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A markovian study of no claim discount system of Insurance Regulatory and Development Authority and its application

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Abstract

No claim discount (NCD) is one of the more controversial areas of automobile insurance, being a topic on which the motorist is liable to hold strong and emotive views from time to time. In this paper we try to find out the probabilities of claims by different categories of policyholders (motorists). The policyholders are divided into two groups viz. 'good drivers' and 'bad drivers' according to their driving experience as well as accident records in last two years. Here, we use a transition probability matrix (TPM) for different discount levels following the Insurance Regulatory and Development Authority (IRDA) rules of NCD using Markov Chains. Using this TPM we obtain the respective amount of premiums to be paid in the long run by different groups of policyholders specially the drivers of the districts of Karimganj, North Tripura and West Tripura. The results of this study show that probability of claims and different NCD rates are not parallel.

Keywords: markov chains, stationary distribution, automobile insurance.

1. Introduction

In automobile insurance, among other general insurance policies, it is quite common tendency to reduce the premium by a factor in case the insured does not make any claim in a given period. This is popularly known as No Claim Discount (NCD). No claim discount systems (sometimes also called Bonus-Malus systems) are experience rating systems which are commonly used in motor insurance. NCD schemes represent an attempt to categorize policyholders into relatively homogeneous risk groups who pay premiums relative to their claims experience. Those who have made few claims in recent years are rewarded with discounts on their initial premium, and hence are enticed to stay with the company. Depending on the rules in the scheme, new policyholders may be required to pay the full premium initially and then will obtain discounts in the future as a results of the claim free years [1]. An NCD can significantly reduce the cost of your car insurance cover. An NCD system discourages small claims. This principle is meant to reward policy holders for not making claims during a year; that is, to grant a bonus to a careful driver. A bonus principle effects the policy holder’s decision whether or not to claim in a particular instance. No claim will be made for some of the accidents where there is only slight damage. Philipson [2] called this phenomenon ‘hunger for bonus’. It reduces claims costs to the insurer which offsets the decrease in premium income from NCD systems.

No claims discounts allow the driver to be more responsible about their vehicle and when driving. If no claims are made, each year the premium reduces. The discount is calculated as a straightforward percentage of the total cost of the insurance premium and will be discounted each year a claim is not made. Every car insurance company has its own method for determining the exact amount to discount off the premium but the commonality is the maximum number of years to accumulate the no claims discount is five years. Also, drivers can insure their no claims discount to protect it once the five year discount maximum has been reached. This charge is added to your insurance policy. If any claim is made or arises on his/her motor insurance policy during the period of cover, his/her NCD for that vehicle will be forfeited and reverts to 0%. The policyholder will have to begin to accumulate his/her discount in a new cycle. So, we can now see how the NCD works for car insurance and exactly how much money a policyholder could save. The difference in premiums can be vast.

In India NCD rewards the policyholder with savings on his/her car or motorcycle insurance for good driving or riding history. The NCD savings start at 20% and go as high as 50%, see Table 1. The insurer calculates the level of NCD based on the number of years the policyholder have been driving or riding, his/her claims and incident history.

Table 1. Levels of NCD system of IRDA**.**

Levels/ Age of Vehicle	No Claim Discount Saving
5	50%
4	45%
3	35%
2	25%
1	20%
0	00%

Each year at renewal, a policyholder automatically moves up to the next level of NCD if he/she haven't made a claim for an accident where he/she was at fault in that year. If policyholder does make a claim for an accident where he/she was at fault, the policyholder will move down zero level of NCD, unless he/she is on maximum NCD for life. If the policyholder makes a claim for something that's not his/her fault, for example, his/her car or motorcycle is stolen or damaged by a storm, or someone scratches the paintwork, his/her NCD level will not change. There is a table fixed by Insurance Regulatory and Development Authority (IRDA) for a NCD and is given Table1. As is apparent therefore a NCD is a special discount given for every claim-free year. This therefore reduces the premium in succeeding years. However a claim in the succeeding years would result in loading, which is the inverse equivalent of NCD. The slabs for a no claim bonus now start at a 20% discount on premium for own damage for no claims in the preceding year and increases to 25% for no claims in the preceding two years, 35% for three years, 45% for four and a maximum of 50% for five years [3].

No Claim Discount (NCD) or Bonus-malus systems (BMSs) are introduced in Europe in the early 1960s, following the seminal works of Delaporte [4], Bichsel [5], and Bühlmann [6]. There exists a vast literature on BMSs in actuarial journals, mainly in the ASTIN Bulletin, the Scandavian Actuarial Journal and the Swiss Actuarial Journal.

Loimaranta [7] develops formulas for some asymptotic properties of bonus systems, where bonus systems are understood as Markov chains. Bonus systems used in Denmark, Norway, Sweden, Finland, Switzerland and West Germany are studied by Vepsäläinen [8] on the basis of the method given by Loimaranta. Lemaire [9] derives an algorithm for obtaining the optimal strategy for a policy holder. Lemaire [10] applies this algorithm to compare bonus systems used in Denmark, Norway, Sweden, Finland, Switzerland and West Germany. Hastings [11] presents a simple model based on a typical British policy, assuming that the number of accidents is Poisson and the amount of damage is negative exponentially distributed. The problem is formulated as a Markov decision problem and is solved by dynamic programming. Lamaire [12] computes a merit-rating system for automobile third party liability insurance. The results are applied to the portfolio of a Belgian company and compared to the premium system provided by the expected value principle. Kolderman and Volgenant [13] present a continuous model based on generalized Markov programming, applicable to bonus-malus systems used by Dutch motor insurance companies. Lemaire [14] obtains the data from the Actuarial Institute of the Republic of China of market wide observed loss severity distributions for property damage and bodily injury for accident years 1987 to 1989. These distributions are very well represented by a lognormal model. Lemaire and Zi [15] analyze 30 bonus-malus systems (BMS) from around the world. All BMS are simulated, assuming that the number of at-fault claims for a given policyholder conforms to a Poisson distribution. Lemaire [16] studies the Markov chain theory for the design, evaluation, and comparison the BMSs of the nations Brazil, Belgium, Japan, Switzerland and Taiwan. The tools are the same, but the assumptions about the probability distributions for the number of claims vary. Pitrebois et al [17] obtain the relativities of the Belgian Bonus-Malus System, including the special bonus rule sending the policyholders in the malus zone to initial level after four claim-free years. The model allows for a priori ratemaking.

All the above mentioned studies are not associated with the NCD of Insurance Regulatory and Development Authority of India.

This paper inspects the desirability of this multi-layer premium system (NCD system); to start with this study works with a given number of levels with fixed gaps. The basic framework considers that of a discrete time parameter Markov chain, where the state space consists of the different levels of the premium, and the state of a particular insured shift randomly from a year to the next. The randomness of the transition is governed by the transition probability of causing an accident in a given year. This study models the probability to be varying depending on quality of the driver. For the most part, it would be considering a finitely many groups of policyholders (drivers) characterized by respective probabilities of getting involved in an accident. A try has been made to obtain the stationary distribution for each group of policyholders. This reflects the distribution of a particular group over the various levels of premium in the long run. For example, one can obtain the percentage of ‘good’ drivers expected to receive the fully discounted rate in the long run. A comparative study of these stationary distributions over the various groups considered, form the basis of appropriateness of the assumed NCD system. Briefly the objectives are as follows:

a) To find the probability distribution(s) of number of accidents/claims in the study area.

b) To study the long run behavior of the claiming process in our study area using IRDA rule.

The article is organized as follows: In Section 2, we describe the NCD model as a discrete state Markov chain. Data analysis and conclusion are given in Section 3.

2. The Model

A posteriori rating is a very efficient way of classifying policyholders into cells according to their risk. Several studies have shown that, if insurers are allowed to use only one rating variable, it should be some form of merit-rating. The best predictor of the number of claims of a driver in the future is not age, car, or the township of residence, but past claims behavior. An insured enters the system, in the initial class, when he or she obtains a driving license. Then, throughout the entire driving lifetime, the transition rules are applied upon each renewal to determine the new class as a function of claims history.

The preceding definition assumes that the NCD forms a Markov chain process. A (first-order) Markov chain is a stochastic process in which the future development depends only on the present state but not on the history of the process or the manner in which the present state was reached. It is a process without memory, such that the states of the chain are the different NCD classes. The knowledge of the present class and the number of claims for the year suffice to determine next year's class. It is not necessary to know how the policy reached the current class. The Markov process, Markov chain, transition probability matrix (TPM) are described in brief in the following sections.

A Markov process is a stochastic process that has a limited form of 'historical' dependency. Let $\{X(t):\ t \in \tau\}$ be defined on the parameter set τ and assume that it represents time. The values that X(t) can obtain are called states, and all together they define the state space S of the process. A stochastic process is a Markov process if it satisfies

$$\begin{aligned} &\mathrm{P}[X(t_0+t_1) \le x \mid X(t_0)=x_0, X(\tau), -\infty<\tau<t_0] \\ &= P[X(t_0+t_1) \le x \mid X(t_0)=x_0], \forall t_0, t_1 > 0 \end{aligned} \qquad (1)$$

Let t_0 be the present time. (1) states that the evolution of a Markov process at a future time, conditioned on its present and past values, depends only on its present value. The condition of (1) is also known as the Markov property. Markov chains are classified as discrete or continuous.

Consider a Markov process as defined by (1) and, without loss of generality, let the state space S be the set of nonnegative integers. The Discrete Time Markov Chains (DTMCs) that characterizes the process captures its evolution among states of S over time t$\in \tau$. The transition probability between state i to state j at time (n -1) is the probability $\mathrm{P}[X_n = j | X_{n-1} = i]$. A DTMC is time homogeneous if

$$\begin{aligned} &\mathrm{P}[X_n = j | X_{n-1} = i] = P[X_{m+n} = j | X_{m+n-1} = i], \\ &n = 1,2,3,........, m \geq 0, i, j \in S \end{aligned} \quad (2)$$

Further we define and the one step probability transition matrix P:

$$\mathbf{P} = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,j} & \cdots \\ p_{1,0} & p_{1,1} & \cdots & p_{1,j} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{i,0} & p_{i,1} & \cdots & p_{i,j} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \quad (3)$$

where, p_{ij} are the probabilities of accidents and hence claims.

Each row of the probability transition matrix represents the transition flow out of the corresponding state. Each column of it represents the transition flow into the state. As the cumulative transition flow out of each state must be 1, the rows of matrix P must sum to 1 and have all non-negative elements (since they are probabilities). A matrix that has non-negative elements and its rows sum to 1 is often called a stochastic matrix. If S is finite, then P has finite dimension.

The probability that the DTMC reaches, on the n^{th} step, state j starting from state i in step 0 is given by the Chapman-Kolmogorov equation:

$$p_{i,j}^{n} = \sum_{k \in S} p_{i,k}^{m} . p_{k,j}^{n-m}, 0 \leq m \leq n. \quad (4)$$

Let $\pi_i^n = (\pi_0^n, \pi_1^n, ...)$ be the probability vector whose element π_i^n denotes the probability that the DTMC is at state i at step n. Since $\pi^n = \pi^0 .P^n$ or $\pi^n = \pi^{n-1}.P$, n = 1, 2,….., then the probability of state i at step n is simply the sum of the probabilities along all sample paths from j to i in n steps weighted by the probability of starting at state j.

A DTMC is irreducible if for each pair of states $(i, j) \in S^2$ there exist an integer n such that $p_{i,j}^n > 0$. State *i* is positive recurrent if $\sum_{n \in N} p_{i,i}^n < \infty$. A state is periodic if $p_{i,i}^n > 0$ iff *n* = *k.d* for some values of *n* and a fixed value of d > 1. If a state is not periodic (d=1), then it is aperiodic, where d is the GCD of n. Clearly, state i is aperiodic if $p_{i,i} \neq 0$. A state is called ergodic if it is positive recurrent and aperiodic.

If P is the probability transition matrix of an irreducible DTMC in an ergodic set of states, the limiting matrix $\lim_{n \to \infty} P^n$ has identical rows that are equal to the stationary probability vector $\pi = \lim_{n \to \infty} \pi^n$.

The stationary probability vector of an irreducible DTMC in an ergodic set of states is unique and satisfies

$$\pi = \pi.P \qquad (5)$$

And the normalization condition

$$\pi.1^T = 1 \qquad (6)$$

This paper always refers to an ergodic and irreducible Markov chain throughout the chapter, and interested on computing the stationary distribution vector π which characterizes the steady state probability distribution of the process. The steady state is reached after the process passes an arbitrary large number of steps. In steady state, the total flow out of a state is equal to the total flow into the state. This property is called flow balance and is expressed in the form of a flow balance equation. The collection of all flow balance equations for a DTMC is formally represented by (5).

A forecast of the future distribution of policies among the classes, say n years from now, can be obtained easily through simulation or by computing the *n*-th power of the transition matrix P. For many purposes, an asymptotic study is sufficient to compare NCDs. An NCD forms a regular Markov chain: all its states are ergodic (it is possible to go from every state to every other state), and the chain is not cyclic.

2.1 IRDA Transition Rule of NCD System

The six levels of discount of IRDA are 0%, 20%, 25%, 35%, 45%, 50%. At the end of each policy year, policyholders change levels according to the following rules:

i) A policyholder who has made no claim(s) during a policy year moves to the next higher discount level or remain at 50 % if already at the highest level.

ii) A policyholder who has made at least one claim during a policy year drops back to zero percent level.

2.2 Transition Matrix

The rules of a NCD system mentioned in section 4 can be summarized in a transition matrix showing probabilities of movements amongst each level, see Figure 1, for the general notation, where p_0 is the probability of no claim and $(1-p_0)$ is the probability of at least one claim. Here,

$P = (P_{ij})_{5x5}$, $P_{i0} = 1 - p_0$, $i = 0,1,\ldots,5$.

$P_{i,i+1} = p_0$, $i = 0,\ldots,4$. and $P_{55} = p_0$

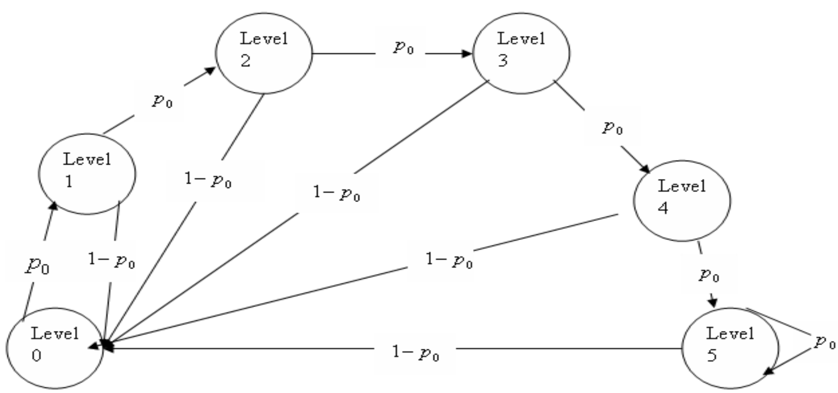

Figure 1. Transition figure of discount levels of IRDA.

Transition Probability Matrix: (7)

Next Discount Level

Existing Discount Level	***0***	***1***	***2***	***3***	***4***	***5***
0	$1-p_0$	p_0	0	0	0	0
1	$1-p_0$	0	p_0	0	0	0
2	$1-p_0$	0	0	p_0	0	0
3	$1-p_0$	0	0	0	p_0	0
4	$1-p_0$	0	0	0	0	p_0
5	$1-p_0$	0	0	0	0	p_0

2.3 General solution of TPM (7) under equilibrium condition

$$\pi_0 = (1-p_0)\,,\ \pi_1 = p_0(1-p_0)\,,\ \pi_2 = p_0^2(1-p_0)\,,\ \pi_3 = p_0^3(1-p_0)$$
$$,\pi_4 = p_0^4(1-p_0)\,, \pi_5 = p_0^5\,. \quad (8)$$

2.4 The average yearly premium paid A(p_0, m), in the steady state in terms of p_0 and m

A(p0,m) = $\mathrm{m}\sum_{i=0}^{5}\pi_i$ x percentage of discount at different levels.

A(p_0,m) **=**

$$\frac{m}{100}(1-p_0)[100+80p_0+75p_0^2+65p_0^3+55p_0^4+50p_0^5/(1-p_0)] \quad (9)$$

where, m is the yearly amount of premium.

3. Data Analysis and Conclusion

The primary data have shown in Tables 2 and 3 are collected from the drivers of the districts of North Tripura, West Tripura and Karimganj during 2007-2008. Table 2 presents the frequency of drivers who made 0, to 8 accidents during their driving life. Here, a maximum of eight accidents are made by drivers during their driving life. Again,

Table 3 describes the two-ways cross-tabulation of the number of accidents made drivers in last 2 years against the driving experience. Here, it is seen that a maximum of two accidents made by drivers. The drivers are divided into six different groups viz. (00 – 05) yrs, (06 – 10) yrs, (11 – 15) yrs, (16 – 20) yrs, (21 – 25) yrs and (25 +) yrs with respect to driving experience.

Table 2. Number of accidents made by drivers during driving life.

No. of Accidents	Frequency	Percent	Cumulative Percent
0	232	44.5	44.5
1	152	29.2	73.7
2	69	13.2	86.9
3	23	4.4	91.4
4	14	2.7	94.0
5	16	3.1	97.1
6	8	1.5	98.7
7	5	1.0	99.6
8	2	.4	100.0
Total	521	100.0	

Table 3. Number of accidents in last 2 years vs. driving experience.

		Driving experience in years						Total
		00 - 05	06 - 10	11 - 15	16 - 20	21 - 25	25 +	
Number of accidents in last 2 years	Zero	11	114	114	22	60	30	351
	One	0	29	45	0	22	27	123
	Two	10	9	10	0	18	0	47
Total		21	152	169	22	100	57	521

Under the assumption of number of accidents indicating number of claims, it is seen that the number of accidents by automobile drivers, see Table 2, have a Poisson distribution with parameter λ =1.14 or Geometric distribution with parameter p = 0.47 or Negative Binomial distribution with parameter r = 1 & p = 0.48, see Figures 2, 3 and 4. So the probability of no claim or accident (p_0) in the aforementioned three cases are 0.32, 0.47 and 0.48 respectively. Again from the classical definition of probability, the probability of no claim or accident (p_0) be 0.45 which is very close to the results obtained from Geometric distribution or Negative Binomial distribution. So, it may be considered that the number of accidents by professional automobile drivers of the study area follows geometric (p = 0.5) or negative binomial (r = 1, p = 0.5) distribution**.** The steady state or equilibrium distribution of policyholders using geometric (p = 0.5) is $\boldsymbol{\pi}$ = (0.5, 0.25, 0.125, 0.0625, 0.03125, 0.03125)**.** The expected amount of premium has to be paid yearly in the long run for the risk is 0.87m where, m is the yearly premium i.e.13% less amount to be paid yearly.

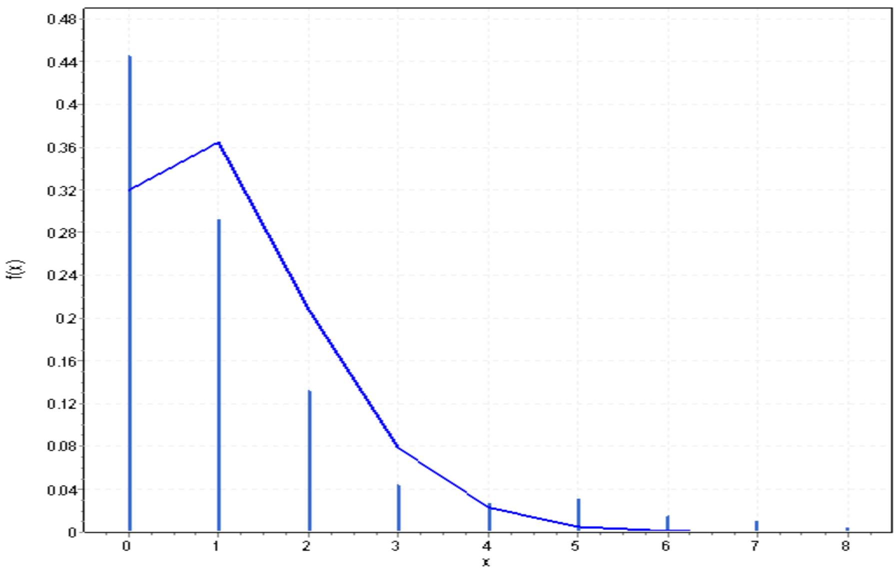

Figure 2. Fitted poisson distribution of number of accidents (presented in Table 2).

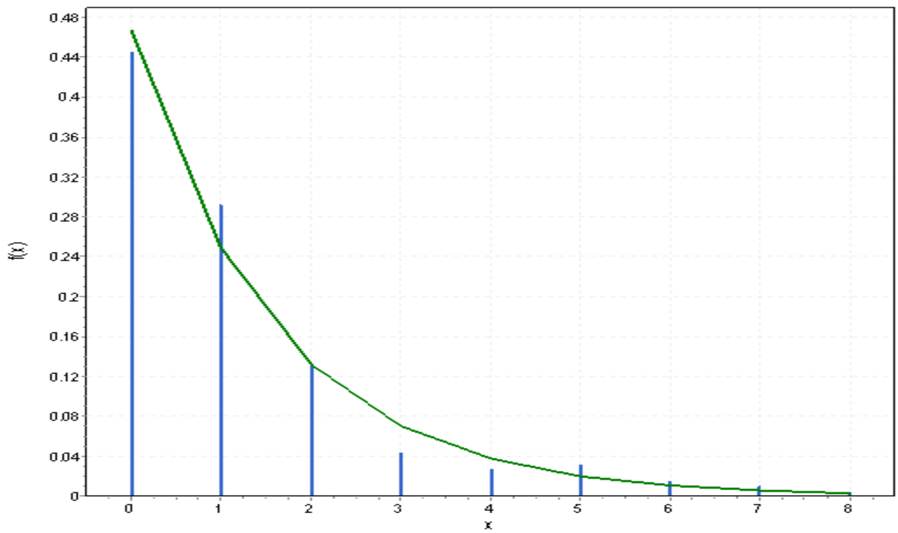

Figure 3. Fitted geometric distribution of number of accidents (presented in Table 2).

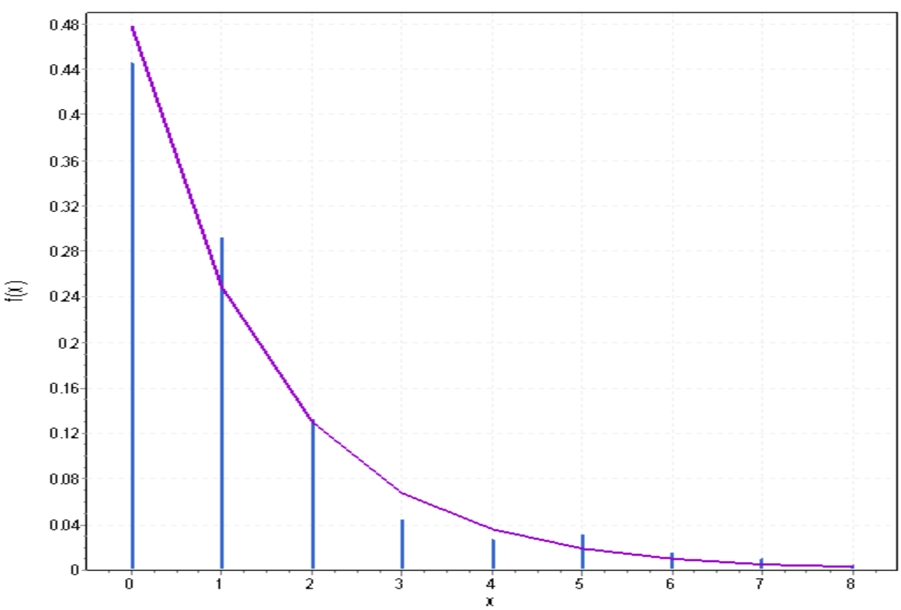

Figure 4. Fitted negative binomial distribution of number of accidents (presented in Table 2).

Again, the probability of making an accident by 'good driver' (06 to 20 years experienced) and 'bad driver' (0 to 5 years and 21 (+) years old i.e. either very young or too old) are 0.20 and 0.45 respectively. So, the probabilities of no claim (p_0) of the two categories of claimant are 0.80 and 0.55 respectively (Table 3). For a good driver, the average premiums paid in the long term is 0.70m and for a bad driver it is 0.85m i.e. they pay 30% and 15% less yearly premiums respectively. So, the bad drivers are almost twice as likely to claim as good drivers but the premium is only, on average marginally higher.

The number of accidents by professional automobile drivers of the study area follows geometric or negative binomial distribution. The bad drivers are almost twice as likely to claim as good drivers but the premium is only, on average marginally higher. Therefore, one could adjust discounts levels of NCD slabs of IRDA so that bad drivers pay a premium doubled than that of good drivers. Alternatively, introduction of different types of discount system like expanding the number of categories of discount levels will be helpful to encourage the good drivers. A longer and more gradual scale might differentiate between the different risk groups more efficiently.

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