Thailand Statistician
January 2014; 12(1): 55-69
http://statassoc.or.th
Contributed paper

Confidence interval estimation of the signal-to-noise ratio using ranked set sampling: a simulation study

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Received: 23 September 2013

Accepted: 12 November 2013

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In this paper, several confidence intervals for estimating the population Signal-to-Noise Ratio (SNR) are compared using simple random sampling (SRS) and ranked set sampling (RSS). A simulation study is conducted to compare the performance of the interval estimators using random data generated from normal distribution with specified population parameters so that the same values of SNR are obtained, with sample sizes $n = 15, 25, 50$. The criteria for performance comparison is based on coverage probability and interval width. From simulation study it is observed that the confidence intervals based on the RSS have the higher coverage probabilities and smaller or equal widths compared to the confidence intervals based on SRS.

Keywords: Coefficient of variation, confidence interval, coverage probability, ranked set sample, signal-to-noise ratio.

1. Introduction

In analog and digital communications, the Signal-to-Noise ratio (SNR) is a measure of signal strength relative to background noise, while in quality control, the SNR represents the magnitude of the mean of a process compared to its variation. The SNR measures how much signal has been corrupted by noise, see McGibney and Smith [1] for a discussion. An alternative definition of the SNR is as the reciprocal of the coefficient of variation, i.e., the ratio of mean to standard deviation of a signal or measurement. Such an alternative definition is only useful for variables that are always non-negative (such as photon counts and luminance). SNR is commonly used in image processing, where the SNR of an image is usually calculated as the ratio of the mean pixel value to the standard deviation of the pixel values over a given neighborhood. For a population with mean μ and standard deviation σ, the SNR is defined as the ratio of the population mean to the population standard deviation, i.e. SNR $= \mu/\sigma$, but in real life situations, the population parameters μ and σ are estimated by $\bar{x}$ and s, respectively. Hence a sample estimate of the population SNR is given by $\widehat{SNR} = \bar{x}/_{S}$. The coefficient of variation (τ) of a distribution is considered as one of the useful descriptive measures of variability and is defined as $\tau = \sigma/\mu$. It is a unit free measure that quantifies the standard deviation as a proportion of the mean and used for comparing data from different distributions or those measured using different scales. Thus the sample estimate of the population SNR is the reciprocal of sample estimate of τ.

It is of great interest to find confidence interval estimate for SNR. Confidence interval estimation allows the researcher to have an idea about the precision of the point estimate rather than only a p value for rejection or no rejection of a specified null hypothesis. Confidence intervals for the SNR are very limited in the literature. The only result is by Sharma and Krishna [2] who developed the asymptotic distribution of the SNR without making any assumption about the distribution. For this reason, confidence intervals available for estimating τ will be used to estimate SNR by simply noting the inverse relationship between τ and SNR. In this regard, George and Kibria [3] performed a simulation study that compares several confidence intervals estimate for estimating SNR. In this paper, rather than using the usual simple random sampling (SRS) technique, the more powerful ranked set sampling (RSS) technique will be used to generate the samples. This makes the current paper very unique in using RSS to estimate SNR rather than SRS.

Recently, RSS gained a solid ground for estimating more efficient population parameter. RSS has been shown to provide more precise and efficient estimates of the mean and variance when measurements on the variable of interest are difficult to obtain or too expensive to get, but ranking the elements in the sample is relatively easy. Moreover, RSS can provide an estimator for the variance which is unbiased and more efficient even for underlying non-normal distribution and where judgment rankings are not perfect. McIntyre [4] was the first to suggest using RSS to estimate the population mean instead of SRS and the idea was later developed by Takahasi and Wakimoto [5] using mathematical theory to support their claim. Takahashi and Wakimoto [5] proved that the sample mean obtained using RSS is unbiased and has smaller variance compared to that obtained using SRS using the same sample size, see Samawi and Muttlak [6], and Samawi [7] for discussion. Stokes [8] proposed an estimator for the variance of a ranked set sample data and showed that the estimator is asymptotically unbiased and asymptotically more efficient than the sample variance of a simple random sample of the same number of observations. MacEachern et al. [9] proposed an alternative estimator for the variance which is unbiased and more efficient than Stokes's estimator even when the underlying distribution is not normal and the ranking of the elements is not perfect. MacEachern et al. [9] estimator of σ^2 performs well for small to moderate sample sizes and is asymptotically equivalent to Stokes [8] estimator.

Many papers compared performance of different confidence intervals for τ under different settings. McKay [10] derived a confidence interval for τ that was modified later by Vangel [11] and shown to be nearly exact under the normality assumption. Also, Verrill [12] discussed confidence intervals for τ when the population is normal or log-normal distributed. Panichkitkosolkul [13] compared the performance of three confidence intervals for estimating τ for normal data. Also, Panichkitkosolkul [14] proposed an asymptotic confidence interval for τ of a Poisson distribution and compared its performance with those of McKay [10] and Vangel [11], while Albrecher et. al. [15] presented Asymptotic of the Sample Coefficient of Variation. Terpstra and Nelson [16] used unbalanced RSS to compare maximum likelihood estimator (MLE) and weighted average (WA) estimate for the population proportion. Later Terpstra and Wang [17] compared several confidence intervals for estimating the population proportion using RSS, which is by far the only work found about implementation and discussing corresponding properties of RSS in interval estimation for the population proportion. On the other hand, Samawi and Muttlak [6] compared RSS to SRS in estimating ratio and proved that the efficiency of the estimator has increased when using RSS relative to SRS.

In this paper, a performance comparison of several confidence intervals for estimating SNR using the concept of RSS compared with the usual SRS technique is performed. Data were simulated from Normal distributions with μ =10 and σ=10, 5, 3, 1 so that their respective SNR are 1, 2, 3.33 and 10. The paper is organized as follows: Section 2 presents balanced RSS sampling, while Section 3 presents statistical methodology and confidence intervals for SNR. Simulation technique and results are presented in Section 4. Finally some concluding remarks are presented in Section 5.

2. Unbiased estimates of μ and σ using RSS

Suppose that we are interested in obtaining a RSS of size k from a population. First, a SRS of size k observations are selected and rank ordered on an attribute of interest. The observation that is determined to be the smallest is the first element of the RSS and is denoted $X_{[1]}$ and the remaining $k-1$ units are discarded. A second SRS of size k is selected from the population and ranked the same way and the second smallest observation is selected and denoted $X_{[2]}$. In a similar fashion, $X_{[3]}, X_{[4]},..., X_{[k]}$ are selected, hence $X_{[1]}, X_{[2]},..., X_{[k]}$ represent our first balanced RSS of size k. To obtain a balanced RSS of size $n = km$, the process is repeated m independent cycles yielding the balanced RSS of size n shown in Table 1.

Table 1. Balanced RSS with m cycles and set size k.

Cycle 1	$X_{[1]1}$	$X_{[1]2}$	$X_{[1]3}$	...	$X_{[1]k}$
Cycle 2	$X_{[2]1}$	$X_{[2]2}$	$X_{[2]3}$	...	$X_{[2]k}$
Cycle 3	$X_{[3]1}$	$X_{[3]2}$	$X_{[3]3}$	...	$X_{[3]k}$
Cycle m	$X_{[m]1}$	$X_{[m]2}$	$X_{[m]3}$	...	$X_{[m]k}$

The complete balanced RSS with set size k and m cycles is given by $\{X_{[r]i} : r = 1,2,..., m; i = 1,2,..., k\}$. The term $X_{[r]i}$ is called the r th judgment order statistic from the i th cycle. It is the observation that is judged to be the r th order statistic from one of the k sets in the i th cycle, see MacEachern et al. [9] for discussion.

Assume that the underlying distribution has finite mean μ and variance σ^2. Stokes [8] proposed an estimator of σ^2 based on RSS given by

$$\hat{\sigma}^2 = \frac{1}{nm-1}\sum_{i=1}^{n}\sum_{r=1}^{m}(X_{[r]i}-\hat{\mu})^2, \quad \text{where} \quad \hat{\mu} = \frac{1}{nm}\sum_{i=1}^{n}\sum_{r=1}^{m}X_{[r]i} \tag{1}$$

Stokes [8] showed that this estimator is a biased estimator of σ^2, but it is asymptotically unbiased as either n or m approach ∞. Moreover, Stokes [8] indicated that the RSS estimator $\hat{\mu}$ has more precision over the sample mean, say $\overline{Y}$ obtained using SRS because of independence of the order statistics composing the ranked set sample. In fact, the author showed that n^2 var($\overline{Y}$) $\geq$ n^2 var($\hat{\mu}$).

The balanced RSS is used in this paper. The estimator of the variance of a RSS proposed by MacEachern et al. [9] given below will be implemented in the simulations.

$$\hat{\sigma}^2 = \frac{1}{nk}\{(k-1)MST + (nk-k+1)MSE\} \tag{2}$$

Where MSE and MST are the mean-square error and mean-square treatment from an analysis of variance performed on the ranked set sample data with the judgment class used as a factor. This estimator has been shown to perform very well for small as well as large ranked set samples, which is an improvement on Stokes [8] estimator. The comparison will be based on the factors: coverage probability and width which seems to be the major factors for comparison, see Mahmoudvand and Hassani [18], Terpstra and Wang [17], Panichkitkosolkul [13], Panichkitkosolkul [14], and Kang and Schmeiser [19] for discussion.

3. Statistical Methodology

Let X_1, X_2, ... X_n be an independently and identically distributed (iid) random sample of size n from a population with finite mean, μ, and finite variance, σ^2. Let $\bar{x}$ be the sample mean and s be the sample standard deviation. Then

$\widehat{SNR} = \frac{\bar{x}}{s}$ would be the estimated value of the population SNR = $(\frac{\mu}{\sigma})$ and $\hat{\tau} = \frac{s}{\bar{x}}$ would be the estimated value of the population coefficient of variation, $\tau = (\frac{\sigma}{\mu})$. The main objective is to estimate $(1-\alpha)100\%$ confidence intervals for the population SNR= $(\tau)^{-1}$ while using the merits of RSS over SRS. George and Kibria [3] have reviewed and proposed several confidence intervals for the SNR parameter. Based on their simulation study they have recommended some promising intervals. In this section we will consider those recommended intervals by George and Kibria and compare the performance of those intervals based on RSS and SRS. The following confidence intervals will be implemented in the simulations.

1. **Method 1**. Miller [20] Confidence Interval for normal distribution(Mill): Miller [14] showed that $\frac{s}{\bar{x}}$ approximates an asymptotic normal distribution with mean, $\frac{\sigma}{\mu}$ and variance, $\frac{1}{(n-1)}(\frac{\sigma}{\mu})^2\left[0.5+(\frac{\sigma}{\mu})^2\right]$. Then the $(1-\alpha)100\%$ approximate confidence interval for the population inverted SNR is $\frac{s}{\bar{x}} \pm Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2\left[0.5+(\frac{s}{\bar{x}})^2\right]}$. Therefore the $(1-\alpha)100\%$ approximate confidence interval for the population SNR can be expressed as

$$\left[\left(\frac{s}{\bar{x}}+Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2\left[0.5+(\frac{s}{\bar{x}})^2\right]}\right)^{-1}, \left(\frac{s}{\bar{x}}-Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2\left[0.5+(\frac{s}{\bar{x}})^2\right]}\right)^{-1}\right].$$

2. **Method 2**. McKay [10] Confidence Interval using Chi-square distribution: The $(1-\alpha)100\%$ approximate confidence interval for the population inverted SNR is

$$\left(\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,1-\alpha/2}}{n}-1\right)(\frac{s}{\bar{x}})^2+\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}\right]},\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,\alpha/2}}{n}-1\right)(\frac{s}{\bar{x}})^2+\frac{\chi^2_{n-1,\alpha/2}}{n-1}\right]}\right)$$

Therefore the $(1-\alpha)100\%$ approximate confidence interval for the population SNR can be expressed as

$$\left(\begin{array}{l}\left[\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,\alpha/2}}{n}-1\right)(\frac{s}{\bar{x}})^2+\frac{\chi^2_{n-1,\alpha/2}}{n-1}\right]}\right]^{-1}\\ ,\left[\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,1-\alpha/2}}{n}-1\right)(\frac{s}{\bar{x}})^2+\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}\right]}\right]^{-1}\end{array}\right).$$

3. **Method 3**: Median Modified Miller Estimator Kibria [21] and Shi and Kibria [22] claimed that for a skewed distribution, the median describes the center of the distribution better than the mean. Thus, for skewed data it makes more sense to measure sample variability in terms of the median rather than the mean. Then following Shi and Kibria [22], the $(1-\alpha)100\%$ CI for the SNR are obtained for four of the existing estimators and provided below. These median modifications are made in attempt to improve the performance of the original intervals. The modified intervals represent both parametric and non-parametric methods.

$$\left[\begin{array}{l}\left(\frac{\tilde{s}}{\bar{x}}+Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2\left[0.5+(\frac{\tilde{s}}{\bar{x}})^2\right]}\right)^{-1}\\ ,\left(\frac{\tilde{s}}{\bar{x}}-Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2\left[0.5+(\frac{\tilde{s}}{\bar{x}})^2\right]}\right)^{-1}\end{array}\right],$$

where $\tilde{s}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(xi-median(x))^2}$ is called modified standard deviation.

4. **Method 4**: Median Modification of McKay

$$\left(\left[\frac{\tilde{s}}{\bar{x}} \sqrt{\left[\left(\frac{\chi^2_{n-1,\alpha/2}}{n} - 1 \right) (\frac{\tilde{s}}{\bar{x}})^2 + \frac{\chi^2_{n-1,\alpha/2}}{n-1} \right]} \right]^{-1} , \left[\frac{\tilde{s}}{\bar{x}} \sqrt{\left[\left(\frac{\chi^2_{n-1,1-\alpha/2}}{n} - 1 \right) (\frac{\tilde{s}}{\bar{x}})^2 + \frac{\chi^2_{n-1,1-\alpha/2}}{n-1} \right]} \right]^{-1} \right).$$

5. **Method 5**: Median Modification of Modified McKay

$$\left(\left[\frac{\tilde{s}}{\bar{x}} \sqrt{\left[\left(\frac{\chi^2_{n-1,\alpha/2} + 2}{n} - 1 \right) (\frac{\tilde{s}}{\bar{x}})^2 + \frac{\chi^2_{n-1,\alpha/2}}{n-1} \right]} \right]^{-1} , \left[\frac{\tilde{s}}{\bar{x}} \sqrt{\left[\left(\frac{\chi^2_{n-1,1-\alpha/2} + 2}{n} - 1 \right) (\frac{\tilde{s}}{\bar{x}})^2 + \frac{\chi^2_{n-1,1-\alpha/2}}{n-1} \right]} \right]^{-1} \right).$$

6. **Method 6**: Median Modified Curto and Pinto[23]

$$\left[\left(\frac{\tilde{s}}{\bar{x}} + Z_{\alpha/2} \sqrt{\frac{1}{n} \left((\frac{\tilde{s}}{\bar{x}})^4 + 0.5 (\frac{\tilde{s}}{\bar{x}})^2 \right)} \right)^{-1} , \left(\frac{\tilde{s}}{\bar{x}} - Z_{\alpha/2} \sqrt{\frac{1}{n} \left((\frac{\tilde{s}}{\bar{x}})^4 + 0.5 (\frac{\tilde{s}}{\bar{x}})^2 \right)} \right)^{-1} \right].$$

where Z(α/2) is the 100(α/2)th percentile of the standard normal distribution . Both of $\bar{x}$ and s will be replaced by $\hat{\mu}$ and $\hat{\sigma}$ in equation (1) to obtain the corresponding confidence intervals for RSS, while regular formulas for $\bar{x}$ and s will be used for SRS.

4. Simulation Study

The objective of the paper is to compare the performance of several interval estimators of SNR using SRS compared to RSS. Since, a theoretical comparison is not possible, a simulation study has been conducted in this section using R © statistical software. Data were randomly generated from normal. The nominal confidence level is set to 95%.

4.1. Simulation Technique

In these simulations, random samples of sizes $n = 15, 25, 50$ are generated with specific parameters from Normal distribution. To better assess and compare the performance of the confidence intervals, comparisons will be made under the same setting. For each combination of n and SNR, 5000 replications were generated.

Normal Distribution

Let $x_1, x_2, \ldots, x_n$ be iid random sample from a Normal distribution with finite mean μ and variance σ^2. For a random variable X such that $X \sim N(\mu, \sigma^2)$, the population SNR $= \mu/\sigma$. Thus, for $\sigma = 10,5,3,1$ and $\mu = 10$ the corresponding population values of SNR are 1, 2, 3.33, and 10 and the corresponding simulated coverage probabilities and widths are presented in Tables 2 to 5 respectively.

Table 2. The estimated coverage probabilities and average widths for Normal $(\mu = 10, \sigma = 10)$ and SNR=1.

Method	One	Two	Three	Four	Five	Six
n=15			**SRS**			
Cover	0.843	0.884	0.844	0.894	0.902	0.853
Lower	0.101	0.112	0.092	0.102	0.094	0.098
Upper	0.052	0.000	0.060	0.000	0.000	0.049
Width	2.203	5.073	2.514	4.992	4.174	1.971
			RSS			
Cover	0.952	0.959	0.951	0.964	0.969	0.958
Lower	0.031	0.041	0.030	0.032	0.031	0.033
Upper	0.012	0.000	0.011	0.000	0.000	0.010
Width	1.882	6.061	2.432	5.962	4.892	1.183
n=25						
			SRS			
Cover	0.912	0.912	0.920	0.921	0.921	0.919
Lower	0.080	0.084	0.072	0.079	0.071	0.077
Upper	0.004	0.000	0.004	0.000	0.000	0.003
Width	1.374	3.060	1.441	3.023	2.640	1.333
			RSS			
Cover	0.973	0.974	0.980	0.972	0.980	0.977
Lower	0.023	0.022	0.020	0.024	0.020	0.023
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	1.324	3.731	1.323	3.693	2.851	1.282
n=50						
			SRS			
Cover	0.934	0.934	0.934	0.933	0.942	0.936
Lower	0.062	0.062	0.062	0.063	0.054	0.064
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	0.789	1.403	0.784	1.399	1.223	0.778
			RSS			
Cover	0.981	0.981	0.983	0.982	0.981	0.982
Lower	0.019	0.019	0.013	0.014	0.011	0.018
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	0.782	1.312	0.782	1.309	1.154	0.772

Table 3. The estimated coverage probabilities and average widths for Normal $(\mu = 10, \sigma = 5)$ and SNR=2.

Method	One	Two	Three	Four	Five	Six
n=15						
			SRS			
Cover	0.909	0.904	0.919	0.911	0.913	0.915
Lower	0.091	0.092	0.081	0.081	0.083	0.085
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	2.384	2.939	2.364	2.883	2.573	2.244
			RSS			
Cover	0.969	0.964	0.971	0.970	0.972	0.971
Lower	0.031	0.032	0.021	0.030	0.024	0.029
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	2.324	2.824	2.301	2.779	2.490	2.188
n=25						
			SRS			
Cover	0.931	0.923	0.933	0.933	0.932	0.934
Lower	0.069	0.073	0.063	0.063	0.064	0.066
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	1.610	1.754	1.600	1.731	1.669	1.560
			RSS			
Cover	0.972	0.964	0.971	0.973	0.974	0.974
Lower	0.024	0.032	0.021	0.023	0.022	0.026
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	1.593	1.731	1.583	1.713	1.649	1.543
n=50						
			SRS			
Cover	0.933	0.933	0.940	0.939	0.941	0.937
Lower	0.061	0.063	0.053	0.061	0.054	0.059
Upper	0.003	0.000	0.003	0.000	0.001	0.004
Width	1.043	1.092	1.040	1.083	1.070	1.028
			RSS			
Cover	0.974	0.972	0.972	0.971	0.972	0.975
Lower	0.021	0.024	0.024	0.021	0.024	0.025
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	1.033	1.082	1.034	1.080	1.064	1.022

Table 4. The estimated coverage probabilities and average widths for Normal $(\mu = 10, \sigma = 3)$ and SNR=3.33 .

Method	One	Two	Three	Four	Five	Six
n=15						
			SRS			
Cover	0.913	0.914	0.923	0.923	0.923	0.922
Lower	0.083	0.082	0.073	0.073	0.073	0.078
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	3.350	3.423	3.304	3.363	3.284	3.151
			RSS			
Cover	0.959	0.952	0.964	0.961	0.962	0.960
Lower	0.041	0.044	0.032	0.039	0.034	0.039
Upper	0.000	0.000	0.000	0.000	0.000	0.000
Width	3.261	3.342	3.222	3.282	3.209	3.073
n=25						
			SRS			
Cover	0.931	0.930	0.932	0.932	0.932	0.933
Lower	0.062	0.064	0.060	0.063	0.061	0.063
Upper	0.003	0.001	0.004	0.002	0.002	0.005
Width	2.070	2.094	2.053	2.073	2.054	2.016
			RSS			
Cover	0.961	0.964	0.964	0.963	0.963	0.966
Lower	0.034	0.031	0.031	0.033	0.032	0.033
Upper	0.000	0.000	0.001	0.001	0.001	0.001
Width	2.044	2.064	2.031	2.052	2.033	1.991
n=50						
			SRS			
Cover	0.939	0.940	0.944	0.942	0.944	0.941
Lower	0.051	0.053	0.050	0.052	0.051	0.052
Upper	0.002	0.004	0.002	0.001	0.001	0.007
Width	1.521	1.533	1.514	1.529	1.521	1.501
			RSS			
Cover	0.972	0.971	0.973	0.973	0.973	0.972
Lower	0.023	0.024	0.021	0.022	0.021	0.026
Upper	0.001	0.001	0.002	0.001	0.001	0.002
Width	1.512	1.521	1.502	1.513	1.509	1.490

Table 5. The estimated coverage probabilities and average widths for Normal $(\mu = 10, \sigma = 1)$ and SNR=10 .

Method	One	Two	Three	Four	Five	Six
n=15						
			SRS			
Cover	0.914	0.913	0.922	0.924	0.924	0.920
Lower	0.081	0.082	0.072	0.073	0.073	0.076
Upper	0.001	0.001	0.002	0.002	0.002	0.003
Width	9.160	9.042	9.004	8.883	8.864	8.605
			RSS			
Cover	0.960	0.954	0.961	0.964	0.964	0.961
Lower	0.040	0.042	0.034	0.032	0.031	0.038
Upper	0.000	0.000	0.001	0.001	0.001	0.001
Width	8.851	8.742	8.702	8.591	8.571	8.321
n=25						
			SRS			
Cover	0.924	0.923	0.931	0.930	0.930	0.928
Lower	0.070	0.071	0.061	0.062	0.062	0.067
Upper	0.002	0.002	0.004	0.004	0.004	0.005
Width	6.423	6.384	6.352	6.312	6.309	6.205
			RSS			
Cover	0.963	0.962	0.970	0.970	0.969	0.968
Lower	0.030	0.031	0.022	0.023	0.023	0.027
Upper	0.003	0.003	0.004	0.004	0.004	0.005
Width	6.282	6.249	6.223	6.181	6.173	6.076
n=50						
			SRS			
Cover	0.933	0.932	0.940	0.939	0.939	0.938
Lower	0.054	0.051	0.049	0.050	0.050	0.050
Upper	0.009	0.009	0.011	0.011	0.011	0.012
Width	4.221	4.211	4.203	4.193	4.190	4.158
			RSS			
Cover	0.972	0.971	0.974	0.974	0.974	0.973
Lower	0.024	0.021	0.021	0.021	0.021	0.022
Upper	0.004	0.004	0.001	0.001	0.001	0.005
Width	4.181	4.171	4.160	4.150	4.144	4.115

4.2. Simulation Results

To compare the performance of the estimators we have generated random samples from Normal. The simulated coverage probabilities and average widths for SNR values of 1, 2, 3.333 and 10 are presented in Tables 2 to 5 respectively, where lower (upper) indicate the proportion of times the estimated SNR is below (above) the lower (upper) confidence limits of the estimated SNR, respectively. From these tables it appears that as sample size increases the coverage probabilities increase while the average widths decreases. This is true as SNR values increase from 1 to 10. For large sample size, the performance of the interval estimators for both RSS and SRS do not differ greatly. However, for small sample sizes, the RSS performed better than SRS in the sense of higher coverage and shorter width for all methods.

5. Conclusion

This paper considered several interval estimators for estimating the population signal to noise ratio (SNR). We have used both SRS and RSS techniques to construct the proposed confidence intervals. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the interval estimators. We have generated data from normal distribution to see the performance of the estimators. Coverage probability and average width are considered as a criterion of a good estimator. When we compare the performance under both sampling techniques, we observed that for large sample sizes, there is not too much difference between the two techniques. However, for small sample sizes, the RSS performed better than the corresponding SRS. We also observed that Methods 1, 3 and 6 performed better than the other methods in most situations. We believe that the findings of the paper will be useful for all practitioners interested in estimating SNR with better accuracy.

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