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Two-Stage Sampling Inspection Plan based on Mean for Lindley Distribution

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Abstract

This study develops Two-Stage Sampling (TSS) inspection plans for lifetime quality characteristics based on the mean, assuming non-normal distributions such as the exponential and the Lindley distribution. The lot mean (μ) is used as the quality measure. Unlike conventional approaches that focus solely on a fixed sample size, the proposed plan emphasizes efficiency by reducing inspection effort through a two-stage decision mechanism, resulting in time and cost savings. The optimal plan parameters are obtained using a non-linear optimization approach under specified acceptable and rejectable quality levels. A comparative analysis of sample size and Average Sample Number (ASN) between the TSS and Single Sampling (SS) plans is presented. An illustrative example is included to demonstrate the applicability of the proposed method. The results indicate that when the underlying data follow the Lindley distribution, the proposed TSS plan provides a more efficient decision-making framework with reduced average inspection requirements. Therefore, the proposed plan is recommended for use in such situations.

Keywords: Acceptance sampling plan, consumer's risk, non-normal distribution, operating characteristics (OC) curve, producer's risk, single sampling inspection plan.

1. Introduction

Quality monitoring is an essential direction for producers and consumers alike. Increasing the productivity of any industry is vital. Therefore, companies worldwide regularly seek to improve product quality using appropriate tools and techniques. An acceptance sampling plan (ASP) is a crucial technique for lot monitoring in the industry. It is of two types: (1) Attribute type and (2) Variable type. For the same level of protection, a variable-type plan uses a smaller sample size than an attribute-type plan to decide whether a lot is accepted or rejected. So, a variable-sampling inspection plan minimizes monitoring costs by enabling more convenient destructive testing of the material.

Several sampling plans are popular in the industry for monitoring the lot, like Quick switching sampling, Double sampling, Multiple dependent state sampling, Two stage sampling, Repetitive sampling, etc. Some recent works include Qomi and Aslam (2026), Qomi and Fernández (2024), Qomi and Tripathi (2025), Tripathi et al. (2024) among others.

Stephens and Dodge (1976) proposed a two-stage chain-based sampling plan based on the cumulative number of defectives to make a decision. Wang and Lu (1992) recommended a two-stage sampling plan for Bogey tests considering a cumulative number of defectives. Aslam et al. (2011) introduced a two-stage group-type sampling plan that considers a general distribution without accounting for the cumulative number of failures. Using the process loss function, Aslam et al. (2012) developed a two-stage acceptance sampling plan that includes specification limits, a process target, a process yield, and a relative expected loss index. Aslam et al. (2014) introduced a skip-lot sampling plan of type SkSP-2 by considering a two-stage group sampling plan under Burr type X and XII distributions. Rao et al. (2014) used the exponentiated Fréchet distribution for a two-stage group-type sampling plan without taking the cumulative number of failures. Ramaswamy and Sowmya (2016) also recommended a two-stage group-type sampling plan, taking a different distribution, Compound Rayleigh, and without taking the cumulative number of failures. Yan et al. (2016) suggested a variable two-stage sampling plan based on the coefficient of variation. Luca et al. (2020) analysed and compared single-stage and two-stage acceptance sampling plans to design a web-based tool. Aslam et al. (2021) demonstrated single-stage and two-stage group sampling plans based on total failure. They used time-truncated life tests, aided by neutrosophic statistics, to estimate the Weibull distribution.

But hardly any article discusses a two-stage variable sampling plan for Lindley-distributed quality characteristics based on mean life. The objective is to design an optimal sampling inspection plan that emphasizes efficiency by reducing inspection effort through a two-stage decision mechanism, resulting in time and cost savings.

In lifetime situations, the exponential distribution is commonly used in industry. Recently, the Lindley distribution has gained significant momentum due to its greater flexibility in modeling mean remaining life, failure rates, and related quantities. A few work relating to this distribution are mentioned as Kantar et al. (2018), Dey et al. (2019), Hassan et al. (2019), El-Morshedy et al. (2019), Altun and Cordeiro (2020), Al-Babtain et al. (2020), Hafez et al. (2020), Al-Babtain et al. (2021), Chesneau et al. (2020), Chesneau et al. (2021). In the work of the acceptance sampling plan, Tripathi et al. (2020), Biswas and Maiti (2022), Biswas et al. (2026), and Maiti and Biswas (2024) applied the Lindley distribution (or, One parameter plynomial exponential distribution) as a lifetime model.

In previous studies, most authors developed a two-stage sampling plan based on the number of defectives of the lot. Some authors take the cumulative number of defectives for the second sample. Some used the two-stage group-type sampling plan for time-truncated tests. Some authors incorporated chain-based or skip-lot sampling with a two-stage sampling plan. Distributions such as exponentiated Fréchet, Compound Rayleigh, Weibull, Burr type X, XII, etc., are used to model the plans. A few used a variable type two-stage sampling plan, but they considered either the coefficient of variation or the process expected loss function for construction. We have used the Lindley distribution in the context of a two-stage sampling plan, based on the mean life of quality characteristics, in the article. The focus on this distribution is justified by both practical and theoretical considerations. From a practical perspective, many real-life lifetime datasets exhibit non-constant hazard rates, for which the exponential distribution may be inadequate. The distribution offers greater flexibility in capturing such behavior, particularly when failure rates increase. From a methodological standpoint, it retains analytical tractability, which is essential for deriving optimal sampling plan parameters under a two-stage framework. Furthermore, several studies have reported its superior goodness-of-fit compared to classical lifetime distributions, making it a suitable and relevant choice for this investigation. It is an entirely different approach from all previous lifetime models; we compare the proposed plan with the single-sampling plan suggested by Mukherjee and Maiti (2014) and found it to be more efficient.

We have focused on the following sections of the article. Section 2 describes the two-stage sampling technique. A short description of the estimation of plan parameters is given in Section 3. We estimate the plan's parameters under both exponential and Lindley distributional assumptions. A brief comparison is made in section 4. Section 5 demonstrates the proposed sampling inspection technique using secondary real-life data. Section 6 ends with concluding remarks.

The remainder of the article is organized as follows. Section 2 describes the two-stage sampling

technique. Section 3 provides a brief account of the estimation procedure for the plan parameters under both exponential and Lindley distributional assumptions. A comparative study of the proposed plans is presented in Section 4. Section 5 illustrates the implementation of the proposed sampling inspection technique using secondary real-life data. Finally, Section 6 concludes the article with relevant remarks.

2. Two-Stage Sampling Inspection Plan

Suppose X is the lifetime quality measure under any distributional assumption, and the quality is measured by the mean (μ). Now, the Two-Stage sampling (TSS) inspection plan for the variable is described as follows:

Let us assume that the lifetime of the units follows a specified probability distribution with an unknown mean μ. We consider that the larger the mean (μ), the better the quality. Suppose μ_0 and μ_1 denote the acceptable quality level (AQL) and limiting quality level (LQL) concerning mean, respectively.

Step 1: Choose the values of μ_0 and μ_1 at producer's risk (α) and consumer's risk (β), respectively.

Step 2: Take a random sample of size n_1, say $(X_{11}, X_{12}, ..., X_{1n_1})$, from the lot. Suppose, X_{1i} be the lifetime of i^{th} unit, and compute the sample mean $\bar{X}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} X_{1i}$.

Step 3: Accept the lot if $\bar{X}_1 \geq c_1$, reject the lot if $\bar{X}_1 < c_2$; and, if $c_2 \leq \bar{X}_1 < c_1$ then, follow Step-4, where, c_1 = 1st acceptance number/critical value for two-stage sampling plan and c_2 = 2nd acceptance number/critical value for two-stage sampling plan.

Step 4: Draw the 2^{nd} random sample of size n_2, say $(X_{21}, X_{22}, ..., X_{2n_2})$ from the lot and compute the sample mean $\bar{X}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} X_{2i}$. Then, accept the lot if $\bar{X}_2 \geq c_2$, reject the lot if $\bar{X}_2 < c_2$.

The OC function of the proposed TSS plan is given by

$$\begin{aligned}
L(\mu) &= \text{Probability of accepting a lot with quality } \mu \\
&= P\left(\bar{X}_1 \geq c_1|\mu\right) + P\left(c_2 \leq \bar{X}_1 < c_1|\mu\right) P\left(\bar{X}_2 \geq c_2|\mu\right) \\
&= P\left(\bar{X}_1 \geq c_1|\mu\right) + \left[P\left(\bar{X}_1 \leq c_1|\mu\right) - P\left(\bar{X}_1 < c_2|\mu\right)\right] P\left(\bar{X}_2 \geq c_2|\mu\right) \\
&= P\left(\bar{X}_1 \geq c_1|\mu\right) + \left[1 - P\left(\bar{X}_1 \geq c_1|\mu\right) - 1 + P\left(\bar{X}_1 > c_2|\mu\right)\right] P\left(\bar{X}_2 \geq c_2|\mu\right) \\
&= P\left(\bar{X}_1 \geq c_1|\mu\right) + \left[P\left(\bar{X}_1 > c_2|\mu\right) - P\left(\bar{X}_1 \geq c_1|\mu\right)\right] P\left(\bar{X}_2 \geq c_2|\mu\right) \\
&= P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_1|\mu\right) + \left[P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_2|\mu\right)\right. \\
&\quad \left. -P\left(\sum_{i=1}^{n_1} X_{1i} > n_1 c_1|\mu\right)\right] P\left(\sum_{i=1}^{n_2} X_{2i} \geq n_2 c_2|\mu\right) \\
&= L_{11}(\mu) + [L_{12}(\mu) - L_{11}(\mu)]L_{22}(\mu), \qquad (1)
\end{aligned}$$

where, $L_{11}(\mu) = P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_1|\mu\right)$, $L_{12}(\mu) = P\left(\sum_{i=1}^{n_1} X_{1i} > n_1 c_2|\mu\right)$, and, $L_{22}(\mu) = P\left(\sum_{i=1}^{n_2} X_{2i} \geq n_2 c_2|\mu\right)$.

A submitted lot may be accepted or rejected in an acceptance sampling plan based on sampled items from the lot(s). The probability of rejecting a good lot by the customer under any plan is called the producer's risk and is denoted by P_p, while the probability of accepting a bad lot by the customer

is called the consumer's risk, which P_c denotes. The producer's and consumer's risks for the proposed plan are given by

$$\begin{aligned} P_p(\mu_0) &= 1 - L(\mu_0) \\ &= 1 - L_{11}(\mu_0) - [L_{12}(\mu_0) - L_{11}(\mu_0)]L_{22}(\mu_0) \\ &= 1 - P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_1 | \mu_0\right) - \left[P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_2 | \mu_0\right)\right. \\ &\quad \left. - P\left(\sum_{i=1}^{n_1} X_{1i} > n_1 c_1 | \mu_0\right)\right] P\left(\sum_{i=1}^{n_2} X_{2i} \geq n_2 c_2 | \mu_0\right), \end{aligned} \tag{2}$$

and

$$\begin{aligned} P_c(\mu_1) &= L(\mu_1) \\ &= L_{11}(\mu_1) + [L_{12}(\mu_1) - L_{11}(\mu_1)]L_{22}(\mu_1) \\ &= P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_1 | \mu_1\right) + \left[P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_2 | \mu_1\right)\right. \\ &\quad \left. - P\left(\sum_{i=1}^{n_1} X_{1i} > n_1 c_1 | \mu_1\right)\right] P\left(\sum_{i=1}^{n_2} X_{2i} \geq n_2 c_2 | \mu_1\right), \end{aligned} \tag{3}$$

where, μ_0 and μ_1 are AQL and LQL respectively.

3. Estimation of the Plan Parameters

The producer often establishes a sampling plan for a continuous supply of raw materials regarding an AQL. AQL represents the minimum level of quality for the supplier's process that the consumer would consider acceptable at a process average. Note that AQL is a property of the supplier's manufacturing process, not a property of the sampling plan. The consumer often designs the sampling procedure so that the OC curve gives a high probability of acceptance at the AQL. The consumer will also be interested in the other end of the OC curve, i.e., the protection obtained for an individual lot of poor quality. In such a situation, the consumer may establish an LQL. It is the minimum level of quality that the consumer is willing to accept in an individual lot. The LQL is also not a characteristic of the sampling plan but is a level of lot quality specified by the consumer. It is possible to design a sampling plan that gives the specified probability of acceptance at LQL. Subsequently, the plan parameters are determined to minimize the Average Sample Number (ASN) such that α is satisfied at AQL(μ_0) and β is satisfied at LQL(μ_1). Thus, the following non-linear optimization problem can be considered:

Minimize $ASN = n_1 + n_2 P(c_2 \leq \bar{X}_1 < c_1)$, subject to

$$1 - P_p(\mu_0) = L(\mu_0) \geq (1 - \alpha), \tag{4}$$

and

$$P_c(\mu_1) = 1 - L(\mu_1) \leq \beta. \tag{5}$$

To determine n_i and c_i for $i = 1, 2$; first we take several combinations of (n_1, n_2, c_1, c_2) for which (4) is true. Thereafter, choose those combinations of (n_1, n_2, c_1, c_2) for which (5) is satisfied, and ultimately consider that combination for which $P_c(\mu_1) = 1 - L(\mu_1)$ is very nearer to β, if not exactly equal. This procedure is to be followed in all the considered models under discussion.

We have used the trial and error method for the calculation of plan parameters (n_1, n_2, c_1, c_2) by the non-linear optimisation technique, and its procedural steps are given below:

Step-1 We have fixed the value of producer's risk, α; consumer's risk, β; acceptable quality level, μ_0 and limiting quality level, μ_1.

Step-2 To conduct the trial and error method, we take a particular n $(= n_1 + n_2)$, and fix $n_1 = r$ and $n_2 = n - r$, and choose a combination of (c_1, c_2) by the trial and error method to satisfy the

inequality in (4). To choose (c_1, c_2) of the TSS plan, one may proceed from a single sampling plan (SSP) using $(c - d_1^0, c - d_2^0)$ for some $d_1^0,\ d_2^0$.

Step-3 We put each combination of $(n_1,\ n_2,\ c_1,\ c_2)$ for checking the inequality in (5) and we will take that particular choice of $(n_1,\ n_2,\ c_1,\ c_2)$ for which it is very nearer and less than β, so that the interest of the consumer is safeguarded and at the same time, the ASN is minimum.

To clarify the whole process of determining optimal plan parameters, we cite Table 2. To find the $(n_1 = 29, n_2 = 30, c_1 = 42.79, c_2 = 37.99)$, we have fixed α=0.02, β=0.01, μ_0=60, μ_1=40. Then we calculate θ_0 and θ_1 from $\mu = \frac{(\theta+2)}{\theta(\theta+1)}$ for Lindley distribution for different values of μ. So, we take the initial choice of $n = 2$, and the initial choice of (c_1, c_2) is $(49.17, 49.17)$ from single sampling plan in the inequality (4) to start the trial and error method, and do it until the calculated value of the left side of the inequality (5) is very near but less than $\beta = 0.01$ and also check that ASN is minimum. The same procedure is to be adopted for the exponential distribution.

To support the use of the proposed plan and facilitate easy, comprehensive understanding among practitioners, we have prepared several Tables for optimal sample size determination and comparison with competitive plans, such as SSP.

3.1. Exponential distribution

Let X be a quality characteristic following an exponential distribution with parameter θ. The probability density function (PDF) of the quality characteristic is given as

$$f(x) = \theta e^{-\theta x}; x > 0, \theta > 0, \tag{6}$$

where θ is the scale parameter. The population mean of the exponential distribution is $\mu = \frac{1}{\theta}$. It is to be noted that, if a 1^{st} random sample $(X_{11}, X_{12}, ..., X_{1n_1})$ of size n_1 is drawn from the exponential distribution, then the distribution of $U_1 = \sum_{i=1}^{n_1} X_{1i}$ is a gamma distribution with shape parameter n_1 and the scale parameter θ. Similarly, if a 2^{nd} random sample $(X_{21}, X_{22}, ..., X_{2n_2})$ of size n_2 is drawn from the exponential distribution, then the distribution of $U_2 = \sum_{i=1}^{n_2} X_{2i}$ is also a gamma distribution with parameters n_2 and θ. For a given value of μ, one can find the value of θ; hence, the OC function can be treated as a function of θ.

We have

$$L_{11}(\theta) = P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_1 | \theta\right) = 1 - \Gamma(\theta n_1 c_1, n_1), \tag{7}$$

$$L_{12}(\theta) = P\left(\sum_{i=1}^{n_1} X_{1i} \geq n_1 c_2 | \theta\right) = 1 - \Gamma(\theta n_1 c_2, n_1), \tag{8}$$

$$L_{22}(\theta) = P\left(\sum_{i=1}^{n_2} X_{2i} \geq n_2 c_2 | \theta\right) = 1 - \Gamma\left(\theta n_2 c_2, n_2\right). \tag{9}$$

Therefore, the OC function for the exponentially distributed quality characteristic of the proposed sampling inspection plan is given by

$$\begin{aligned} L(\theta) &= L_{11}(\theta) + [L_{12}(\theta) - L_{11}(\theta)] L_{22}(\theta) \\ &= 1 - \Gamma(\theta n_1 c_1, n_1) + [1 - \Gamma(\theta n_1 c_2, n_1) - 1 + \Gamma(\theta n_1 c_1, n_1)] \{1 - \Gamma\left(\theta n_2 c_2, n_2\right)\} \\ &= 1 - \Gamma(\theta n_1 c_1, n_1) + [\Gamma(\theta n_1 c_1, n_1) - \Gamma(\theta n_1 c_2, n_1)] \{1 - \Gamma\left(\theta n_2 c_2, n_2\right)\}, \end{aligned} \tag{10}$$

where, $\Gamma(\nu, n) = \frac{1}{\Gamma(n)} \int_0^{\nu} e^{-t} t^{n-1} dt$.

The producer's risk and the consumer's risk are:

$$P_p(\theta_0) = 1 - L(\theta_0), \tag{11}$$

and

$$P_c(\theta_1) = L(\theta_1), \tag{12}$$

respectively.

The parameters of the plan are determined so as to minimize the ASN by the following non-linear optimization problem:
Minimize ASN, subject to

$$P_p(\theta_0) \geq (1-\alpha), \tag{13}$$

and

$$P_c(\theta_1) \leq \beta. \tag{14}$$

3.2. Lindley distribution

Let X be a quality characteristic following the Lindley distribution with parameter θ. The probability density function (PDF) of the quality characteristic is given as

$$f(x) = \frac{\theta^2(1+x)e^{-\theta x}}{\theta+1}; x>0, \theta>0, \tag{15}$$

where θ is the scale parameter of the distribution. The population mean of the Lindley distribution is $\mu = \frac{(\theta+2)}{\theta(\theta+1)}$ [see, Lindley (1958) and Ghitany et al. (2008)].

If $(X_{j1}, X_{j2}, ..., X_{jn_j})$ be a j^{th} random sample of size n_j, for $j = 1, 2$ from the Lindley with parameter θ, then the pdf of $U_j = \sum_{i=1}^{n_1} X_{ji}$, for $j = 1, 2$ is

$$g(u_j; n_j, \theta) = \sum_{k=0}^{n_j} P_{(k,n_j)}(\theta).f_{(GA)}(u_j; 2n_j - k, \theta)$$

where

$$P_{(k,n_j)}(\theta) = \binom{n_j}{k} \frac{\theta^k}{(1+\theta)^{n_j}}$$

and

$$f_{(GA)}(u_j; 2n_j - k, \theta) = \frac{\theta^{(2n_j-k)}}{\Gamma(2n_j-k)} {u_j}^{(2n_j-k)-1} e^{-\theta u_j}.$$

Hence, U_j have the probability density function of gamma distribution with shape and scale parameters $(2n_j - k)$ and θ, respectively, for j=1,2. [see in Al-Mutairi et al. (2013)].

Again, for $j = 1, 2$ and, $h = 1, 2$, we can write,

$$\begin{aligned} L_{jh}(\theta) &= P\left(\sum_{i=1}^{n_j} X_{ji} \geq n_j c_h | \theta\right) \\ &= 1 - \int_0^{n_j c_h} g(u_j; n_j, \theta) du_j \\ &= 1 - \sum_{k=0}^{n_j} \binom{n_j}{k} \frac{\theta^k}{(1+\theta)^{n_j}} \Gamma(\theta n_j c_h, 2n_j - k). \end{aligned} \tag{16}$$

Hence, the OC function for the Lindley distributed quality characteristic of the proposed TSS plan is given by

$$
\begin{aligned}
L(\theta) &= L_{11}(\theta) + [L_{12}(\theta) - L_{11}(\theta)]L_{22}(\theta) \\
&= 1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_1, 2n_1 - k) + \left[1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_2, 2n_1 - k) \right. \\
&\quad \left. -1 + \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_1, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta^k}{(1+\theta)^{n_2}} \Gamma(\theta n_2 c_2, 2n_2 - k) \right\} \\
&= 1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_1, 2n_1 - k) + \left[\sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_1, 2n_1 - k) \right. \\
&\quad \left. - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta^k}{(1+\theta)^{n_1}} \Gamma(\theta n_1 c_2, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta^k}{(1+\theta)^{n_2}} \Gamma(\theta n_2 c_2, 2n_2 - k) \right\}. \qquad (17)
\end{aligned}
$$

Now, the producer's risk and the consumer's risk are given as

$$
\begin{aligned}
P_p(\theta_0) &= 1 - L(\theta_0) \\
&= \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_1, 2n_1 - k) - \left[\sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_1, 2n_1 - k) \right. \\
&\quad \left. - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_2, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_2}} \Gamma(\theta_0 n_2 c_2, 2n_2 - k) \right\}, \qquad (18)
\end{aligned}
$$

and

$$
\begin{aligned}
P_c(\theta_1) &= L(\theta_1) \\
&= 1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_1, 2n_1 - k) + \left[\sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_1, 2n_1 - k) \right. \\
&\quad \left. - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_2, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_2}} \Gamma(\theta_1 n_2 c_2, 2n_2 - k) \right\}. \qquad (19)
\end{aligned}
$$

The parameters of the plan are determined so as to minimize the ASN by the following non-linear optimization problem:
Minimize ASN, subject to

$$
\begin{aligned}
&1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_1, 2n_1 - k) + \left[\sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_1, 2n_1 - k) \right. \\
&\left. - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_1}} \Gamma(\theta_0 n_1 c_2, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta_0^k}{(1+\theta_0)^{n_2}} \Gamma(\theta_0 n_2 c_2, 2n_2 - k) \right\} \geq (1 - \alpha), \qquad (20)
\end{aligned}
$$

and

$$
\begin{aligned}
&1 - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_1, 2n_1 - k) + \left[\sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_1, 2n_1 - k) \right. \\
&\left. - \sum_{k=0}^{n_1} \binom{n_1}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_1}} \Gamma(\theta_1 n_1 c_2, 2n_1 - k) \right] \left\{ 1 - \sum_{k=0}^{n_2} \binom{n_2}{k} \frac{\theta_1^k}{(1+\theta_1)^{n_2}} \Gamma(\theta_1 n_2 c_2, 2n_2 - k) \right\} \leq \beta. \qquad (21)
\end{aligned}
$$

We have presented plan parameter values (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for single sampling plan [n = sample size for single sampling plan, and c = acceptance number/critical value for single sampling plan] with respect to some specified values of θ_0, θ_1, α and β in the Table 1 5. We have used R-software for the calculation of parameters by the non-linear optimisation method, and its procedural steps are given below.

Step-1 We have fixed the value of θ_0 (acceptable quality level) and θ_1 (limiting quality level) and the corresponding value of producer's risk, α, and consumer's risk, β.

Step-2 We take different choices of (n_1, n_2) and (c_1, c_2) satisfying the inequality in (20).

Step-3 We choose those pairs of (n_1, n_2, c_1, c_2) satisfying the inequality in (21) and finally, take that particular choice of (n_1, n_2, c_1, c_2) which is very nearer to β.

To minimize risk, either from the producer's side of rejecting a good lot or from the consumer's side of accepting a bad one, inspect a large sample to ensure the quality. For example, in Table 2, for $\mu_0 = 60$ and $\mu_1 = 40$, the values of (n_1, n_2, c_1, c_2) for TSS Plan is (57, 56, 43.01, 37.41) for the exponential distribution and is (29, 30, 42.79, 37.99) for Lindley distribution, whereas, the corresponding (n, c) values for the exponential and the Lindley distribution for Single Sampling Plan (SSP) are (117, 49.17) and (61, 49.22), respectively.

So, a proper sampling plan and model selection for the quality characteristic are vital for saving time and costs in industry. Suppose we select the Single Sampling Plan (common in industry) instead of TSS. In that case, there is a significant loss to the organization in terms of cost and time when they consider the quality characteristic data fits the exponential distribution.

Table 1 Values of (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for SS Plan in Lindley and Exponential Distribution respectively, for given, $\alpha = 0.01,\ \beta = 0.01$

Lindley Distribution										Exponential Distribution							
μ		θ		TSS Plan				SS Plan		θ		TSS Plan				SS Plan	
μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
70	30	0.0282	0.0646	17	(8,9)	30.69	20.59	17	44.76	0.0143	0.0333	29	(15,14)	31.2	21.1	31	44.10
70	35	0.0282	0.0556	23	(11,12)	35.83	25.83	24	48.33	0.0143	0.0286	44	(22,22)	36.96	27.96	46	48.25
70	40	0.0282	0.0488	36	(18,18)	42.67	35.57	37	52.19	0.0143	0.0250	67	(34,33)	42.65	34.65	70	52.02
60	40	0.0328	0.0488	67	(33,34)	42.25	37.75	69	48.59	0.0167	0.0250	128	(64,64)	42.36	36.86	133	48.56
60	35	0.0328	0.0556	39	(19,20)	37.08	31.08	40	45.28	0.0167	0.0286	73	(37,36)	37.42	30.92	76	45.16
60	30	0.0328	0.0646	23	(11,12)	30.63	21.63	24	41.42	0.0167	0.0333	44	(22,22)	31.67	23.67	46	41.35

Table 2 Values of (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for SS Plan in Lindley and Exponential Distribution respectively, for given, $\alpha = 0.02,\ \beta = 0.01$

Lindley Distribution										Exponential Distribution							
μ		θ		TSS Plan				SS Plan		θ		TSS Plan				SS Plan	
μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
70	30	0.0282	0.0646	15	(7,8)	31.27	21.27	15	45.96	0.0143	0.0333	26	(13,13)	31.74	20.74	27	45.18
70	35	0.0282	0.0556	20	(10,10)	37.09	28.09	21	49.33	0.0143	0.0286	39	(20,19)	38.23	29.23	40	49.19
70	40	0.0282	0.0488	31	(15,16)	42.70	34.70	32	52.96	0.0143	0.0250	60	(30,30)	43.45	35.45	62	52.97
60	40	0.0328	0.0488	59	(29,30)	42.79	37.99	61	49.22	0.0167	0.0250	113	(57,56)	43.01	37.41	117	49.17
60	35	0.0328	0.0556	34	(17,17)	37.89	31.79	35	45.99	0.0167	0.0286	64	(32,32)	37.89	30.99	66	45.82
60	30	0.0328	0.0646	20	(10,10)	31.71	23.61	21	42.28	0.0167	0.0333	39	(20,19)	32.76	24.76	40	42.16

Table 3 Values of (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for SS Plan in Lindley and Exponential Distribution respectively, for given, $\alpha = 0.05,\ \beta = 0.01$

Lindley Distribution										Exponential Distribution							
μ		θ		TSS Plan				SS Plan		θ		TSS Plan				SS Plan	
μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
70	30	0.0282	0.0646	12	(6,6)	33.42	23.42	12	47.94	0.0143	0.0333	21	(11,10)	33.80	22.80	22	47.39
70	35	0.0282	0.0556	17	(8,9)	38.10	28.10	17	51.20	0.0143	0.0286	32	(16,16)	39.41	29.41	33	51.23
70	40	0.0282	0.0488	25	(12,13)	43.84	35.84	26	54.59	0.0143	0.0250	48	(24,24)	44.58	35.58	50	54.55
60	40	0.0328	0.0488	48	(24,24)	43.98	39.38	50	50.34	0.0167	0.0250	92	(46,46)	43.91	37.81	95	50.25
60	35	0.0328	0.0556	27	(13,14)	38.34	31.34	28	47.26	0.0167	0.0286	52	(26,26)	39.00	31.50	54	47.23
60	30	0.0328	0.0646	17	(8,9)	32.65	24.65	17	43.89	0.0167	0.0333	32	(16,16)	33.79	25.29	33	43.91

Table 4 Values of (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for SS Plan in Lindley and Exponential Distribution respectively, for given, $\alpha = 0.02,\ \beta = 0.02$

Lindley Distribution										Exponential Distribution							
μ		θ		TSS Plan				SS Plan		θ		TSS Plan				SS Plan	
μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
70	30	0.0282	0.0646	13	(7,6)	31.21	19.51	13	44.39	0.0143	0.0333	24	(12,12)	30.42	18.82	25	44.33
70	35	0.0282	0.0556	19	(9,10)	35.38	23.88	19	48.37	0.0143	0.0286	35	(18,17)	36.71	26.71	36	48.17
70	40	0.0282	0.0488	28	(14,14)	41.74	31.74	29	52.16	0.0143	0.0250	53	(27,26)	42.14	33.14	55	52.01
60	40	0.0328	0.0488	52	(26,26)	41.85	36.35	54	48.57	0.0167	0.0250	100	(50,50)	41.92	35.32	104	48.54
60	35	0.0328	0.0556	30	(15,15)	36.49	28.49	31	45.18	0.0167	0.0286	57	(29,28)	36.86	28.86	59	45.07
60	30	0.0328	0.0646	19	(9,10)	30.26	20.26	19	41.46	0.0167	0.0333	35	(18,17)	31.45	22.45	36	41.29

Table 5 Values of (n_1, n_2, c_1, c_2) for TSS Plan and (n, c) for SS Plan in Lindley and Exponential Distribution respectively, for given, $\alpha = 0.05,\ \beta = 0.02$

Lindley Distribution										Exponential Distribution							
μ		θ		TSS Plan				SS Plan		θ		TSS Plan				SS Plan	
μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
70	30	0.0282	0.0646	10	(5,5)	29.78	15.78	10	46.04	0.0143	0.0333	19	(10,9)	32.19	20.19	19	45.84
70	35	0.0282	0.0556	15	(7,8)	35.88	24.08	15	50.08	0.0143	0.0286	28	(14,14)	37.46	26.46	29	50.08
70	40	0.0282	0.0488	22	(11,11)	42.57	32.57	23	53.67	0.0143	0.0250	42	(21,21)	42.93	32.93	44	53.59
60	40	0.0328	0.0488	41	(20,21)	42.34	35.84	43	49.61	0.0167	0.0250	80	(40,40)	42.78	35.78	83	49.59
60	35	0.0328	0.0556	24	(12,12)	37.42	29.42	25	46.55	0.0167	0.0286	45	(23,22)	37.74	29.24	47	46.37
60	30	0.0328	0.0646	15	(7,8)	30.62	19.12	15	42.93	0.0167	0.0333	28	(14,14)	32.08	22.08	29	42.92

4. Comparative Study

The proposed TSS plan is based on the sample mean, which is different from other TSS plans (based on the number of defective, coefficient of variation, and process loss function) in the literature. Therefore, there is hardly any scope to compare the current TSS plan with other TSS plans. However, we have compared the plan with the corresponding SS Plan. A comparative study is made for the Lindley and the exponentially distributed quality characteristic.

The features of the proposed TSS plan are studied under different conditions of the plan parameters. Tables 15 display the plan parameters ($n = n_1 + n_2,\ c_1,\ c_2$) for TSS plan and $(n,\ c)$ for the Lindley as well as the exponentially distributed quality characteristic for different producer's risk, α and consumer's risk, β. By studying the nature of the distribution of the quality characteristic through model selection methods and with the help of the Tables, practitioners can determine the size of a sample required for inspection and the associated rules for lot sentencing. It is observed from the Tables that (i) the required sample size, n, decreases as α and/or β increases. For example, a sample of size $n = 23$ is required for $(\alpha,\ \beta) = (0.01,\ 0.01)$, $n = 20$ for $(\alpha,\ \beta) = (0.02,\ 0.01)$, $n = 19$ for $(\alpha,\ \beta) = (0.02,\ 0.02)$ under the same quality conditions $(\mu_0,\ \mu_1) = (60,\ 30)$. The figures indicate that a relatively smaller sample size is needed as long as the producer and/or consumer is ready to suffer larger risks for making incorrect decisions. (ii) The required sample size, n, decreases as the difference between μ_0 and μ_1 increases. For example, a sample of size $n = 48$ is required for $(\mu_0,\ \mu_1) = (60,\ 40)$ and a sample of size $n = 17$ is required for $(\mu_0,\ \mu_1) = (60,\ 30)$ under the same risk conditions, i.e., $(\alpha,\ \beta) = (0.05,\ 0.01)$. The situation indicates that smaller sample size is required because it becomes easier to make correct decisions if the difference between two specified quality levels increases. (iii) The Lindley distributed quality characteristic's sample sizes are smaller than that for the exponentially distributed one. (iv) The sample sizes for the TSS plan are smaller than that for SS Plan.

5. Example and Data Analysis

The following data represents the strength of glass of the aircraft window (31 units) reported by Fuller et al. (1994):

18.830, 20.800, 21.657, 23.030, 23.230, 24.050, 24.321, 25.500,
25.520, 25.800, 26.690, 26.770, 26.780, 27.050, 27.670, 29.900,

31.110, 33.200, 33.730, 33.760, 33.890, 34.760, 35.750, 35.910,
36.980, 37.080, 37.090, 39.580, 44.045, 45.290, 45.381.

From Table 6, we find that the Lindley distribution is the best fit for the data as the value of AIC is the least. The histogram and fitted distributions have been shown in Figure 1.

Table 6 Comparison between the exponential and the Lindley distribution for Data Set

Distribution	**Estimate of θ**	**Negative Log-likelihood**	**AIC**
Exponential	0.0324592	137.2644	276.5289
Lindley	0.0630000	126.9942	255.9884

Table 7 Plan Parameter Estimation for the Data Set, using exponential and Lindley Distribution, for given $\alpha = 0.02,\ \beta = 0.01$, where, $\mu_0 = 44,\ \mu_1 = 20$

Distribution	μ		θ		TSS plan				SS plan	
	μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
Exponential	44	20	0.0227	0.0500	25	(13,12)	19.92	11.42	31	29.33
Lindley			0.0445	0.0956	15	(7,8)	19.4	7.9	17	29.62

Table 8 Plan Parameter Estimation for the Data Set, using the exponential and the Lindley distribution, for given $\alpha = 0.05,\ \beta = 0.02$, where, $\mu_0 = 50,\ \mu_1 = 25$

Distribution	μ		θ		TSS plan				SS plan	
	μ_0	μ_1	θ_0	θ_1	n	(n_1, n_2)	c_1	c_2	n	c
Exponential	50	25	0.0200	0.0400	23	(12,11)	24.99	13.99	29	35.77
Lindley			0.0392	0.0771	14	(7,7)	25.49	16.69	15	35.77

Table 9 Sampling Inspection plans for Data Set, at level $\alpha = 0.02,\ \beta = 0.01$

Distribution	Sampling Plan	(n, n_1, n_2)	c_1	c_2	n	c	Sample mean	Decision
Exponential	Single Sampling Inspection Plan	—	—	—	31	29.33	30.81142	Accept the lot
	Two Stage Sampling Inspection Plan (1st Sample)	(25,13,12)	19.92	11.42	—	—	24.07523	Accept the lot on the basis of 1st sample. No need to draw 2^{nd} sample of size $n_2 = 12$.
Lindley	Single Sampling Inspection Plan	—	—	—	17	29.62	25.21812	Reject the lot
	Two Stage Sampling Inspection Plan (1st Sample)	(15,7,8)	19.40	7.90	—	—	22.274	Accept the lot on the basis of 1st sample. No need to draw 2^{nd} sample of size $n_2 = 8$.

Table 7 indicates the estimation of plan parameters for the Data Set, using exponential and Lindley distribution if we consider the producer's value, $\mu_0 = 44$ and producer's risk, $\alpha = 0.02$, and for consumer's value, $\mu_1 = 20$, and consumer's risk, $\beta = 0.01$. We summarize the analysis and decision for this data set in Table 9. The decision is the same for a single sampling plan and a two-stage sampling plan (based on the 1st sample) for the exponential distribution, and also the same for the two-stage sampling plan (based on the 1st sample) for the Lindley distribution. However, the decision differs only for the Single sampling plan for the Lindley distribution. The sample required for inspection is less for the Lindley distribution fitting.

In this situation, the most crucial benefit of a two-stage sampling inspection plan is that there is no need to draw 2^{nd} sample. Here, the sample size is minimum(n_1, n_2), i.e., we need to draw just $n_1 = 7$ sample observations for the two-stage sampling inspection plan for the Lindley distribution (which is a better fit in this case), which is the least.

Table 10 Sampling Inspection plans for Data Set, at level $\alpha = 0.05,\ \beta = 0.02$

Distribution	Sampling Plan	(n, n_1, n_2)	c_1	c_2	n	c	Sample mean	Decision
Exponential	Single Sampling Inspection Plan	—	—	—	29	35.77	29.80976	Reject the lot
	Two Stage Sampling Inspection Plan (1st Sample)	(23,12,11)	24.99	13.99	—	—	23.84983	Need to draw 2^{nd} sample of size $n_2 = 11$
	Two Stage Sampling Inspection Plan (2nd Sample)	(23,12,11)	24.99	13.99	—	—	31.60000	Accept the lot on the basis of 2nd sample
Lindley	Single Sampling Inspection Plan	—	—	—	15	35.77	24.51320	Reject the lot
	Two Stage Sampling Inspection Plan (1st Sample)	(14,7,7)	25.49	16.69	—	—	22.27400	Need to draw 2^{nd} sample of size $n_2 = 11$
	Two Stage Sampling Inspection Plan (2nd Sample)	(14,7,7)	25.49	16.69	—	—	26.30143	Accept the lot on the basis of 2nd sample

Table 8 indicates the estimation of plan parameters for the Data Set, using exponential and Lindley distribution if we consider the producer's value, $\mu_0 = 50$ and producer's risk, $\alpha = 0.05$, and for consumer's value, $\mu_1 = 25$, and consumer's risk, $\beta = 0.02$. We also summarize the analysis and decision for the same data set in Table 10. The decision is acceptance of the lot for a two-stage sampling plan in case of both exponential (based on 2nd sample) and Lindley distribution (based on 2nd sample). However, the decision is the rejection of the lot for the Single sampling plan for the exponential and Lindley distribution.

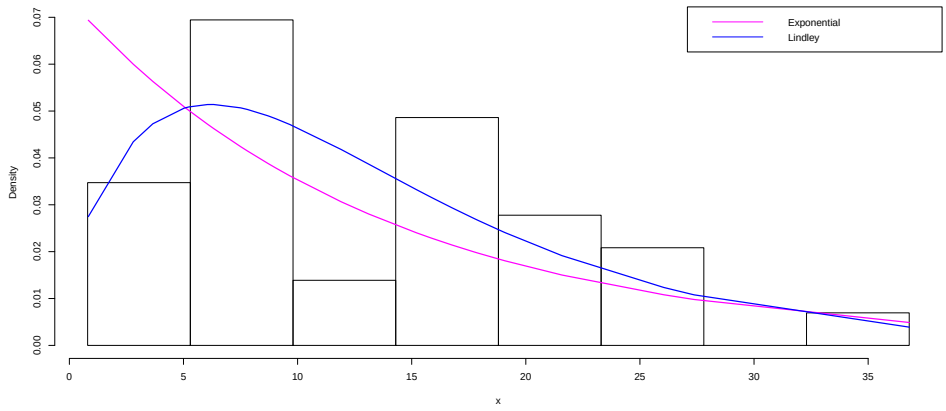


Figure 1 Histogram and fitted distributions for the Data Set

In this situation, also two-stage sampling inspection plan provides more benefits than a single sampling plan because we need to draw $n = 14$ sample observations for the two-stage sampling inspection plan for the Lindley distribution (that is a better fit in this case), which is, again, least.
OC curves for Exponential and Lindley distributions have been presented respectively in Figure 2, and it is observed that the OC curve under the two-stage sampling inspection plan for the Lindley distribution assumption is steeper than the others. This fact also justifies more protection for the customers.

6. Concluding Remarks

This article discusses the TSS sampling inspection plans for the variable for the exponential and Lindley distributed quality characteristics. Estimating the plan parameters is based on a two-point OC

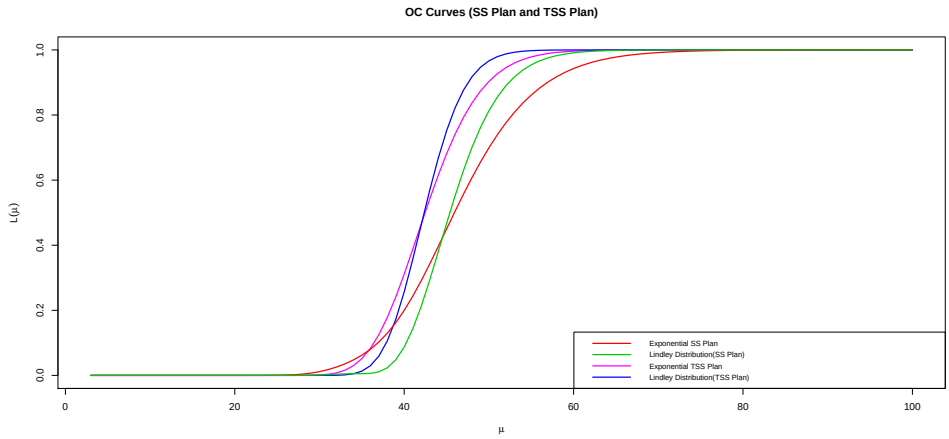


Figure 2 OC curve for the Single Sampling Plan and Two Stage Sampling Plan for both Exponential and Lindley distributed quality characteristic for the Data Set

curve approach, the acceptable quality level (AQL) and the limiting quality level (LQL). In a comparison study, we have observed that the TSS sampling inspection plans for both the exponential and the Lindley distribution work more efficiently in terms of sample size rather than a single sampling inspection plan. In other words, the plan for the two-stage sampling required smaller sample size to draw a verdict about acceptance or rejection of the lot compared to a single sampling inspection plan. The TSS plan is recommended to perform using the Lindley distribution. Tables of plan parameters have obtained through computational technique at various values of $[AQL(\mu_0), \alpha]$, $[LQL(\mu_1), \beta]$ for ready reference for practitioners. Also, the sampling plan has been demonstrated through a real-life example. We expect that the industrial engineers and statisticians can effectively implement the TSS sampling approach to Lindley distributed lifetime data.

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