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Pair Copula Construction for Dependencies Structure in Cryptocurrencies Trading Volumes

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Abstract

Understanding how cryptocurrency trading volumes co-move remains challenging, especially in high dimensions where bivariate models fall short. Exploring these dependencies aids risk management, diversification, and policy oversight. This study investigates monthly trading volumes for Binance Coin (BNB), Bitcoin (BTC), TRON (TRX), Ethereum (ETH), and Dogecoin (DOGE) from October 2017 to July 2021. Monthly aggregation dampens microstructure noise and short-lived bursts that distort daily data, while providing far more observations and modeling stability than annual series, which are too coarse for reliable vine estimation. After transforming margins to uniform, we model dependence using pair-copula constructions within D-vine, C-vine, and R-vine frameworks, fitting Student's t, Clayton, Gumbel, and Frank copulas via sequential estimation. Log-likelihood, BIC evaluates model fit, and dependence is summarized via Kendall's τ. We also assess trading volume volatility using the mean and standard deviation of monthly changes. Across vine structures, the Gumbel copula consistently provides the best fit, indicating pronounced upper-tail dependence among cryptocurrencies. Conditioning reduces τ at higher vine trees, showing that conditional links capture much of the residual dependence. BTC exhibits the highest volatility risk, followed by DOGE, ETH, TRX, and BNB. These results support stress-testing for simultaneous surges, inform portfolio construction, diversification with lower-dependence pairs, and offer practical guidance for traders and regulators monitoring systemic risk.

Keywords: Copula model, pair copula construction, D-vine, C-vine, R-vine, dependence measure.

1. Introduction

Cryptocurrencies have rapidly evolved into a major component of the global financial system, reshaping how digital assets are traded, valued, and managed. Since the introduction of Bitcoin in 2009, the cryptocurrency market has expanded to thousands of coins and tokens with a combined market capitalization exceeding trillions of dollars. Their growing economic relevance stems not only from speculative trading but also from their potential roles in payment systems, decentralized finance, and portfolio diversification. However, this expansion has also brought pronounced volatility, extreme

price fluctuations, and surging trading activities across multiple exchanges. Understanding the dependence structure among cryptocurrency trading volumes is therefore vital for assessing market dynamics, liquidity conditions, investor behavior, and systemic risk.

Trading volume, often viewed as a measure of market participation and information flow, reflects collective investor sentiment and liquidity. Modeling its interdependencies can help reveal how trading behavior in one cryptocurrency influences others, offering more profound insights into market contagion and co-movement. In this study, we focus on monthly trading volumes, which provide a balanced temporal resolution, and smooth short-term volatility and noise inherent in daily data while preserving the medium-term dynamics essential for detecting structural dependencies and risk trends. This aggregation facilitates stable model estimation and a more straightforward interpretation of systemic relationships across major cryptocurrencies.

Prior studies have examined various aspects of cryptocurrency dynamics, including price formation, return volatility, and spillover effects. For example, Dyhrberg (2016), Guesmi et al. (2019), Genest and Favre (2007), Aas et al. (2009), and Kajtazi and Moro (2019) showed that Bitcoin can serve as a hedge, while Bouoiyour and Selmi (2015) argued that its uncertainty undermines its role as a haven. Brauneis and Mestel (2018) analyzed predictability under liquidity constraints, finding that greater liquidity reduces efficiency. Other works, such as Rehman et al. (2023), explored tail dependence between Bitcoin and traditional currencies using copula models, and Liu and Serletis (2024) applied semiparametric GARCH-in-Mean copula models to study volatility linkages between cryptocurrencies and financial markets. However, these studies have advanced our understanding of crypto risk behavior, with most focusing on prices or returns rather than trading volumes and overlooking how participation intensity and liquidity co-move across coins.

Furthermore, while traditional models (e.g., ARIMA, GARCH, or VAR) capture linear or time-based dependencies, they fail to characterize the nonlinear and asymmetric relationships that dominate cryptocurrency markets. Copula-based models have emerged as powerful alternatives, offering flexibility in modeling tail dependencies and complex joint distributions. Yet, most prior copula studies rely on bivariate frameworks, which cannot adequately capture high-dimensional relationships or conditional dependencies among multiple assets.

Despite extensive literature on cryptocurrency price behavior, a remarkable gap remains in understanding how trading volumes jointly evolve across major cryptocurrencies. Traditional dependence models are limited in their ability to handle high-dimensional, nonlinear, and tail-dependent relationships that characterize these markets. Moreover, simplifying assumptions such as conditional independence in pair-copula constructions (PCCs) often yield unrealistic representations of real-world market interactions. This study addresses these gaps by developing and applying advanced vine copula models to characterize the high-dimensional dependence structures among leading cryptocurrencies' trading volumes.

Early studies on cryptocurrency dependence primarily relied on bivariate copula models to capture joint relationships between pairs of assets, such as Bitcoin and Ethereum or Bitcoin and Ripple. These approaches, although effective for two-dimensional dependence, fail to adequately represent the complex and asymmetric dependence structures among multiple cryptocurrencies traded simultaneously. To overcome these limitations, the development of PCCs and vine copula frameworks provided a natural extension of the bivariate case, enabling the modeling of higher-dimensional dependence while preserving the interpretability of bivariate copulas. This study follows this pathway, beginning from the analysis of bivariate dependence in cryptocurrency trading volumes and extending it to a comprehensive vine copula framework capable of modeling intricate multi-asset relationships.

This study aims to model the dependence structure among selected cryptocurrencies' monthly trading volumes using vine copula models, specifically D-vine, C-vine, and R-vine structures. These models are chosen for their flexibility in capturing complex, asymmetric, and high-dimensional dependencies that traditional bivariate or elliptical copulas cannot represent. The D-vine effectively models sequential or chain-like dependencies among cryptocurrencies, the C-vine highlights systems where one dominant cryptocurrency exerts central influence over others, and the R-vine provides a flexible framework, allowing arbitrary pairwise connections without imposing a specific order. The study further evaluates and compares various copula families (Student's t, Clayton, Gumbel, and Frank) based on their goodness of fit and analyzes the volatility and risk levels of these cryptocurrencies to identify the most influential and risk-prone assets.

The key contributions are threefold. First, it provides a comprehensive dependence analysis of cryptocurrency trading volumes using advanced vine copula methods, extending existing research beyond price-based dependence studies. Second, it demonstrates the effectiveness of the Gumbel copula in capturing upper-tail dependence, crucial for understanding simultaneous surges in trading activity. Third, it offers practical implications for traders, portfolio managers, and regulators in assessing contagion risk, optimizing diversification, and designing stress-testing frameworks.

The empirical analysis uses monthly trading volumes for five major cryptocurrencies: Binance Coin (BNB), Bitcoin (BTC), TRON (TRX), Ethereum (ETH), and Dogecoin (DOGE), obtained from Yahoo Finance for the period October 2017 to July 2021. These cryptocurrencies were selected due to their high liquidity, diverse functional roles in the crypto ecosystem, and consistent data availability. The monthly aggregation ensures stability in parameter estimation and captures medium-term dependency dynamics relevant for systemic risk analysis. A detailed description of the dataset and pre-processing steps is provided in the numerical application section.

The remainder of this paper is organized as follows. Section 1.2 presents the theoretical background on copula functions and PCCs. Section 2 introduces the vine copula frameworks (D-vine, C-vine, and R-vine) employed in this study. Section 3 outlines the dependence measures used for model evaluation, while Section 4 presents the empirical analysis, results, and discussion. Finally, Section 5 concludes with key findings, implications, and directions for future research.

1.1. Copula function

A copula function C links a multivariate distribution function of several random variables to their respective marginal distribution functions. Formally,

$$C : I^n \to I \quad \left(I \in [0,1]\right),$$

where each marginal variable follows a uniform distribution on $[0,1]$. The function C possesses the following key properties:

1. Groundedness: For every $u = u_i \in I^n, \ \left(i = 1,2,...,n\right),$ if at least one coordinate u_i is 0, then $C(u) = 0.$

2. Monotonicity: C is non-decreasing. For every $u, v \in I^n$ such that $u_i \le u_j$ and $v_i \le v_j$ for $i = 1,2,...,n, \ j = 2,3,...,n$ and $i \le j.$ Then,

$$C(v) - C(u) \ge 0.$$

Specifically, for the bivariate case $(n = 2),$

$$C\left(u_2, v_2\right) - C\left(u_2, v_1\right) - C\left(u_1, v_2\right) + C\left(u_1, v_1\right) \ge 0 \text{ for } \left(u_1, u_2\right), \left(v_1, v_2\right) \in I^2.$$

3. Uniform Margins: $\forall u_i \in I^n,\ C(1,...,1,u_i,1...,1) = u_i$.

The primary role of copula functions is to separate the dependence structure of multivariate data from their marginal behavior. According to Sklar's Theorem (see Sklar (1959)), any joint distribution function G_i with continuous margins $F_i(x_i)$ can be expressed as:

$$F(x_1, x_2, ..., x_n) = C(F_1(x_1), F_2(x_2), ..., F_n(x_n)).$$

This property makes copulas particularly powerful in modeling complex dependencies among variables whose marginal distributions differ in form or scale.

Traditionally, dependence among random variables has been measured using linear correlation coefficients, which capture only linear associations. However, copula-based models provide a more comprehensive framework, allowing for the modeling of nonlinear, asymmetric, and tail-dependent relationships that are prevalent in financial and economic data. As demonstrated by Embrechts et al. (2003), copulas outperform correlation-based methods when analyzing variables that exhibit non-normality or heavy tails, features commonly observed in cryptocurrency markets.

An essential concept in copula theory is tail dependence, introduced by Embrechts et al. (2001), which quantifies the probability that one variable exceeds a high quantile given that another does the same. A copula exhibits upper-tail dependence if the likelihood that both variables simultaneously take extremely large values remains positive as the quantile level tends to one. Conversely, lower-tail dependence characterizes joint occurrences of extremely small values. These features enable copulas to model co-movements in extreme market conditions, such as sudden surges or crashes, where traditional correlation metrics fail.

In the context of this study, copula functions serve as the foundational tool for constructing higher-dimensional dependence models through PCCs. These allow the decomposition of multivariate relationships into hierarchies of bivariate copulas, enabling flexible and accurate modeling of complex interactions among multiple cryptocurrencies. This framework forms the basis for the vine copula models developed in the subsequent sections.

1.2. PCC of multivariate distribution

Every joint distribution function inherently contains information about the marginal behavior of individual variables and the dependence structure linking them. Copula theory provides a powerful framework for isolating and analyzing this dependence structure, allowing for flexible modeling of complex relationships among multiple variables. Specifically, PCC offers a systematic way to decompose a high-dimensional dependence structure into a sequence of simpler bivariate copulas, making it highly suitable for modeling nonlinear and asymmetric dependencies observed in financial and economic systems.

Let $X_1, X_2, ..., X_n$ be continuous random variables with a joint density function $g(x_1, ..., x_n)$. This joint density can be expressed as the product of its conditional and unconditional components:

$$g(x_1, ..., x_n) = g(x_n)\, g(x_{n-1} \mid x_n)\, g(x_{n-2} \mid x_{n-1}, x_n), ..., g(x_1 \mid x_2, ..., x_n). \quad (1)$$

Suppose $X_1, X_2, ..., X_n$ have an absolutely continuous and strictly increasing joint distribution function $G(x_1, ..., x_n)$ with corresponding marginal distributions $G_1(x_1), ..., G_n(x_n)$. Then, by Sklar's Theorem, there exists an appropriate n-dimensional copula C such that:

$$G(x_1, ..., x_n) = C(G_1(x_1), G_2(x_2), ..., G_n(x_n)). \quad (2)$$

Equation (2) can be re-expressed in terms of uniform marginals as:

$$C(u_1,...,u_n) = G\left(G_1^{-1}(u_1), G_2^{-1}(u_2),..., G_n^{-1}(u_n)\right),$$

where G_i^{-1} represents the quasi-inverses of the marginal functions G_i.

For some n-dimensional copula density $c_{i,j|D}(.)$ the joint density function can be expressed as:

$$g(x_1,...,x_n) = c_{12...n}\left(G_1(x_1), G_2(x_2),..., G_n(x_n)\right) g_1(x_1)...g_n(x_n) \quad (3)$$

for some n-dimensional copula density $c_{12...n}$. A simple form of equation (3) can be obtained for smaller n. The $c_{12...n}(.,.)$ is the pair copula density for the pair of n-dimensional variables. Furthermore, equation (1) can be expressed as $g(x|u) = c_{xu_i|u_{-i}}\left(G(x|u_{-i}), G(u_i|u_{-i})\right) g(x|u_{-i})$, $i = 1,2,...,n$ the decomposition of the pair copula multiplies the conditional marginal density.

According to Joe (1996), for every i, the conditional distribution can be expressed as:

$$G(x|u) = \frac{\delta C_{x,u_i|u_{-i}}\left(G(x|u_{-i}), G(u_i|u_{-i})\right)}{\delta G(u_i|u_{-i})} \text{ such that } 0 \le \frac{\delta C_{x,u_i|u_{-i}}\left(G(x|u_{-i}), G(u_i|u_{-i})\right)}{\delta G(u_i|u_{-i})} \le 1.$$

The significance of this bivariate inflated joint modeling framework lies in its exceptional flexibility to capture complex, nonlinear, and asymmetric dependencies that conventional correlation-based or purely multivariate models often overlook. By decomposing high-dimensional dependencies into a hierarchy of interconnected bivariate components, the PCC framework enhances interpretability and computational tractability while allowing the selection of specific copula families to model different tail behaviors. This modular design enables a more accurate representation of tail dependence, particularly useful for financial applications such as cryptocurrency trading volumes, which often exhibit non-normal, heavy-tailed distributions. The PCC approach thus provides a robust foundation for constructing vine copula models, discussed in the next section, to effectively model intricate interdependencies among multiple assets.

2. Vine Copula Models

This section presents the vine copula models that will be explored in this study to examine the dependence structures among selected cryptocurrency trading index volumes. While bivariate copula models are widely applied in dependence modeling, they face significant challenges when handling complex, high-dimensional relationships among variables with heterogeneous characteristics, particularly in achieving complete and accurate inference. To overcome these limitations, graphical statistical models known as vine copulas have been developed. Vine copulas decompose a multivariate copula into a hierarchical structure of interconnected bivariate copulas, allowing for flexible and effective modeling of intricate dependencies that traditional bivariate copulas cannot adequately capture.

Previous studies have demonstrated the versatility and robustness of vine copula models in financial dependence analysis. For instance, Boako et al. (2019) utilized the vine copula approach to analyze the co-dependence and portfolio value-at-risk of six major cryptocurrencies over three years. The study revealed that Ethereum provides an optimal and economically favorable risk–reward trade-off for portfolio investors, especially under a no-shorting constraint, as evidenced by the efficient frontier analysis. Similarly, Syuhada et al. (2022) employed the vine copula method to investigate the safe-haven properties of Bitcoin and gold related to energy commodities such as oil and petroleum during the COVID-19 pandemic. The results showed that gold effectively reduces downside portfolio risk, confirming its safe-haven role, whereas Bitcoin exhibited inconsistent safe-haven behavior. Furthermore, Mba (2024) examined the sensitivity and robustness of systemic risk measures and

conditional value-at-risk using a combination of vine copula and APARCH-DCC models. Their findings indicated that the vine copula model captures complex dependence structures and tail dependencies, while the APARCH-DCC model effectively accounts for volatility clustering, kurtosis, and skewness.

Vine copulas are characterized by a series of nested trees that unconditionally describe pair copula functions in the first tree T_q for finite q and conditionally in subsequent trees. The construction and inference of a vine copula model rely on three key components: the vine structure, the parametric families associated with each copula pair, and their corresponding copula parameters.

This study gives particular attention to the following types of vine structures for the variable of interest: the D-vine, C-vine, and R-vine. The choice of vine copula models, specifically D-vine, C-vine, and R-vine, was driven by their flexibility and ability to capture complex dependence structures in high-dimensional data, essential for analyzing the interdependencies among cryptocurrencies. These models decompose a multivariate density into a series of bivariate copulas, allowing for the modeling of intricate dependence patterns that traditional bivariate copula models do not easily capture.

The D-vine copula model is particularly suitable for datasets where the order of variables plays a substantial role in defining the dependence structure. This model was selected to represent scenarios where the relationship between pairs of cryptocurrencies can be modeled sequentially, reflecting a chain-like dependence structure.

Similarly, the C-vine copula model is characterized by a central node connected to all other nodes, making it ideal for datasets where one cryptocurrency substantially influences the others. This structure is suitable for scenarios where a single cryptocurrency plays a central or dominant role in defining the system's dependence relationships. Additionally, the R-vine copula model appears more flexible, allowing for arbitrary pairings of variables without imposing a specific order or centrality. This model was chosen to explore scenarios where the dependence structure does not adhere to a sequential or star-shaped pattern, offering a more generalized framework for capturing complex dependencies among cryptocurrencies.

Vine copula models provide a hierarchical approach to modeling dependencies, making them suitable for high-dimensional data where dependencies are not uniformly distributed or linear. They also allow for the incorporation of bivariate copulas with specific tail dependence properties (e.g., Student's t-copula, Clayton copula), which is crucial for accurately modeling the extreme co-movements observed in cryptocurrency markets. Furthermore, their structures balance model complexity and interpretability, allowing for a nuanced understanding of the dependencies among high-dimensional datasets. Using Bayesian Information Criteria (BIC) and log-likelihood (LL) values helped assess the goodness of fit, ensuring that the selected models robustly represent the data.

An n-dimensional vine tree structure has a series of $n-1$ trees, such that the tree T_q $\left(q=1,2,...,n-1\right)$ has $n+1-q$ nodes, $n-q$ edges, and edges in the tree T_q become nodes in the tree T_{q+1}. For any n-dimensional variables, there are $n(n-1)/2$ possible edges. Furthermore, based on the proximity principle, two nodes in the tree T_{q+1} are joined by an edge if and only if two boundaries in the tree T_q have a common node. Each edge in a tree represents a pair of copula densities. This study constructed the D-vine, C-vine, and R-vine structures for the selected cryptocurrencies: Binance Coin (BNB, as x_1), Bitcoin (BTC, as x_2), TRON (TRX, as x_3), ETHEREUM (ETH, as x_4), and DOGECOIN (DOGE, as x_5).

The selection of Binance Coin (BNB), Bitcoin (BTC), TRON, Ethereum (ETH), and Dogecoin (DOGE) for this study was based on several specific considerations relevant to the research objectives and the evolving landscape of the cryptocurrency market. The primary factor guiding the choice of these cryptocurrencies was the availability of monthly trading volume data over a substantial period (October 1, 2017, to July 15, 2021). Reliable and comprehensive data are crucial for robust statistical analysis, particularly when employing advanced modeling techniques like vine copulas. These cryptocurrencies represent a diverse cross-section of the market, each with unique characteristics and considerable trading volumes. Bitcoin, as the first and most widely recognized cryptocurrency, serves as a benchmark for the entire market. Ethereum is notable for its smart contract functionality, which has made it the foundation for many decentralized applications. Binance Coin is essential as it is the native currency of one of the largest cryptocurrency exchanges, Binance. Though created as a meme coin, Dogecoin gained substantial popularity and trading volume, especially during the study period. On the other hand, TRON represents a platform-focused cryptocurrency aimed at decentralized content distribution, offering a distinct use case in the market.

The selected cryptocurrencies exhibit various distributional characteristics, such as positive skewness and kurtosis, which are essential for exploring different dependency structures using copula models. The diversity in their behavior, particularly in trading volume distributions, provides a rich dataset for analyzing dependence and volatility, which are critical aspects of the study. The study aims to explore the dependence structures among monthly cryptocurrencies' trading volumes using vine copula models and their volatility. The selected cryptocurrencies' varied market positions and trading behaviors are well-suited to this analysis. The study aims to understand how these cryptocurrencies influence each other and the implications for trading strategies, making them ideal candidates for the applied models.

2.1. D-vine

The D-vine is a specific type of vine copula model that can be visually represented. It falls under the category of regular vine copulas and can decompose a multivariate density into a nested arrangement of trees. Unlike other vine models, the D-vine exhibits a distinctive pattern in which no node within any tree is connected to more than two edges. The D-vine model was utilized by Mendes and Carneiro (2020) to investigate the dependence relationships among several cryptocurrencies, including BTC, ETH, XRP, LTC, XLM, and XMR. This approach allowed the study to build reliable bivariate copula models for these digital currencies, revealing a substantial increase in their dependence strength over time within the cryptocurrency market. Wang (2024) introduced a mixed D-vine copula-based conditional quantile model in a separate study to analyze dependence using 55 years of monthly streamflow data from 1956 to 2010. They employed an up-to-down sequential technique, integrating the maximum weight method with the Akaike information criterion and the maximum likelihood approach to define the D-vine model structures and estimate parameters. The findings indicated that this model outperformed the widely used bivariate copula model in capturing streamflow variability. Bedford and Cooke (2001) introduced the density function of an n-dimensional distribution corresponding to the D-vine as follows:

$$g(x_1, x_2, ..., x_n) = \prod_{p=1}^{n} h(x_p) \prod_{q=1}^{n-1} \prod_{s=1}^{n-q} c_{s,s+q|s+1,...,s+q-1} \left(H(x_s \mid x_{s+1}, ..., x_{s+q-1}), H(x_{s+q} \mid x_{s+1}, ..., x_{s+q-1}) \right), \quad (4)$$

where the index q shows the position of the tree T_q in the D-vine and s signifies the edges in each tree T_q, $h(x_p)$ denotes the marginal densities, and $c_{s,s+q|s+1,...,s+q-1}$ represents the bivariate copula

density functions with parameter(s) $\Theta_{q,s}$. Figure 1 presents a visualized $n-1$ trees D-vine structure for 5-dimensional variables. Each tree has $n+1-q$ nodes and $n-q$ edges. Figure 1 presents the graphical representation of the D-vine structure with the edges and nodes for the selected cryptocurrencies and their pair copula density (see also Friday and Çelebioğlu (2023)).

The corresponding log-likelihood function for the D-vine copula model in equation (4) is expressed as

$$\ln\left(Lg(x_1,x_2,...,x_n)\right)=\prod_{q=1}^{n-1}\prod_{s=1}^{n-q}\prod_{t=1}^{T}\ln\left(c_{s,s+q|s+1,...,s+q-1}\left(H(x_{s,t}\mid x_{s+1,t},...,x_{s+q-1,t}),H(x_{s+q,t}\mid x_{s+1,t},...,x_{s+q-1,t})\right)\right). \quad (5)$$

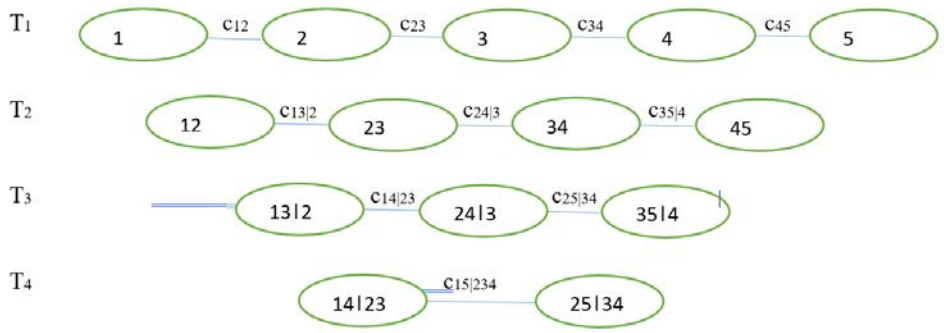


Figure 1. The visualized D-vine structure with 10 edges and 14 nodes represents the dependence among selected cryptocurrencies

The D-vine structure models sequential dependencies, reflecting a chain-like relationship between the cryptocurrencies. By employing equation (4), the density function of the visualized D-vine structure, which consists of 10 edges and 14 nodes, as depicted in Figure 1, can be described as follows:

$$\begin{aligned}g(x_1,x_2,x_3,x_4,x_5)=\;&g_1(x_1)g_2(x_2)g_3(x_3)g_4(x_4)g_5(x_5)c_{12}\left(G_1(x_1),G_2(x_2)\right)c_{23}\left(G_2(x_2),G_3(x_3)\right)\\&c_{34}\left(G_3(x_3),G_4(x_4)\right)c_{45}\left(G_4(x_4),G_5(x_5)\right)c_{13|2}\left(\left(G_1(x_1),G_3(x_3)\right)|\,G_2(x_2)\right)\\&c_{24|3}\left(\left(G_2(x_2),G_4(x_4)\right)|\,G_3(x_3)\right)c_{35|4}\left(\left(G_3(x_3),G_5(x_5)\right)|\,G_4(x_4)\right)c_{14|23}\left(\left(G_1(x_1),G_4(x_4)\right)|\,G_2(x_2),G_3(x_3)\right)\\&c_{25|34}\left(\left(G_2(x_2),G_5(x_5)\right)|\,G_3(x_3),G_4(x_4)\right)c_{15|234}\left(\left(G_1(x_1),G_5(x_5)\right)|\,G_2(x_2),G_3(x_3),G_4(x_4)\right).\end{aligned}$$

2.2. C-vine

The C-vine is a canonical vine and a specific type of regular vine copula characterized by a star-shaped structure. In the C-vine, each tree exhibits a unique node interacting with other nodes through conditional and unconditional pair copulas. This structure illustrates common dependence structures, such as family or football match dependencies. For example, in a family, the entire family's livelihood may depend solely on the head of the family, or in a football match, a specific player (e.g., the goalkeeper) may have strong interactions with other players in their respective positions. Similarly, in cryptocurrency trading, a particular cryptocurrency may serve as a pilot variable, or within an organization, additional units may be interconnected with a central unit. In a study by Li et al. (2023), the authors introduced a transfer learning method based on the C-vine model. The study first constructs the structure and parameters of the C-vine model using the study's data and subsequently applies the model for fault detection. The results indicate that the introduced C-vine-based model achieves higher

detection accuracy and more clearly and accurately expresses the dependence between variables compared to traditional machine learning and deep transfer models. Furthermore, Yang and Chang (2023) employed the C-vine copula model to examine wind power generation data, aiding in the optimization of the location and capacity of a STATCOM device to improve power system stability. Moreover, the C-vine model was utilized in Mwambi (2022) to investigate the dynamic tail dependence risk between BRICS economies and the global energy market during the financial crisis of the COVID-19 pandemic in 2020, to determine optimal investment decisions based on stock and global crude oil price datasets. The study's findings, derived from the C-vine model, reveal the presence of both symmetric and lower tail dependence among different countries, demonstrating the model's ability to capture complex interdependencies between these variables. Bedford and Cooke (2001) formulated the density function for the C-vine structure as follows:

$$g(x_1, x_2, ..., x_n) = \prod_{p=1}^{n} h(x_p) \prod_{q=1}^{n-1} \prod_{s=1}^{n-q} c_{q,q+s|1,...,q-1}\left(H(x_q \mid x_1, ..., x_{q-1}), H(x_{q+s} \mid x_1, ..., x_{q-1})\right), \tag{6}$$

where q, s, $h(x_p)$, $c_{q,q+s|1,...,q-1}$, and $\Theta_{q,s}$ denote the same meaning as in D-vine (see also Bedford and Cooke (2002)). The corresponding log-likelihood function to equation (6) is given as

$$\ln\left(Lg(x_1, x_2, ..., x_n)\right) = \prod_{q=1}^{n-1} \prod_{s=1}^{n-q} \prod_{t=1}^{T} \ln\left(c_{q,q+s|1,...,q-1}\left(H(x_{q,t} \mid x_{1,t}, ..., x_{q-1,t}), H(x_{q+s,t} \mid x_{1,t}, ..., x_{q-1,t})\right)\right). \tag{7}$$

Figure 2 presents a visualized graphical representation of the $n-1$ tree C-vine structure for the selected cryptocurrencies with the associated edges and nodes.

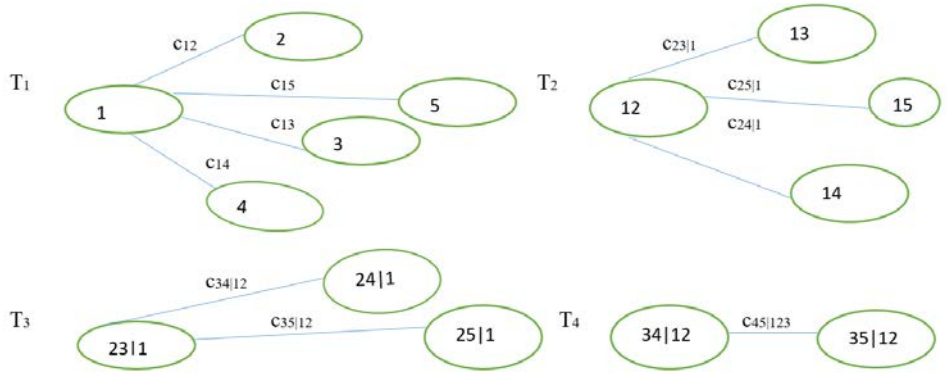


Figure 2. The visualized C-vine structure, comprising 10 edges and 14 nodes, is used for selected cryptocurrencies

The C-vine structure highlights a central cryptocurrency influencing others, ideal for scenarios where one cryptocurrency plays a dominant role in defining market dependencies. The density function of the visualized C-vine structure for the selected cryptocurrencies is given as

$$\begin{aligned} g(x_1, x_2, x_3, x_4, x_5) = {} & g_1(x_1) g_2(x_2) g_3(x_3) g_4(x_4) g_5(x_5) c_{12}(G_1(x_1), G_2(x_2)) c_{15}(G_1(x_1), G_5(x_5)) c_{13}(G_1(x_1), G_3(x_3)) \\ & c_{14}(G_1(x_1), G_4(x_4)) c_{23|1}\left((G_2(x_2), G_3(x_3)) \mid G_1(x_1)\right) c_{25|1}\left((G_2(x_2), G_5(x_5)) \mid G_1(x_1)\right) \\ & c_{24|1}\left((G_2(x_2), G_4(x_4)) \mid G_1(x_1)\right) c_{34|12}\left((G_3(x_3), G_4(x_4)) \mid G_1(x_1), G_2(x_2)\right) c_{35|12}\left((G_3(x_3), G_5(x_5)) \mid G_1(x_1), G_2(x_2)\right) \\ & c_{45|123}\left((G_4(x_4), G_5(x_5)) \mid G_1(x_1), G_2(x_2), G_3(x_3)\right). \end{aligned}$$

2.3. R-vine

R-vine is a regular vine. All the vines are used to specify necessary copulas for pair copula construction. This vine has been used to explore the interdependencies among Dow Jones stocks across different economic periods, particularly around the global financial crisis Allen et al. (2013). The findings indicate that dependencies among the stocks change in a complex manner across these periods, with a notable increase in reliance on the Student-t copula during the global financial crisis, reflecting the presence of heavier tails in return distributions. This highlights the flexibility of regular vine copulas in modeling co-dependencies and demonstrates their practical application in calculating portfolio value at risk. While numerous studies have focused on exploring C-vine and D-vine copulas, Dissmann et al. (2013) introduced a novel model selection and parameter estimation technique for R-vine copulas using a graphical approach. This method was applied to simulated data and a 16-dimensional financial dataset collected over the past decade. The study offers substantial insights into the dependence structure among the variables. In a similar effort to explore the R-vine structure, Peng et al. (2022) analyze three key indicators: the overall monthly loss ratio of the insurance industry, the monthly yield of the Shanghai Stock Exchange Treasury Bond Index, and the monthly yield of the Shanghai Composite Index. These indicators are used to assess the three primary risks faced by insurance companies in China: insurance risk, credit risk, and market risk. After data collection, the authors applied an R-vine structure and a GARCH model to evaluate the value at risk (VaR) of these indicators. The findings demonstrate that the R-vine model effectively captures the relationships between these risks and their interdependencies based on its graphical representation. The visualized graphical representation of the R-vine for the selected cryptocurrencies is presented in Figure 3.

The density function of the R-vine is given as

$$g(x_1, x_2, ..., x_n) = \prod_{p=1}^{n} h(x_p) \prod_{q=1}^{n-1} \prod_{e \in E_s} c_{j(e),k(e)|D(e)} \left(H(x_{p,j(e)} \mid x_{p,D(e)}), H(x_{p,k(e)} \mid x_{p,D(e)}) \right), \qquad (8)$$

where c denotes a bivariate copula density associated with an edge e and parameter(s) $\Theta_{j(e),k(e)|D(e)}$. The corresponding log-likelihood function for equation (8) can be expressed as

$$\ln\left(Lg(x_1, x_2, ..., x_n)\right) = \prod_{p=1}^{n} \prod_{q=1}^{n-1} \prod_{e \in E_s} \ln\left(c_{j(e),k(e)|D(e)} \left(H(x_{p,j(e)} \mid x_{p,D(e)}), H(x_{p,k(e)} \mid x_{p,D(e)}) \right) \right). \qquad (9)$$

For instance, $n = 5$, the tree structure of the R-vine can take the following structure. Similarly, the density function of the visualized R-vine structure corresponding to the structure described above can be expressed as follows:

$$\begin{aligned} g(x_1,x_2,x_3,x_4,x_5) = {} & g_1(x_1) g_2(x_2) g_3(x_3) g_4(x_4) g_5(x_5) c_{12}(G_1(x_1), G_2(x_2)) c_{25}(G_2(x_2), G_5(x_5)) c_{35}(G_3(x_3), G_5(x_5)) \\ & c_{45}(G_4(x_4), G_5(x_5)) c_{15|2}\left((G_1(x_1), G_5(x_5)) \mid G_2(x_2)\right) c_{23|5}\left((G_2(x_2), G_3(x_3)) \mid G_5(x_5)\right) c_{24|5}\left((G_2(x_2), G_4(x_4)) \mid G_5(x_5)\right) \\ & c_{13|25}\left((G_1(x_1), G_3(x_3)) \mid G_2(x_2), G_5(x_5)\right) c_{34|25}\left((G_3(x_3), G_4(x_4)) \mid G_2(x_2), G_5(x_5)\right) c_{14|235}\left((G_1(x_1), G_4(x_4)) \mid G_2(x_2), G_3(x_3), G_5(x_5)\right). \end{aligned}$$

The R-vine structure in Figure 3 allows for an arbitrary dependency pattern, capturing complex interrelationships without imposing a specific order.

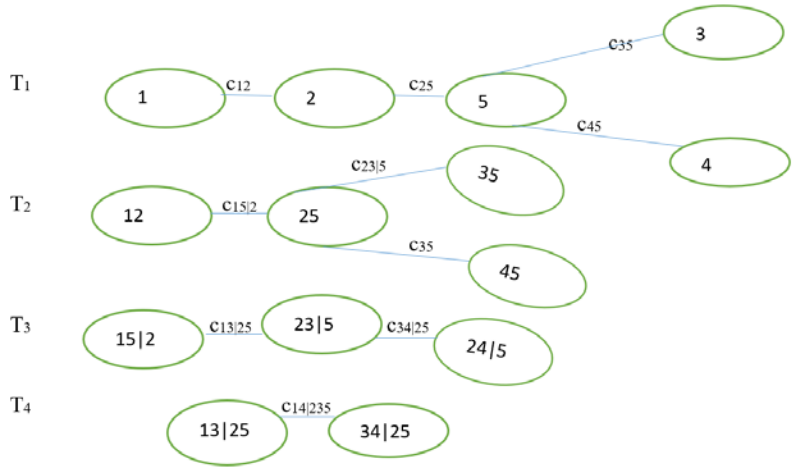


Figure 3. The visualized R-vine structure with 10 edges and 14 nodes illustrates flexible pairings among the selected cryptocurrencies

3. Dependence Measure in Copula

Using copulas for examining the dependence structure among continuous random variables has gained increasing significance, particularly when addressing n-dimensional systems outside the realm of elliptical distributions (non-normal systems). In such cases, traditional correlation models may not be suitable. Consequently, there has been a shift toward alternative rank-based correlation measures defined in the context of copula functions. Kendall's tau (τ) is one of the most commonly used rank-based correlation measures.

If (X,Y) is a vector of continuous random variables with two independent pairs (x_1, y_1) and (x_2, y_2) as observations, each with a joint distribution function G. Then, the pairs (x_1, y_1) and (x_2, y_2) are

1. Concordant if and only if $(x_1 - x_2)(y_1 - y_2) > 0$
2. Discordant if and only if $(x_1 - x_2)(y_1 - y_2) < 0.$

The Kendall's tau (τ) is defined as

$$\tau = P\left[(x_1 - x_2)(y_1 - y_2) > 0\right] - P\left[(x_1 - x_2)(y_1 - y_2) < 0\right]. \tag{10}$$

If C is the copula function associated with (X,Y) and G is the joint distribution function. Then, Kendall's tau measure of dependence function is expressed as

$$\tau = 4\int_0^1\int_0^1 C(u,v)\delta C(u,v) - 1. \tag{11}$$

4. Application: Cryptocurrency Trading Volumes

This study investigates the dependence measures within the dynamic and rapidly evolving cryptocurrency market, focusing on a selected group of cryptocurrencies. The analysis utilized vine structures to explore the relationships between the trading volumes of the following cryptocurrencies:

Binance Coin (BNB, as x_1), Bitcoin (BTC, as x_2), TRON (TRX, as x_3), ETHEREUM (ETH, as x_4), and DOGECOIN (DOGE, as x_5).

The dataset comprises monthly trading volumes for these cryptocurrencies, covering the period from October 1, 2017, to July 15, 2021. These data were sourced from Yahoo Finance (https://finance.yahoo.com/), a reputable and widely used platform that provides historical financial data for various assets, including cryptocurrencies. The selected time frame captures substantial market events and fluctuations, offering a comprehensive view of the market dynamics over nearly four years.

The empirical implementation began by transforming each cryptocurrency's trading volume data to uniform margins using the probability integral transform. Pairwise dependencies were first analyzed using traditional bivariate copulas, and these results informed the selection of copula families within the vine structures. The models were estimated using the sequential Dissmann algorithm with maximum likelihood estimation, implemented in the Vine copula R package. Model performance was assessed through the log-likelihood and Bayesian information criterion (BIC) values to ensure an optimal balance between fit and complexity. This stepwise approach reflects the progression from bivariate joint modeling toward high-dimensional dependence analysis suitable for cryptocurrency markets.

The raw data were extracted directly from Yahoo Finance, ensuring consistency and reliability. Each cryptocurrency's trading volume data was obtained separately and then compiled into a unified dataset for analysis.

Since the cryptocurrencies had different starting points for their trading data availability, the dataset was carefully aligned to ensure a consistent time frame across all assets. The time series for each cryptocurrency was truncated to match the common period from October 1, 2017, to July 15, 2021. The dataset was reviewed for any missing values, but none were found. This approach was chosen to maintain the continuity of the time series data and avoid potential biases. Given that the study investigates dependence measures using copula models, the data were transformed to uniform margins before assessing the dependence among variables and fitting the copula models. This step ensures that each variable adheres to a uniform distribution within the interval [0,1], thereby meeting the necessary conditions for copula models. This transformation to uniform margins is crucial for accurately applying vine copula and other bivariate copula models, which require uniform marginals to analyze the dependence structure among the variables effectively.

The study examines the monthly trading volumes of selected cryptocurrencies over a specified period, influenced by several key factors aligned with its objectives and methodology. While daily trading volume data is significant in financial research due to its high frequency, this study prioritizes monthly data to focus on long-term dependencies rather than short-term fluctuations. This approach is particularly relevant for analyzing systemic risk and tail dependence, where monthly data can offer a clearer view of sustained trends and correlations. The selected cryptocurrencies demonstrate positive skewness and under-dispersion in their monthly trading volumes, indicating distinctive behavioral patterns over extended periods. These characteristics make them well-suited for the copula models employed, which are adept at capturing complex dependencies that the noise in higher-frequency data might obscure. Daily trading volumes in the cryptocurrency market tend to be highly volatile and subject to short-term noise, potentially masking the true dependence structures to uncover. Monthly data helps smooth these fluctuations, enabling a more robust and stable analysis of the underlying relationships among the cryptocurrencies. The study investigates the dependence among these cryptocurrencies and their volatility, which is more accurately captured over longer periods. To

accurately identify substantial patterns and dependencies that persist over time, which is crucial for understanding risk in the cryptocurrency market, a monthly financial dataset could be utilized. The selection of monthly data is also motivated by practical considerations, including data availability and the computational challenges associated with fitting high-dimensional copula models to extensive datasets. This approach strikes a balance between capturing meaningful patterns and maintaining computational feasibility, particularly when analyzing a large number of periods.

Figures 1, 2, and 3 visually represent the PCCs, which delineate the intricate relationships between the selected cryptocurrencies and showcase their associations across various vine structures. These structures, D-vine, C-vine, and R-vine, provide a robust framework for understanding the dependencies within the data. The corresponding density and log-likelihood functions for each structure further enhance the comprehension of these relationships, offering a quantitative measure of the model's fit to the observed data.

Table 1 presents a detailed summary of the statistical properties of the selected cryptocurrencies, including their mean, variance, skewness, and kurtosis. These statistics are crucial in understanding the underlying distributional characteristics of the data. The positive skewness observed in the monthly trading volumes across all selected cryptocurrencies indicates that their distributions are heavily skewed to the right, meaning that most trading volumes are concentrated at lower values with a long tail extending towards higher values. This is particularly evident in Ethereum (ETH), which exhibits the most significant skewness and kurtosis values. The high kurtosis indicates that the distribution has fatter tails and a more peaked distribution than a normal distribution, which implies a higher probability of extreme trading volumes. Though slightly less, Dogecoin (DOGE) and TRON (TRX) follow this trend. These statistical properties underscore the unique trading behaviors of these cryptocurrencies, which are further explored using copula models. These findings are significant because they have implications for risk and volatility analysis. The positive skewness and high kurtosis of the distributions indicate that these cryptocurrencies experience frequent low trading volumes with occasional substantial spikes. Such behavior makes these assets potential candidates for copula models, which are well-suited to capture the dependence structure in data with non-normal distributions.

Table 1. Summary statistics of the selected cryptocurrencies, including their mean, variance, skewness, and kurtosis, provide insight into their trading volumes' distributional characteristics

Crypto coins	Mean	Variance	Skewness	Kurtosis
BNB	1.897e+10	1.577e+21	2.856	10.016
BTC	6.750e+11	3.486e+23	1.120	3.4910
TRON	2.796e+10	1.009e+21	2.739	12.882
ETHEREUM	3.051e+10	2.588e+21	4.985	30.537
DOGECOIN	2.539e+10	6.503e+21	3.978	18.108

Table 2. Estimating the pairwise dependence measure for selected cryptocurrencies

Crypto coins	BNB	BTC	TRON	DOGECOIN
BNB	1.000	0.802	0.478	0.536
BTC	0.802*	1.000	0.583	0.642
TRON	0.478***	0.583	1.000	0.676**
DOGECOIN	0.536	0.642	0.676	1.000

Key: * the highest dependence measure value, ** high dependence measure value, and *** the least dependent measure value.

Table 2 illustrates the strength and direction of dependencies between cryptocurrency pairs, where higher values denote stronger positive relationships, which are crucial for predicting how changes in one cryptocurrency's trading volume may impact others.

Figure 4 provides a visual representation through histograms of the monthly trading volumes of the selected cryptocurrencies. The histogram plot offers a more tangible understanding of the skewness and concentration of data points within each distribution. For instance, Bitcoin (BTC) exhibits a distinct pattern, characterized by a heavy concentration of trading volumes on the right side of the histogram, which indicates that larger trading volumes occur more frequently than others in the long-term dynamic. This pattern is followed by TRON, Ethereum, Dogecoin, and Binance Coin (BNB), each exhibiting unique distribution shapes that reinforce the need to analyze their interdependencies.

Table 2 illustrates the pairwise Kendall's tau dependence measures, which quantify the strength and direction of the relationships between the monthly trading volumes of the selected cryptocurrencies. The positive dependence measures, notably between BNB and BTC, indicate that an increase in one's trading volume is likely to result in a significant increase in the other. Such insights are critical for traders, as they signify potential co-movements in the market, which can be exploited for trading strategies. Similar patterns of positive dependence are observed between other pairs, including BNB and Dogecoin, BTC and TRON, TRON and Dogecoin, and Dogecoin and BTC, further highlighting the interconnectedness of these digital assets. To further explore these relationships, Figure 5 displays pair plots that graphically represent the fundamental trading volumes of the selected cryptocurrencies. These plots are instrumental in visually assessing the relationships between pairs of cryptocurrencies. Although the direct nature of interdependence is not immediately apparent from these plots, the subsequent transformation of the data to copula form, as detailed theoretically in Section 1.2, provides a more precise understanding of the dependence structure. This transformation involves ranking and normalizing the trading volumes, aligning the data with the principles of copula functions. This process enables the application of rank-based correlation measures, as depicted in Figure 6.

Figure 6 showcases pair plots of the transformed data, revealing the presence of upper tail dependence among the cryptocurrencies. This is evidenced by the concentration of data points in the upper regions of the plots for all the cryptocurrencies, indicating stronger dependencies during periods of extreme market movements. These findings are pivotal as they indicate that during significant market upswings, the trading volumes of these cryptocurrencies are likely to increase in tandem, which is critical for understanding long-term market dynamics and making informed trading decisions.

Figure 4 illustrates the distribution of trading volumes, with BTC showing a heavy concentration to the right, indicating its dominance and higher trading volume than other cryptocurrencies. Figure 5 visually represents the relationships between cryptocurrency pairs, revealing potential dependencies. Figure 6 indicates a concentration of data points in the upper regions, suggesting stronger dependencies during extreme market movements.

Furthermore, the study assesses the efficacy of the pair-constructed vine copula models for the selected cryptocurrencies by employing four distinct bivariate copula models: the Student t, Clayton, Gumbel, and Frank copula models. The selection of these models is predicated on their distinctive tail dependence characteristics, which are aligned with the characteristics of the transformed data. Specifically, the Student t copula represents an implicit model encompassing lower and upper tail dependence features, whereas the Clayton, Gumbel, and Frank models are classified as Archimedean copulas. The Clayton copula exclusively exhibits lower tail dependence, the Gumbel copula solely displays upper tail dependence, and the Frank copula possesses a distinctive attribute characterized by a positive slope and a diffuse yet uniform distribution along the entire correlation spectrum.

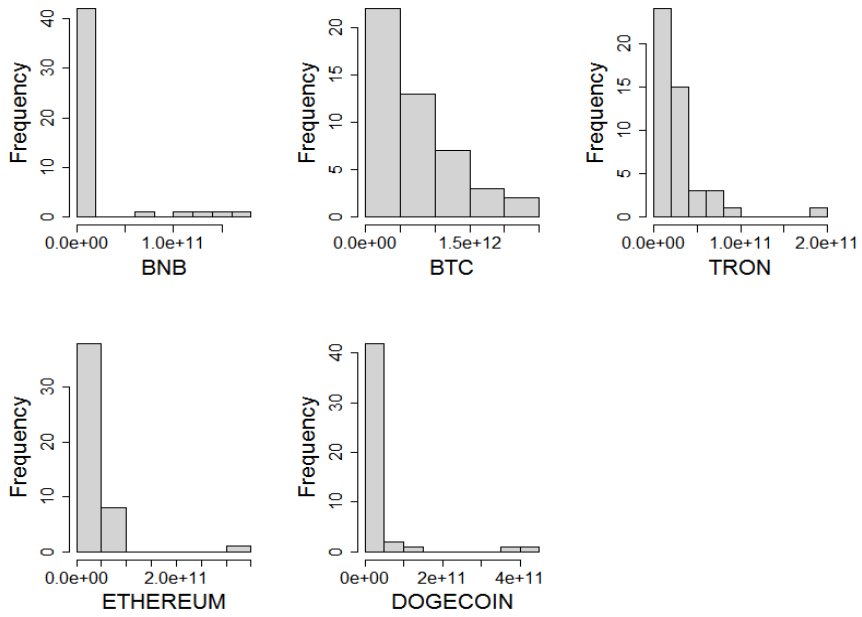


Figure 4. The histogram plot for the monthly trading volumes of the selected cryptocurrencies

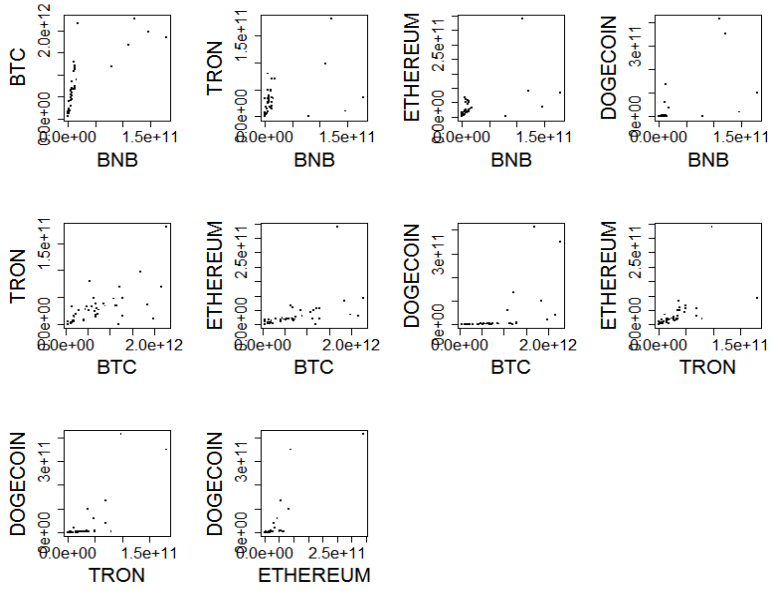


Figure 5. The pair plots for the fundamental values of the trading volumes of the selected cryptocurrencies

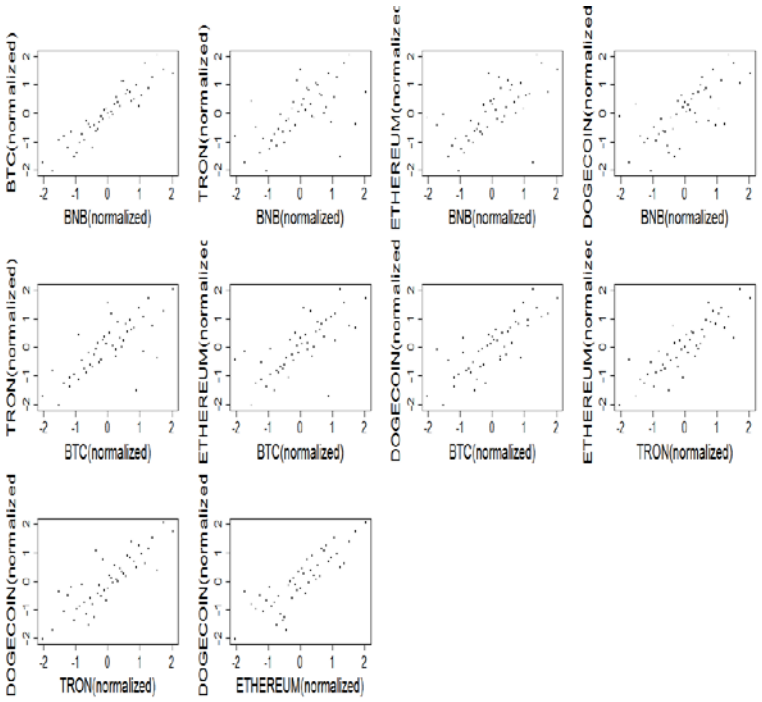


Figure 6. The pair plots for the transformed cryptocurrencies highlight upper tail dependence

In this study, the copula models were individually fitted to the pair-constructed vine structures using the Dissmann algorithm, which employs a sequential top-down estimation approach introduced by Dissmann et al. (2013). This algorithm allows for the efficient fitting of copula models to each constructed pair within the vine structure in each tree, thereby facilitating the determination of goodness of fit and the calculation of Kendall's tau values as measures of dependence. The output from this algorithm also yields the parameter estimate for the fitted model. The association parameters (Par1 and Par2: the parameters of the copula models) and their corresponding Kendall's tau values were estimated for each tree and edge. To evaluate the performance of the fitted copula models, the study employed only the Bayesian information criteria (BIC) and log-likelihood (LL) values. The BIC balances the goodness of fit and complexity in data and identifies the best model suitable for the data. The copula model, characterized by the lowest BIC values, is deemed the most ideal for the analyzed data. The fitting of copula models was conducted on both D-vine, C-vine, and R-vine structures, and the outcomes are presented in Tables 3, 4, and 5, respectively. Notably, some bivariate copula models did not yield estimated association parameters and corresponding Kendall's tau values for certain edges within the vine trees and are consequently not reported.

Generally, Tables 3, 4, and 5 collectively illustrate a notable trend: Kendall's tau values exhibit a consistent decrease as one progresses from the lowest-order trees to the higher-order trees across all fitted copula models. This consistent reduction in Kendall's tau values indicates that the conditioning set employed in the higher-order trees effectively captures a substantial portion of the dependence between the nodes within the respective vine structures. For instance, when fitting the Student's t copula to T2 of the D-vine structure involving edge 6, Kendall's tau value indicates a 32% dependence rate between BTC and ETH when conditioned on TRON during trading. This discovery holds promise

as an informative indicator for traders, aiding them in determining which cryptocurrency pairs to consider during trading, considering their trading volumes and dependence patterns.

Table 3. Fitting of bivariate copula models to the D-vine structure

Copula model	Tree	Edge	Par1	Par2	Kendall's tau
Student's t	T1	1	0.94	30.00	0.77
		2	0.88	3.52	0.69
		3	0.84	8.78	0.64
		4	0.87	3.09	0.67
	T2	6	0.41	11.58	0.27
Frank	T1	1	17.83	-	0.80
		2	10.83	-	0.69
		3	8.88	-	0.63
		4	10.47	-	0.68
	T2	5	-1.47	-	-0.16
		6	2.53	-	0.26
		7	2.03	-	0.21
	T3	8	-0.67	-	-0.07
		9	1.46	-	0.15
	T4	10	0.66	-	0.07
Clayton	T1	1	4.02	-	0.67
		2	2.91	-	0.59
		3	2.50	-	0.56
		4	2.98	-	0.60
	T2	5	-0.17	-	-0.08
		6	0.79	-	0.28
		7	0.42	-	0.17
		8	-0.39	-	-0.16
Clayton	T1	1	4.02	-	0.67
		2	2.91	-	0.59
		3	2.50	-	0.56
		4	2.98	-	0.60
	T2	5	-0.17	-	-0.08
		6	0.79	-	0.28
		7	0.42	-	0.17
		8	-0.39	-	-0.16
Gumbel	T1	1	3.99	-	0.75
		2	3.05	-	0.67
		3	2.65	-	0.62
		4	3.03	-	0.67
	T2	6	1.38	-	0.28

Table 3 presents the association parameters and Kendall's Tau values for different edges within the D-vine structure, offering insights into the strength of dependencies between cryptocurrency pairs.

Table 4. Fitting of bivariate copula models to the C-vine structure

Copula model	Tree	Edge	Par1	Par2	Kendall's tau
Student's t	T1	1	0.94	30.00	0.77
		2	0.80	3.00	0.59
		3	0.82	2.00	0.61
		4	0.84	8.78	0.64
	T2	6	0.69	4.04	0.49
		7	0.69	30.00	0.48
Frank	T1	1	17.83	-	0.80
		2	8.19	-	0.61
		3	7.75	-	0.59
		4	8.88	-	0.63
	T2	5	-2.04	-	-0.21
		6	5.47	-	0.48
		7	6.03	-	0.51
	T3	8	-0.85	-	-0.09
		9	2.22	-	0.23
	T4	10	0.62	-	0.07
Clayton	T1	1	4.02	-	0.67
		2	2.39	-	0.54
		3	1.91	-	0.49
		4	2.50	-	0.56
	T2	5	-0.17	-	-0.08
		6	0.86	-	0.30
		7	1.55	-	0.44
		8	0.50	-	0.20
Gumbel	T1	1	3.99	-	0.75
		2	2.32	-	0.57
		3	2.52	-	0.60
		4	2.65	-	0.62
	T2	5	-1.25	-	-0.20
		6	1.90	-	0.47
		7	1.87	-	0.46

Table 4 details the association parameters and Kendall's Tau values, highlighting how the central cryptocurrency in the C-vine influences others.

Table 5. Fitting of bivariate copula models to the R-vine structure

Copula model	Tree	Edge	Par1	Par2	Kendall's tau
Student's t	T1	1	0.94	30.00	0.77
		2	0.88	3.52	0.69
		3	0.84	8.78	0.64
		4	0.87	3.09	0.67
	T2	6	0.41	11.58	0.27
Frank	T1	1	17.83	-	0.80
		2	10.83	-	0.69
		3	8.88	-	0.63
		4	10.47	-	0.68
	T2	5	-1.47	-	-0.16
		6	2.53	-	0.26
		7	2.03	-	0.21
	T3	8	-0.67	-	-0.07
		9	1.46	-	0.15
	T4	10	0.66	-	0.07
Clayton	T1	1	4.02	-	0.67
		2	2.91	-	0.59
		3	2.50	-	0.56
		4	2.98	-	0.60
	T2	5	-0.17	-	-0.08
		6	0.79	-	0.28
		7	0.42	-	0.17
		8	-0.39	-	-0.16
Gumbel	T1	1	3.99	-	0.75
		2	3.05	-	0.67
		3	2.65	-	0.62
		4	3.03	-	0.67
	T2	6	1.38	-	0.28

Table 5 captures the flexibility of R-vine in modeling dependencies among cryptocurrencies with varying association parameters and Kendall's Tau values.

Tables 6, 7, and 8 compare the performance of the employed bivariate copula models within the D-vine, C-vine, and R-vine structures. In each instance, the Gumbel copula model, distinguished by its upper tail dependence characteristic, yielded the most favorable Bayesian Information Criterion (BIC) values, establishing it as the optimal model for data fitting. The selection of the Gumbel copula, with its upper tail dependence feature, not only signifies its superior fit to the dataset but also implies the presence of upper tail dependence phenomena among the cryptocurrencies under investigation in this study. Detailed representations of these chosen bivariate copula models are visually presented in Figures 7 and 8, providing further insights into their characteristics.

Table 6. The goodness of fit of the bivariate copula models fitted to the D-vine structure

Copula models	LL	BIC
Student's t	142	-246
Frank	145	-252
Clayton	127	-223
Gumbel	139	-256

Table 6 compares log-likelihood (LL) and Bayesian Information Criterion (BIC) values, identifying the Gumbel copula model as the most suitable for capturing the dependencies within this vine structure.

Table 7. The goodness of fit of the bivariate copula models fitted to the C-vine structure, with the Gumbel copula emerging as the best fit due to its ability to model upper tail dependence, as indicated by the lowest BIC values

Copula models	LL	BIC
Student's t	151	-256
Frank	149	-259
Clayton	125	-219
Gumbel	146	-266

Table 8. The goodness of fit of the bivariate copula models fitted to the R-vine structure

Copula models	LL	BIC
Student's t	143	-247
Frank	144	-250
Clayton	127	-223
Gumbel	140	-260

Table 8 highlights the superior performance of the Gumbel copula model in capturing the complex dependencies among the selected cryptocurrencies.

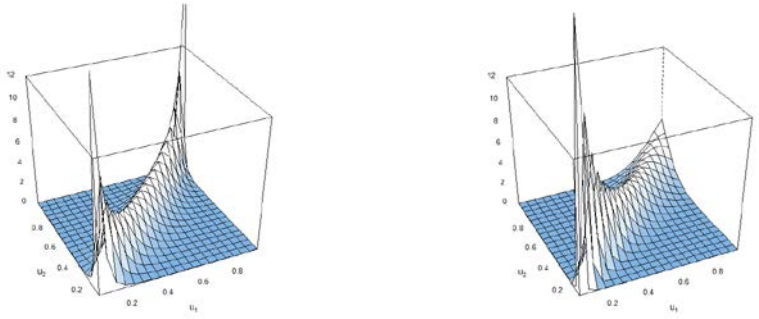


Figure 7. The 3D surface plot of the bivariate density for Student's t copula with df = 3 and ρ = 0.8 (left) and Clayton copula with ρ = 0.8 (right)

Figure 7 visualizes the dependence structures, with Student's t copula capturing lower and upper tail dependencies, while Clayton's copula emphasizes lower tail dependence.

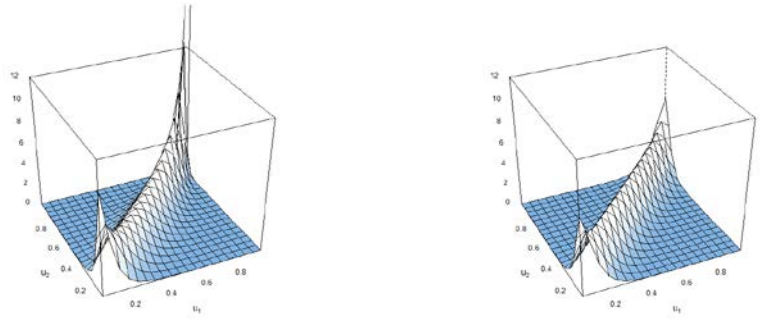


Figure 8. The 3D surface plot of the bivariate density for the Gumbel copula with $\rho = 0.8$ (left) and the Frank copula with $\rho = 0.8$ (right).

The Gumbel copula plot illustrates strong upper tail dependence, making it suitable for modeling simultaneous extreme events in cryptocurrency trading volumes.

4.1. Volatility analysis of the cryptocurrencies trading volumes

This study conducts a volatility analysis of selected cryptocurrencies' monthly trading volumes to assess the associated risk levels. This task involves calculating the average and standard deviation of the monthly volume change. These calculations reveal how volume fluctuations deviate from the mean, thus providing insights into trading volume volatility.

This study initially derives the monthly trading volume changes to compute the average monthly volume changes. Subsequently, it exclusively employs the average and standard deviation changes to examine the volatility of cryptocurrencies. Table 9 presents the average and standard deviation of volume changes for the selected cryptocurrencies. Furthermore, Figure 9 visually represents the variation in the observed trading volume changes of these cryptocurrencies.

The tabulated data and visual representations reveal that BTC exhibits the highest standard deviation in trading volumes, followed by DOGE, ETH, TRON, and BNB. This result aligns with the description of BTC in Figure 4. Furthermore, this observation suggests that while a higher average volume change in a cryptocurrency's trading may signify increasing interest and demand, it may also indicate a bullish market sentiment. In this context, BTC exhibits the highest volatility in trading volumes, making it the most susceptible to risk among the analyzed coins.

The standard deviation value measures cryptocurrency volatility or risk. A higher standard deviation signifies substantial variability in monthly volume changes, implying greater fluctuation in trading activity. This fluctuation can present both opportunities and risks. Consequently, crypto traders can tailor their strategies based on risk tolerance and potential reward.

Based on the findings presented in Table 9 and Figure 9, a conservative trader may prefer to invest in BNB, which has the lowest standard deviation, indicating lower price volatility and potentially lower risk. Conversely, an aggressive trader may seek cryptocurrencies with higher average monthly volume changes, suggesting price momentum and trading opportunities. However, they should be prepared for higher price volatility and associated risks, especially if the standard deviation is high.

Table 9. Volatility analysis of the average changes in the volume of the selected cryptocurrencies

Cryptocurrencies	Average volume changes	Standard deviation
BNB	1668604341	21511973649
BTC	23967063476	256813737514
TRON	14007550	27098688928
ETHEREUM	13890820	55180303905
DOGECOIN	19379917	70055383302

Table 9 provides insights into the volatility and risk associated with each cryptocurrency, with higher standard deviation values indicating greater fluctuations and potential trading risks.

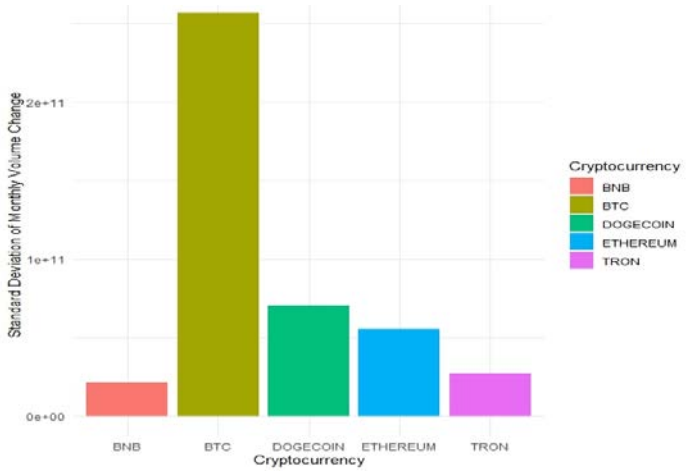


Figure 9. The standard deviation of monthly volume change for each cryptocurrency

BTC and DOGE show the highest volatility, with BTC showing the most substantial fluctuations, indicating the highest risk profile among the analyzed cryptocurrencies

4.2. Discussion

The study's application of vine copula models (D-vine, C-vine, and R-vine) has proven effective in capturing the intricate dependencies among selected cryptocurrencies. This supports earlier research that emphasizes the flexibility and capability of vine copulas in modeling high-dimensional data, as demonstrated in studies such as Joe (1996) and others who have extended these models to higher dimensions. The observed decline in Kendall's tau values from lower to higher-order trees indicates that the conditioning process effectively captures dependencies, consistent with findings reported in studies like Dissmann et al. (2013).

Our findings, which reveal substantial dependencies among cryptocurrencies, align with the analysis of tail dependence and risk spillovers between Bitcoin and the currencies of BRICS and G7 economies, as reported by Rehman et al. (2023). The Gumbel copula's ability to capture upper tail dependence complements Syuhada et al. (2022)’s findings of stronger lower tail dependence between Ethereum and Litecoin, indicating that specific copulas are well-suited for modeling distinct tail behaviors. Furthermore, our insights into Bitcoin's high volatility resonate with Bouoiyour and Selmi (2015)’s assertion that Bitcoin exhibits substantial uncertainty and does not serve as a safe haven. The

inconsistent performance of Bitcoin as a safe haven during the COVID-19 pandemic, as reported by Syuhada et al. (2022), further supports our findings of Bitcoin's elevated volatility risk levels. Additionally, Brauneis and Mestel (2018) observed that increased liquidity makes cryptocurrencies less predictable. Our analysis of Bitcoin's high standard deviation in trading volumes corroborates this assertion by highlighting the remarkable variability and risk associated with Bitcoin, making it less predictable.

The dominance of the Gumbel copula model, characterized by its upper tail dependence, signifies that extreme positive movements in one cryptocurrency's trading volume are likely to coincide with similar movements in others. This finding is crucial for risk management strategies, emphasizing the need for robust risk measures to account for potential simultaneous surges in trading volumes across cryptocurrencies. Traders and risk managers should consider employing stress testing and scenario analysis to prepare for such events. The study also indicates that Ethereum offers an optimal risk-reward trade-off for portfolio investors, as noted in Boako et al. (2019)'s research. This implies that diversifying portfolios with cryptocurrencies exhibiting lower dependence measures (e.g., TRON and BNB) could effectively mitigate risk. Incorporating cryptocurrencies with lower tail dependencies, as identified using the Clayton copula, can further enhance diversification strategies. Moreover, the high volatility of Bitcoin, as evidenced by its standard deviation in trading volumes, underscores the importance of incorporating volatility forecasting into trading strategies. Traders may benefit from adopting volatility-adjusted position sizing to manage risk exposure, while conservative traders might prefer cryptocurrencies like BNB, which exhibit lower volatility, to reduce risk.

The study also reveals that conditioning sets in higher-order trees capture substantial portions of the dependence between nodes within vine structures. This insight can inform dynamic trading strategies, where traders adjust their positions based on the conditional relationships among cryptocurrencies. For instance, knowing that Bitcoin and Ethereum exhibit a 32% dependence when conditioned on TRON can guide trading decisions involving these cryptocurrencies.

Finally, the superior performance of the Gumbel copula model in fitting the data, as indicated by the lowest BIC values, underscores its utility for strategic decision-making. The results also suggest that a bivariate inflated joint model may achieve even better fitting performance under certain data conditions. Specifically, such models could outperform standard copulas when the dataset exhibits a high frequency of identical or clustered observations, typical of over-dispersed or zero-inflated financial data. This extension allows more flexibility in capturing mixed discrete, continuous behaviors or concentration in specific regions of the joint distribution. Future studies could explore whether incorporating an inflated joint copula structure enhances model accuracy for cryptocurrency trading data or other financial series with similar characteristics. Traders and analysts can utilize this model to gain a deeper understanding of and predict cryptocurrency trading volumes, thereby enabling more accurate trading signals and investment decisions.

Although this study primarily focused on traditional bivariate copula models within the vine structures, future research could examine whether a bivariate inflated joint model provides a better fit for certain types of data, especially those characterized by excess zeros, overdispersion, or structural breaks. Such a model could capture mixed dependence behaviors that standard copulas may not adequately represent.

5. Conclusion

This study uses vine copula models to analyze the dependence structures among selected cryptocurrency trading volumes. By employing advanced statistical methods, the study identifies the

intricacies of these dependencies, offering critical insights for cryptocurrency traders, investors, and policymakers.

The findings highlight the significant role of tail dependence in cryptocurrency trading volumes, particularly the effectiveness of the Gumbel copula model in capturing upper tail dependence. This implies that extreme positive movements in the trading volume of one cryptocurrency are likely to coincide with similar movements in others. Traders can utilize this knowledge to anticipate market movements and make informed decisions. For instance, the study indicates that Bitcoin exhibits the highest volatility, indicating a higher risk profile. Traders should consider adopting volatility-adjusted position sizing and robust risk management strategies, such as stress testing and scenario analysis, to manage exposure to sudden market surges. Furthermore, the study identifies Ethereum as offering an optimal risk-reward trade-off for portfolio investors, suggesting that diversification with lower dependence cryptocurrencies like TRON and Binance Coin (BNB) could mitigate risk.

The results have implications for regulatory policies, particularly in understanding the systemic risk within the cryptocurrency market. The strong dependencies observed among cryptocurrencies signify that market disturbances in one asset could rapidly spread to others, amplifying systemic risk. Policymakers should consider implementing regulations that promote transparency and stability in the cryptocurrency market. Additionally, the findings highlight the importance of monitoring the volatility of key cryptocurrencies, such as Bitcoin, which can serve as indicators of broader market risks. Developing frameworks to monitor and manage these risks could enhance market resilience.

Despite the valuable insights, the study has some limitations that should be acknowledged. First, the analysis is based on monthly trading volume data, which helps capture long-term dependencies among variables in long-term market dynamics. It does not consider daily or intraday data analysis for a short-term market system that could provide additional short-term insights. Second, the study is limited to a selection of five cryptocurrencies, which, while representative and valid, may not capture the full spectrum of dependencies present in the broader cryptocurrency market. Finally, some bivariate copula models failed to yield estimated association parameters for certain edges within the vine trees.

Future research could explore several avenues to build on the findings of this study. One area of interest could be applying different copula models or developing new models better suited to capturing the complexities of high-dimensional cryptocurrency data. Additionally, expanding the analysis to include a broader range of cryptocurrencies and incorporating higher frequency data, such as daily or intraday trading volumes, could provide a more detailed understanding of the short-term market system to complement this study's long-term market dynamics findings. Another promising direction would be to investigate the factors that lead to the failure of specific copula models to estimate parameters in some scenarios, potentially leading to improved model selection and fitting techniques.

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Conflict of Interest

There is no conflict of interest.

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