Thailand Statistician
January 2015; 13(1): 79-96
http://statassoc.or.th
Contributed paper

The MCSP-F-L Fractional Continuous Sampling Plan

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Received: 30 May 2014

Accepted: 8 August 2014

Abstract

This paper presents the MCSP-F-L for the concept of a fractional sampling plan that has been developed from the CSP-F-L continuous sampling plan. The attractive feature of the MCSP-F-L is that addition a maximum allowable number of inspected units for prevention long length of inspection at level 2 in the procedure of CSP-F-L plan. The conventional measures of performance for continuous sampling plans have been derived using a Markov Chain model, namely average fraction inspected (AFI), average fraction of total produced accepted on sampling basis (Pa(p)), average outgoing quality (AOQ) and average outgoing quality limit (AOQL). The accuracy of all performance measures has been verified by extensive simulations. The performance measures of the proposed plan were compared with the CSP-F-L plan and the Modified MLP-T-2 plan and a numerical comparison at various levels of incoming quality levels and plan parameters is illustrated in this paper.

Keywords: modified fractional continuous sampling plan, average fraction inspected, average outgoing quality.

1. Introduction

A continuous sampling plan (CSP) is used for inspecting units that produced item by item on a continuous process and the result of the inspection is either conforming or nonconforming. CSPs alternate between two phases of inspection, i.e. screening and sampling inspections. There are two types of CSPs: single-level continuous sampling plans and multi-level continuous sampling plans. Both types of CSPs are different at sampling inspection in that single-level continuous sampling plans are only one level of sampling inspection but multi-level continuous sampling plans are more than one level of sampling inspection. The original CSP-1 was a single-level continuous sampling plan developed by Dodge [1]. The procedure of CSP-1 starts with 100% inspection until i successive conforming units are found and then starts randomly inspection with a rate f of the units. The procedure continues until a nonconforming sample unit is found then the procedure switches back to 100% inspection and the nonconforming units are replaced by conforming units. The procedure of Dodge CSP-1 is the simplest and its later modifications CSP-2, CSP-3 and several plans have been developed. A review of various CSPs can be seen in many statistical quality control textbooks.

Derman et al. [2] developed three tightened multi-level plans. The tightest, namely MLP-T, provides for an infinite number of sampling inspection levels. The modified of the MLP-T plan with two sampling inspection levels is designated as MLP-T-2. The procedure of the MLP-T-2 plan alternates between screening inspection and sampling inspection with two sampling levels. Kandaswamy and Govindaraju [3] derived the performance measures of the MLP-T-2 plan using the Markov Chain approach. Balamurali and Govindaraju [4] have developed the Modified MLP-T-2 plan that the operating procedure start with 100% inspection. When the first i consecutive conforming units are found, then switch to the sampling inspection at level 2 (f_2). Otherwise, the 100% inspection is continued until any run of i successive conforming units are found and then switch to the sampling inspection at level 1 (f_1, $f_1 > f_2$). When a nonconforming unit is found on either sampling level, immediately revert to the 100% inspection. A prominent point of the Modified MLP-T-2 plan over MLP-T-2 plan is that one cannot go from one level of sampling inspection to another without going back to 100% inspection.

Guayjarernpanishk [5] developed a fractional sampling plan, namely CSP-F-L, based on Modified MLP-T-2. The purpose of developing CSP-F-L is to reduce the number of units inspected of Modified MLP-T-2. The difference between the two plans is in the beginning of inspection which the Modified MLP-T-2 starts with 100% inspection but the procedure of CSP-F-L starts by sampling inspection at level 1 with a rate f_1 of the units. For the CSP-F-L plan, the inspection is continued *k* consecutive units. If the first *k* consecutive units are found clear of nonconforming, then switch to sampling inspection at level 2 (f_2, $f_2<f_1$). Otherwise, switch to 100% inspection of units in the order of production. During at the 100% inspection, if the first *i* consecutive units are found clear of nonconforming discontinue 100% inspection and switch to sampling inspection at level 2. Otherwise, continue 100% inspection until *i* successive units are found clear of nonconforming then proceed to sampling inspection at level 1 begins. When a nonconforming unit is found at level 2, immediately revert to the sampling inspection at level 1. Guayjarernpanishk derived the performance measures of the CSP-F-L plan using the Markov Chain approach.

In this paper, a modification is proposed on the CSP-F-L plan and the resultant plan is designated as a Modified CSP-F-L (MCSP-F-L) plan. The operating procedure of the proposed plan, a detailed derivation of the performance measure formulas, a testing of the accuracy formulas, and a comparison of the proposed plan with other existing continuous sampling plans are presented in a separate section.

2. Materials and Methods

2.1 The operating procedure of MCSP-F-L plan

The MCSP-F-L uses five parameters for inspection of the units being produced on the production line, namely two fractions f_1 and f_2, and three positive integers *i*, *k* and *l*, which are defined by:

f_1 = the sampling inspection at level 1,
f_2 = the sampling inspection at level 2,
i = the clearance number,
k = the number of conforming units to be found in the sampling inspection at level 1,
l = the number of conforming units to be found in the sampling inspection at level 2.

The MCSP-F-L plan starts with sampling inspection at level 1 with a rate f_1 of the units as the CSP-F-L plan. The MCSP-F-L is characterized by a maximum allowable number of inspected units (*l*) in the phases of sampling inspection at level 2 for deciding when switch from the phases of sampling inspection at level 2 to the phases of sampling inspection at level 1. During the inspection at level 2 are found clear of nonconforming, the inspection is continued until *l* sampled units have been inspected before switching to sampling inspection at level 1. The operating procedure of the MCSP-F-L plan is given below.

(1) The procedure starts with sampling inspection at level 1 with a rate f_1 of the units, selecting individual units one at a time in the order of production randomly.

(1.1) If the first k consecutive units are found clear of nonconforming unit, then the inspection switch to sampling inspection at level 2 with a rate f_2 of the units ($f_2 < f_1$).

(1.2) Otherwise, switch to 100% inspection of units in the order of production.

(2) During the inspection at the 100% inspection.

(2.1) If the first i consecutive units are found clear of nonconforming discontinue 100% inspection and switch to sampling inspection at level 2.

(2.2) Otherwise, continue 100% inspection until any run of i successive units found clear of nonconforming then proceed to sampling inspection at level 1 and then continues as in (1).

(3) During the sampling inspection at level 2, count the number of inspected units.

(3.1) If a nonconforming unit is found then revert immediately to sampling inspection at level 1 and then continues as in (1).

(3.2) Otherwise, continue sampling inspection at level 2 till l conforming units are found then revert to sampling inspection at level 1 and then continues as in (1).

(4) Replace or correct all the nonconforming units found with conforming units.

The operating procedure of the MCSP-F-L plan may be represented schematically as in Figure 1.

In this paper, the formulation of the MCSP-F-L procedure as a Markov Chain is given, assuming that the production process is in statistical control. The performance measures such as the average fraction inspected (AFI), the average fraction of the total produced accepted on sampling basis (Pa(p)) and the average outgoing quality (AOQ) that we are derived and given in the following.

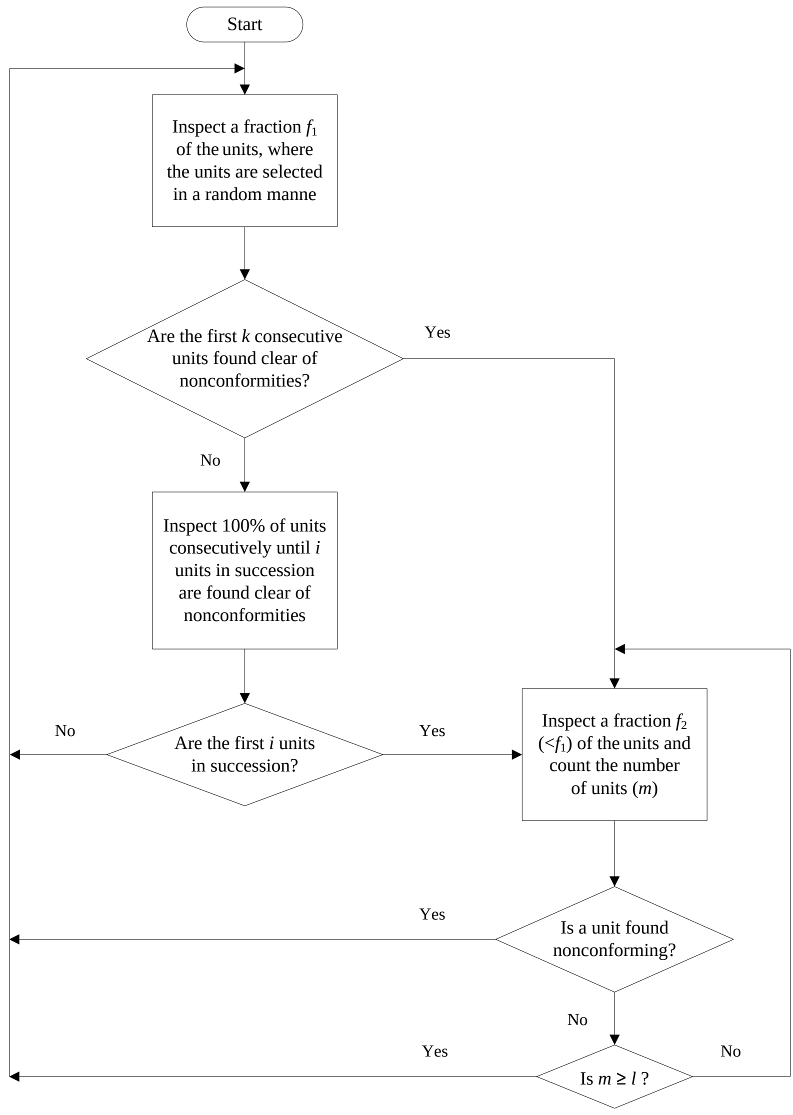

Figure 1. Flow diagram for the operating procedure of MCSP-F-L plan.

2.2 The MCSP-F-L procedure as a Markov Chain

Let [X_t] (t=1, 2, …) denote a discrete-parameter Markov Chain with finite state space (S_n), n = 1, 2, …, $3k+2i+3l+1$. The states of the process are defined, in a same way Roberts [6] and Lasater [7], as follows:

$S_{3n+1} \quad = \ f_1 N_{n+1}$ (n = 0, 1, 2,…, k–1)

= Sampling inspection at level 1 is in effect and the n units submitted for inspection were all found to be conforming but the last unit was not selected for inspection.

S_{3n+2} = f_1In_{n+1} (n = 0, 1, 2,…, k–1)

= Sampling inspection at level 1 is in effect and the n+1 units submitted for inspection were all found to be conforming.

S_{3n+3} = f_1Id_{n+1} (n = 0, 1, 2,…, k–1)

= Sampling inspection at level 1 is in effect, the n+1 units submitted for inspection and only unit n + 1 was found to be nonconforming.

S_{3k+1} = A_0

= Nonconforming unit is found on 100% inspection.

S_{3k+n+1} = A_n (n = 1, 2,…, i)

= n consecutive conforming units found during 100% inspection after having a nonconforming unit is found on 100% inspection.

$S_{3k+i+n+1}$ = B_n (n = 1, 2,…, i)

= n consecutive conforming units found during 100% inspection following its commencement.

$S_{3k+2i+3n+2}$ = f_2N_{n+1} (n = 0, 1, 2,…, l–1)

= Sampling inspection at level 2 is in effect and the n units submitted for inspection were all found to be conforming but the last unit was not selected for inspection.

$S_{3k+2i+3n+3}$ = f_2In_{n+1} (n = 0, 1, 2,…, l–1)

= Sampling inspection at level 2 is in effect and the n+1 units submitted for inspection were all found to be conforming.

$S_{3k+2i+3n+4}$ = f_2Id_{n+1} (n = 0, 1, 2,…, l–1)

= Sampling inspection at level 2 is in effect, the n+1 units submitted for inspection and only unit n+1 was found to be nonconforming.

The set of (3k+2i+3l+1) states defined above completely describe the mutually exclusive phases of inspection for the MCSP-F-L procedure. A flow chart showing the description of the process by means of states and transition is given in Figure 2 and the one-step transition probability matrix for the process is given in Table 1. The transition probability matrix reveals that the process is a discrete-parameter, finite, recurrent, irreducible, aperiodic (DFRIA) Markov Chain (see Karlin [8] and Lasater [7]).

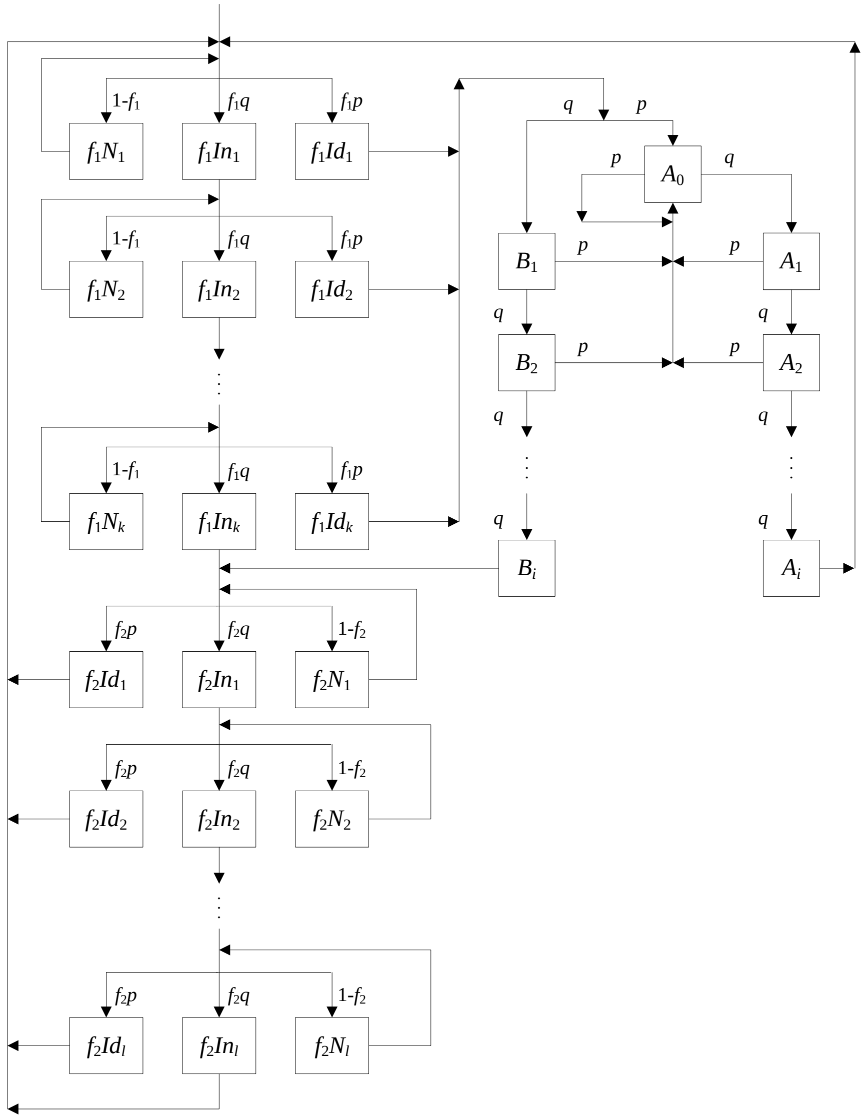

Figure 2. States transitions of the MCSP-F-L procedure.

Table 1. One-step transition probability matrix of the MCSP-F-L plan.

	f_1N_1	f_1In_1	f_1Id_1	f_1N_2	…	f_1N_k	f_1In_k	f_1Id_k	A_0	A_1	…	A_i	B_1	B_2	…	B_i	f_2N_1	f_2In_1	f_2Id_1	f_2N_2	…	f_2N_l	f_2In_l	f_2Id_l
f_1N_1	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_1In_1	0	0	0	$1-f_1$	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_1Id_1	0	0	0	0	…	0	0	0	p	0	…	0	q	0	…	0	0	0	0	0	…	0	0	0
f_1N_2	0	0	0	$1-f_1$	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_1In_2	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_1Id_2	0	0	0	0	…	0	0	0	p	0	…	0	q	0	…	0	0	0	0	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
f_1N_k	0	0	0	0	…	$1-f_1$	f_1q	f_1p	0	0	…	0	0	0	…						…	0	0	0
f_1In_k	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	$1-f_2$	f_2q	f_2p	0	…	0	0	0
f_1Id_k	0	0	0	0	…	0	0	0	p	0	…	0	q	0	…	0	0	0	0	0	…	0	0	0
A_0	0	0	0	0	…	0	0	0	p	q	…	0	0	0	…	0	0	0	0	0	…	0	0	0
A_1	0	0	0	0	…	0	0	0	p	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
A_i	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
B_1	0	0	0	0	…	0	0	0	p	0	…	0	0	q	…	0	0	0	0	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
B_i	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	$1-f_2$	f_2q	f_2p	0	…	0	0	0
f_2N_1	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	$1-f_2$	f_2q	f_2p	0	…	0	0	0
f_2In_1	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	$1-f_2$	…	0	0	0
f_2Id_1	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0		…	0	0	0
f_2N_2	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	$1-f_2$	…	0	0	0
f_2In_2	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_2Id_2	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
f_2N_l	0	0	0	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	$1-f_2$	f_2q	f_2p
f_2In_l	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0
f_2Id_l	$1-f_1$	f_1q	f_1p	0	…	0	0	0	0	0	…	0	0	0	…	0	0	0	0	0	…	0	0	0

2.3 The Performance Measures of the MCSP-F-L

Let p be the probability of a unit produced by the process being nonconforming and q be the probability of a unit produced by the process being conforming (=1 − p), the following performance measures may be obtained:

The average number of units inspected under the 100% inspection, u:

$$u = \frac{(1-q^k)(1-q^i)}{pq^i} \tag{1}$$

The average number of units passed under the sampling inspection, v:

$$v = \frac{f_1(1-q^l)(q^k-q^{k+i}+q^i)+f_2(1-q^k)}{f_1 f_2 p} \tag{2}$$

The average fraction inspected, AFI:

$$\text{AFI} = \frac{f_1 f_2[(1-q^k)+q^i(1-q^l)(q^k-q^{k+i}+q^i)]}{f_1q^i(1-q^l)(q^k-q^{k+i}+q^i)+f_1 f_2(1-q^k)(1-q^i)+f_2q^i(1-q^k)} \tag{3}$$

The average fraction of the total produced accepted on sampling basis, Pa(p):

$$\text{Pa(p)} = \frac{q^i[f_1(1-q^l)(q^k-q^{k+i}+q^i)+f_2(1-q^k)]}{f_1q^i(1-q^l)(q^k-q^{k+i}+q^i)+f_1 f_2(1-q^k)(1-q^i)+f_2q^i(1-q^k)} \tag{4}$$

The average outgoing quality, AOQ:

$$\text{AOQ} = \frac{pq^i\left[f_2(1-f_1)(1-q^k)+f_1(1-f_2)(1-q^l)(q^k-q^{k+i}+q^i)\right]}{f_1q^i(1-q^l)(q^k-q^{k+i}+q^i)+f_1 f_2(1-q^k)(1-q^i)+f_2q^i(1-q^k)} \tag{5}$$

The average outgoing quality limit, AOQL:

$$\text{AOQL} = \max_{\text{all } p} (\text{AOQ}) \tag{6}$$

A detailed derivation of these measures, based on Markov Chain formulation of the plan, is given in the Appendix.

2.4 Test of the Accuracy of Performance Measures for the MCSP-F-L

For testing the accuracy of the performance measure formulas that defined for the MCSP-F-L, the results from the formulas were compared with the values obtained from extensive simulations. Three different levels were examined for the probability p of nonconforming units produced on the line 0.005, 0.02 and 0.03. For each p, we were defined values of i=k=50, 100 and 150, values of f_1=1/2 and 1/6, values of f_2=f_1/2, and values of l=i and 2i. For each set of values of l, p, i, k and f_1, a simulation was carried out to compute the fraction of units inspected, the fraction of the total produced accepted on sampling basis and the fraction of outgoing nonconforming units. The simulation was repeated 250 different product lines and the values of the average fraction inspected (AFI), the average fraction of the total produced accepted on sampling basis (Pa(p)) and the average outgoing quality (AOQ) were calculated and then compared with the values of AFI, Pa(p) and AOQ computed from the formulas given in equations (3), (4) and (5), respectively.

When DAFI, DPa and DAOQ were defined by

$$\text{DAFI} = |\text{AFI\_S} - \text{AFI\_F}| \tag{7}$$

$$\text{DPa} = |\text{Pa(p)\_S} - \text{Pa(p)\_F}| \tag{8}$$

and

$$\text{DAOQ} = |\text{AOQ\_S} - \text{AOQ\_F}| \tag{9}$$

where

AFI_S = the AFI values from the simulation,
AFI_F = the AFI values from the formula,
Pa(p)_S = the Pa(p) values from the simulation,
Pa(p)_F = the Pa(p) values from the formula,
AOQ_S = the AOQ values from the simulation,
AOQ_F = the AOQ values from the formula.

The AFI and Pa(p) formulas are accepted as the accurate formulas if DAFI and DPa were less than or equal to 0.02. The AOQ formula is accepted as an accurate formula if DAOQ was less than or equal to 0.002. The accuracy of the formulas was then compared for each set of values of l, p, i, k and f_1 and the results are presented in Section 3.1.

2.5 Comparison of the performance measure of MCSP-F-L plan with CSP-F-L and Modified MLP-T-2 plans

The ultimate aim of developing the MCSP-F-L plan from the CSP-F-L plan is to decrease an opportunity in the sampling inspection at level 2 from the ending of procedure of CSP-F-L plan by presented stopping rule to prevent long length of inspection at level 2 for ensure the protection to the consumer. The results of a modification decrease in terms of the Pa(p) and AOQ values. To reduce the Pa(p) and AOQ values, one need to tighten the plan inspection by involve larger AFI value. The MCSP-F-L plan is a modification of both CSP-F-L and Modified MLP-T-2 plans therefore in this section, the AFI values and the Pa(p) values for MCSP-F-L were compared with AFI and Pa(p) values respectively obtained for CSP-F-L and Modified MLP-T-2 when the values of i=k=50, 75, 100, 125 and 150, and values of f_1=1/2 and 1/6, and values of f_2=f_1/2, and values of p=0.005, 0.02, 0.03 and 0.05, and values of l=i and 2i. The results are presented in Section 3.2.

3. Results

3.1 The Accuracy Performance Measures for MCSP-F-L

The difference of the AFI values from the simulations and the AFI values from the formula (DAFI), the difference of the Pa(p) values from the simulations and the Pa(p) values from the formula (DPa) and the difference of the AOQ values from the simulations and the AOQ values from the formula (DAOQ) for each set of l, p, i and k values are shown in Table 2 and 3 for f_1=1/2 and 1/6, respectively. It was found that the DAFI and DPa values were less than 0.02 for all sets of l, p, i, k and f_1 values, and it was also found that the DAOQ values was less than 0.002 for all sets of l, p, i, k and f_1 values. So the simulations signified that the AFI, the Pa(p) and the AOQ formulas are accurate.

3.2 The Comparison of the performance measures

Figure 3 and 4 show a comparison of the AFI curves for the MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for all sets of p when f_1=1/2 and 1/6, respectively. At the same values of p for all of p values, it is observed that the shape of AFI curves at f_1=1/2 and f_1=1/6 are similar. At good quality levels (p=0.005) when i is less than or equal to 75, the AFI values of MCSP-F-L plan using l=i are slightly higher than the other two plans but when i is more than or equal to 100, the AFI values of the MCSP-F-L plan using l=i and 2i are clearly lower than the Modified MLP-T-2 and higher than the CSP-F-L plan, and the AFI values of the MCSP-F-L using l=2i are obviously seen lower than the MCSP-F-L plan using l=i. For the case of moderate quality levels (p=0.02 and 0.03) for all sets of *i* values, it can be found that the AFI values of the MCSP-F-L plan using *l*=*i* and 2*i* are lower than the Modified MLP-T-2 and a little higher than the CSP-F-L plan. However, the difference of the AFI values between the three plans become small as the value of *i* is increased. For the case of poor quality levels (*p*=0.05), it is observed that when *i* is less than 75, the AFI values of the three plans are slightly different but when *i* is more than or equal to 75, the AFI values are similar.

Table 2. The DAFI, DPa and DAOQ values of MCSP-F-L for f_1=1/2.

l	*p*	*i*=*k*	AFI_S	AFI_F	DAFI	Pa(p)_S	Pa(p)_F	DPa	AOQ_S	AOQ_F	DAOQ
=*i*	0.005	50	0.3640	0.3672	0.0032	0.9567	0.9532	0.0035	0.0028	0.0032	0.0004
		100	0.4106	0.4139	0.0032	0.8966	0.8921	0.0045	0.0027	0.0029	0.0003
		150	0.4707	0.4733	0.0027	0.8207	0.8133	0.0074	0.0025	0.0026	0.0002
	0.02	50	0.5405	0.5452	0.0047	0.7165	0.7151	0.0013	0.0091	0.0091	0.0000
		100	0.8230	0.8171	0.0058	0.3055	0.3138	0.0083	0.0033	0.0037	0.0004
		150	0.9381	0.9419	0.0038	0.1139	0.1076	0.0063	0.0012	0.0012	0.0000
	0.03	50	0.6903	0.6967	0.0064	0.5037	0.4978	0.0059	0.0089	0.0091	0.0002
		100	0.9366	0.9429	0.0063	0.1126	0.1059	0.0068	0.0022	0.0017	0.0005
		150	0.9890	0.9891	0.0001	0.0217	0.0214	0.0003	0.0004	0.0003	0.0000
=2*i*	0.005	50	0.3265	0.3289	0.0024	0.9709	0.9685	0.0024	0.0031	0.0034	0.0004
		100	0.3690	0.3724	0.0034	0.9234	0.9194	0.0040	0.0030	0.0031	0.0003
		150	0.4308	0.4321	0.0013	0.8531	0.8478	0.0053	0.0027	0.0028	0.0002
	0.02	50	0.5004	0.5086	0.0083	0.7598	0.7505	0.0094	0.0099	0.0098	0.0001
		100	0.8186	0.8094	0.0092	0.3171	0.3231	0.0060	0.0033	0.0038	0.0005
		150	0.9367	0.9413	0.0046	0.1158	0.1084	0.0074	0.0012	0.0012	0.0000
	0.03	50	0.6731	0.6764	0.0034	0.5277	0.5206	0.0072	0.0094	0.0097	0.0003
		100	0.9368	0.9424	0.0055	0.1129	0.1066	0.0063	0.0022	0.0017	0.0005
		150	0.9890	0.9891	0.0001	0.0217	0.0214	0.0003	0.0004	0.0003	0.0000

Table 3. The DAFI, DPa and DAOQ values of MCSP-F-L for f_1=1/6.

l	*p*	*i*=*k*	AFI_S	AFI_F	DAFI	Pa(p)_S	Pa(p)_F	DPa	AOQ_S	AOQ_F	DAOQ
		50	0.1274	0.1263	0.0010	0.9833	0.9839	0.0006	0.0041	0.0044	0.0003
	0.005	100	0.1541	0.1487	0.0054	0.9557	0.9612	0.0055	0.0040	0.0043	0.0002
		150	0.1800	0.1802	0.0002	0.9292	0.9289	0.0003	0.0040	0.0041	0.0001
		50	0.2167	0.2244	0.0077	0.8916	0.8828	0.0089	0.0154	0.0155	0.0001
=*i*	0.02	100	0.5105	0.5021	0.0084	0.5715	0.5784	0.0068	0.0095	0.0100	0.0005
		150	0.7799	0.7751	0.0049	0.2593	0.2657	0.0064	0.0041	0.0045	0.0004
		50	0.3421	0.3491	0.0070	0.7543	0.7483	0.0060	0.0201	0.0195	0.0006
	0.03	100	0.7719	0.7782	0.0062	0.2679	0.2621	0.0058	0.0076	0.0067	0.0009
		150	0.9433	0.9486	0.0053	0.0675	0.0614	0.0060	0.0017	0.0015	0.0002
		50	0.1125	0.1120	0.0006	0.9890	0.9893	0.0003	0.0043	0.0044	0.0002
	0.005	100	0.1343	0.1312	0.0031	0.9681	0.9716	0.0035	0.0042	0.0043	0.0001
		150	0.1591	0.1603	0.0012	0.9446	0.9435	0.0010	0.0041	0.0042	0.0001
		50	0.1959	0.2034	0.0075	0.9096	0.9002	0.0094	0.0160	0.0159	0.0000
=2*i*	0.02	100	0.5001	0.4917	0.0085	0.5833	0.5888	0.0055	0.0097	0.0102	0.0005
		150	0.7799	0.7737	0.0063	0.2615	0.2672	0.0057	0.0041	0.0045	0.0005
		50	0.3237	0.3314	0.0077	0.7718	0.7651	0.0067	0.0205	0.0201	0.0005
	0.03	100	0.7708	0.7768	0.0060	0.2661	0.2635	0.0026	0.0074	0.0067	0.0007
		150	0.9433	0.9486	0.0053	0.0675	0.0615	0.0060	0.0017	0.0015	0.0002

Figure 5 and 6 show a comparison of the Pa(p) curves for the MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for all sets of *p* when f_1=1/2 and 1/6, respectively. It is observed that the feature of Pa(p) curves at f_1=1/2 and f_1=1/6 are similar at the same values of *p* for all of *p* values. At good quality levels (*p*=0.005), the Pa(p) values of the MCSP-F-L plan using *l*=*i* and 2*i* are lower than the CSP-F-L and clearly higher than the Modified MLP-T-2 plan, and the Pa(p) values of the MCSP-F-L using *l*=*i* are lower than the MCSP-F-L plan using *l*=2*i*. For the case of moderate quality levels (*p*=0.02 and 0.03) for all sets of *i* values, it can be found that the Pa(p) values of the MCSP-F-L plan using *l*=*i* and 2*i* and the CSP-F-L plan are slightly different and a little higher than the Modified MLP-T-2 plan. However, the difference of the Pa(p) values between the three plans relatively small when the value of *i* increases. For the case of poor quality levels (*p*=0.05), it is observed that when *i* is less than 75, the Pa(p) values of the Modified MLP-T-2 plan are slightly lower than the other plans but when *i* is more than or equal to 75, the Pa(p) values of the three plans are similar.

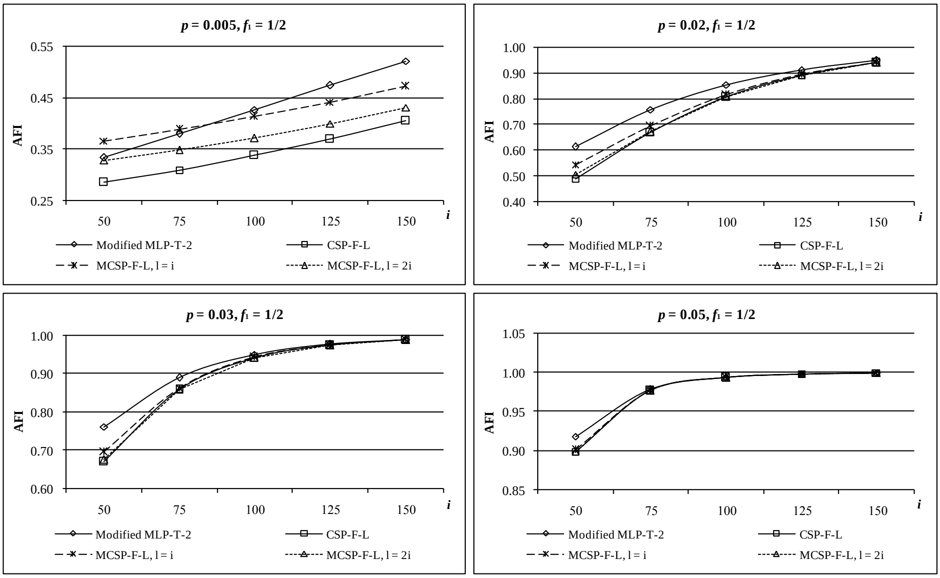

Figure 3. AFI curves of MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for f_1=1/2.

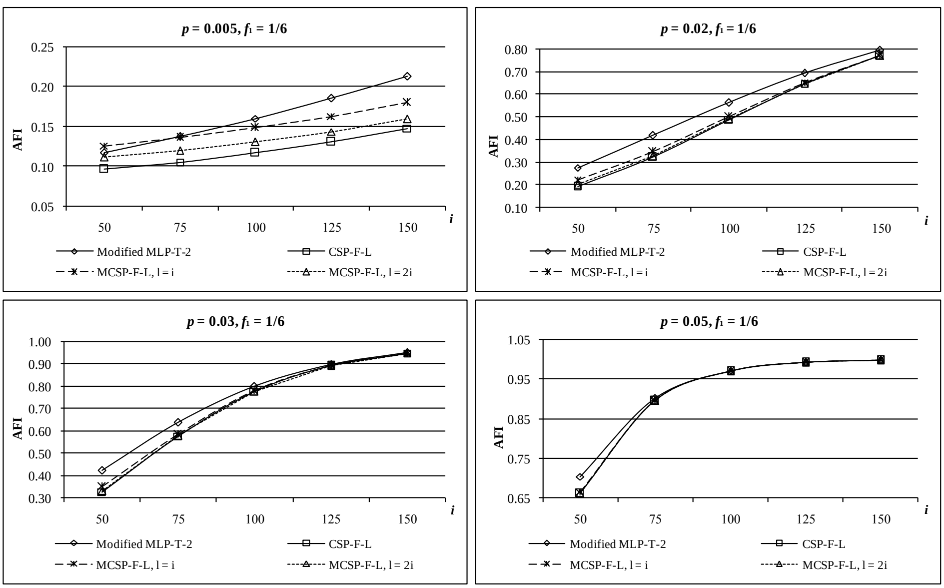

Figure 4. AFI curves of MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for f_1=1/6.

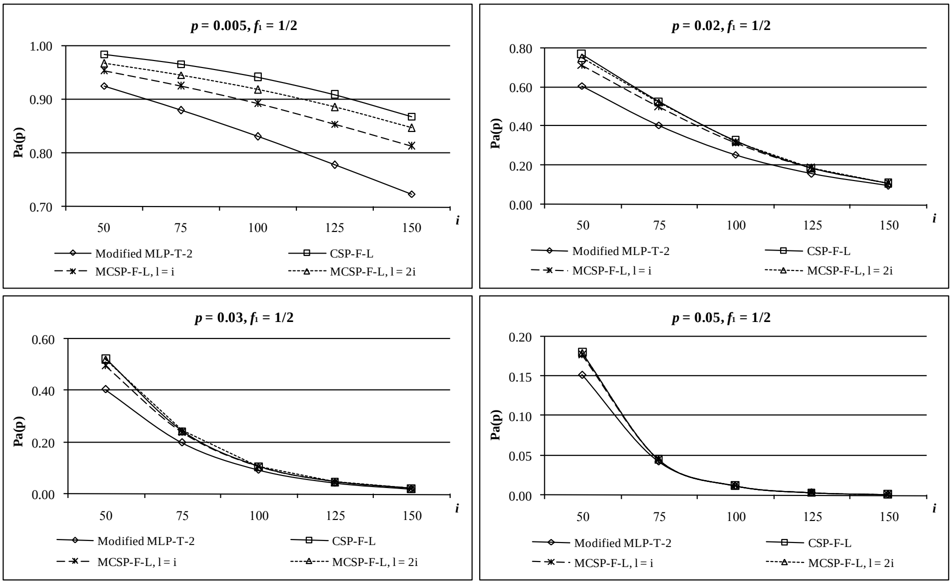

Figure 5. Pa(p) curves of MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for f_1=1/2.

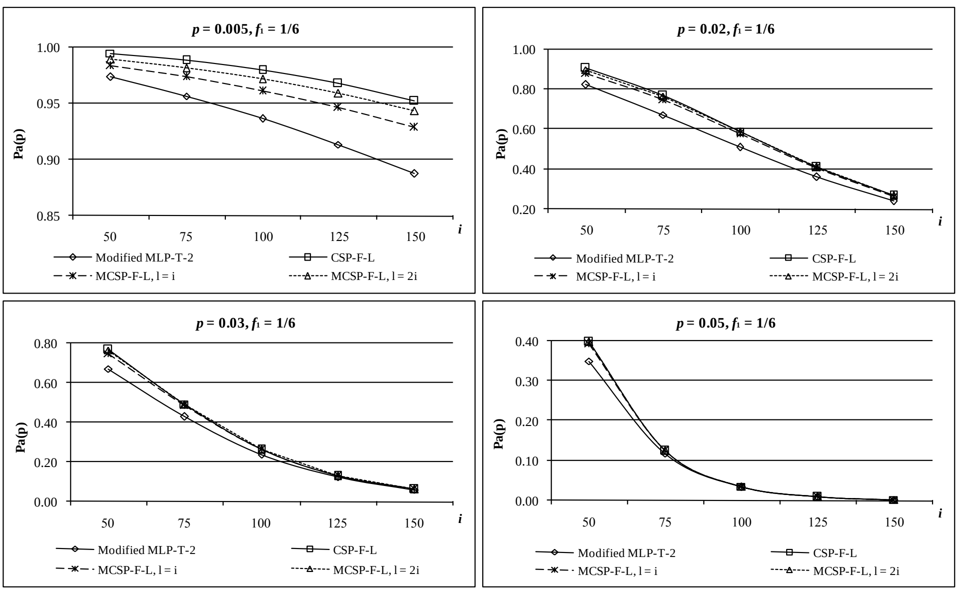

Figure 6. Pa(p) curves of MCSP-F-L, CSP-F-L and Modified MLP-T-2 plans for f_1=1/6.

4. Conclusions

In this paper, a modification has been proposed on the CSP-F-L continuous sampling plan. The resultant plan is designated as a MCSP-F-L plan. The important feature of the MCSP-F-L plan is that added stopping rule to limit sampling inspection at level 2 at the ending procedure of CSP-F-L plan. If the number of consecutive conforming units reaches some specified value the inspector has to take special action by proceeds to sampling inspection at level 1. This action prevents long length of inspection at level 2 for ensure the protection to the consumer. Theirs measures of performance have been derived using a Markov Chain model such as the average fraction inspected (AFI), the average fraction of total produced accepted on sampling basis (Pa(p)), the average outgoing quality (AOQ) and the average outgoing quality limit (AOQL). The accuracy of all the above performance measures has been tested by extensive simulations. The difference of the AFI, Pa(p) and AOQ values from the formula and from the simulations were found to agree within target values in all simulations. The attractive feature of the MCSP-F-L plan is that a smaller inspection effort is required at good incoming quality levels (p) and at high level of i, and at moderate and high level of p for low level of i when compared to the Modified MLP-T-2 plan. From an important property of the MCSP-F-L plan that modified from the CSP-F-L plan, the AFI values of the MCSP-F-L plan are greater than or equal to the AFI values of the CSP-F-L plan for all of incoming quality levels and parameters. When considering the results of the probability of acceptance (Pa(p)) comparisons, the MCSP-F-L plan gives the Pa(p) values lower than or equal to the Pa(p) values of the CSP-F-L plan and greater than or equal to the Pa(p) values of the Modified MLP-T-2 plan for all of incoming quality levels and parameters. However, the differences of the performance measures of the three plans are quite small as the value of p is increased, especially by high level of i. For values of f_1, there was a little influence on the differences of the performance measures of the three plans. Figures provided in this paper will be useful for selecting the plan parameters and comparing for given various AFI and Pa(p) values.

Acknowledgements

The authors would like to thank for the Khon Kaen University for the financial support this research.

Appendix

Glossary of symbols

S_n = the n^{th} state of the process,

$P(S_n)$ = the steady-state probability for the state S_n,

p_{in} = the probability that the process transits from state S_i to S_n in one step.

Derivation of Performance Measures of the MCSP-F-L plan

The formulation of the MCSP-F-L using the Markov Chain development is similar to Stephens [9]. Let [X_t] (t = 1, 2,…) denote a discrete-parameter Markov Chain with finite state space (S_n), n = 1,…, 3k+2i+3l+1. The states of the process are defined, in a way similar to that of Roberts [6].

These steady-state probabilities $P(S_n)$ satisfy the following conditions:

$$P(S_n) \geq 0 \quad \text{for } n = 1, 2, \ldots, 3k+2i+3l+1 \tag{10}$$

$$P(S_n) = \sum_{x=1}^{3k+2i+3l+1} P(S_x)\, p_{xn} \quad \text{for } n = 1, 2, \ldots, 3k+2i+3l+1 \tag{11}$$

$$\sum_{\text{all}\, n} P(S_n) = 1 \tag{12}$$

From conditions (11), the $P(S_n)$ can be found with the one-step transition probability matrix of the MCSP-F-L plan that showed in Table 1 for all values of n, we acquire the following:

$$P(f_1N_n) = \frac{(1-f_1)q^{n-1}}{f_1}[P(A_i) + P(f_2In_l) + \sum_{n=1}^{l} P(f_2Id_n)];\ n = 1, 2, \ldots, k \tag{13}$$

$$P(f_1In_n) = q^n[P(A_i) + P(f_2In_l) + \sum_{n=1}^{l} P(f_2Id_n)];\ n = 1, 2, \ldots, k \tag{14}$$

$$P(f_1Id_n) = pq^{n-1}[P(A_i) + P(f_2In_l) + \sum_{n=1}^{l} P(f_2Id_n)];\ n = 1, 2, \ldots, k \tag{15}$$

$$P(A_0) = p[\sum_{n=1}^{k} P(f_1Id_n) + \sum_{n=0}^{i-1} P(A_n) + \sum_{n=1}^{i-1} P(B_n)] \tag{16}$$

$$P(A_n) = q^n P(A_0);\ n = 1, 2, \ldots, i \tag{17}$$

$$P(B_n) = q^n \sum_{n=1}^{k} P(f_1Id_n);\ n = 1, 2, \ldots, i \tag{18}$$

$$P(f_2N_n) = \frac{(1-f_2)q^{n-1}}{f_2}[P(f_1In_k) + P(B_i)];\ n = 1, 2, \ldots, l \tag{19}$$

$$P(f_2In_n) = q^n[P(f_1In_k) + P(B_i)];\ n = 1, 2, \ldots, l \tag{20}$$

$$P(f_2 Id_n) = pq^{n-1}[P(f_1 In_k) + P(B_i)];\ n = 1, 2, ..., l \quad (21)$$

and from conditions (12), we get

$$\sum_{n=1}^{k}\left[P(f_1N_n) + P(f_1In_n) + P(f_1Id_n)\right] + \sum_{n=0}^{i} P(A_n) + \sum_{n=1}^{i} P(B_n) + \sum_{n=1}^{l}\left[P(f_2N_n) + P(f_2In_n) + P(f_2Id_n)\right] = 1 \quad (22)$$

By equations (13) to (22), (16) can be written as

$$P(A_0) = \frac{f_1 f_2 p(1-q^k)(1-q^i)}{D}$$

Where

$$D = f_1 q^i (1-q^l)(q^k - q^{k+i} + q^i) + f_1 f_2 (1-q^k)(1-q^i) + f_2 q^i (1-q^k)$$

The steady-state probabilities can be written as follows:

$$P(A_n) = \frac{f_1 f_2 pq^n (1-q^k)(1-q^i)}{D};\ n=1, 2, ..., i$$

$$P(B_n) = \frac{f_1 f_2 pq^{i+n} (1-q^k)}{D};\ n=1, 2, ..., i$$

$$P(f_1 N_n) = \frac{f_2 (1-f_1) pq^{i+n-1}}{D};\ n = 1, 2, ..., k$$

$$P(f_1 In_n) = \frac{f_1 f_2 pq^{i+n}}{D};\ n = 1, 2, ..., k$$

$$P(f_1 Id_n) = \frac{f_1 f_2 p^2 q^{i+n-1}}{D};\ n = 1, 2, ..., k$$

$$P(f_2 N_n) = \frac{f_1 (1-f_2) pq^{i+n-1} (q^k - q^{k+i} + q^i)}{D};\ n = 1, 2, ..., l$$

$$P(f_2 In_n) = \frac{f_1 f_2 pq^{i+n} (q^k - q^{k+i} + q^i)}{D};\ n = 1, 2, ..., l$$

$$P(f_2 Id_n) = \frac{f_1 f_2 p^2 q^{i+n-1} (q^k - q^{k+i} + q^i)}{D};\ n = 1, 2, ..., l$$

Then

$$u = \frac{\sum_{n=0}^{i} P(A_n) + \sum_{n=1}^{i} P(B_n)}{P(f_1 In_k) + P(A_i) + P(B_i)}$$

$$v = \frac{\sum_{n=1}^{k}\left[P(f_1N_n) + P(f_1In_n) + P(f_1Id_n)\right] + \sum_{n=1}^{l}\left[P(f_2N_n) + P(f_2In_n) + P(f_2Id_n)\right]}{P(f_1 In_k) + P(A_i) + P(B_i)}$$

AFI $= 1-\sum_{n=1}^{k} P(f_1 N_n) - \sum_{n=1}^{l} P(f_2 N_n)$

Pa(p) $= 1-\sum_{n=0}^{i} P(A_n) - \sum_{n=1}^{i} P(B_n)$

AOQ $= p\sum_{n=1}^{k} P(f_1 N_n) + p\sum_{n=1}^{l} P(f_2 N_n)$

By simplifying the above equations, we can get the performance measures of a MCSP-F-L plan which are given in equations (1) to (6).

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