Thailand Statistician
January 2015; 13(1): 111-124
http://statassoc.or.th
Contributed paper

An Approximation of ARL for Poisson GWMA Using Markov Chain Approach

Yada Phengsalae, Yupaporn Areepong and Saowanit Sukparungsee*
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand.
*Corresponding author; e-mail: swns@kmutnb.ac.th

Received: 26 June 2014
Accepted: 10 October 2014

Abstract

The objective of this research is to propose an approximation of Average Run Length (ARL) by Markov Chain Approach (MCA) for Generally Weighted Moving Average Control Chart (GWMA) when observations are from Poisson distribution. The numerical results obtained from MCA are compared with the results obtained from Monte Carlo Simulation (MC). The performance of control charts are compared in term of monitoring of a change in the process mean defined by out-of-control Average Run Length (ARL_1). The results found that the numerical results obtained from MCA are as good as from MC, however, MCA is very time saving. Furthermore, the performance of GWMA chart is superior to EWMA chart when the magnitudes of changes are small $(\delta \leq 0.20)$.

Keywords: generally weighted moving average, exponentially weighted moving average, monitoring, average run length, markov chain approach, monte carlo simulation.

1. Introduction

Statistical Process Control (SPC) is the method for monitoring process quality characteristic. Through control charts is one of important tool of SPC, because it can detect whether the present manufacturing process. So, the processes detect the occurrence of process variations. When the process is in-control it should have minimize false alarm rate and maximize true alarm rate when the process is out-of-control. The performance of control charts are measured by the Average Run Length (ARL). The ARL_0 is defined as in-control ARL and the ARL_1 is defined as out-of-control ARL.

A Poisson distribution is often employed to control manufacturing processes when the quality measure X is the number of nonconformities or defects per unit from process. For example, number of nonconformities of produce buttons or weaving clothes. Assume that $X_1, X_2,...$ are independent and identically distributed with a mean $\alpha.$ When the process is in-control define $\alpha = \alpha_0$ and $\alpha = \alpha_1$ when the process is out-of-control. The process is out-of-control when the mean changes to some another value, say, $\alpha_1 > \alpha_0$ or $\alpha_1 < \alpha_0$. Generally, the control charts have proposed to fast detect of changes early in a process. These changes probably occur from new controller that they have not sufficient experience.

Usually, the c chart is a chart for monitoring Poisson observations. However, it is insensitive to small process changes such $\delta < 3\sigma.$ In literatures, various control charts have been developed to enhance the ability of detection small process changes. Roberts [1] proposed Exponentially Weighted Moving Average Control Chart (EWMA). Borror et al. [2] presented EWMA chart for monitoring Poisson observations showed that the performance of EWMA chart is superior to the c chart. Zhang et al. [3] introduced Double Exponentially Weighted Moving Average Control Chart (DEWMA) for Poisson observations and showed that this chart is more sensitive to small process changes than the EWMA chart, however, it has a larger than standard deviation of in-control average run length ($SARL_0$) than the EWMA chart. Sheu and Lin [4] developed Generally Weighted Moving Average Control Chart (GWMA) for monitoring process changes. This chart has a better than other control charts especially sensitive for detecting small process changes. Sheu and Yang [5] proposed GWMA chart for monitoring Poisson observations. The results found that GWMA chart perform better than c and EWMA charts for large process changes.

In this paper, we propose Markov Chain Approach (MCA) for evaluating Average Run Length (ARL) of Generally Weighted Moving Average Control Chart

(GWMA) for Poisson observations and compared the performance of GWMA and EWMA charts.

2. Control Chart

2.1 Generally Weighted Moving Average Control Chart: GWMA

The GWMA chart was first published by [5] is weighted moving average of sequential historical observations. Each observation is a different weight that decreases from the present period to past periods such that it can reflect the important observations on recent process. This chart is developed and implemented method from EWMA chart by adding an adjustment smoothing constant (w). If the weighted historical observation constant equal to $q=1-\lambda$ and $w=1$, then the GWMA chart coincides the EWMA chart.

The statistic of GWMA chart is as following

$$Y_t=\sum_{i=1}^{t}(q^{(i-1)^w}-q^{i^w})X_{t-i+1}+q^{t^w}Y_0. \quad (1)$$

By using geometric series can be rewritten as

$$Y_t=\frac{(1-q)(q-1)-(q-1)q(q-1)}{(q-1)(1-q)}X_{t-i+1}+q^wY_0 \quad (2)$$

where

Y_t is the GWMA statistic at time t^{th}, where the initial statistic value $Y_0=\alpha_0$

X_{t-i+1} is the Poisson observations at the $t-i+1^{th}$; t = 2, 3, …

q is a weighted historical observations constant $(0\le q\le 1)$

w is an adjustment smoothing constant $(w>0)$

Mean and variance of GWMA statistic are $E(Y_t)=\alpha_0$ and $Var(Y_t)=\sigma_{Y_t}^2=Q_t\sigma^2$, respectively. Therefore, the control limits of GWMA chart are

Upper control limit: $UCL=\alpha_0+L\sigma\sqrt{Q_t}=h_U$ (3)

Center line: $CL=\alpha_0$

Lower control limit: $LCL=\alpha_0-L\sigma\sqrt{Q_t}=h_L=0$ (4)

where

$Q_t = \sum_{i=1}^{t} (q^{(i-1)^w} - q^{i^w})^2$ and L is the width of control limit.

We let $LCL = h_L = 0$ as we considered GWMA chart for monitoring the case of increasing of process mean and the number of nonconformities cannot be less than 0.

2.2 Exponentially Weighted Moving Average Control Chart: EWMA

The EWMA chart was first introduced by [2] is a weighted moving average of sequential historical observations same GWMA chart but the weighted is less than GWMA chart. It can detect the process mean changes are small $(\delta < 1.5\sigma)$ [6].

The statistic of EWMA chart is as following

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1} \tag{5}$$

where

Z_t is the EWMA statistic at time t^{th}, where the initial statistic value $Z_0 = \alpha_0$

X_t is the Poisson observations at the t^{th} time; t = 1, 2, …

λ is a weighted historical observations constant $(0 \le \lambda \le 1)$.

Mean and variance of EWMA statistic are $E(Z_t) = \alpha_0$ and $Var(Z_t) = \sigma^2_{Z_t} = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right]$, respectively. Therefore, the control limits of EWMA chart are

Upper control limit: $$UCL = \alpha_0 + H\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]} = h_U \tag{6}$$

Center line: $$CL = \alpha_0$$

Lower control limit: $$LCL = \alpha_0 - H\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]} = h_L = 0 \tag{7}$$

where L is the width of control limit and let $LCL = h_L = 0$ as we considered EWMA chart for monitoring the case of increasing of mean and the number of nonconformities cannot be less than 0.

3. Average Run Length: ARL

Average Run Length (ARL) is the expected number of samples obtained before a change in process is detected. The ARLs have two values, first, ARL before an out-of-control condition is detected when the process is in control defined as ARL_0 and second, ARL before an out-of-control condition is detected after process mean changed defined as ARL_1.

3.1 Approximation of ARL using Markov Chain Approach: MCA

Lucas and Saccucci [7] proposed Markov Chain Approach for approximate ARL *t* state is in-control process where they assume that observation x_j ; $j = 1, 2, ..., n$ is in-control state and $j = n+1$ is out-of-control state. The transition probability, P_{ij} , is the probability of moving from state i to state j in one step and is given by

$$P_{ij} = (X_{ij} = x_j \,|\, X_t = x_i). \tag{8}$$

We can replace to the transition matrix $(\mathbf{P})$ and element of matrix (P_{ij}) is

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1n} & | & P_{1,n+1} \\ \vdots & \ddots & \vdots & | & \vdots \\ P_{n1} & \cdots & P_{nn} & | & P_{n,n+1} \\ --- & --- & --- & | & --- \\ 0 & \cdots & 0 & | & 1 \end{bmatrix} \text{ or } \mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1(n+1)} \\ \vdots & \ddots & \vdots \\ P_{(n+1)1} & \cdots & P_{(n+1)(n+1)} \end{bmatrix} \text{ or}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{R} & (\mathbf{I}-\mathbf{R})\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix} \tag{9}$$

where

$\mathbf{R}$ is the $n \times n$ transition probability matrix among the in-control states

$\mathbf{I}$ is the $n \times n$ identity matrix

$\mathbf{1}$ is the $n \times 1$ column vector of ones

$\mathbf{0}$ is the $1 \times n$ row vector of zeros

1 is the scalar of one.

An approximation of ARL by using MCA for detecting mean changes of process is in interval of lower control limit and upper control limit. The region of in-control state divided into n subintervals.

The j^{th} subinterval of upper control limit (U_j), j^{th} subinterval of lower control limit (L_j) and the i^{th} subinterval of midpoint (m_i) are given by

$$U_j = h_L + \frac{j(h_U - h_L)}{n}$$

$$m_i = h_L + \frac{(2i-1)(h_U - h_L)}{2n}$$

$$L_j = h_L + \frac{(j-1)(h_U - h_L)}{n}.$$

Consequently, the transition probability equation (P_{ij}) can be rewritten as

$$P_{ij} = P(L_j \le Z_t \le U_j \,|\, Z_{t-1} = m_i) \qquad (10)$$

and substitute GWMA statistic (Y_t), L_j, U_j and m_i into Eq. (10). This transition probability equation is

$$P_{ij} = P(L_j < \frac{(1-q)(q-1)-(q-1)q(1-q)}{(q-1)(1-q)} X_{t-i+1} + q^w Y_{t-1} < U_j \,|\, Y_{t-1} = m_i)$$

$$= P\left(L_j < \frac{(1-q)(q-1)-(q-1)q(1-q)}{(q-1)(1-q)} X_{t-i+1} + q^w m_i < U_j \right)$$

$$= P\left(\begin{array}{l} \dfrac{[2nh_L + 2(j-1)(h_U - h_L) - 2nq^w h_L - q^w(2i-1)(h_U - h_L)](q-1)(1-q)}{2n[(1-q)(q-1)-(q-1)q(1-q)]} < X_{t-i+1} \\ < \dfrac{[2nh_L + 2j(h_U - h_L) - 2nq^w h_L - q^w(2i-1)(h_U - h_L)](q-1)(1-q)}{2n[(1-q)(q-1)-(q-1)q(1-q)]} \end{array} \right). \qquad (11)$$

Besides, we substitute EWMA statistic (Z_t), L_j, U_j and m_i into Eq. (10). This transition probability equation can be written as

$$P_{ij} = P(L_j < \lambda X_t + (1-\lambda) Z_{t-1} < U_j \,|\, Z_{t-1} = m_i)$$

$$= P(L_j < \lambda X_t + (1-\lambda) m_i < U_j)$$

$$= P\left(h_L + \frac{h_U - h_L}{2n\lambda}(2(j-1) - (1-\lambda)(2i-1)) < X_t < h_L + \frac{h_U - h_L}{2n\lambda}(2j - (1-\lambda)(2i-1)) \right). \qquad (12)$$

We define the transition probability matrix from state i to state j in i^{th} order as

$$\mathbf{P}^i = \begin{bmatrix} \mathbf{R}^i & (\mathbf{I}-\mathbf{R}^i)\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix} \tag{13}$$

where

$(\mathbf{I}-\mathbf{R}^i)\mathbf{1}$	is the $n \times n$ transition probability vector state $i \le n+1$ in i^{th} order
$\mathbf{R}^i$	is the $n \times n$ transition probability matrix among the in-control states in i^{th} order
$\mathbf{0}$	is the $n \times 1$ column vector of ones
1	is the scalar of one

The approximation ARL of MCA is given by

$$ARL(t) = \sum_{i=1}^{\infty} iP(RL = i) \tag{14}$$

and then substitute $P(RL=i) = \mathbf{p}^{(i)T}(\mathbf{R}^{i-1} - \mathbf{R}^i)\mathbf{1}$ in Eq. (14). The ARL can be rewritten as

$$\begin{aligned} ARL(t) &= \sum_{i=1}^{\infty} i\mathbf{P}^{(i)T}(\mathbf{R}^{i-1} - \mathbf{R}^i)\mathbf{1} \\ &= \sum_{i=1}^{\infty} \mathbf{P}^{(i)T}\mathbf{R}^{i-1}\mathbf{1} \\ &= \mathbf{P}^{(i)T}(\mathbf{I}-\mathbf{R})^{-1}\mathbf{1} \end{aligned} \tag{15}$$

where $\mathbf{P}^{(i)T}$ is the initial probability vector $\left[0, \quad \ldots, \quad 0, \quad 1, \quad 0, \quad \ldots, \quad 0\right]_{1\times n}$.

3.2 Approximation of ARL using Monte Carlo Simulation: MC

The Monte Carlo Simulation is the classical method to evaluate the ARL values which the closed-form formula and the explicit expression are not exist. In addition, the results obtained from MC use for checking an accuracy the results from other approaches.

The approximation ARL by MC is given by

$$ARL = \frac{\sum_{t=1}^{N} RL_t}{N}.$$

The standard deviations of ARL *(SARL)* as

$$SARL = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(RL_t - ARL)^2}.$$

The time used for simulation ARL (CPU Times) is

$$CPUTimes = \sum_{t=1}^{N} T_t$$

where

RL_t is the number of observations used to monitoring before out-of-control in simulation t^{th} round

N is the number simulation each situations, in this paper we assume that N = 50,000 runs

T_t is CPU Times for simulation ARL t^{th} round with CPU i-core 5.

4. Numerical Results

In this section, an approximation ARL of GWMA chart using MCA and MC approaches and comparison of performance between GWMA and EWMA charts for Poisson observations are presented.

Table 1 to 3 show the accuracy of the numerical results of ARL for GWMA chart obtained from MCA and MC when observations are from Poisson distribution. We assumed that the ARL_0 values are 300, 370 and 500 the mean of process $\alpha_0 = 1$ and the magnitudes of change in the process mean δ = 0.00, 0.01, 0.05, 0.1 and 0.2, respectively. The results found that the numerical results obtained from MCA are in good agreement with the results obtained from MC. Then, we use MCA for evaluating the ARL of GWMA and EWMA charts.

Table 4 to 6 show comparison of performance of GWMA and EWMA charts by ARL_1. We assume that ARL_0 values are 300, 370 and 500, the mean of process $\alpha_0 = 1$ and the magnitudes of change in the process mean δ = 0.00, 0.01, 0.05, 0.1 and 0.2, respectively. The results found that the performance of GWMA chart is superior to EWMA chart for all levels of δ.

Table 1. ARL of GWMA chart by MCA and MC when $q = 0.10$ and ARL_0 = 300.

w	δ	Methods	
		MCA	MC
0.10 (h_U = 8.664)	0.00	300.036 (79.264)	299.432 $\pm$ 1.3045 (2580.190)
	0.01	278.175 (81.370)	277.661 $\pm$ 1.2147 (2382.460)
	0.05	209.293 (80.371)	208.219 $\pm$ 0.9065 (1801.830)
	0.10	151.452 (79.093)	151.506 $\pm$ 0.6489 (1325.010)
	0.20	87.149 (79.498)	87.059 $\pm$ 0.3584 (783.141)
0.20 (h_U = 6.174)	0.00	300.343 (80.761)	299.412 $\pm$ 1.3171 (2608.210)
	0.01	282.168 (81.792)	281.098 $\pm$ 1.2437 (2423.730)
	0.05	221.942 (81.183)	221.776 $\pm$ 0.9719 (1935.080)
	0.10	167.785 (80.606)	165.598 $\pm$ 0.7306 (1446.270)
	0.20	101.919 (80.668)	100.770 $\pm$ 0.4368 (908.238)
0.40 (h_U = 4.900)	0.00	300.360 (83.851)	300.098 $\pm$ 1.3402 (2578.030)
	0.01	285.290 (83.820)	285.260 $\pm$ 1.2719 (2439.480)
	0.05	233.732 (84.132)	233.063 $\pm$ 1.0364 (2014.130)
	0.10	184.733 (84.256)	183.950 $\pm$ 0.8118 (1641.270)
	0.20	120.297 (85.317)	120.245 $\pm$ 0.5339 (1081.660)
0.50 (h_U = 4.658)	0.00	299.590 (86.503)	301.865 $\pm$ 1.3414 (2579.320)
	0.01	285.338 (85.676)	284.551 $\pm$ 1.2634 (2435.680)
	0.05	236.186 (85.582)	237.150 $\pm$ 1.0579 (2086.750)
	0.10	188.817 (85.722)	190.765 $\pm$ 0.8481 (1691.850)
	0.20	125.253 (84.974)	125.207 $\pm$ 0.5601 (1131.270)
0.60 (h_U = 4.561)	0.00	299.928 (85.926)	301.592 $\pm$ 1.3461 (2594.260)
	0.01	286.416 (85.894)	288.255 $\pm$ 1.2908 (2468.540)
	0.05	239.544 (86.455)	241.904 $\pm$ 1.0832 (2086.110)
	0.10	193.891 (86.440)	194.687 $\pm$ 0.8634 (1703.390)
	0.20	131.582 (86.393)	131.486 $\pm$ 0.5844 (1178.170)
0.80 (h_U = 4.508)	0.00	301.700 (91.495)	303.884 $\pm$ 1.3587 (2576.650)
	0.01	288.621 (90.169)	287.895 $\pm$ 1.2911 (2443.960)
	0.05	243.068 (90.277)	243.884 $\pm$ 1.0888 (2090.160)
	0.10	198.376 (90.714)	199.787 $\pm$ 0.8888 (1754.650)
	0.20	136.707 (89.014)	137.859 $\pm$ 0.6139 (1232.110)

Number in parenthesis () is CPU Times (sec.)

Table 2. ARL of GWMA chart by MCA and MC when $q = 0.50$ and ARL_0 = 370.

w	δ	Methods	
		MCA	MC
0.10 (h_U = 10.794)	0.00	370.184 (78.468)	369.181 ± 1.5054 (3161.110)
	0.01	334.621 (79.732)	333.794 ± 1.3285 (2843.350)
	0.05	231.905 (79.622)	230.702 ± 0.8876 (1986.140)
	0.10	157.985 (79.139)	157.686 ± 0.5694 (1376.490)
	0.20	88.823 (77.922)	88.957 ± 0.2744 (793.125)
0.20 (h_U = 6.679)	0.00	369.632 (79.249)	367.825 ± 1.6015 (3128.330)
	0.01	338.097 (78.718)	336.884 ± 1.4490 (2920.790)
	0.05	242.089 (78.376)	239.974 ± 1.0237 (2089.620)
	0.10	167.176 (78.921)	167.537 ± 0.6969 (1468.270)
	0.20	91.215 (78.406)	91.091 ± 0.3525 (819.146)
0.40 (h_U = 4.443)	0.00	369.866 (81.307)	371.634 ± 1.6438 (3170.350)
	0.01	343.294 (79.919)	344.644 ± 1.5129 (2927.300)
	0.05	258.265 (80.746)	257.763 ± 1.1366 (2209.910)
	0.10	186.242 (80.075)	185.911 ± 0.8112 (1616.730)
	0.20	105.492 (80.075)	105.675 ± 0.4504 (959.796)
0.50 (h_U = 3.979)	0.00	369.728 (80.450)	371.266 ± 1.6548 (3136.380)
	0.01	344.658 (81.042)	343.774 ± 1.5252 (2913.150)
	0.05	263.327 (81.059)	259.685 ± 1.1519 (2238.650)
	0.10	192.829 (80.902)	191.553 ± 0.8461 (1665.750)
	0.20	111.355 (81.480)	111.381 ± 0.4869 (991.589)
0.60 (h_U = 3.661)	0.00	371.327 (81.276)	369.511 ± 1.6469 (3143.420)
	0.01	347.374 (81.667)	348.615 ± 1.5540 (3020.160)
	0.05	268.826 (81.854)	268.098 ± 1.1903 (2375.160)
	0.10	199.476 (80.684)	196.936 ± 0.8678 (1713.160)
	0.20	117.323 (81.650)	116.510 ± 0.5092 (1038.780)
0.80 (h_U = 3.255)	0.00	370.145 (83.975)	368.887 ± 1.6526 (3104.800)
	0.01	348.180 (82.993)	346.909 ± 1.5630 (2942.690)
	0.05	274.972 (82.696)	275.213 ± 1.2169 (2375.180)
	0.10	208.505 (82.041)	210.662 ± 0.9346 (1853.120)
	0.20	126.667 (83.383)	126.820 ± 0.5588 (1131.010)

Number in parenthesis () is CPU Times (sec.)

5. Conclusion

The numerical results of ARL for GWMA chart obtained from MCA and MC approaches when observations have Poisson distribution are compared. The results found that the numerical results obtained from those methods are in good agreement however, MCA is very time saving with CPU Times about 1 minute whereas MC consumes CPU Times between 10 minutes to 1 hour per case study. For the case of ARL_0 = 500 the GWMA chart is superior to EWMA chart when the magnitudes of changes are small $(\delta \le 0.20)$ and for the case of ARL_0 = 300 and 370, the GWMA chart is superior to EWMA chart when the magnitudes of changes are small $(\delta \le 0.10).$ When the magnitudes of changes are increased $(\delta > 0.10),$ the EWMA chart is superior to GWMA chart. Therefore, in order to select the control chart for monitoring change in process mean we conclude the optimal control chart when the process mean changes are small as shown on Table 7.

Table 3. Evaluation of ARL of GWMA chart by MCA and MC when $q = 0.90$ and ARL_0 = 500.

w	δ	Methods	
		MCA	MC
0.10 (h_U = 10.192)	0.00	500.338 (79.763)	500.198 ± 1.1630 (4252.810)
	0.01	458.746 (79.358)	461.519 ± 1.0060 (3932.080)
	0.05	346.863 (80.247)	349.485 ± 0.6199 (2988.180)
	0.10	270.236 (78.859)	271.548 ± 0.3889 (2364.370)
	0.20	193.555 (79.451)	194.082 ± 0.2056 (1733.190)
0.20 (h_U = 5.666)	0.00	499.857 (78.750)	492.795 ± 1.6564 (4160.970)
	0.01	444.847 (80.106)	440.027 ± 1.4271 (3732.780)
	0.05	301.272 (79.420)	299.717 ± 0.8529 (2566.200)
	0.10	210.715 (81.105)	210.895 ± 0.5071 (1826.520)
	0.20	132.153 (81.651)	132.416 ± 0.2359 (1178.990)
0.40 (h_U = 3.229)	0.00	498.748 (78.718)	488.930 ± 1.9461 (4149.500)
	0.01	442.061 (80.013)	434.498 ± 1.6998 (3775.880)
	0.05	288.259 (78.749)	285.461 ± 1.0655 (2482.960)
	0.10	187.765 (78.344)	187.805 ± 0.6354 (1702.480)
	0.20	102.408 (79.514)	102.979 ± 0.2908 (929.719)
0.50 (h_U = 2.717)	0.00	501.973 (77.829)	487.429 ± 2.0097 (4193.700)
	0.01	445.880 (78.172)	435.815 ± 1.8018 (3722.220)
	0.05	290.991 (79.545)	285.632 ± 1.1204 (2452.040)
	0.10	187.409 (80.559)	186.960 ± 0.6858 (1625.410)
	0.20	98.335 (79.357)	98.020 ± 0.3078 (870.127)
0.60 (h_U = 2.368)	0.00	499.354 (77.454)	489.319 ± 2.0964 (4126.990)
	0.01	444.881 (77.953)	434.565 ± 1.8241 (3661.560)
	0.05	292.069 (77.112)	289.313 ± 1.1842 (2532.100)
	0.10	187.537 (78.640)	187.140 ± 0.7250 (1657.510)
	0.20	96.081 (77.423)	95.905 ± 0.3286 (867.147)
0.80 (h_U = 1.922)	0.00	502.733 (76.628)	490.821 ± 2.1456 (4236.850)
	0.01	450.252 (78.375)	437.800 ± 1.8994 (3929.920)
	0.05	299.390 (77.766)	295.342 ± 1.2758 (2625.120)
	0.10	192.373 (77.563)	191.769 ± 0.8054 (1672.360)
	0.20	95.709 (77.564)	95.441 ± 0.3694 (852.187)

Number in parenthesis () is CPU Times (sec.)

Table 4. Evaluation of ARL of GWMA chart and EWMA chart when $q = 0.10, \lambda = 0.90$ and ARL_0 = 300.

charts	GWMA						EWMA
w	0.10	0.20	0.40	0.50	0.60	0.80	
h_U / δ	8.664	6.174	4.900	4.658	4.561	4.508	4.502
0.00	300.036 (79.264)	300.343 (80.761)	300.360 (83.851)	299.590 (86.503)	299.928 (85.926)	301.700 (91.495)	298.578 (89.903)
0.01	278.175* (81.370)	282.168 (81.792)	285.290 (83.820)	285.338 (85.676)	286.416 (85.894)	288.621 (90.169)	285.699 (91.932)
0.05	209.293* (80.371)	221.942 (81.183)	233.732 (84.132)	236.186 (85.582)	239.544 (86.455)	243.068 (90.277)	240.816 (92.071)
0.10	151.452* (79.093)	167.785 (80.606)	184.733 (84.256)	188.817 (85.722)	193.891 (86.440)	198.376 (90.714)	196.737 (92.493)
0.20	87.149* (79.498)	101.919 (80.668)	120.297 (85.317)	125.253 (84.974)	131.582 (86.393)	136.707 (89.014)	135.819 (91.183)

* Minimize ARL_1 in each δ level
Number in parenthesis () is CPU Times (sec.)

Table 5. Evaluation of ARL of GWMA chart and EWMA chart when $q = 0.50, \lambda = 0.50$ and ARL_0 = 370.

Charts	GWMA						EWMA
w	0.10	0.20	0.40	0.50	0.60	0.80	
h_U / δ	10.794	6.679	4.443	3.979	3.661	3.255	3.009
0.00	370.184 (78.468)	369.632 (79.249)	369.866 (81.307)	369.728 (80.450)	371.327 (81.276)	370.145 (83.975)	369.764 (83.258)
0.01	334.621* (79.732)	338.097 (78.718)	343.294 (79.919)	344.658 (81.042)	347.374 (81.667)	348.180 (82.993)	349.431 (83.273)
0.05	231.905* (79.622)	242.089 (78.376)	258.265 (80.746)	263.327 (81.059)	268.826 (81.854)	274.972 (82.696)	280.772 (83.898)
0.10	157.985* (79.139)	167.176 (78.921)	186.242 (80.075)	192.829 (80.902)	199.476 (80.684)	208.505 (82.041)	217.015 (82.930)
0.20	88.823* (77.922)	91.215 (78.406)	105.492 (80.075)	111.355 (81.480)	117.323 (81.650)	126.667 (83.383)	135.978 (83.929)

* Minimize ARL_1 in each δ level
Number in parenthesis () is CPU Times (sec.)

Table 6. Evaluation of ARL of GWMA chart and EWMA chart when $q = 0.90, \lambda = 0.10$ and ARL_0 = 500.

charts	GWMA						EWMA
w	0.10	0.20	0.40	0.50	0.60	0.80	
h_U / δ	10.192	5.666	3.229	2.717	2.368	1.922	1.647
0.00	500.338 (79.763)	499.857 (78.750)	498.748 (78.718)	501.973 (77.829)	499.354 (77.454)	502.733 (76.628)	503.373 (77.548)
0.01	458.746 (79.358)	444.847 (80.106)	442.061* (80.013)	445.880 (78.172)	444.881 (77.953)	450.252 (78.375)	452.895 (78.437)
0.05	346.863 (80.247)	301.272 (79.420)	288.259* (78.749)	290.991 (79.545)	292.069 (77.112)	299.390 (77.766)	305.064 (78.235)
0.10	270.236 (78.859)	210.715 (81.105)	187.765 (78.344)	187.409* (80.559)	187.537 (78.640)	192.373 (77.563)	197.133 (78.329)
0.20	193.555 (79.451)	132.153 (81.651)	102.408 (79.514)	98.335 (79.357)	96.081 (77.423)	95.709* (77.564)	96.810 (77.704)

* Minimize ARL_1 in each δ level
Number in parenthesis () is CPU Times (sec.)

Table 7. Optimal parameter of control charts for the process mean change.

ARL_0	The process mean change level (δ)	Optimal Control Chart
300	0.01	GWMA (q = 0.90, w = 0.60)
	0.05	
	0.10	GWMA (q = 0.90, w = 0.80)
	0.20	EWMA (λ = 0.10)
370	0.01	GWMA (q = 0.90, w = 0.60)
	0.05	
	0.10	
	0.20	EWMA (λ = 0.10)
500	0.01	GWMA (q = 0.90, w = 0.40)
	0.05	
	0.10	GWMA (q = 0.90, w = 0.50)
	0.20	GWMA (q = 0.90, w = 0.80)

References

[1] Roberts, S.W., Control Chart Tests Based on Geometric Moving Average, *Technometrics,* 1959; 1: 239–250.

[2] Borror, C.M., Champ, C.W. and Rigdon, S.E., Poisson EWMA Control Charts, *Journal of Quality Technology*, 1998; 30: 352-361.

[3] Zhang, L., Govindaraju, K., Lai, C.D. and Bebbington, M.S., Poisson GWMA Control Chart, *Communications in Statistics - Simulation and Computation*, 2003; 32 (4): 1265-1283.

[4] Sheu, S.H. and Lin, T.C., The Generally Weighted Moving Average Control Chart for Detecting Small Shifts in the Process Mean, Quality Engineering, 2003; 16: 209-231. cited in Cheng, S.R. Yeh, Y.H. and Shue, M.H., A Comparative Study of the Monitoring Performance for Weighted Control Charts, *Journal of Quality Technology*.

[5] Sheu, S.H. and Yang, L., The Generally Weighted Moving Average Control Chart for Detecting Small Shifts in the Process Mean, *Quality Engineering*, 2006; 18: 333-344.

[6] Montgomery, D.C., *Introduction to Statistical Quality Control*. New York, Chichester: Wiley, 2005.

[7] Lucas, J.M. and Saccucci, M.S., Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements, *Technometrics,* 1990; 32: 1-29.