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A Note on a Measure of Rotatability for Second Order Response Surface Designs Using Balanced Incomplete Block Designs

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Abstract

In this paper, a measure of rotatability for second order response surface designs using balanced incomplete block designs is suggested which enables us to assess the degree of rotatability for a given response surface design.

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Keywords: second order response surface designs, second order rotatable designs, measure of rotatability.

1. Introduction

Response surface methodology is a statistical technique that is very useful in design and analysis of scientific experiments. In many experimental situations the experimenter is concerned with explaining certain aspects of a functional relationship $Y=f(x_1,x_2,...,x_v)+e$, where Y is the response, $x_1,x_2,...,x_v$ are the levels of v-quantitative variables or factors and e is the random error. Response surface methods are useful where several independent variables influence a dependent variable. The independent variables are assumed to be continuous and controlled by the experimenter. The response is assumed to be a random variable. For example, if a chemical engineer wishes to find the temperature (x_1) and pressure (x_2) that maximizes the yield (response) of his process, the observed response Y may be written as a function of the levels of the temperature (x_1) and pressure (x_2) as $Y=f(x_1,x_2)+e$.

The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter [1]. Das and Narasimham [2] constructed second order rotatable designs (SORD) using balanced incomplete block designs. Draper and Guttman [3] suggested an index of rotatability. Khuri [4] introduced a measure of rotatability for response surface designs. Park et al. [5] introduced a new measure of rotatability for second order response surface designs and illustrated it for 3^k factorial and central composite designs.

2. Conditions for Second Order Rotatable Designs

Suppose we want to use the second order response surface design D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u \tag{1}$$

where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment and the e_u's are uncorrelated random errors with mean zero and variance σ^2. Then D is said to be second order rotatable design (SORD) if the variance of the estimate of $Y_u(x_1,x_2,\ldots,x_v)$ with respect to each of independent variables (x_i) is only a function of the distance ($d^2=\sum_{i=1}^{v} x_i^2$) of the point ($x_1,x_2,\ldots,x_v$) from the origin (center) of the design. Such a spherical variance function for estimation of responses in the second order response surface is achieved if the design points satisfy the following conditions.

$$\sum x_{iu} = 0, \sum x_{iu}x_{ju} = 0, \sum x_{iu}x_{ju}^2 = 0, \sum x_{iu} x_{ju} x_{ku} = 0,$$

$$\sum x_{iu}^3 = 0, \sum x_{iu}x_{ju}^3 = 0, \sum x_{iu}x_{ju} x_{ku}^2 = 0, \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0;$$

for $i \neq j \neq k \neq l$; (2)

(i) $\sum x_{iu}^2 =$ constant = $N\lambda_2$; (ii) $\sum x_{iu}^4 =$ constant = $cN\lambda_4$; for all i (3)

$\sum x_{iu}^2 x_{ju}^2 =$ constant = $N\lambda_4$; for $i \neq j$ (4)

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \tag{5}$$

$$\sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2 \tag{6}$$

where c, λ_2 and λ_4 are constants.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]} \ ,$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii}) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]} \ ,$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj}) = \frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]}$$ and other covariances vanish. (7)

The variance of the estimated response at the point $(x_{10},x_{20},\ldots,x_{v0})$ is

$$V(\hat{y}_0) = V(\hat{b}_0) + \left[V(\hat{b}_i) + 2cov(\hat{b}_o,\hat{b}_{ii})\right]d^2 + V(\hat{b}_{ii})d^4 + \sum x_{i0}^2 x_{j0}^2\left[V(\hat{b}_{ij}) + 2cov(\hat{b}_{ii},\hat{b}_{jj}) - 2V(\hat{b}_{ii})\right] \tag{8}$$

The coefficient of $\sum x_{i0}^2 x_{j0}^2$ in the above equation (8) is simplified to $(c-3)\sigma^2/(c-1)N\lambda_4$.

A second order response surface design D is said to be a SORD, if in this design c=3 and all the other conditions (2) to (7) hold.

3. SORDs Using BIBDs

Balanced Incomplete Block Design:

A BIBD denote by (v,b,r,k,λ) is an arrangement of v treatments in b blocks each containing k(<v) treatments, if (i) every treatment occurs at most once in each block, (ii) every treatment occurs in exactly r blocks and (iii) every pair of treatments occurs together in λ blocks.

Incidence Matrix:

Let (v,b,r,k,λ) denote a BIBD. Associated with any design D is the incidence matrix $N=((n_{ij})), (i=1,2,...,v; j=1,2,...,b)$ where n_{ij} denotes the number of times the i^{th} treatment occurs in the j^{th} block.

Where,

n_{ij} = 1, if i^{th} treatment occurs in the j^{th} block,

= 0, otherwise.

Fractional Replication:

A suitable choice of fractional replication of 2^k factorials, in which no interaction with less than five factors is confounded, plays an important role in the construction of rotatable designs. The notation is explained below.

I= Truncated integer space

= [k: k is a positive integer, k ≥2].

The function t(k) is defined as

t: I→I where $2^{t(k)}$ is the smallest fractional replicate of 2^k design with levels +1 or -1 such that no interaction with less than five factors is confounded.

Let (v,b,r,k,λ) denote a BIBD, $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors are confounded. $\left[1-(v,b,r,k,\lambda)\right]$ denote the design points generated from the transpose of the incidence matrix of a BIBD. $[1-(v,b,r,k,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from a BIBD by "multiplication" (c.f. Raghavarao,[7]), 2v the axial points have coordinates of the form (±a,0,0,…,0), (0, ±a,0,…,0),…, (0,0,…, ±a) etc., $(a,0,0...,0)2^1$ denote the 2v design points generated from (a,0,0...,0) point set. Let $\bigcup$ denote the union of the design points generated from different sets of points, (n_a) denote the number of replications of the axial points and n_0 denote the number of central points. The method of construction of a SORD using a BIBD is given in the following result (cf. Das and Narasimham [2]).

Result: The design points, $\left[1-\left(v,b,r,k,\lambda\right)\right]2^{t(k)} \cup n_a\,(a,0,\ldots,0)\,2^1 \cup (n_0)$ will give a v-dimensional SORD in $N = b2^{t(k)}+2vn_a+n_0$ design points, with $a^4 = \frac{(3\lambda\text{-}r)2^{t(k)}}{2n_a}$.

Note: "Multiplication" in Das and Narasimham [2] sense:

To explain Das and Narasimham 'multiplication' which is very useful in the construction of rotatable designs, we adopt notations given by Das and Narasimham (Das and Narasimham [2]; Das and Giri [6]; Raghavarao [7]), namely (i) Factorial combinations of levels α, β, etc., together with zero; (ii) Factorial combinations of levels ±1; (iii) Combinations when each of o, α, β, etc., is associated with ±1, through 'multiplication'. The first type of factorial combinations is called combinations of unknown levels. The second type of combinations is called associated combinations. The third type of combinations generated through 'multiplication' will actually constitute the design points.

(i) Generation of Combinations of Unknown Levels Through BIBDs:

Let us consider a BIBD with parameters $\left(v,b,r,k,\lambda\right)$. Let the design be in the form of a b×v matrix, the elements of which are one and zero. If in any block i a particular treatment j occurs, the element n_{ij} in that block corresponding to that treatment will be α (an unknown level) and will be zero, if otherwise. Each row of the matrix corresponding to a block of the BIBD can be considered to give a combination of the unknown level α and zero. The b combinations of the unknown levels thus obtained from a BIBD are denoted by $\alpha\text{-}\left(v,b,r,k,\lambda\right)$.

(ii) Associate Combinations:

If $\left(v,b,r,k,\lambda\right)$ is the BIBD under consideration, then the combinations in the smallest fractional replicate of 2^k design with levels ±1, in which no interaction with less than five factors is confounded, are called associate combinations.

(iii) Generation of Design Points From BIBD Through 'Multiplication' :

By multiplying each of the $\alpha\text{-}\left(v,b,r,k,\lambda\right)$ combinations from the BIBD with $2^{t(k)}$ associate combinations, we get $b\times 2^{t(k)}$ design points. The $b\times 2^{t(k)}$ design points are denoted as $\left[\alpha\text{-}\left(v,b,r,k,\lambda\right)\right]\times 2^{t(k)}$.

4. Conditions of A Measure of Rotatability for Second Order Response Surface Designs

Following Box and Hunter [1], Das and Narasimham [2], Park et al [5], conditions (2) to (6) and (7) give the necessary and sufficient conditions for a measure of rotatability for any general second order response surface designs. Further we have,

$\mathrm{V(b_i)}$ are equal for i,

$\mathrm{V(b_{ii})}$ are equal for i,

$\mathrm{V(b_{ij})}$ are equal for $\mathrm{i, j}$, where $i \neq j$,

$$\mathrm{Cov(b_i,b_{ii})=Cov(b_i,b_{ij})=Cov(b_{ii},b_{ij})=Cov(b_{ij},b_{il})=0}$$

for all $i \neq j, j \neq l, l \neq i$. (9)

Park et al [5] suggested that if the conditions in (2) to (6) together with (7) and (9) are met, then the following measure $(\mathrm{P_v(D)})$ given below can be used to assess the degree of rotatability for any general second order response surface design (cf. Park et al., [5], page 661).

$$\mathrm{P_v(D)}=\frac{1}{1+\mathrm{R_v(D)}}. \tag{10}$$

where

$$\mathrm{R_V(D)}=\left[\frac{\mathrm{N}}{\sigma^2}\right]^2 \frac{6\mathrm{v}(\mathrm{V}(\hat{\mathrm{b}}_{\mathrm{ij}})+2\mathrm{cov}(\hat{\mathrm{b}}_{\mathrm{ii},}\hat{\mathrm{b}}_{\mathrm{jj}})-2\mathrm{V}(\hat{\mathrm{b}}_{\mathrm{ii}}))^2(\mathrm{v}-1)}{(\mathrm{v}+2)^2(\mathrm{v}+4)(\mathrm{v}+6)(\mathrm{v}+8)\mathrm{g}^8}, \tag{11}$$

and g is the scaling factor.

On simplification of $\mathrm{V}(\hat{\mathrm{b}}_{\mathrm{ij}})+2\mathrm{cov}(\hat{\mathrm{b}}_{\mathrm{ii},}\hat{\mathrm{b}}_{\mathrm{jj}})-2\mathrm{V}(\hat{\mathrm{b}}_{\mathrm{ii}})$ becomes $\frac{(c-3)\sigma^2}{(c-1)N\lambda_4}$.

Thus, $\mathrm{R_v(D)}$ becomes

$$\mathrm{R_V(D)}=\left[\frac{\mathrm{c-3}}{\mathrm{c-1}}\right]^2 \frac{6\mathrm{v}(\mathrm{v}-1)}{\lambda_4^{\,2}(\mathrm{v}+2)^2(\mathrm{v}+4)(\mathrm{v}+6)(\mathrm{v}+8)\mathrm{g}^8}. \tag{12}$$

5. A Measure of Rotatability for Second Order Response Surface Designs Using BIBD

In this section the method of a measure of rotatability for second order response surface designs using BIBD is suggested below.

Let (v,b,r,k,λ) denote a BIBD, $2^{t(k)}$ denote a fractional replicate of 2^k in +1 and -1 levels, in which no interaction with less than five factors is confounded. $[1-(v,b,r,k,\lambda)]$ denote the design points generated from the transpose of incidence matrix of BIBD. $[1-(v,b,r,k,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from BIBD by "multiplication", $(a,0,0,\ldots,0)2^1$ denote the design points generated from (a,0,0...,0) point set, and $\bigcup$ denotes union of the design points generated from different sets of points. Let (n_a) denote the number of replications of axial points, n_0 denote the number of central points. Thus, we have the total number of experimental points $N=b2^{t(k)}+2vn_a+n_0$. The method of construction of a measure of rotatability for second order response surface designs using BIBD is given in the following result.

Result: The design points, $\left[1-\left(v,b,r,k,\lambda\right)\right]2^{t(k)}\bigcup n_a\,(a,0,\ldots,0)\,2^1\bigcup(n_0)$ will give a v-dimensional measure of rotatability for second order response surface designs using BIBD in N design points, with level 'a' pre-fixed and $c=\dfrac{r2^{t(k)}+2n_a a^4}{\lambda 2^{t(k)}}$.

From (10) we can obtain the measure of rotatability values for second order response surface designs using BIBD. From (12) we have

$$R_V(D)=\left[\frac{c-3}{c-1}\right]^2\frac{6v(v-1)}{\lambda_4{}^2(v+2)^2(v+4)(v+6)(v+8)g^8}$$

where

$$g=\begin{cases}\dfrac{1}{a},\ if\ a<\sqrt{2^{t(k)-1}(b-r)+v}\\[2ex] \dfrac{1}{\sqrt{2^{t(k)-1}(b-r)+v}},\ otherwise\end{cases}$$

The following table gives the values of a measure of rotatability for second order response surface designs using BIBD. It can be verified that $P_v(D)$ is 1 if and only if the design is rotatable, and it is smaller than one for a non-rotatable designs.

Table 1. Values of a measure of rotatability for second order response surface designs using BIBD**.**

(3,3,2,2,1), N=19, a* = 1.1892				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.5	1.000000	5.209235209×10^{-3}	0.9948
1.1	2.7321	0.909090909	2.405163647×10^{-3}	0.9976
1.1892	2.99998	0.840901446	2.6852345×10^{-11}	1.0000
1.2	3.0368	0.833333333	6.580597223×10^{-5}	0.9999
1.3	3.42805	0.769230769	0.011886036	0.9883
1.6	5.2768	0.625	0.636669281	0.6367
1.9	8.51605	0.526315789	4.288667252	0.1891
2.2	13.7128	0.454545454	18.26928236	0.0519
2.5	21.53125	0.447213595	23.87124893	0.0402
2.8	32.7328	0.447213595	25.72475955	0.0374
3.1	48.17605	0.447213595	26.87013539	0.0359
3.4	68.8168	0.447213595	27.59913264	0.0350
3.7	95.70805	0.447213595	28.07744578	0.0344
4.0	130.0000	0.447213595	28.40040383	0.0340
4.3	172.94005	0.447213595	28.62423443	0.0338
4.6	225.8728	0.447213595	28.78304771	0.0336
4.9	290.24005	0.447213595	28.89812235	0.0334
(4,4,3,3,2), N=41, a* = 1.8612				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	1.6250	1.0000000	0.066211263	0.9379
1.1	1.6830	0.909090909	0.109026982	0.9017
1.2	1.7592	0.833333333	0.157118617	0.8642
1.3	1.8570	0.769230769	0.198491529	0.8344
1.6	2.3192	0.625	0.156481925	0.8647
1.8612	2.99996	0.537287771	4.833404515×10^{-3}	1.0000
1.9	3.1290	0.526315789	8.531442691×10^{-3}	0.9915
2.2	4.4282	0.454545454	1.302907889	0.4342
2.5	6.3828	0.4	8.244112871	0.1082
2.8	9.1832	0.357142857	29.50740509	0.0328
3.1	13.0440	0.35355339	38.96893641	0.0251
3.4	18.2042	0.35355339	43.76275366	0.0223
3.7	24.9270	0.35355339	47.05745484	0.0208
4.0	33.5	0.35355339	49.34912032	0.0199
4.3	44.2350	0.35355339	50.96916773	0.0192
4.6	57.4682	0.35355339	52.1344285	0.0188
4.9	73.5600	0.35355339	52.98696677	0.0185

Table 1. (Continued).

(5,5,4,4,3), N=91, a* = 2.5149				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	1.3750	1.00000000	0.128425458	0.8862
1.1	1.3943	0.909090909	0.243064079	0.8045
1.2	1.4197	0.833333333	0.416841294	0.7058
1.3	1.4523	0.769230769	0.653102099	0.6049
1.6	1.6064	0.625	1.551403191	0.3919
1.9	1.8763	0.526315789	1.909698112	0.3437
2.2	2.3094	0.454545454	1.04399123	0.4892
2.5	2.9609	0.4	4.141136656×10^{-3}	0.9959
2.5149	3.0001	0.397630124	2.111539992×10^{-8}	0.99999
2.8	3.8944	0.357142857	2.467275589	0.2884
3.1	5.1813	0.322580645	15.8750742	0.0593
3.4	6.9014	0.294117647	53.37883854	0.0184
3.7	9.1423	0.277350098	111.1602436	8.9158×10^{-3}
4.0	12.0000	0.277350098	130.761528	7.5895×10^{-3}
4.3	15.5783	0.277350098	145.4155414	6.8299×10^{-3}
4.6	19.9894	0.277350098	156.3557783	6.3550×10^{-3}
4.9	25.3533	0.277350098	164.5690318	6.0398×10^{-3}
(6,10,5,3,2), N=93, a* = 1.4142				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.6250	1.00000000	3.012069483×10^{-3}	0.9970
1.1	2.6830	0.909090909	4.300908991×10^{-3}	0.9957
1.2	2.7592	0.833333333	4.556608144×10^{-3}	0.9955
1.3	2.8570	0.769230769	2.735410385×10^{-3}	0.9973
1.4142	2.99998	2.414292612×10^{-4}	$8.322262087\times10^{-11}$	1.0000
1.6	3.3192	0.625	0.04601701	0.9560
1.9	4.1290	0.526315789	1.250607109	0.4443
2.2	5.4282	0.454545454	9.332652216	0.0968
2.5	7.3828	0.4	40.69217092	0.0240
2.8	10.1832	0.357142857	130.7438481	7.5905×10^{-3}
3.1	14.0440	0.322580645	345.8072532	2.8834×10^{-3}
3.4	19.2042	0.294117647	800.2994357	1.2480×10^{-3}
3.7	25.9270	0.27027027	1680.650644	5.9465×10^{-4}
4.0	34.5	0.25	3277.333259	3.0503×10^{-4}
4.3	45.2350	0.232558139	6026.563977	1.6590×10^{-4}
4.6	58.4682	0.217391304	10563.42593	9.4657×10^{-5}
4.9	74.5600	0.204081632	17788.3247	5.6213×10^{-5}

Table 1. (Continued).

(7,7,4,4,2), N=127, a* = 2.0000				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.0625	1.0000000	0.017786085	0.9825
1.1	2.0915	0.909090909	0.033925685	0.9672
1.2	2.1296	0.833333333	0.058322223	0.9449
1.3	2.1785	0.769230769	0.090549754	0.9170
1.6	2.4096	0.625	0.172129967	0.8531
1.9	2.8145	0.526315789	0.040547692	0.9610
2.0000	3.0000	0.5	0	1.0000
2.2	3.4641	0.454545454	0.444716903	0.6922
2.5	4.4414	0.4	6.11528491	0.1405
2.8	5.8416	0.357142857	29.73089869	0.0325
3.1	7.7720	0.322580645	96.75124637	0.0102
3.4	10.3521	0.294117647	252.134093	3.9505×10^{-3}
3.7	13.7135	0.27027027	569.8257569	1.7518×10^{-3}
4.0	18.0000	0.25	1165.628927	8.5717×10^{-4}
4.3	23.3675	0.232558139	2214.032222	4.5146×10^{-4}
4.6	29.9841	0.217391304	3969.671786	2.5185×10^{-4}
4.9	38.0300	0.204081632	6794.182523	1.4716×10^{-4}
(8,14,7,4,3), N=241, a* = 2.0000				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.3750	1.00000000	6.510528761×10^{-3}	0.9935
1.1	2.3943	0.909090909	0.012744671	0.9874
1.2	2.4197	0.833333333	0.022633581	0.9779
1.3	2.4523	0.769230769	0.036551007	0.9647
1.6	2.6064	0.625	0.081250169	0.9249
1.9	2.8763	0.526315789	0.023245803	0.9773
2.0000	3.0000	0.5	0	1.0000
2.2	3.3094	0.454545454	0.310374023	0.7631
2.5	3.9609	0.4	5.064230864	0.1649
2.8	4.8944	0.357142857	28.17008329	0.0343
3.1	6.1813	0.322580645	101.3190011	9.7734×10^{-3}
3.4	7.9014	0.294117647	283.8303636	3.5109×10^{-3}
3.7	10.1423	0.27027027	675.5253528	1.4781×10^{-3}
4.0	13.0000	0.25	1434.098766	6.9682×10^{-4}
4.3	16.5783	0.232558139	2798.080788	3.5726×10^{-4}
4.6	20.9894	0.217391304	5116.324916	1.9541×10^{-4}
4.9	26.3533	0.204081632	8885.009566	1.1254×10^{-4}

Table 1. (Continued).

(9,18,8,4,3), N=307, a* = 1.6818				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.7083	1.00000000	1.284214256×10^{-3}	0.9987
1.1	2.7277	0.909090909	2.3464803×10^{-3}	0.9977
1.2	2.7531	0.833333333	3.758561512×10^{-3}	0.9963
1.3	2.7857	0.769230769	5.177447206×10^{-3}	0.9948
1.6	2.9397	0.625	1.826582389×10^{-3}	0.9982
1.6818	3.000006	1.466705779×10^{-4}	$2.277485897\times10^{-11}$	1.0000
1.9	3.2097	0.526315789	0.067368798	0.9369
2.2	3.6427	0.454545454	1.430031252	0.4115
2.5	4.2943	0.4	10.37673954	0.0879
2.8	5.2277	0.357142857	46.21517532	0.0212
3.1	6.5147	0.322580645	152.6275018	6.5093×10^{-3}
3.4	8.2347	0.294117647	411.8931087	2.4219×10^{-3}
3.7	10.4757	0.27027027	963.1708307	1.0372×10^{-3}
4.0	13.3333	0.25	2026.791107	4.9315×10^{-4}
4.3	16.9117	0.232558139	3936.260021	2.5398×10^{-4}
4.6	21.3227	0.217391304	7179.398474	1.3927×10^{-4}
4.9	26.6867	0.204081632	12450.02344	8.0315×10^{-5}
(10,18,9,5,4), N=309, a* = 2.2134				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.2813	1.0000000	6.822691177×10^{-3}	0.9932
1.1	2.2958	0.909090909	0.01372823	0.9865
1.2	2.3148	0.833333333	0.025318212	0.9753
1.3	2.3393	0.769230769	0.043048643	0.9587
1.6	2.4548	0.625	0.130777408	0.8843
1.9	2.6573	0.526315789	0.157494675	0.8639
2.2	2.9821	0.454545454	9.757732223×10^{-4}	0.9990
2.2134	3.00005	0.45179362	7.500791242×10^{-9}	0.99999
2.5	3.4707	0.4	1.200717551	0.4544
2.8	4.1708	0.357142857	11.16759703	0.0822
3.1	5.1360	0.322580645	49.31776159	0.0199
3.4	6.4261	0.294117647	154.3543603	6.4369×10^{-3}
3.7	8.1068	0.27027027	393.2138588	2.5367×10^{-3}
4.0	10.25	0.25	872.8512405	1.1444×10^{-3}
4.3	12.9338	0.232558139	1755.852485	5.6920×10^{-4}
4.6	16.2421	0.217391304	3280.606126	3.0473×10^{-4}
4.9	20.2650	0.204081632	5786.707761	1.7278×10^{-4}

Table 1. (Continued).

(11,11,5,5,2), N=199, a* = 1.6818				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.5625	1.00000000	2.443912813×10^{-3}	0.9976
1.1	2.5915	0.909090909	4.402154315×10^{-3}	0.9956
1.2	2.6296	0.833333333	6.924677278×10^{-3}	0.9931
1.3	2.6785	0.769230769	9.328581446×10^{-3}	0.9908
1.6	2.9096	0.625	3.000416575×10^{-3}	0.9970
1.6818	3.000009	0.594601022	3.62564502×10^{-11}	1.0000
1.9	3.3145	0.526315789	0.097755296	0.9109
2.2	3.9641	0.454545454	1.809712371	0.3559
2.5	4.9414	0.4	11.54038135	0.0797
2.8	6.3416	0.357142857	46.08929813	0.0212
3.1	8.2720	0.322580645	139.7354321	7.1055×10^{-3}
3.4	10.8521	0.294117647	353.6022365	2.8201×10^{-3}
3.7	14.2135	0.27027027	788.5516563	1.2665×10^{-3}
4.0	18.5000	0.25	1602.643311	6.2358×10^{-4}
4.3	23.8675	0.232558139	3034.036366	3.2949×10^{-4}
4.6	30.4841	0.217391304	5430.247049	1.8412×10^{-4}
4.9	38.5300	0.204081632	9284.779197	1.0769×10^{-4}
(12,22,11,6,5), N=729, a* = 2.8284				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.2125	1.00000000	6.143266397×10^{-3}	0.9939
1.1	2.2183	0.909090909	0.012852057	0.9873
1.2	2.2259	0.833333333	0.024966583	0.9756
1.3	2.2357	0.769230769	0.045447276	0.9565
1.6	2.2819	0.625	0.196266146	0.8359
1.9	2.3629	0.526315789	0.540474554	0.6492
2.2	2.4928	0.454545454	0.922470557	0.5202
2.5	2.6882	0.4	0.757561761	0.5690
2.8	2.9683	0.357142857	0.014252954	0.9859
2.8284	2.99997	0.353556781	1.40431554×10^{-8}	0.99999
3.1	3.3544	0.322580645	2.814393373	0.2622
3.4	3.8704	0.294117647	23.91442167	0.0401
3.7	4.5427	0.27027027	96.99963775	0.0102
4.0	5.4000	0.25	283.9610538	3.5093×10^{-3}
4.3	6.4735	0.232558139	685.5100696	1.4566×10^{-3}
4.6	7.7968	0.217391304	1454.185059	6.8720×10^{-4}
4.9	9.4060	0.204081632	2810.771207	3.5565×10^{-4}

Table 1. (Continued).

(13,26,12,6,5), N=859, a* = 2.6321				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.4125	1.00000000	3.058135363×10^{-3}	0.9970
1.1	2.4183	0.909090909	6.374096688×10^{-3}	0.9937
1.2	2.4259	0.833333333	0.01232033	0.9878
1.3	2.4357	0.769230769	0.022276897	0.9782
1.6	2.4819	0.625	0.092794422	0.9151
1.9	2.5629	0.526315789	0.234824137	0.8098
2.2	2.6928	0.454545454	0.319421122	0.7579
2.5	2.8883	0.4	0.094418417	0.9137
2.6321	2.99996	0.379924774	1.952204209×10^{-8}	0.99999
2.8	3.1683	0.357142857	0.402445263	0.7130
3.1	3.5544	0.322580645	7.10201511	0.1234
3.4	4.0704	0.294117647	38.36737347	0.0254
3.7	4.7427	0.27027027	134.6192694	7.3736×10^{-3}
4.0	5.6000	0.25	370.1083124	2.6946×10^{-3}
4.3	6.6735	0.232558139	866.2108006	1.1531×10^{-3}
4.6	7.9968	0.217391304	1807.451019	5.5296×10^{-4}
4.9	9.6060	0.204081632	3461.473486	2.8881×10^{-4}
(15,15,7,7,3), N=991, a* = 2.8284				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.3438	1.00000000	3.018689892×10^{-3}	0.9970
1.1	2.3486	0.909090909	6.330214105×10^{-3}	0.9937
1.2	2.3549	0.833333333	0.012335027	0.9878
1.3	2.3631	0.769230769	0.022541456	0.9780
1.6	2.4016	0.625	0.099085858	0.9098
1.9	2.4691	0.526315789	0.280739268	0.7808
2.2	2.5774	0.454545454	0.498659245	0.6673
2.5	2.7402	0.4	0.430313296	0.6991
2.8	2.9736	0.357142857	8.555997594×10^{-3}	0.9915
2.8284	2.99997	0.353556781	8.475302457×10^{-9}	0.99999
3.1	3.2953	0.322580645	1.787090102	0.3588
3.4	3.7254	0.294117647	16.0102761	0.0588
3.7	4.2856	0.27027027	68.06228677	0.0145
4.0	5.0000	0.25	207.3656233	4.7993×10^{-3}
4.3	5.8946	0.232558139	517.3747802	1.9291×10^{-3}
4.6	6.9974	0.217391304	1127.210533	8.8636×10^{-4}
4.9	8.3383	0.204081632	2225.881372	4.4906×10^{-4}

'*' indicates values of SORD using BIBD.

6. Conclusion

The concept of rotatability has been very important criteria in response surface designs. If rotatability in response surface designs is unachievable, it is good to achieve nearly rotatability in response surface designs. In such situations we study sensitivity of the spherical variance function for the disturbance that occur due to the modifications in these values of a and c.

In this paper, a note on a measure of rotatability for second order response surface designs using BIBD for 3≤v≤15 has been proposed which enables us to assess the degree of rotatability for a given second order response surface design. This measure, $P_v(D)$ has the value one if and only if the design D is rotatable, and it is smaller than one for a non-rotatable designs.

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