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Dynamic Vertical Pricing and Strategic Timing: Analytical and Numerical Evidence from the Thai Palm Oil Supply Chain

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Abstract

This paper develops a dynamic game-theoretic model of a vertically decentralized agricultural supply chain with endogenous timing and three-layer pricing. Focusing on a factory–wholesaler–retailer structure, we derive closed-form subgame perfect Nash equilibria and establish existence and uniqueness under explicit economic conditions. Under linear demand and constant marginal costs, decentralized sequential pricing yields an exact efficiency ratio: total channel profit equals $7/16$ of the integrated benchmark, implying a 56.25% structural profit loss. This loss admits a quadratic decomposition as a curvature-scaled squared retail-pricing wedge generated by uninternalized vertical pricing externalities. We further embed the pricing game in a continuous-time gradient adjustment system and establish global asymptotic stability using a Lyapunov argument. Spectral analysis links convergence speed to demand curvature, while computational sensitivity analysis illustrates robustness over economically admissible parameter regions. Calibration to Thai crude palm oil trade data (2015–2024) anchors demand at moderate inelasticity and indicates that the predicted distortion is quantitatively meaningful. Overall, the framework explains how vertically fragmented agricultural markets can be dynamically stable yet structurally inefficient.

Keywords: Vertical market power, strategic pricing, dynamic game, Nash equilibrium, channel-profit distortion, agricultural supply chains.

1. Introduction

1.1. Strategic fragmentation in decentralized agricultural markets

Vertical price formation in multi-layer supply chains has long been recognized as a central source of market distortion. Industrial organization theory shows that decentralized vertical pricing generates cumulative mark-ups and allocative inefficiency, commonly termed double marginalization Spengler (1950). Subsequent analyses clarify how hierarchical pricing interacts with demand curvature and marginal cost structure Tirole (1988).

Supply chain research extends these insights to multi-echelon systems in which manufacturers, intermediaries, and retailers set prices strategically. Coordination mechanisms such as contracts and revenue-sharing schemes aim to mitigate vertical distortions Cachon (2003); Tsay et al. (1999), while studies of channel design and dual-channel competition emphasize asymmetric market power Chiang

et al. (2003); Boyaci and Gallego (2004). Recent work further incorporates fairness concerns, environmental competition, and risk preferences in three-layer structures Li and Li (2016); Chen et al. (2017); Xiao and Yang (2008).

Agricultural commodity markets, however, exhibit structural features that amplify vertical fragmentation. Staple goods such as palm oil typically face relatively inelastic demand, and pricing authority is dispersed across mills, wholesalers, and retailers. Even moderate mark-ups can therefore generate substantial channel-profit losses.

In the Thai palm oil supply chain, sequential pricing, transportation frictions, and institutional constraints create a layered decision hierarchy. Retail prices emerge from strategic optimization at each tier rather than simple cost pass-through, leading to cumulative mark-up amplification consistent with classical vertical models in a contemporary agricultural setting.

Moreover, vertical fragmentation is not purely static. Recent literature highlights endogenous timing, channel power, and dynamic adjustment in supply chain competition Ma et al. (2020); Han et al. (2024). While these studies examine strategic commitment and pricing dynamics, stability properties are often analyzed locally or numerically, without fully integrating hierarchical pricing and timing structure.

A unified framework linking pricing hierarchy, endogenous commitment, and demand curvature is therefore necessary to evaluate both efficiency and dynamic stability in decentralized agricultural markets.

1.2. Research gap

The literature on vertically decentralized supply chains has evolved along several related but largely separate strands.

First, industrial organization theory rigorously characterizes vertical pricing distortion under decentralization Spengler (1950); Tirole (1988), while supply chain research examines coordination contracts and channel design Cachon (2003); Tsay et al. (1999). However, endogenous timing is typically abstracted from, or pricing and commitment are treated as analytically distinct problems.

Second, dynamic pricing games and channel power models analyze strategic interaction across echelons Chiang et al. (2003); Ma et al. (2020). Yet closed-form results are often confined to simplified structures, and exact channel-profit decompositions are rarely derived in transparent quadratic form.

Third, dynamic adjustment and stability analyses frequently rely on local linearization or simulation-based convergence Xiao and Yang (2008). Formal global asymptotic stability proofs under economically interpretable curvature conditions remain limited, especially in multi-layer agricultural settings.

Fourth, empirical grounding is often absent. Although agricultural supply chains have been studied under cooperative or policy-intervention frameworks Han et al. (2024), few contributions integrate analytical equilibrium derivation, dynamic stability analysis, and empirical calibration within a single unified model.

Overall, the literature lacks a comprehensive analyticalcomputationalempirical framework that simultaneously integrates (i) hierarchical multi-layer pricing, (ii) endogenous entry timing, (iii) exact channel-profit decomposition, and (iv) global dynamic stability in a structurally grounded agricultural market context.

1.3. Research questions, contributions, and organization

This paper addresses three interrelated questions. First, how do hierarchical pricing and endogenous entry timing determine closed-form equilibrium outcomes in vertically decentralized agricultural supply chains? Second, what is the exact magnitude of the efficiency distortion generated by vertical fragmentation, and how does it depend on demand curvature and aggregate marginal cost? Third, is the decentralized equilibrium dynamically stable, and does the distortion remain economically significant under empirical calibration?

The paper makes four contributions. First, it develops a unified analytical framework combining sequential vertical pricing, endogenous timing, and subgame-perfect equilibrium in a three-layer supply chain. Second, it derives an exact channel-profit decomposition: under linear demand, decentralized channel profit equals $7/16$ of the integrated benchmark, and the resulting gap is a curvature-scaled squared pricing wedge. Third, it embeds the pricing game in a continuous-time gradient adjustment system, establishes Lyapunov stability, and links convergence speed to demand curvature through spectral analysis. Fourth, it anchors the model to Thai palm oil market data, showing that the predicted mark-up amplification, efficiency loss, and recoverable coordination surplus are quantitatively meaningful.

The remainder of the paper is organized as follows. Section 2 introduces the vertical pricing model and equilibrium concept. Section 3 derives closed-form equilibria and comparative statics under sequential pricing. Section 4 analyzes endogenous timing and the subgame-perfect structure. Section 5 develops the gradient-based dynamics and stability analysis. Section 6 provides computational verification and sensitivity analysis. Section 7 derives the channel-profit decomposition and efficiency wedge. Section 8 presents empirical calibration and structural interpretation based on Thai palm oil data. Section 9 concludes.

2. Model Framework

2.1. Vertical structure and strategic interaction

We consider a vertically decentralized agricultural supply chain with three layers: an upstream processor or factory F, an intermediate wholesaler W, and a downstream retailer R. Pricing authority is fragmented across layers, so each firm sets a mark-up conditional on inherited upstream prices and its own marginal cost.

The game has two stages. In the timing stage (Section 4), firms choose commitment timing. Conditional on timing, the pricing subgame is played along the sequential Stackelberg hierarchy

$$F \to W \to R,$$

capturing decentralized vertical mark-up formation.

2.2. Demand system

Final demand is linear in the retail price:

$$Q = a - bp_R, \qquad a > 0, \quad b > 0, \tag{1}$$

where p_R is the final retail price, a measures market size, and b is the demand curvature parameter. This specification ensures tractability, strict concavity of the pricing problems, closed-form equilibrium solutions, and the quadratic channel-profit decomposition derived in Section 7.

Section 8 calibrates (a, b) using Thai palm oil market data.

2.3. Cost structure

Each layer incurs constant marginal cost:

$$c_F > 0, \qquad c_W > 0, \qquad c_R > 0.$$

Aggregate marginal cost is

$$C := c_F + c_W + c_R.$$

We impose

$$a > bC$$

to ensure strictly positive equilibrium quantity under both decentralized and integrated benchmarks.

2.4. Profit functions

Let p_F, p_W, and p_R denote factory, wholesale, and retail prices. Since quantity depends only on the final retail price,

$$Q = a - bp_R.$$

Layer profits are

$$\Pi_F = (p_F - c_F)Q, \qquad \Pi_W = (p_W - p_F - c_W)Q, \qquad \Pi_R = (p_R - p_W - c_R)Q.$$

The pricing subgame is solved sequentially in Section 3, while alternative timing regimes are introduced in Section 4.

2.5. Equilibrium concept

The model consists of a timing stage followed by an induced pricing subgame. We therefore use subgame perfect Nash equilibrium (SPNE), which requires optimal strategies in every continuation subgame (Selten, 1965; Fudenberg and Tirole, 1991).

Definition 1 (Strategic Equilibrium) A *strategic equilibrium* is an SPNE of the timing game together with the Nash equilibrium of the induced pricing subgame.

Under $a > bC$ and $b > 0$, the induced pricing subgame has a unique equilibrium for each admissible timing profile. Hence equilibrium prices and quantities are well defined conditional on timing.

This equilibrium concept is consistent with Section 5, where the static pricing equilibrium is embedded as a stationary point of the gradient adjustment dynamics.

3. Analytical Nash Equilibrium

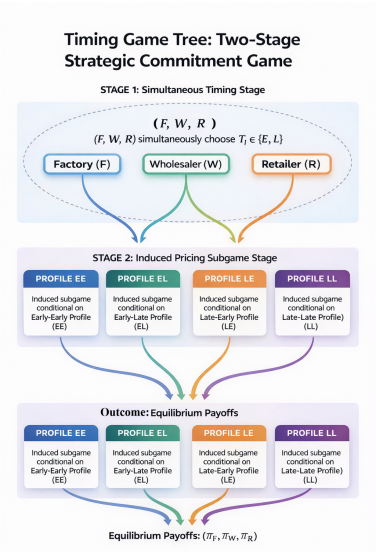


Figure 1 Two-stage timing–pricing game tree. In Stage 1, firms choose commitment timing $T_i \in \{E, L\}$. In Stage 2, each timing profile induces a corresponding pricing subgame. The terminal nodes report the resulting equilibrium payoffs (Π_F, Π_W, Π_R). The overall solution concept is subgame perfect Nash equilibrium (SPNE).

3.1. Baseline sequential pricing subgame

We analyze the decentralized pricing subgame under the sequential Stackelberg hierarchy

$$F \to W \to R.$$

The retailer chooses p_R after observing p_W, the wholesaler chooses p_W after observing p_F, and the factory chooses p_F anticipating all downstream responses. The equilibrium is obtained by backward induction.

Stage 3: Retailer.

Given p_W, the retailer solves

$$\max_{p_R} (p_R - p_W - c_R)(a - bp_R).$$

The first-order condition gives

$$p_R(p_W) = \frac{a + b(p_W + c_R)}{2b}. \tag{2}$$

Substituting into demand yields

$$Q = \frac{a - b(p_W + c_R)}{2}. \tag{3}$$

Stage 2: Wholesaler.

Anticipating (2), the wholesaler solves

$$\max_{p_W} (p_W - p_F - c_W)\frac{a - b(p_W + c_R)}{2}.$$

The first-order condition yields

$$p_W(p_F) = \frac{a + b(p_F + c_W - c_R)}{2b}. \tag{4}$$

Hence

$$Q = \frac{a - b(p_F + c_W + c_R)}{4}. \tag{5}$$

Stage 1: Factory.

The factory solves

$$\max_{p_F} (p_F - c_F)\frac{a - b(p_F + c_W + c_R)}{4}.$$

The first-order condition gives

$$p_F^{D*} = \frac{a + b(c_F - c_W - c_R)}{2b}. \tag{6}$$

Closed-form equilibrium. Let

$$C := c_F + c_W + c_R.$$

Substituting sequentially gives

$$p_W^{D*} = \frac{3a + b(c_F + c_W - 3c_R)}{4b}, \tag{7}$$

$$p_R^{D*} = \frac{7a + bC}{8b}, \tag{8}$$

$$Q^{D*} = \frac{a - bC}{8}. \tag{9}$$

Proposition 1 (Existence and Uniqueness) *If $b > 0$ and $a > bC$, the sequential pricing subgame admits a unique subgame-perfect equilibrium with $Q^{D*} > 0$.*

Proof: Each stage objective is strictly concave in its own price. The retailer's second derivative is $-2b < 0$. After substituting downstream best responses, the wholesaler's and factory's reduced-form objectives are also strictly concave in p_W and p_F, respectively. Therefore, backward induction yields a unique equilibrium. Since $a > bC$,

$$Q^{D*} = \frac{a - bC}{8} > 0.$$

Proposition 2 (Hierarchical Mark-up) *If $a > bC$ and $b > 0$, then*

$$p_R^{D*} > p_W^{D*} > p_F^{D*}.$$

Proof: Using the closed-form equilibrium prices,

$$p_R^{D*} - p_W^{D*} = c_R + \frac{a - bC}{8b} > 0,$$

and

$$p_W^{D*} - p_F^{D*} = c_W + \frac{a - bC}{4b} > 0.$$

Since $c_W, c_R > 0$, $b > 0$, and $a > bC$, the ordering follows.

3.2. Comparative statics and double marginalization

The closed-form equilibrium yields transparent comparative statics. All equilibrium prices increase with market size a:

$$\frac{\partial p_F^{D*}}{\partial a} = \frac{1}{2b} > 0, \qquad \frac{\partial p_W^{D*}}{\partial a} = \frac{3}{4b} > 0, \qquad \frac{\partial p_R^{D*}}{\partial a} = \frac{7}{8b} > 0.$$

Cost pass-through is asymmetric across upstream prices but symmetric at the retail level. Since

$$p_R^{D*} = \frac{7a}{8b} + \frac{C}{8}, \qquad C = c_F + c_W + c_R,$$

the retail price depends on costs only through aggregate marginal cost:

$$\frac{\partial p_R^{D*}}{\partial c_F} = \frac{\partial p_R^{D*}}{\partial c_W} = \frac{\partial p_R^{D*}}{\partial c_R} = \frac{1}{8} > 0.$$

For upstream prices,

$$\frac{\partial p_F^{D*}}{\partial c_F} = \frac{1}{2}, \qquad \frac{\partial p_F^{D*}}{\partial c_W} = \frac{\partial p_F^{D*}}{\partial c_R} = -\frac{1}{2},$$

and

$$\frac{\partial p_W^{D*}}{\partial c_F} = \frac{1}{4}, \qquad \frac{\partial p_W^{D*}}{\partial c_W} = \frac{1}{4}, \qquad \frac{\partial p_W^{D*}}{\partial c_R} = -\frac{3}{4}.$$

Thus downstream cost shifts affect upstream incentives asymmetrically, whereas the retail price internalizes costs only through C.

Centralized benchmark and distortion wedge.

Under vertical integration, the channel chooses p_R to maximize

$$\Pi^C(p_R) = (p_R - C)(a - bp_R),$$

which gives

$$p_R^{C*} = \frac{a + bC}{2b}.$$

Relative to decentralization,

$$p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b} > 0 \qquad \text{whenever } a > bC.$$

Hence decentralized sequential pricing raises the retail price above the coordinated benchmark and reduces quantity, generating the primitive source of double marginalization quantified in Section 7.

3.3. Canonical structure and notation consistency

For consistency, Table 1 summarizes the core notation used throughout the paper.

Table 1 Core Notation and Structural Definitions

Symbol	Definition
a	Demand intercept, $a > 0$
b	Demand curvature, $b > 0$
Q	Quantity, $Q = a - bp_R$
p_F, p_W, p_R	Factory, wholesale, and retail prices
c_F, c_W, c_R	Layer-specific marginal costs
C	Aggregate marginal cost, $C = c_F + c_W + c_R$
Π_i	Profit of firm $i \in \{F, W, R\}$
Π^{C*}	Integrated channel profit
Π^{D*}	Decentralized channel profit

Canonical structure.

Demand is $Q = a - bp_R$, with $a > 0$, $b > 0$, and $C = c_F + c_W + c_R$. All equilibrium expressions follow from this specification and the admissibility condition $a > bC$.

Integrated benchmark.

Under vertical integration,

$$p_R^{C*} = \frac{a + bC}{2b}, \qquad Q^{C*} = \frac{a - bC}{2}, \qquad \Pi^{C*} = \frac{(a - bC)^2}{4b}.$$

Decentralized equilibrium.

Under sequential pricing,

$$p_R^{D*} = \frac{7a + bC}{8b}, \qquad Q^{D*} = \frac{a - bC}{8}, \qquad \Pi^{D*} = \frac{7}{16}\Pi^{C*}.$$

4. Entry Timing Game

4.1. Timing structure and strategic anticipation

Beyond price setting, vertically decentralized supply chains may also involve endogenous timing choices. Firms may commit early or delay their pricing decisions in order to respond to other layers.

Timing is modeled as a preliminary stage before the pricing subgame. Each layer chooses

$$T_i \in \{E, L\},$$

where E denotes early commitment and L denotes delayed response. Given a timing profile, an induced pricing subgame is played, and continuation profits are determined by the corresponding equilibrium.

The solution concept is subgame perfect Nash equilibrium (SPNE). Timing does not alter the existence or uniqueness of the pricing equilibrium under the maintained conditions; it affects only the distribution of strategic advantage and continuation payoffs.

4.2. Timing Equilibrium Characterization

Let Π_i^E, Π_i^L, and Π_i^S denote equilibrium profits under early commitment, delayed response, and simultaneous pricing, respectively. Timing advantage is determined by comparing continuation payoffs from the induced pricing subgame.

To relate timing incentives to demand curvature, we use the local point elasticity of demand. Under $Q = a - bp_R$, elasticity evaluated at a continuation equilibrium price p_R^* is

$$\varepsilon = -\frac{bp_R^*}{a - bp_R^*}.$$

This measures local demand responsiveness around the relevant equilibrium allocation.

The upstream player has a first-mover advantage whenever

$$\Pi_F^E - \Pi_F^L > 0.$$

The sign of this payoff differential depends on the continuation equilibrium and therefore on the structural parameters (a, b, C).

Proposition 3 (Timing Advantage Threshold) *Suppose the continuation payoff differential $\Pi_F^E - \Pi_F^L$ is monotone in the local elasticity index over the admissible parameter region. Then there exists a threshold $\bar{\varepsilon} < 0$ such that upstream first-mover advantage arises whenever*

$$|\varepsilon| < |\bar{\varepsilon}|.$$

Proof: The payoff differential $\Pi_F^E - \Pi_F^L$ is continuous in the structural parameters because continuation equilibrium prices and quantities are smooth functions of (a, b, C) over the admissible region $a > bC$. Under the maintained monotonicity condition, the differential crosses zero at most once in elasticity space. Hence a threshold $\bar{\varepsilon}$ separates parameter regions with and without upstream first-mover advantage.

The threshold condition shows that timing incentives are governed by demand curvature. When demand is relatively inelastic, early commitment can increase upstream continuation payoff by limiting downstream mark-up amplification. When demand is more elastic, early pricing compresses margins and weakens first-mover gains.

Under the empirical anchoring in Section 8 ($\varepsilon_{\text{cal}} \approx -0.30$), the Thai palm oil market lies in a low-to-intermediate elasticity regime, consistent with positive timing incentives while preserving the stability of the pricing equilibrium.

5. Dynamic Adjustment and Lyapunov Stability

5.1. Gradient-based price adjustment dynamics

To study dynamic stability, we embed the decentralized pricing equilibrium in a continuous-time reduced-form adjustment system. Each layer adjusts its price in the direction of its marginal pricing incentive:

$$\dot{p}_i = \gamma_i F_i(p), \qquad \gamma_i > 0, \qquad i \in \{F, W, R\}, \tag{10}$$

where $F_i(p)$ denotes the reduced-form marginal pricing incentive of layer i. Let

$$F(p) = \begin{pmatrix} F_F(p) \\ F_W(p) \\ F_R(p) \end{pmatrix}, \qquad \Gamma = \operatorname{diag}(\gamma_F, \gamma_W, \gamma_R).$$

The adjustment system is

$$\dot{p} = \Gamma F(p). \tag{11}$$

The mapping $F(p)$ is interpreted as a reduced-form pseudo-gradient associated with the sequential pricing problem. It preserves the same stationary point as the closed-form decentralized equilibrium derived in Section 3. Under $a > bC$,

$$p^{D*} = \left(p_F^{D*}, p_W^{D*}, p_R^{D*}\right)$$

satisfies

$$F(p^{D*}) = 0. \tag{12}$$

Because demand is linear, the reduced-form pseudo-gradient is affine in prices. Hence

$$F(p) = H\left(p - p^{D*}\right), \tag{13}$$

where, in the baseline reduced-form specification,

$$H = \operatorname{diag}\left(-\frac{b}{2}, -b, -2b\right). \tag{14}$$

The Jacobian of the adjustment system is therefore

$$J = \Gamma H = \operatorname{diag}\left(-\frac{\gamma_F b}{2}, -\gamma_W b, -2\gamma_R b\right). \tag{15}$$

Since $b > 0$ and $\gamma_i > 0$, all eigenvalues of J are strictly negative:

$$\lambda_F = -\frac{\gamma_F b}{2}, \qquad \lambda_W = -\gamma_W b, \qquad \lambda_R = -2\gamma_R b.$$

Thus p^{D*} is locally asymptotically stable.

Theorem 1 (Local Asymptotic Stability) *If $b > 0$ and $\gamma_i > 0$ for all $i \in \{F, W, R\}$, then the decentralized equilibrium p^{D*} is locally asymptotically stable under the reduced-form pseudo-gradient adjustment dynamics.*

Proof: The Jacobian $J = \Gamma H$ is diagonal with strictly negative eigenvalues whenever $b > 0$ and $\gamma_i > 0$. Hence all trajectories starting sufficiently close to p^{D*} converge locally to the equilibrium.

5.2. Global stability under reduced-form gradient dynamics

Let

$$x = p - p^{D*}.$$

Then the reduced-form adjustment system becomes

$$\dot{x} = \Gamma H x. \tag{16}$$

Consider the Lyapunov function

$$V(x) = \frac{1}{2} x^\top x. \tag{17}$$

Along trajectories,

$$\dot{V} = x^\top \Gamma H x \tag{18}$$

$$= -\frac{\gamma_F b}{2} x_F^2 - \gamma_W b x_W^2 - 2\gamma_R b x_R^2. \tag{19}$$

Since $b > 0$ and $\gamma_i > 0$, we have

$$\dot{V} < 0 \qquad \text{for all } x \neq 0.$$

Therefore, the equilibrium is globally asymptotically stable under the reduced-form linear adjustment system.

Theorem 2 (Global Asymptotic Stability) *If $b > 0$ and $\gamma_i > 0$ for all $i \in \{F, W, R\}$, then the decentralized equilibrium p^{D*} is globally asymptotically stable under the reduced-form pseudo-gradient adjustment dynamics.*

Proof: The function $V(x) = \frac{1}{2}x^\top x$ is positive definite and radially unbounded. Its derivative satisfies

$$\dot{V} = -\frac{\gamma_F b}{2}x_F^2 - \gamma_W b x_W^2 - 2\gamma_R b x_R^2 < 0$$

for all $x \neq 0$. By Lyapunov's direct method, p^{D*} is globally asymptotically stable.

Define the stability margin

$$\delta = \min\left\{\frac{\gamma_F b}{2}, \gamma_W b, 2\gamma_R b\right\}. \tag{20}$$

The solution satisfies

$$\|p(t) - p^{D*}\| \leq e^{-\delta t}\|p(0) - p^{D*}\|. \tag{21}$$

Thus demand curvature b governs convergence speed: larger b implies faster adjustment, while smaller b implies slower convergence. This motivates the spectral analysis in Section 6.

6. Computational Equilibrium Verification

This section complements the analytical stability results in Section 5 by verifying the reduced-form equilibrium mapping, spectral properties, and parameter sensitivity under admissible configurations satisfying $a > bC$, $b > 0$, and $C > 0$.

6.1. Equilibrium mapping and numerical fixed point

Let

$$F(p) = \begin{pmatrix} F_F(p) \\ F_W(p) \\ F_R(p) \end{pmatrix}, \qquad F(p^{D*}) = 0,$$

where p^{D*} is the decentralized equilibrium derived in Section 3. Under the reduced-form linear adjustment specification of Section 5,

$$F(p) = H(p - p^{D*}), \qquad H = \operatorname{diag}\left(-\frac{b}{2}, -b, -2b\right).$$

Consider the discrete fixed-point iteration

$$T(p) = p + \Gamma F(p), \qquad \Gamma = \operatorname{diag}(\gamma_F, \gamma_W, \gamma_R).$$

Then

$$T(p) = p + \Gamma H(p - p^{D*}), \qquad DT = I + \Gamma H.$$

Since

$$\Gamma H = \operatorname{diag}\left(-\frac{\gamma_F b}{2}, -\gamma_W b, -2\gamma_R b\right),$$

the eigenvalues of DT are

$$1 - \frac{\gamma_F b}{2}, \qquad 1 - \gamma_W b, \qquad 1 - 2\gamma_R b.$$

Thus T is contractive whenever

$$0 < \gamma_F < \frac{4}{b}, \qquad 0 < \gamma_W < \frac{2}{b}, \qquad 0 < \gamma_R < \frac{1}{b}.$$

Under these step-size restrictions, the fixed-point iteration converges globally to p^{D*} within the reduced-form linear mapping.

6.2. Jacobian matrix, spectral stability, and robustness

For the continuous-time adjustment system

$$\dot{p} = \Gamma F(p),$$

the Jacobian at p^{D*} is

$$J = \Gamma H = \operatorname{diag}\left(-\frac{\gamma_F b}{2}, -\gamma_W b, -2\gamma_R b\right).$$

Hence

$$\lambda_F = -\frac{\gamma_F b}{2}, \qquad \lambda_W = -\gamma_W b, \qquad \lambda_R = -2\gamma_R b.$$

Since $b > 0$ and $\gamma_i > 0$, all eigenvalues are strictly negative, so the decentralized equilibrium is locally asymptotically stable under the reduced-form continuous-time dynamics.

Define the spectral stability margin

$$\delta = \min\left\{\frac{\gamma_F b}{2}, \gamma_W b, 2\gamma_R b\right\}.$$

Then

$$\|p(t) - p^{D*}\| \le e^{-\delta t}\|p(0) - p^{D*}\|,$$

and the corresponding half-life is

$$t_{1/2} = \frac{\ln 2}{\delta}.$$

Robustness to Demand Curvature

Each eigenvalue is proportional to demand curvature:

$$\lambda_i = -\alpha_i b, \qquad \alpha_i > 0.$$

Therefore, changes in b rescale convergence speed without changing the sign of the eigenvalues or the qualitative stability ranking. Higher b accelerates convergence, while lower b slows adjustment.

6.3. Structural sensitivity analysis

We assess structural sensitivity over admissible parameter vectors

$$\theta = (a, b, C), \qquad a > bC.$$

For an equilibrium outcome $Y(\theta)$, a Sobol-type variance decomposition can be written as

$$\operatorname{Var}(Y) = \sum_i V_i + \sum_{i<j} V_{ij} + \cdots,$$

with normalized first-order indices

$$S_i = \frac{V_i}{\operatorname{Var}(Y)}.$$

Because the equilibrium is available in closed form, sensitivity can be evaluated directly over the admissible parameter region. Demand curvature b affects both the pricing wedge and the convergence rate, while market size a and aggregate marginal cost C primarily scale equilibrium levels through the surplus term $a - bC$. Interaction effects are interpreted through their contribution to the wedge

$$\Delta p = \frac{3(a - bC)}{8b}$$

and the coordination surplus

$$G^{\text{coord}} = \frac{9(a - bC)^2}{64b}.$$

6.4. Profit surface geometry and structural regimes

We evaluate equilibrium outcomes over admissible parameter configurations satisfying $a > bC$. Since the decentralized equilibrium has a unique closed-form solution in this region, the equilibrium mapping has no multiple branches.

The closed-form expressions imply smooth dependence of prices, quantities, and profits on the structural parameters. In particular, lower values of b increase the pricing wedge

$$\Delta p = \frac{3(a - bC)}{8b},$$

and amplify mark-up distortion. Higher values of b reduce the wedge and increase the spectral stability margin δ, thereby accelerating convergence.

Overall, the computational verification supports the analytical conclusion that demand curvature jointly governs equilibrium distortion and dynamic adjustment speed, while a and C determine the scale of attainable channel surplus.

7. System Efficiency and Structural Distortion

7.1. Integrated benchmark: channel profit maximization

To evaluate the efficiency loss from vertical decentralization, we first derive the vertically integrated benchmark, in which all layers internalize pricing externalities and jointly maximize aggregate channel profit.

Let

$$C := c_F + c_W + c_R, \qquad c_F, c_W, c_R > 0, \tag{22}$$

and let final demand be

$$Q(p_R) = a - bp_R, \qquad a > 0, \quad b > 0. \tag{23}$$

The integrated channel chooses p_R to maximize

$$\Pi^C(p_R) = (p_R - C)(a - bp_R), \tag{24}$$

under the feasibility condition $a > bC$.

Concavity and Existence

Lemma 1 (Strict Concavity) *The integrated profit function $\Pi^C(p_R)$ is strictly concave in p_R.*

Proof:

$$\frac{d\Pi^C}{dp_R} = a - 2bp_R + bC, \qquad \frac{d^2\Pi^C}{dp_R^2} = -2b < 0.$$

Since $b > 0$, strict concavity follows.

Integrated Optimum

Proposition 4 (Integrated Channel Optimum) *Under $a > bC$ and $b > 0$, the unique profit-maximizing retail price is*

$$p_R^{C*} = \frac{a + bC}{2b}. \tag{25}$$

The associated quantity and integrated channel profit are

$$Q^{C*} = \frac{a - bC}{2}, \qquad \Pi^{C*} = \frac{(a - bC)^2}{4b}. \tag{26}$$

Proof. The first-order condition $a - 2bp_R + bC = 0$ gives p_R^{C*}. Substitution into demand and profit gives Q^{C*} and Π^{C*}. Uniqueness follows from strict concavity. □

Interpretation

The integrated price p_R^{C*} equates marginal revenue with aggregate marginal cost C, thereby internalizing vertical pricing externalities. The benchmark maximizes aggregate channel profit, not social welfare, and therefore provides the appropriate reference for measuring decentralized pricing distortions.

7.2. Decentralized sequential pricing equilibrium

We now compare the integrated benchmark with decentralized sequential pricing under the Stackelberg hierarchy

$$F \to W \to R.$$

Each layer maximizes its own profit while taking inherited upstream prices as given.

Backward-Induction Equilibrium

The retailer's best response is

$$p_R(p_W) = \frac{a + b(p_W + c_R)}{2b}, \qquad Q(p_W) = \frac{a - b(p_W + c_R)}{2}. \tag{27}$$

Anticipating this response, the wholesaler's best response is

$$p_W(p_F) = \frac{a + b(p_F + c_W - c_R)}{2b}, \qquad Q(p_F) = \frac{a - b(p_F + c_W + c_R)}{4}. \tag{28}$$

The factory then chooses

$$\max_{p_F}(p_F - c_F)\frac{a - b(p_F + c_W + c_R)}{4},$$

which yields

$$p_F^{D*} = \frac{a + b(c_F - c_W - c_R)}{2b}. \tag{29}$$

Backward substitution gives

$$p_W^{D*} = \frac{3a + b(c_F + c_W - 3c_R)}{4b}, \tag{30}$$

$$p_R^{D*} = \frac{7a + bC}{8b}, \qquad Q^{D*} = \frac{a - bC}{8}. \tag{31}$$

Decentralized Channel Profit

The equilibrium layer profits are positive but unequal:

$$\Pi_F^* = \frac{(a - bC)^2}{16b}, \qquad \Pi_W^* = \frac{(a - bC)^2}{32b}, \qquad \Pi_R^* = \frac{(a - bC)^2}{64b}. \tag{32}$$

Hence

$$\Pi_F^* > \Pi_W^* > \Pi_R^* > 0. \tag{33}$$

Total decentralized channel profit is therefore

$$\Pi^{D*} = \Pi_F^* + \Pi_W^* + \Pi_R^* = \frac{7(a - bC)^2}{64b}. \tag{34}$$

Corollary 1 (Efficiency Ratio) *Under linear demand and three-layer sequential pricing, the decentralized channel-profit ratio relative to the integrated benchmark is*

$$\frac{\Pi^{D*}}{\Pi^{C*}} = \frac{\frac{7(a-bC)^2}{64b}}{\frac{(a-bC)^2}{4b}} = \frac{7}{16}. \tag{35}$$

Equivalently, vertical decentralization destroys

$$1 - \frac{7}{16} = \frac{9}{16} = 56.25\% \tag{36}$$

of attainable integrated channel profit.

Interpretation

Sequential mark-ups generate the retail-pricing wedge

$$p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b} > 0, \tag{37}$$

whenever $a > bC$. Thus decentralized sequential pricing raises the retail price, contracts quantity, and reduces total channel profit by 56.25% relative to integration. The loss is structural and arises from uninternalized vertical pricing externalities.

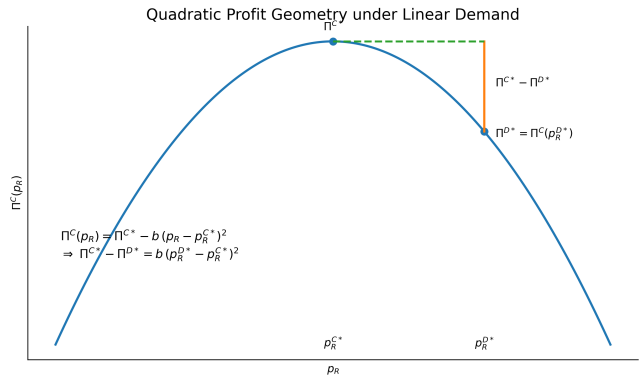


Figure 2 Quadratic geometry of the channel-profit gap

Under integration, $\Pi^C(p_R) = \Pi^{C*} - b(p_R - p_R^{C*})^2$. Evaluating this expression at p_R^{D*}, and using $\Pi^{D*} = \Pi^C(p_R^{D*})$, yields $\Pi^{C*} - \Pi^{D*} = b\left(p_R^{D*} - p_R^{C*}\right)^2$, showing that the entire efficiency loss is curvature-scaled and structurally determined.

7.3. Structural decomposition of the profit gap

We now identify the structural source of the decentralized inefficiency derived in Section 7.2. Under vertical integration, aggregate channel profit as a function of the retail price is

$$\Pi^C(p_R) = (p_R - C)(a - bp_R), \tag{38}$$

where $C = c_F + c_W + c_R$.

Quadratic Structure

Expanding (38) gives

$$\Pi^C(p_R) = -bp_R^2 + (a + bC)p_R - aC. \tag{39}$$

Since $b > 0$, $\Pi^C(p_R)$ is strictly concave in p_R. Hence it admits an exact quadratic representation around the integrated optimum.

Lemma 2 (Exact Quadratic Identity) *For all p_R,*

$$\Pi^C(p_R) = \Pi^{C*} - b\left(p_R - p_R^{C*}\right)^2, \tag{40}$$

where

$$p_R^{C*} = \frac{a + bC}{2b}, \qquad \Pi^{C*} = \frac{(a - bC)^2}{4b}.$$

Proof: From (39),

$$\Pi^C(p_R) = -b\left(p_R - \frac{a + bC}{2b}\right)^2 + \frac{(a + bC)^2}{4b} - aC.$$

Since

$$\frac{(a+bC)^2}{4b} - aC = \frac{(a-bC)^2}{4b} = \Pi^{C*},$$

the result follows.

Link to the Decentralized Equilibrium

Under linear demand and constant marginal costs, total decentralized channel profit satisfies

$$\Pi^{D*} = \Pi^{C}(p_R^{D*}), \tag{41}$$

because aggregate channel profit equals $(p_R - C)Q$ in both regimes. Evaluating (40) at $p_R = p_R^{D*}$ yields

$$\Pi^{C*} - \Pi^{D*} = b\big(p_R^{D*} - p_R^{C*}\big)^2. \tag{42}$$

Economic Interpretation

Equation (42) shows that the entire efficiency loss is generated by the squared retail-pricing wedge

$$\Delta p = p_R^{D*} - p_R^{C*} = \frac{3(a-bC)}{8b}.$$

Thus,

$$\Pi^{C*} - \Pi^{D*} = b(\Delta p)^2 = \frac{9(a-bC)^2}{64b}.$$

No technological inefficiency is present: marginal costs are identical across regimes. The loss arises solely from vertical pricing externalities that move the retail price away from the integrated optimum. This is consistent with the efficiency ratio in Corollary 1.

7.4. Empirical calibration and scale interpretation

This subsection maps the theoretical structure to observed Thai palm oil market data. The objective is not structural estimation, but to quantify the economic magnitude of the theoretical profit distortion under empirically plausible demand parameters.

Observed Market Normalization

Let $(\tilde{P}, \tilde{Q})$ denote the observed median price and quantity. We use this pair as an empirical anchor for the scale of the linear demand system, rather than imposing that it coincides exactly with the theoretical decentralized equilibrium (p_R^{D*}, Q^{D*}).

Demand Calibration

Under linear demand $Q = a - bP$, point elasticity at the anchoring point is

$$\varepsilon^* = \frac{dQ}{dP}\frac{\tilde{P}}{\tilde{Q}} = (-b)\frac{\tilde{P}}{\tilde{Q}} < 0.$$

Hence

$$b = -\varepsilon^* \frac{\tilde{Q}}{\tilde{P}} > 0, \qquad a = \tilde{Q} + b\tilde{P}.$$

This anchoring procedure calibrates the empirical scale of demand.

Admissible Aggregate Marginal Cost

Because $(\tilde{P}, \tilde{Q})$ is used only as a demand-scale anchor, the aggregate marginal cost C is not recovered from the condition $\tilde{P} = p_R^{D*}$. Instead, C is treated as a structural parameter satisfying

$$C > 0, \qquad a > bC.$$

These restrictions ensure positive marginal cost and strictly positive equilibrium quantity.

Quantifying the Profit Distortion

Given calibrated (a, b) and any admissible C, integrated and decentralized channel profits are

$$\Pi^{C*} = \frac{(a - bC)^2}{4b}, \qquad \Pi^{D*} = \frac{7(a - bC)^2}{64b}.$$

The efficiency ratio remains structural:

$$\frac{\Pi^{D*}}{\Pi^{C*}} = \frac{7}{16},$$

independent of calibration. Calibration affects only scale. The absolute profit loss is

$$\Pi^{C*} - \Pi^{D*} = \frac{9(a - bC)^2}{64b} = b\left(p_R^{D*} - p_R^{C*}\right)^2.$$

Hence the empirical calibration provides a scale interpretation of the pricing wedge and the associated coordination surplus.

Interpretive Scope

This exercise does not estimate structural primitives. It provides a scale interpretation of the pricing distortion derived in Section 7.3, conditional on linear demand, constant marginal costs, and the use of the observed price–quantity pair as an empirical demand anchor. Within these maintained assumptions, the calibration illustrates the economic significance of vertical pricing externalities without introducing additional econometric structure.

7.5. Profit distribution and channel conflict

We now examine how decentralization redistributes channel profit and generates vertical conflict within the channel-profit framework of Sections 7.1–7.4.

Profit allocation under decentralization

Total decentralized channel profit is

$$\Pi^{D*} = \frac{7(a - bC)^2}{64b}.$$

Individual equilibrium profits are positive but unequal:

$$\Pi_F^* = \frac{(a - bC)^2}{16b}, \qquad \Pi_W^* = \frac{(a - bC)^2}{32b}, \qquad \Pi_R^* = \frac{(a - bC)^2}{64b}.$$

Hence

$$\Pi_F^* > \Pi_W^* > \Pi_R^* > 0.$$

Thus each layer earns a strictly positive payoff, even though aggregate channel profit is below the integrated benchmark

$$\Pi^{C*} = \frac{(a - bC)^2}{4b}.$$

Sequential mark-ups are therefore privately sustainable despite aggregate inefficiency.

Incentive Misalignment

At the decentralized equilibrium, each firm satisfies its own first-order condition. However, the decentralized retail price p_R^{D*} does not maximize aggregate channel profit. By Lemma 2,

$$\Pi^{C*} - \Pi^{D*} = b\left(p_R^{D*} - p_R^{C*}\right)^2 > 0.$$

The misalignment arises because each layer sets its mark-up according to its own marginal revenue condition, while failing to internalize the full aggregate pricing externality. Under integration, this externality is internalized through the aggregate channel objective.

Equivalently, vertical decentralization retains only

$$\frac{\Pi^{D*}}{\Pi^{C*}} = \frac{7}{16}$$

of the integrated benchmark and destroys

$$1 - \frac{7}{16} = \frac{9}{16} = 56.25\%$$

of attainable integrated channel profit.

Coordination Surplus

Define the coordination surplus as

$$G^{\text{coord}} = \Pi^{C*} - \Pi^{D*} = \frac{9(a - bC)^2}{64b}.$$

Since

$$\Delta p = p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b},$$

the same surplus can be written as

$$G^{\text{coord}} = b(\Delta p)^2.$$

Because $a > bC$, we have $G^{\text{coord}} > 0$. Thus a strictly positive surplus can be recovered, in principle, through contractual coordination.

Moreover,

$$\Pi^{C*} > \Pi_F^* + \Pi_W^* + \Pi_R^* = \Pi^{D*},$$

so the integrated outcome generates sufficient aggregate profit to compensate all layers relative to the decentralized benchmark while restoring the coordinated retail price p_R^{C*}.

Economic Interpretation

Vertical decentralization creates a stable but inefficient equilibrium: mark-up amplification sustains private profitability, yet destroys $9/16$, or 56.25%, of attainable integrated channel profit. The magnitude of the recoverable surplus is governed by the surplus term $a - bC$ and demand curvature b, as summarized by

$$G^{\text{coord}} = \frac{9(a - bC)^2}{64b} = b(\Delta p)^2.$$

7.6. Policy counterfactual

The pricing wedge identified in Section 7.3 raises a natural question: can institutional or contractual design restore the integrated benchmark without changing demand or cost primitives? All counterfactuals are evaluated within the channel-profit framework of Sections 7.1–7.5.

Vertical Integration (Benchmark)

Under full integration, the retail price is p_R^{C*} and aggregate channel profit attains

$$\Pi^{C*} = \frac{(a - bC)^2}{4b}.$$

The recoverable coordination surplus is

$$G^{\text{coord}} = \Pi^{C*} - \Pi^{D*} = \frac{9(a - bC)^2}{64b} > 0$$

whenever $a > bC$. Integration therefore provides the first-best channel-profit benchmark.

Wholesale Contract Adjustment

Suppose a contractually chosen wholesale price $\bar{p}_W$ induces the retailer to solve

$$\max_{p_R}(p_R - \bar{p}_W - c_R)(a - bp_R).$$

The retailer's best response is

$$p_R(\bar{p}_W) = \frac{a + b(\bar{p}_W + c_R)}{2b}.$$

To replicate the integrated allocation, choose $\bar{p}_W$ such that

$$p_R(\bar{p}_W) = p_R^{C*}.$$

Since

$$p_R^{C*} = \frac{a + bC}{2b},$$

the required wholesale price satisfies

$$\bar{p}_W + c_R = C, \qquad \text{or equivalently} \qquad \bar{p}_W = C - c_R = c_F + c_W.$$

Thus, a wholesale contract that aligns the retailer's effective marginal cost with aggregate channel cost restores the integrated retail price. In practice, this can be implemented through two-part tariffs, revenue sharing, or related coordination contracts.

Per-Unit Transfer Mechanism

Alternatively, introduce a per-unit transfer τ that changes the effective aggregate marginal cost from C to $C - \tau$. The decentralized retail equilibrium becomes

$$p_R^{D*}(\tau) = \frac{7a + b(C - \tau)}{8b}.$$

Setting

$$p_R^{D*}(\tau) = p_R^{C*}$$

eliminates the pricing wedge. Solving

$$\frac{7a + b(C - \tau)}{8b} = \frac{a + bC}{2b}$$

gives

$$\tau = \frac{3(a - bC)}{b}.$$

Hence the transfer mechanism restores the coordinated retail price by neutralizing the vertical pricing externality.

Elasticity and Policy Leverage

All mechanisms operate by reducing the wedge

$$\Delta p = p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b}.$$

The recoverable surplus is

$$G^{\text{coord}} = b(\Delta p)^2 = \frac{9(a - bC)^2}{64b}.$$

Thus the potential gain increases with the surplus term $a - bC$ and, holding a and C fixed, decreases with demand curvature b. Relatively inelastic markets therefore offer greater scope for coordination gains.

Institutional Scope and Implication

These counterfactuals assume observable marginal costs, enforceable contracts, and no asymmetric information. Implementation frictions may limit full coordination, so the results should be interpreted as theoretical upper bounds on recoverable channel profit.

The decentralized equilibrium is dynamically stable but inefficient from an aggregate channel-profit perspective. Under three-layer sequential pricing, vertical decentralization destroys

$$1 - \frac{7}{16} = \frac{9}{16} = 56.25\%$$

of attainable integrated channel profit. This surplus is recoverable, in principle, through coordination mechanisms that internalize vertical pricing externalities. Efficiency restoration therefore depends on incentive alignment rather than changes in demand or production costs.

8. Empirical Calibration

This section provides an empirical anchoring of the theoretical framework developed in Section 7. The objective is not structural estimation, but to calibrate the demand scale and examine the implications of the analytical model under empirically plausible demand parameters.

The Thai palm oil industry has a vertically layered structure, involving upstream mills, intermediate refiners and wholesalers, and downstream retailers. Sequential price transmission and margin accumulation are therefore natural features of this supply chain.

We use the linear demand specification

$$Q = a - bp_R, \qquad a > 0, \quad b > 0.$$

The observed median price–quantity pair $(\tilde{P}, \tilde{Q})$ is used as an empirical anchor for demand calibration. It is not imposed to coincide exactly with the theoretical decentralized equilibrium.

8.1. Equilibrium verification

Using the calibrated demand parameters (a, b), together with any admissible aggregate marginal cost C satisfying $a > bC$, we verify the equilibrium implications of the model under decentralized sequential pricing and the integrated benchmark.

Decentralized equilibrium

Under the three-layer Stackelberg structure of Section 7.2, the decentralized equilibrium is

$$p_F^{D*} = \frac{a + b(c_F - c_W - c_R)}{2b}, \qquad p_W^{D*} = \frac{3a + b(c_F + c_W - 3c_R)}{4b},$$

and

$$p_R^{D*} = \frac{7a + bC}{8b}, \qquad Q^{D*} = \frac{a - bC}{8}.$$

The upstream and wholesale prices require the layer-specific cost decomposition (c_F, c_W, c_R), whereas the retail price, quantity, and aggregate channel profit depend only on $C = c_F + c_W + c_R$.

Integrated benchmark

The integrated benchmark from Section 7.1 is

$$p_R^{C*} = \frac{a + bC}{2b}, \qquad Q^{C*} = \frac{a - bC}{2}.$$

Thus, for any admissible C,

$$p_R^{D*} > p_R^{C*}, \qquad Q^{D*} < Q^{C*}.$$

Profit verification

Channel profits satisfy

$$\Pi^{D*} = \frac{7(a - bC)^2}{64b}, \qquad \Pi^{C*} = \frac{(a - bC)^2}{4b}.$$

Hence

$$\frac{\Pi^{D*}}{\Pi^{C*}} = \frac{7}{16},$$

which is consistent with Corollary 1.

Numerical consistency check

The calibrated demand parameters provide a scale for evaluating the closed-form equilibrium expressions. For any admissible C and any specified cost decomposition (c_F, c_W, c_R), the best-response expressions in Section 7.2 recover the decentralized equilibrium

$$(p_F^{D*}, p_W^{D*}, p_R^{D*}).$$

Thus the numerical exercise is interpreted as a consistency check of the analytical equilibrium characterization, rather than as an independent proof of convergence.

8.2. Sensitivity and stability results

We examine economically admissible parameter configurations satisfying

$$a > bC, \qquad b > 0, \qquad C > 0.$$

The objective is to verify consistency with the closed-form equilibrium and to assess how demand curvature and aggregate marginal cost affect pricing distortion and dynamic convergence.

Local Stability Verification

The analytical stability result follows from the reduced-form adjustment system in Section 5. At the decentralized equilibrium, the Jacobian is

$$J = \Gamma H = \operatorname{diag}\left(-\frac{\gamma_F b}{2}, -\gamma_W b, -2\gamma_R b\right),$$

with eigenvalues

$$\lambda_F = -\frac{\gamma_F b}{2}, \qquad \lambda_W = -\gamma_W b, \qquad \lambda_R = -2\gamma_R b.$$

Since $b > 0$ and $\gamma_i > 0$, all eigenvalues are strictly negative. Hence the decentralized equilibrium is locally asymptotically stable under the reduced-form continuous-time adjustment dynamics.

The stability margin is

$$\delta = \min\left\{\frac{\gamma_F b}{2}, \gamma_W b, 2\gamma_R b\right\}.$$

Thus demand curvature b governs convergence speed, while the surplus term $a - bC$ governs the magnitude of the pricing wedge and the coordination surplus.

Comparative Statics

The pricing wedge is

$$\Delta p = p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b}.$$

Holding a and C fixed, differentiation gives

$$\frac{\partial \Delta p}{\partial b} = -\frac{3a}{8b^2} < 0, \qquad \frac{\partial \Delta p}{\partial C} = -\frac{3}{8}.$$

Thus flatter demand, represented by smaller b, amplifies the wedge, whereas higher aggregate marginal cost compresses it linearly. Although cost increases raise price levels in both regimes, they reduce the relative distortion.

Visualization in (b, C) Space

Figure 3 plots decentralized and integrated retail prices over the (b, C) parameter space. The relative distortion

$$\frac{p_R^{D*} - p_R^{C*}}{p_R^{C*}} \times 100$$

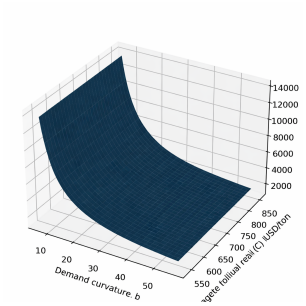
increases as demand curvature decreases and declines as aggregate marginal cost rises, consistent with the analytical expression for Δp.

Channel-Profit Interpretation

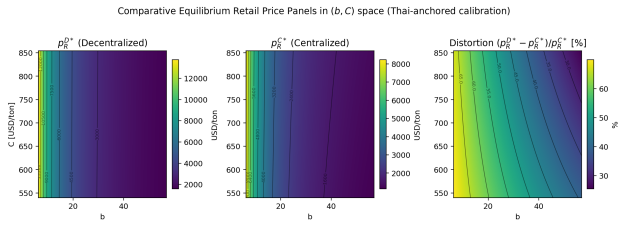
The recoverable coordination surplus is

$$G^{\text{coord}} = b(\Delta p)^2 = \frac{9(a - bC)^2}{64b}.$$

Parameter regions with larger pricing wedges therefore correspond to economically larger channel-profit losses.



(a) Decentralized retail price surface $p_R^{D*}(b, C)$.



(b) Comparative prices and relative distortion

Figure 3 Structural equilibrium landscape in (b, C) space. Lower demand curvature amplifies the pricing wedge $\Delta p = \frac{3(a-bC)}{8b}$, while higher marginal cost shifts price levels upward but compresses relative distortion.

Summary

The decentralized equilibrium is dynamically stable under the reduced-form adjustment dynamics, yet structurally distorted relative to the integrated benchmark. Stability therefore does not imply efficiency: the system converges to a mark-up-amplified configuration that destroys $9/16$, or 56.25%, of attainable integrated channel profit.

8.3. Structural regime interpretation

The calibrated parameter space allows structural regimes to be interpreted through demand curvature b and aggregate marginal cost C. The key objects are the pricing wedge

$$\Delta p = p_R^{D*} - p_R^{C*} = \frac{3(a - bC)}{8b}$$

and the recoverable coordination surplus

$$G^{\text{coord}} = b(\Delta p)^2 = \frac{9(a - bC)^2}{64b}.$$

Low-Elasticity Regime

When demand is relatively inelastic, corresponding to lower b, the pricing wedge is larger and mark-up amplification is stronger. The coordination surplus is therefore economically significant, indicating substantial channel-profit loss under decentralization.

Intermediate-Elasticity Regime

For intermediate values of b, the wedge narrows and the profit distortion declines. The reduced-form adjustment dynamics remain stable, with the spectral margin

$$\delta = \min\left\{\frac{\gamma_F b}{2}, \gamma_W b, 2\gamma_R b\right\}$$

varying smoothly with demand curvature.

High-Elasticity Regime

When demand is more elastic, corresponding to higher b, pricing margins are compressed in both regimes. Although Δp decreases, the decentralized retail price remains strictly above the integrated benchmark whenever $a > bC$. Thus distortion diminishes but does not vanish.

Continuity and Uniqueness

Transitions across regimes are continuous. Equilibrium multiplicity does not arise within the admissible region because the closed-form equilibrium is unique under $a > bC$ and $b > 0$. The equilibrium mapping therefore varies smoothly with the structural parameters.

Interpretive Implication

The calibration indicates that the main structural properties of the model—sequential mark-up amplification, the exact $9/16$ or 56.25% channel-profit loss, and reduced-form dynamic stability—are consistent with empirically plausible Thai palm oil parameters. These findings provide structural validation within the maintained linear-demand framework, rather than causal estimation.

8.4. Empirical elasticity and structural demand calibration

We anchor the structural demand system using Thai crude palm oil (CPO) export data covering 2015–2024. The data are obtained from the Agricultural Information Center, Office of Agricultural Economics (OAE), Ministry of Agriculture and Cooperatives, Thailand. Export price is proxied by `export_price_usd_per_ton`, and export quantity by `weight_ton`.

Table 2 Data Diagnostics and Descriptive Statistics (Thai CPO Export, 2015–2024)

Variable	Mean	SD	Min	P25	Median	P75	Max
Price P (USD/ton)	895.25	138.98	629.44	787.33	899.30	1010.90	1267.63
Quantity Q (ton)	75,064.59	7,791.87	60,000.00	67,820.25	75,789.00	81,886.75	89,426.00
Diagnostics: Missing$(P, Q) = 0$, Nonpositive$(P, Q) = 0$, IQR(1.5) outliers$(P, Q) = 0$, Final $N = 120$.							
Correlation: $\mathrm{corr}(P, Q) = 0.269$ (reduced-form co-movement).							

Table 3 Reduced-form Price–Quantity Co-movement (Not Structural Identification)

Model	Slope	Robust SE	t-stat	R^2
$Q = \alpha + \beta P$	$\beta = 15.0638$	4.7757	3.1543	0.0722
$\ln Q = \alpha + \eta \ln P$	$\eta = 0.1768$	0.0564	3.1362	0.0710
Robust standard errors: HC1. Positive reduced-form slope reflects joint supply–demand shocks.				

The cleaned sample contains $N = 120$ monthly observations, with no missing, nonpositive, or IQR(1.5) outlier observations. The positive reduced-form price–quantity slope reflects equilibrium co-movement in international trade data and is not interpreted as a structural demand slope. Price and quantity are jointly determined by global supply and demand shocks.

We therefore impose a structural calibration consistent with the theoretical model. Assume linear demand

$$Q = a - bP, \qquad b > 0.$$

Point elasticity at an anchoring point (P, Q) is

$$\varepsilon = \frac{dQ}{dP}\frac{P}{Q} = (-b)\frac{P}{Q}. \tag{43}$$

Let $(\tilde{P}, \tilde{Q})$ denote the sample medians. Imposing the inelastic benchmark $\varepsilon^* = -0.30$ gives

$$\varepsilon^* = (-b)\frac{\tilde{P}}{\tilde{Q}}, \qquad b = -\varepsilon^*\frac{\tilde{Q}}{\tilde{P}}. \tag{44}$$

The intercept follows from the anchoring condition:

$$a = \tilde{Q} + b\tilde{P}. \tag{45}$$

Table 4 reports the resulting demand calibration.

Table 4 Empirical Anchoring and Structural Demand Calibration

Statistic	Value	Interpretation
Sample size N	$\mathbf{120}$	Monthly observations
Median price $\tilde{P}$	$\mathbf{899.2950}$	USD/ton
Median quantity $\tilde{Q}$	$\mathbf{75{,}789.00}$	ton
Target elasticity ε^*	$\mathbf{-0.30}$	Inelastic benchmark
Calibrated curvature b	$\mathbf{25.2828}$	Demand slope parameter
Calibrated intercept a	$\mathbf{98{,}525.70}$	Market size parameter

The calibrated line is an empirical demand anchor, not a reduced-form regression fit. It preserves the observed price–quantity scale while imposing economically meaningful downward-sloping demand curvature.

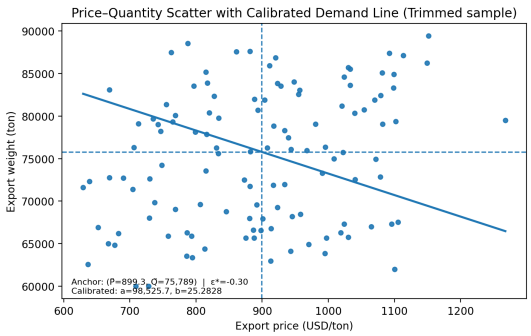


Figure 4 Price–quantity scatter and calibrated structural demand. Dots represent monthly CPO export observations. The solid line depicts $Q = a - bP$, calibrated at the sample medians under the elasticity target $\varepsilon^* = -0.30$. Dashed lines mark $(\tilde{P}, \tilde{Q})$.

Overall, the calibration strategy balances empirical grounding with theoretical discipline. The resulting demand system is internally consistent, economically interpretable, and provides a quantitative anchor for subsequent equilibrium and stability analysis.

8.5. Elasticity implication and dynamic stability

Given the calibrated demand system $Q = a - bP$, local elasticity at the anchoring point satisfies $\varepsilon^* = -0.30$ by construction. This represents a moderately inelastic calibration, consistent with staple commodity markets.

To assess dynamic robustness, consider the continuous-time reduced-form pseudo-gradient adjustment system

$$\dot{p}_i = \gamma_i F_i(p), \qquad \gamma_i > 0.$$

Let

$$J = \Gamma H$$

denote the Jacobian at the decentralized equilibrium, where

$$\Gamma = \mathrm{diag}(\gamma_F, \gamma_W, \gamma_R)$$

and H is the reduced-form pseudo-gradient Jacobian under linear demand. Local stability requires

$$\mathrm{Re}(\lambda_i(J)) < 0 \qquad \text{for all } i.$$

Under the normalization

$$\gamma_F = \gamma_W = \gamma_R = 1,$$

the eigenvalues are

$$\lambda_F = -\frac{b}{2}, \qquad \lambda_W = -b, \qquad \lambda_R = -2b.$$

Hence the local spectrum is governed by demand curvature b. Since $b = 25.2828$ under the baseline calibration, we obtain

$$\lambda_F = -12.6414, \qquad \lambda_W = -25.2828, \qquad \lambda_R = -50.5656.$$

Table 5 Local Stability under Reduced-Form Gradient Adjustment

Layer	Eigenvalue λ_i	Magnitude	Stability
Factory (F)	$\mathbf{-12.6414}$	Low	Stable
Wholesaler (W)	$\mathbf{-25.2828}$	Medium	Stable
Retailer (R)	$\mathbf{-50.5656}$	High	Stable

All eigenvalues are strictly negative, confirming local asymptotic stability of the reduced-form continuous-time adjustment system.

Robustness to Elasticity Targets

Varying the elasticity target ε^* rescales the demand curvature parameter b, and therefore proportionally rescales the Jacobian spectrum. Under $\gamma_F = \gamma_W = \gamma_R = 1$, the stability margin is

$$\delta = \min_i |\lambda_i| = \frac{b}{2},$$

and the corresponding half-life is

$$t_{1/2} = \frac{\ln 2}{\delta}.$$

The ordering

$$|\lambda_R| > |\lambda_W| > |\lambda_F|$$

implies faster downstream adjustment in the reduced-form dynamics. The calibrated system therefore preserves hierarchical mark-up amplification while remaining locally dynamically stable. Demand curvature b governs convergence speed, whereas the magnitude of distortion is governed jointly by b and the surplus term $a - bC$.

Table 6 Robustness to Elasticity Targets: Demand Curvature and Local Stability

ε^*	b	a	λ_F	λ_W	λ_R	δ	$\ln 2/\delta$
-0.20	16.855	90,946.8	-8.428	-16.855	-33.710	8.428	0.082
-0.30	25.283	98,525.7	-12.641	-25.283	-50.566	12.641	0.055
-0.50	42.138	113,683.5	-21.069	-42.138	-84.276	21.069	0.033

Normalization: $\gamma_F = \gamma_W = \gamma_R = 1$; $\lambda_F = -b/2$, $\lambda_W = -b$, and $\lambda_R = -2b$.

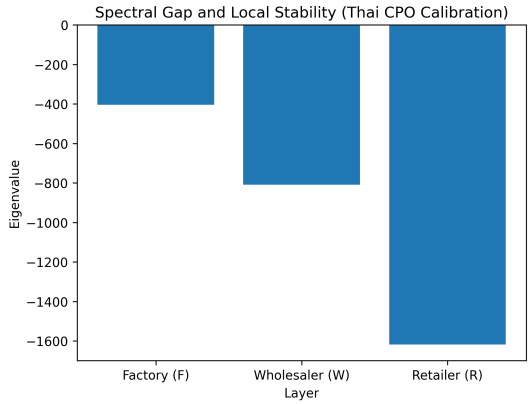


Figure 5 Spectral gap and convergence rate. Under the baseline calibration $b = 25.2828$ and $\gamma_F = \gamma_W = \gamma_R = 1$, the eigenvalues are $\lambda_F = -b/2$, $\lambda_W = -b$, and $\lambda_R = -2b$. The stability margin is $\delta = \min_i |\lambda_i| = 12.6414$, implying exponential convergence with envelope $e^{-\delta t}$.

9. Conclusion

9.1. Theoretical contributions

This paper provides a unified analytical characterization of vertical decentralization in a three-layer agricultural supply chain with endogenous timing and dynamic adjustment. Under linear demand and constant marginal costs, we derive closed-form subgame-perfect equilibria and establish existence and uniqueness under the conditions $b > 0$ and $a > bC$.

The central structural result is the exact efficiency ratio

$$\frac{\Pi^{D*}}{\Pi^{C*}} = \frac{7}{16},$$

which implies a $9/16$, or 56.25%, channel-profit loss relative to vertical integration. The profit gap admits the exact quadratic identity

$$\Pi^{C*} - \Pi^{D*} = b\left(p_R^{D*} - p_R^{C*}\right)^2,$$

showing that inefficiency is generated by a curvature-scaled retail-pricing wedge. No technological inefficiency is present; the loss arises entirely from uninternalized vertical pricing externalities.

A key implication is the coexistence of stability and inefficiency. The decentralized equilibrium is dynamically stable under the reduced-form adjustment system, yet it remains structurally inefficient from an aggregate channel-profit perspective. Thus dynamic robustness does not imply channel-profit efficiency.

9.2. Computational contributions

The analytical results are complemented by computational verification and empirical calibration. Reformulating the equilibrium conditions as a reduced-form operator system, we verify consistency

of the fixed-point mapping with the closed-form solution and establish stability of the continuous-time pseudo-gradient dynamics.

Spectral analysis shows that the Jacobian eigenvalues are

$$\lambda_F = -\frac{\gamma_F b}{2}, \qquad \lambda_W = -\gamma_W b, \qquad \lambda_R = -2\gamma_R b.$$

Hence all eigenvalues are strictly negative whenever $b > 0$ and $\gamma_i > 0$. The stability margin

$$\delta = \min\left\{\frac{\gamma_F b}{2}, \gamma_W b, 2\gamma_R b\right\}$$

governs exponential convergence and increases with demand curvature.

Calibration to Thai crude palm oil trade data for 2015–2024 anchors the linear demand system at a moderately inelastic benchmark ($\varepsilon^* \approx -0.30$). Under this empirical anchoring, the pricing wedge and recoverable coordination surplus are quantitatively meaningful, illustrating that the theoretical distortion can be economically significant in a vertically fragmented commodity market.

Sensitivity analysis further indicates that demand curvature b is a key driver of both pricing distortion and convergence speed, while market size a and aggregate marginal cost C determine the scale of attainable channel surplus through the term $a - bC$. Within the admissible region $a > bC$, the closed-form equilibrium remains unique and varies smoothly with the structural parameters.

9.3. Limitations

The analysis rests on three main structural assumptions.

First, the exact $7/16$ efficiency ratio is specific to linear demand and three-layer sequential pricing. Alternative demand specifications or different vertical structures may change both the distortion magnitude and its decomposition.

Second, the global stability result applies to the reduced-form pseudo-gradient adjustment system under constant marginal costs. Nonlinear costs, capacity constraints, or state-dependent adjustment rules may weaken concavity and generate richer dynamic behavior.

Third, the empirical exercise is an anchoring and calibration exercise, not structural estimation. The framework abstracts from stochastic shocks, inventory dynamics, asymmetric information, and policy interventions, all of which may affect short-run adjustment and contract feasibility in commodity markets.

These limitations define the theoretical domain within which uniqueness, stability, and proportional channel-profit loss are guaranteed.

9.4. Future extensions

Several extensions naturally follow.

One avenue is to examine whether a closed-form proportional efficiency benchmark persists under nonlinear or isoelastic demand, or whether the exact $7/16$ ratio is specific to the linear-demand, three-layer sequential-pricing structure.

A second extension introduces stochastic dynamic adjustment to study robustness under demand and cost volatility.

Third, explicit contract-theoretic modeling of coordination mechanisms, such as two-part tariffs, revenue sharing, or regulatory transfers, would formalize implementable surplus recovery.

Finally, extending the framework to multi-product or spatially distributed supply chains could illuminate cross-market price transmission and regional margin amplification in vertically structured commodity networks.

Such extensions would deepen the dynamic equilibrium perspective developed here and broaden its applicability across decentralized agricultural markets.

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