset. From February 2nd, 2023, to April 28th, 2023, daily observations were made. The Kolmogorov-Smirnov test was used to confirm that the white noise distribution is asymptotically exponential. As shown in Table 5, the results demonstrate that the white noise strongly fitted an exponential distribution (p-value = 0.292 > 0.05) with the mean and standard deviation for this dataset being 2.379 and 0.229, respectively, for the $MA(1)$ model.

Table 2 The ARL values for $MA(1)$ processes running on the DMEWMA control chart using the exact formula and NIE approach when $b = 0.682643, \lambda_1 = 0.2, \lambda_2 = 0.5, k_1 = 1, k_2 = 2, \mu = 2, \theta = -0.1$

δ		Exact	NIE approach				APRE(%)
			Midpoint	Trapezoidal	Simpson	Gauss	
0.000	ARL_0	370.068439	370.068436	370.068444	370.068439	370.068439	7.458×10^{-9}
	Time	<0.001	11.437	12.422	49.438	56.702	
0.001	ARL_1	234.717094	234.717093	234.717097	234.717094	234.717094	6.305×10^{-9}
	Time	<0.001	12.203	12.329	49.953	57.047	
0.003	ARL_1	135.723158	135.723157	135.723159	135.723158	135.723158	5.452×10^{-9}
	Time	<0.001	12.421	12.781	45.375	55.313	
0.005	ARL_1	95.570815	95.570815	95.570816	95.570815	95.570815	5.127×10^{-9}
	Time	<0.001	11.702	12.375	46.157	55.438	
0.007	ARL_1	73.817174	73.817174	73.817175	73.817174	73.817174	4.877×10^{-9}
	Time	<0.001	12.454	12.781	51.5	53.172	
0.01	ARL_1	55.096760	55.096759	55.096760	55.096760	55.096760	4.719×10^{-9}
	Time	<0.001	11.703	12.922	48.687	52.031	
0.03	ARL_1	20.766805	20.766805	20.766805	20.766805	20.766805	3.852×10^{-9}
	Time	<0.001	12.25	12.907	46.734	56.438	
0.05	ARL_1	12.988986	12.988986	12.988986	12.988986	12.988986	3.849×10^{-9}
	Time	<0.001	12.062	12.86	46.703	53.812	
0.07	ARL_1	9.552804	9.552804	9.552805	9.552804	9.552804	3.140×10^{-9}
	Time	<0.001	12.266	12.86	46.625	51.094	
0.1	ARL_1	6.937162	6.937162	6.937162	6.937162	6.937162	2.883×10^{-9}
	Time	<0.001	12.297	12.359	48.594	54.672	
0.3	ARL_1	2.834658	2.834658	2.834658	2.834658	2.834658	3.528×10^{-9}
	Time	<0.001	11.344	11.719	47.219	53.328	
0.5	ARL_1	2.033311	2.033311	2.033311	2.033311	2.033311	0
	Time	<0.001	11.156	11.938	48.812	54.266	
0.7	ARL_1	1.702003	1.702003	1.702003	1.702003	1.702003	0
	Time	<0.001	10.453	11.641	48.141	55.844	
1	ARL_1	1.463819	1.463819	1.463819	1.463819	1.463819	0
	Time	<0.001	10.5	11.172	48.125	56.719	

Table 3 Comparison of the ARL and SDRL values of the EWMA, MEWMA, and DMEWMA control chart for $MA(1)$ process with $\theta = 0.1, k_1 = 1$ and $\mu = 2$

δ	Control Chart	$\lambda = \lambda_1 = 0.05$				
		EWMA	MEWMA	DMEWMA $\lambda_2 = 0.1$		
				$k_2 = 1$	$k_2 = 3$	$k_2 = 5$
	UCL	0.0000006913	0.408731	0.436645	1.305332	2.17474
0.000	ARL_0	370.08814	370.08071	370.16112	370.17953	370.09550
	$SDRL_0$	369.58780	369.58037	369.66078	369.67919	369.59516
0.001	ARL_1	363.47412	274.59216	265.50425	226.27007	**217.05661**
	$SDRL_1$	362.97378	274.09170	265.00378	225.76952	216.55603
0.003	ARL_1	350.63629	181.122851	169.64045	127.52935	**119.07734**
	$SDRL_1$	350.13593	180.62216	169.13971	127.02837	118.57629
0.005	ARL_1	338.30020	135.12644	124.66755	88.93547	**82.21591**
	$SDRL_1$	337.79983	134.62551	124.16654	88.43406	81.71438
0.007	ARL_1	326.44452	107.76106	98.56197	63.36224	**62.88156**
	$SDRL_1$	325.94414	107.25989	98.06070	62.86025	62.37956

0.01	ARL_1	309.51742	82.65462	75.02448	50.84265	**46.58621**
	$SDRL_1$	309.01702	82.15310	74.52280	50.34017	46.08350
0.03	ARL_1	218.76547	32.39434	29.05176	19.14855	**17.49425**
	$SDRL_1$	218.26490	31.89042	28.54738	18.64185	16.98689
0.05	ARL_1	156.70451	20.17506	18.09755	12.04222	**11.03794**
	$SDRL_1$	156.20371	19.66871	17.59045	11.53139	10.52607
0.07	ARL_1	113.69509	14.67585	13.19421	8.90966	**8.19899**
	$SDRL_1$	113.19399	14.16703	12.68436	8.39478	7.68274
0.1	ARL_1	71.89363	10.45213	9.43804	6.52636	**6.04139**
	$SDRL_1$	71.39188	9.93956	8.92404	6.00558	5.51879
0.3	ARL_1	6.48776	3.80211	3.53094	2.77521	**2.64410**
	$SDRL_1$	5.96685	3.26404	2.98941	2.21959	2.08499
0.5	ARL_1	**1.82331**	2.52581	2.39153	2.02994	1.96507
	$SDRL_1$	1.22521	1.96314	1.82425	1.44593	1.37711
0.7	ARL_1	**1.18999**	2.01003	1.92805	1.71612	1.67708
	$SDRL_1$	0.47549	1.42485	1.33766	1.10858	1.06561
1	ARL_1	**1.03572**	1.64821	1.60063	1.48586	1.46409
	$SDRL_1$	0.19234	1.07115	0.98050	0.84966	0.8243
RMI		5.038	0.635	0.510	0.130	0.069

Table 3 (Continued)

δ	Control Chart	$\lambda = \lambda_1 = 0.05$				
		EWMA	MEWMA	DMEWMA $\lambda_2 = 0.1$		
				$k_2 = 1$	$k_2 = 3$	$k_2 = 5$
	UCL	0.0000006913	0.408731	0.436645	1.305332	2.17474
0.01	ARL_1	309.51742	82.65462	75.02448	50.84265	**46.58621**
	$SDRL_1$	309.01702	82.15310	74.52280	50.34017	46.08350
0.03	ARL_1	218.76547	32.39434	29.05176	19.14855	**17.49425**
	$SDRL_1$	218.26490	31.89042	28.54738	18.64185	16.98689
0.05	ARL_1	156.70451	20.17506	18.09755	12.04222	**11.03794**
	$SDRL_1$	156.20371	19.66871	17.59045	11.53139	10.52607
0.07	ARL_1	113.69509	14.67585	13.19421	8.90966	**8.19899**
	$SDRL_1$	113.19399	14.16703	12.68436	8.39478	7.68274
0.1	ARL_1	71.89363	10.45213	9.43804	6.52636	**6.04139**
	$SDRL_1$	71.39188	9.93956	8.92404	6.00558	5.51879
0.3	ARL_1	6.48776	3.80211	3.53094	2.77521	**2.64410**
	$SDRL_1$	5.96685	3.26404	2.98941	2.21959	2.08499
0.5	ARL_1	**1.82331**	2.52581	2.39153	2.02994	1.96507
	$SDRL_1$	1.22521	1.96314	1.82425	1.44593	1.37711
0.7	ARL_1	**1.18999**	2.01003	1.92805	1.71612	1.67708
	$SDRL_1$	0.47549	1.42485	1.33766	1.10858	1.06561
1	ARL_1	**1.03572**	1.64821	1.60063	1.48586	1.46409
	$SDRL_1$	0.19234	1.07115	0.98050	0.84966	0.8243
RMI		5.038	0.635	0.510	0.130	0.069

Table 3 (Continued)

δ	Control Chart	$\lambda=\lambda_1=0.1$				
		EWMA	MEWMA	DMEWMA $\lambda_2=0.2$		
				$k_2=1$	$k_2=3$	$k_2=5$
	UCL	0.03414	0.413936	0.475993	1.40258	2.33227
0.000	ARL_0	370.05883	370.04923	370.14661	370.47840	370.45449
	$SDRL_0$	369.55849	369.54889	369.64627	369.97806	369.95415
0.001	ARL_1	367.16092	268.20079	252.23648	223.57938	**216.58707**
	$SDRL_1$	366.66058	267.70032	251.73598	223.07882	216.08649
0.003	ARL_1	361.44912	173.00675	154.17499	124.93872	**118.58010**
	$SDRL_1$	360.94877	172.50603	153.67418	124.43772	118.07904
0.005	ARL_1	355.84721	127.70543	111.08175	86.84809	**81.81373**
	$SDRL_1$	355.34686	127.20445	110.58062	86.34664	81.31219
0.007	ARL_1	350.35273	101.21777	86.85840	66.64825	**62.55171**
	$SDRL_1$	349.85237	100.71653	86.35695	66.14636	62.04970
0.01	ARL_1	342.30715	77.21411	65.49481	49.50385	**46.32929**
	$SDRL_1$	341.80678	76.71248	64.99289	49.00130	45.82656
0.03	ARL_1	294.15541	29.99556	25.01592	18.62122	**17.39467**
	$SDRL_1$	293.65498	29.49132	24.51082	18.11432	16.88727
0.05	ARL_1	254.16605	18.67931	15.60403	11.71995	**10.97791**
	$SDRL_1$	253.66556	18.17243	15.09575	11.20880	10.46597
0.07	ARL_1	220.75480	13.60626	11.41790	8.68031	**8.15684**
	$SDRL_1$	220.25423	13.09672	10.90644	8.16502	7.64050
0.1	ARL_1	180.32866	9.71737	8.22114	6.36859	**6.01297**
	$SDRL_1$	179.82796	9.20380	7.70493	5.84725	5.49025
0.3	ARL_1	58.85977	3.60123	3.19962	2.73026	**2.63711**
	$SDRL_1$	58.35763	3.06066	2.65291	2.17349	2.07780
0.5	ARL_1	25.70563	2.42426	2.22461	2.00647	**1.96199**
	$SDRL_1$	25.20067	1.85816	1.65054	1.42107	1.37383
0.7	ARL_1	13.67516	1.94682	1.82464	1.70118	**1.67548**
	$SDRL_1$	13.16567	1.35768	1.22665	1.09217	1.06384
1	ARL_1	6.84574	1.61055	1.53953	1.47679	**1.46242**
	$SDRL_1$	6.32601	0.99163	0.91139	0.83912	0.82235
RMI		11.882	0.471	0.288	0.048	0

Table 4 Comparison of the ARL and SDRL values of the EWMA, MEWMA, and DMEWMA control chart for $MA(1)$ process with $\theta=-0.1, k_1=1$ and $\mu=2$

δ	Control Chart	$\lambda=\lambda_1=0.05$				
		EWMA	MEWMA	DMEWMA $\lambda_2=0.1$		
				$k_2=1$	$k_2=3$	$k_2=5$
	UCL	0.000000845	0.33399	0.356206	1.06469	1.77376
0.000	ARL_0	370.36920	370.57033	370.13135	369.54835	369.96452
	$SDRL_0$	369.86886	370.06999	369.63101	369.04801	369.46418
0.001	ARL_1	363.82262	269.99360	260.22170	219.81501	**210.64260**
	$SDRL_1$	363.32228	269.49314	259.72122	219.31444	210.14201

Table 4 (Continued)

δ	Control Chart	$\lambda=\lambda_1=0.05$				
		EWMA	MEWMA	DMEWMA $\lambda_2=0.1$		
				$k_2=1$	$k_2=3$	$k_2=5$
	UCL	0.000000845	0.33399	0.356206	1.06469	1.77376
0.003	ARL_1	351.11191	174.99104	163.29580	121.65189	**113.44295**
	$SDRL_1$	350.61155	174.49032	162.79503	121.15086	112.94184
0.005	ARL_1	338.89311	129.43829	119.00428	84.24374	**77.79333**
	$SDRL_1$	338.39274	128.93732	118.50323	83.74225	77.29171
0.007	ARL_1	327.14546	102.70050	93.62931	64.51679	**59.28967**
	$SDRL_1$	326.64508	102.19928	93.12797	64.01484	58.78754
0.01	ARL_1	310.36436	78.40429	70.95803	47.83371	**43.79732**
	$SDRL_1$	309.86396	77.90269	70.45626	47.33107	43.29443
0.03	ARL_1	220.20452	30.42949	27.23629	17.92100	**16.37071**
	$SDRL_1$	219.70395	29.92531	26.73161	17.41382	15.86283
0.05	ARL_1	158.31421	18.89945	16.92761	11.26175	**10.32451**
	$SDRL_1$	157.81342	18.39266	16.42000	10.75013	9.81178
0.07	ARL_1	115.26625	13.72860	12.32779	8.33229	**7.67095**
	$SDRL_1$	114.76516	13.21915	11.81722	7.81631	7.15350
0.1	ARL_1	73.24939	9.76598	8.81127	6.10647	**5.65672**
	$SDRL_1$	72.74767	9.25248	8.29622	5.58413	5.13242
0.3	ARL_1	6.75135	3.55410	3.30326	2.61327	**2.49369**
	$SDRL_1$	6.23132	3.01289	2.75831	2.05327	1.92997
0.5	ARL_1	1.88074	2.37317	2.25041	1.92434	**1.86589**
	$SDRL_1$	1.28703	1.80521	1.67748	1.33370	1.27108
0.7	ARL_1	**1.20646**	1.89989	1.82561	1.63634	1.60151
	$SDRL_1$	0.49908	1.30755	1.22770	1.02043	0.98149
1	ARL_1	**1.03950**	1.57080	1.52813	1.42677	1.40755
	$SDRL_1$	0.20263	0.946896	0.898360	0.780322	0.757395
RMI		5.481	0.615	0.491	0.118	0.052

δ	Control Chart	$\lambda=\lambda_1=0.1$				
		EWMA	MEWMA	DMEWMA $\lambda_2=0.2$		
				$k_2=1$	$k_2=3$	$k_2=5$
	UCL	0.0435	0.337684	0.38708	1.1398	1.89499
0.000	ARL_0	370.25853	370.00629	370.18122	370.05309	370.23781
	$SDRL_0$	369.75819	369.50595	369.68088	369.55275	369.73747
0.001	ARL_1	367.45204	263.00308	246.35373	216.73490	**209.66625**
	$SDRL_1$	366.95170	262.50260	245.85322	216.23432	209.16565
0.003	ARL_1	361.91830	166.64724	147.69981	118.76718	**112.55406**
	$SDRL_1$	361.41795	166.14649	147.19896	118.26612	112.05294
0.005	ARL_1	356.48829	121.97844	105.53071	81.94859	**77.09633**
	$SDRL_1$	355.98794	121.47741	105.02952	81.44706	76.59470
0.007	ARL_1	351.15971	96.202402	82.13263	62.64519	**58.72590**
	$SDRL_1$	350.65935	95.70110	81.63110	62.14318	58.22375
0.01	ARL_1	343.35219	73.06188	61.67696	46.38055	**43.36273**
	$SDRL_1$	342.85183	72.56016	61.17492	45.87783	42.85981

Table 4 (Continued)

δ	Control Chart	$\lambda = \lambda_1 = 0.1$				
		EWMA	MEWMA	DMEWMA $\lambda_2 = 0.2$		
				$k_2 = 1$	$k_2 = 3$	$k_2 = 5$
	UCL	0.0435	0.337684	0.38708	1.1398	1.89499
0.03	ARL_1	296.49571	28.12617	23.37539	17.35519	**16.20407**
	$SDRL_1$	295.99529	27.62164	22.86992	16.84777	15.69611
0.05	ARL_1	257.39397	17.47288	14.55541	10.91706	**10.22359**
	$SDRL_1$	256.89348	16.96551	14.04651	10.40505	9.71073
0.07	ARL_1	224.57197	12.71257	10.64381	8.08741	**7.59957**
	$SDRL_1$	224.07141	12.20233	10.13148	7.57092	7.08194
0.1	ARL_1	184.63940	9.07106	7.66204	5.93832	**5.60799**
	$SDRL_1$	184.13872	8.55646	7.14457	5.41529	5.08346
0.3	ARL_1	62.43232	3.36747	2.99551	2.56561	**2.48038**
	$SDRL_1$	61.93030	2.82354	2.44491	2.00418	1.91622
0.5	ARL_1	27.94293	2.27994	2.09711	1.89947	**1.85921**
	$SDRL_1$	27.43837	1.70827	1.51683	1.30710	1.26390
0.7	ARL_1	15.10971	1.84239	1.73146	1.62051	**1.59742**
	$SDRL_1$	14.60115	1.24580	1.12539	1.00277	0.97690
1	ARL_1	7.65880	1.53689	1.47302	1.41712	**1.40521**
	$SDRL_1$	7.14132	0.908373	0.834726	0.768836	0.754589
RMI		13.196	0.477	0.291	0.048	0

Table 5 The applicability of an exponential distribution for the exponential white noise is tested

One-sample Kolmogorov-Smirnov test*	0.9800
Exponential parameter	0.1722
Asymp. Sig (two-tailed)	0.2920

*Testing whether the white noise is exponentially distributed. Alternative hypothesis: two-sided

For the Table 6, we set up the EWMA control chart with $\lambda = 0.05$ the MEWMA chart with $\lambda = 0.05, k = 1$ and the DMEWMA chart with $\lambda_1 = 0.05, \lambda_2 = 0.1, k_1 = 1$ and $k_2 = 1,3,5$ while an ARL_0 = 370, Figures 1, 2, 3(a), and 3(b) display the ARL and RMI values from Table 6 for the three control charts, respectively. From the result of the table and figures, it is clear that all three charts are extremely effective at detecting early changes. In all circumstances, the suggested DMEWMA chart detects them the fastest, as indicated by lower ARL and RMI values than the other charts. Consequently, it can be noted that the DMEWMA chart that is being displayed is actually a control chart that has been more skillfully constructed than the other two charts.

Table 6 Comparative ARL and SDRL performance of EWMA, MEWMA and DMEWMA control charts for the $MA(1)$ process using the daily natural gas price data when $\beta = 1.722$, $\mu = 2.380$ and $\theta = -0.590$ various shifts

$\lambda_1 = 0.05$ and $\lambda_2 = 0.1$

δ	EWMA (0.000000414)		MEWMA (0.0000009712)		DMEWMA $k_2 = 1$ (0.0000009286)		DMEWMA $k_2 = 3$ (0.0000049644)		DMEWMA $k_2 = 5$ (0.0000088279)	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.000	370.0992	369.5989	370.0740	369.5736	369.9394	369.4391	370.2009	369.7006	**370.1101**	369.6097
0.001	335.8697	335.3693	293.7178	293.2174	282.2717	281.7713	63.49944	62.99746	**35.74845**	35.2449
0.003	277.5323	277.0319	197.9125	197.4119	181.3022	180.8015	22.73827	22.23265	**12.50333**	11.99291
0.005	230.3171	229.8166	141.4401	140.9393	126.0856	125.5847	13.34105	12.83131	**7.49934**	6.981458
0.007	191.9357	191.4351	105.0826	104.5815	92.08886	91.5875	9.21731	8.702959	**5.33756**	4.811651
0.01	147.1297	146.6288	70.74352	70.24174	61.04109	60.53903	6.13547	5.613245	**3.73086**	3.191936
0.03	30.91301	30.4089	10.59939	10.08701	9.04164	8.526993	1.79138	1.190656	**1.44825**	0.805716
0.05	9.07457	8.55998	3.22535	2.679092	2.86292	2.309414	1.23251	0.535323	**1.13937**	0.39849
0.07	3.68581	3.146329	1.67463	1.06290	1.56600	0.941465	1.08765	0.30876	**1.05513**	0.241183
0.1	1.68999	1.07985	1.15716	0.42645	1.13233	0.387093	1.02689	0.166172	**1.01795**	0.135175
0.3	1.00596	0.077431	1.00095	0.030837	1.00081	0.028472	1.00043	0.020741	**1.00035**	0.018712
0.5	1.00072	0.026842	1.00011	0.010489	1.00009	0.009487	1.00007	0.008367	**1.00006**	0.007746
0.7	1.00022	0.014834	1.00003	0.005477	1.00003	0.005477	**1.00002**	0.004472	**1.00002**	0.004472
1	1.00007	0.008367	1.00001	0.003162	1.00001	0.003162	1.00000	0.00000	**1.00000**	0.00000
RMI	12.551		6.571		5.783		0.316		0	

$\lambda_1 = 0.1$ and $\lambda_2 = 0.2$

δ	EWMA (0.000903)		MEWMA (0.0000009147)		DMEWMA $k_2 = 1$ (0.0000008973)		DMEWMA $k_2 = 3$ (0.0000046331)		DMEWMA $k_2 = 5$ (0.0000085896)	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.000	370.00429	369.50395	370.07302	369.57268	369.92240	369.42206	370.01503	369.51469	**370.30481**	369.80447
0.001	349.58034	349.07998	271.85545	271.355	213.92439	213.4238	48.58898	48.08638	**30.33587**	29.83168
0.003	312.63762	312.13722	167.59451	167.0938	108.01275	107.5116	17.08430	16.57676	**10.62095**	10.10859
0.005	280.27958	279.77913	114.12328	113.6222	67.58931	67.08745	10.08976	9.576716	**6.43091**	5.909796
0.007	251.86295	251.36245	82.35029	81.84876	46.70675	46.20404	7.04847	6.529354	**4.62582**	4.095411
0.01	215.46248	214.96190	54.05604	53.55371	29.72881	29.22453	4.78453	4.255255	**3.28552**	2.740278
0.03	85.42085	84.91938	8.05444	7.537875	4.72602	4.196337	1.59261	0.971492	**1.37875**	0.722635
0.05	39.95521	39.45204	2.65226	2.093376	1.89993	1.307595	1.17655	0.455763	**1.11867**	0.364352
0.07	21.27317	20.76715	1.50785	0.875078	1.28548	0.605788	1.06731	0.268031	**1.04722**	0.222373
0.1	10.01006	9.49691	1.12055	0.367535	1.07058	0.274885	1.02093	0.146178	**1.01548**	0.125378
0.3	1.48374	0.84720	1.00078	0.027939	1.00053	0.023028	1.00035	0.018712	**1.00031**	0.01761
0.5	1.12997	0.38323	1.00009	0.009487	**1.00006**	0.007746	**1.00006**	0.007746	**1.00006**	0.007746
0.7	1.05954	0.25117	1.00003	0.005477	**1.00002**	0.004472	**1.00002**	0.004472	**1.00002**	0.004472
1	1.02832	0.17065	1.00001	0.003162	**1.00001**	0.003162	**1.00001**	0.003162	**1.00001**	0.003162
RMI	24.932		6.038		3.483		0.230		0	

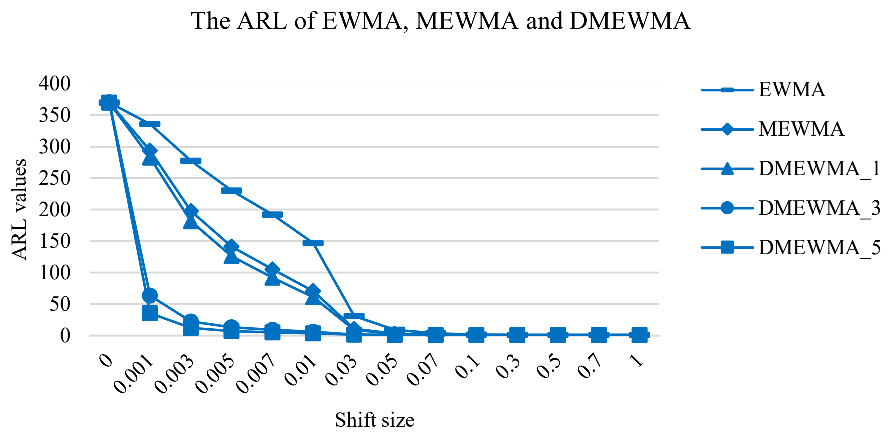

Figure 1 The ARL of EWMA, MEWMA and DMEWMA when $\lambda = 0.05$

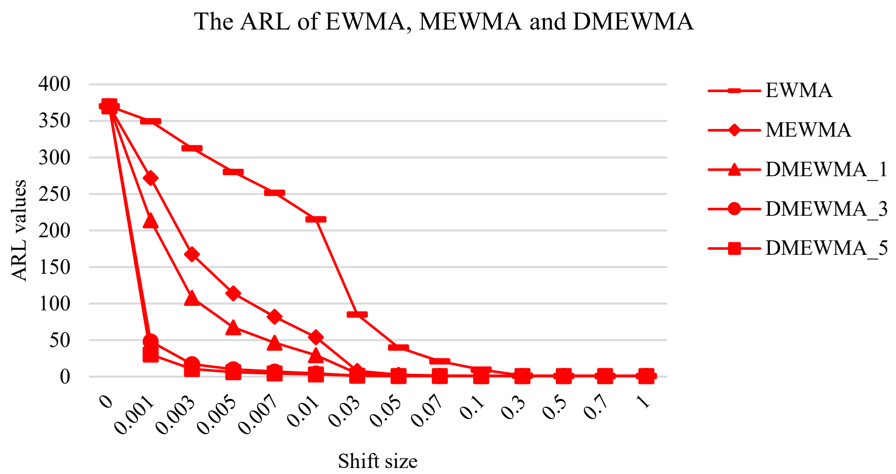

Figure 2 The ARL of EWMA, MEWMA and DMEWMA when $\lambda = 0.1$

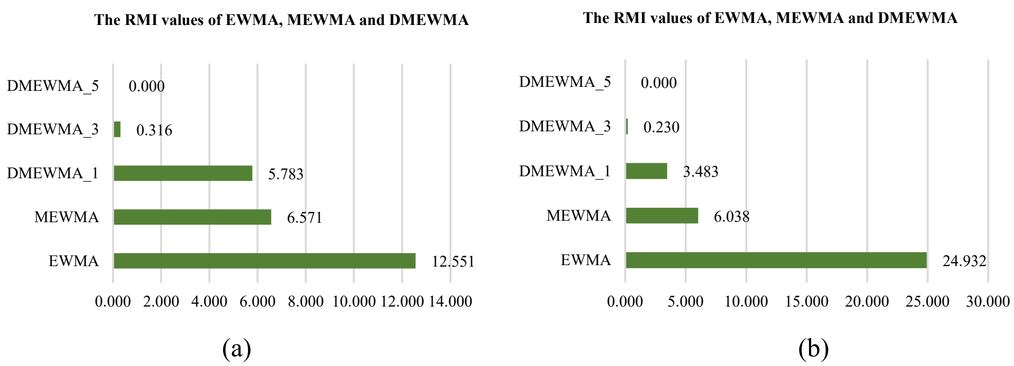

Figure 3 The RMI values of EWMA, MEWMA and DMEWMA when (a) $\lambda = 0.05$ and (b) $\lambda = 0.1$

7. Discussion and Conclusions

In the present article, the performance of ARL in $MA(1)$ utilizing the explicit versus numerical formula was compared, which examined the explicit formula of the DMEWMA chart. It was discovered that all approaches yield about the same values of ARL, except for the CPU time required to run the results, using the CPU time in seconds. In every situation, the midpoint formula requires the least amount of time, then the midpoint formula is the chosen method for comparison with the explicit formula by using absolute percentage relative error (APRE) as criteria and it was discovered that for both approaches, the out-of-control ARL results dropped quickly and in the same direction.

For performance comparative of control charts, it was determined to utilize EWMA and MEWMA to compare the suggested control charts' performances. Additionally, the relative mean index (RMI) is used to further examine how well ARL performs comparably in the three different control charts. In terms of additional parameter (k) in the DMEWMA method, increasing k_2 causes the ARL value to decrease rapidly. The DMEWMA control chart regularly provides better performance at lower ARL levels. Adjusting the smoothing constant to higher values gives consistent results; the DMEWMA chart shows lesser out-of-control ARLs. The relative mean index (RMI) further underlines the DMEWMA control chart's efficiency, as it produces the lowest values. Finally, when the suggested DMEWMA chart, EWMA chart, and MEWMA chart are applied to the monthly stock price data of PTT Company Limited (Public Company Limited) in Thailand, the same results are obtained: DMEWMA charts have the fastest detection performance. The experiment's accuracy will increase if an appropriate chart is chosen under the specified circumstances. However, using an improper chart for the experiment could lead to less accurate results or more inefficient results.

For the future work, we can expand an order of moving average process by expanding $MA(1)$ to $MA(q)$, which investigates the case $q = 2,3$ and we can also utilize other time series such as ARMA or ARIMA etc.

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