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An empirical study on second order slope rotatable designs with intraclass correlation structure of errors using symmetrical unequal block arrangements with two unequal block sizes

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Abstract

In this paper, following the works of Das [11-12], an empirical study on second order slope rotatable designs with intra-class correlation structure of errors using symmetrical unequal block arrangements (SUBAs) with two unequal block sizes is suggested. The variance of the estimated slopes for different values of the intra-class correlation coefficient (ρ) and the distance from the centre (d) for “v factors (6≤v≤12)” are studied empirically.

Keywords: Second order slope rotatable designs (SOSRDs), robust SOSRDs, correlated errors.

1. Introduction

Box and Hunter [1] introduced rotatable designs for the exploration of response surface designs. Das and Narasimham [2] constructed second order rotatable designs (SORDs) through balanced incomplete block designs (BIBDs). Raghavarao [3] constructed symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Raghavarao [4] constructed SORDs using incomplete block designs. Panda and Das [5] introduced first order rotatable designs with correlated errors. Das [6] introduced robust second order rotatable designs (RSORDs).

In response surface methodology, good estimation of the derivatives of the response function may be as important or perhaps more important than estimation of mean response. Estimation of differences in responses at two different points in the factor space will often be of great importance. If difference in responses at two points close together is of interest then estimation of local slope (rate of change) of the response is required. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in animal etc, [7].

Hader and Park [8] introduced slope rotatable central composite designs (SRCCDs). Park [7] studied a class of multifactor designs for estimating the slope of response surfaces. Victorbabu and Narasimham [9] constructed second order slope rotatable designs (SOSRDs) using balanced incomplete block designs. Victorbabu [10] constructed SOSRDs using SUBAs with two unequal block sizes. Das [11] introduced slope rotatability with correlated errors.

2. Conditions for SOSRDs with intra-class correlated errors (cf. [11], pp. 66)

A second order response surface design D = ((x_{iu})) for fitting

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i=1}^{v-1}\sum_{j=i+1}^{v} b_{ij} x_{iu} x_{ju} + e_u \tag{2.1}$$

where x_{iu} denotes the level of the i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are correlated random errors, is said to be a SOSRDs with intra-class correlated errors, if the variance of the estimate of first order partial derivative of $Y_u(x_1, x_2, \ldots, x_v)$ with respect to each of independent variable (x_i) is only a function of the distance ($d^2 = \sum_{i=1}^{v} x_i^2$) of the point ($x_1, x_2, \ldots, x_v$) from the origin (centre of the design).

Such a spherical variance function for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions (cf. [11]).

1. $\sum_{u=1}^{N}\prod_{i=1}^{v} x_{iu}^{\alpha_i} = 0$ if any α_i is odd, for $\sum \alpha_i \le 4$
2. $\sum_{u=1}^{N} x_{iu}^2 = \text{ constant} = N\lambda_2$
3. $\sum_{u=1}^{N} x_{iu}^4 = \text{constant} = \ cN\lambda_4$, for all i
4. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \text{constant} = \ N\lambda_4$, for all $i \neq j$
5. $\sum_{u=1}^{N} x_{iu}^4 = c\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2$
6. $\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)}$ (non − singularity condition)
7. $$\left(\frac{AcN\lambda_4 - B}{(1-\rho)}\right)\left[4N - \left(\frac{AcN\lambda_4 - B}{A\lambda_4}\right) + v\left(\frac{N\lambda_2^2(1-\rho)}{A\lambda_4}\right) - (v-2)\left(\frac{AN\lambda_4 - B}{A\lambda_4}\right)\right] + \frac{N[AN\lambda_4 - B]}{(1-\rho)}\left[4(v-2) + (v-1)\left(\frac{AN\lambda_4 - B}{AN\lambda_4}\right)\right] - N^2\lambda_2^2\left[4(v-1) + v\left(\frac{AN\lambda_4 - B}{AN\lambda_4}\right)\right] = 0. \tag{2.2}$$

where c, λ_2 and λ_4 are constants and the summation is over the design points.

The variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{[\lambda_4(c+v-1)A - v\rho N\lambda_2^2]\sigma^2 A}{N\Delta}$$

$$V(\hat{b}_i) = \frac{\sigma^2(1-\rho)}{N\lambda_2}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2(1-\rho)}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2(1-\rho)[\lambda_4(c+v-2)A - (v-1)\rho N\lambda_2^2 - (v-1)\lambda_2^2(1-\rho)]}{(c-1)N\lambda_4\Delta}$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = -\frac{\lambda_2\sigma^2(1-\rho)A}{N\Delta}$$

$$\mathrm{Cov}(\hat{b}_{ii}, \hat{b}_{jj}) = \frac{\sigma^2(1-\rho)[\lambda_2^2(1-\rho) - \lambda_4 A + N\rho\lambda_2^2]}{(c-1)N\lambda_4\Delta} \tag{2.3}$$

where $A = \{1 + (N-1)\rho\}$, $B = \rho N^2\lambda_2^2$.

$\Delta = [\lambda_4(c+v-1)A - v\rho N\lambda_2^2 - v\lambda_2^2(1-\rho)]$ and other covariances are zero.

$$\text{Further, } V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right) = \frac{(1-\rho)}{N}\left(\frac{1}{\lambda_2} + \frac{d^2}{\lambda_4}\right)\sigma^2 \tag{2.4}$$

Note: For SOSRD with uncorrelated errors case, simply set ρ=0.

Some Preliminaries:

(i) Intra-class correlation structure: The intra-class correlation structure is defined as the variance-covariance structure, when errors of any two observations have the same correlation and each has the same variance. It is known that the SORDs preserve the property of rotatability under the intra-class structure of errors (c.f.[12], p.1273).

(ii) SUBA with two unequal block sizes (cf. [3]) :

The arrangement of v-treatments in b blocks where b_1 blocks of size k_1, and b_2 blocks of size k_2 is said to be a symmetrical unequal block arrangement with two unequal block sizes, if

(i) every treatment occurs $\frac{b_i k_i}{v}$ blocks of size k_i (i = 1,2), and

(ii) every pair of first associate treatments occurs together in μ blocks of size k_1 and in (λ-μ) blocks of size k_2 while every pair of second associate treatments occurs together in λ blocks of size k_2.

From (i) each treatment occurs in $\left(\frac{b_1k_1}{v} + \frac{b_2k_2}{v}\right) = r$ blocks in all. (v, b, r, k_1, k_2, b_1, b_2, λ) are known as the parameters of the SUBA with two unequal block sizes.

Let (v, b, r, k_1, k_2, b_1, b_2, λ), k = sup(k_1, k_2) and $b = b_1 + b_2$, be a SUBA with two unequal block sizes. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. n_0 denotes the number of centre points in the design. [1-(v, b, r, k_1, k_2, b_1, b_2, λ)] denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes, [1-(v, b, r, k_1, k_2, b_1, b_2, λ)]$2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by multiplication (c.f. Raghavarao [13], pp. 298-300),

$(a,0,0,\ldots,0)2^1$ denote the design points generated from (a,0,0...,0) point set, and ∪ denotes combination of the design points generated from different sets of points.

- **(iii) Second order rotatable design (SORD):** A second order response surface design as given in equation (2.1) where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, $e_u's$ are uncorrelated random errors with mean zero and variance σ^2. D is said to be a SORD, if the variance of the estimated response of $\widehat{Y}_u$ from the fitted surface is only a function of the distance ($d^2 = \sum_{i=1}^{v} x_i^2$) of the point ($x_{1u}, x_{2u}, \ldots, x_{vu}$) from the origin (centre) of the design (c.f. [1-2]).
- **(iv) Robust second order rotatable design (RSORD):** A second order response surface design as given in equation (2.1) where x_{iu} denotes the level of the i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are correlated random errors. A second order response surface design D is said to be a RSORD, if the variance of the estimated response of $\widehat{Y}_u$ from the fitted surface is only a function of the distance, ($d^2 = \sum x_i^2$) of the point $(x_{1u}, x_{2u}, \ldots, x_{vu})$ from the origin (centre) of the design (cf. [6]).

3. An empirical study of SOSRDs with intra-class correlation structure of errors using SUBAs with two unequal block sizes

Following the methods of construction of [6, 10-12] here an empirical study on SOSRDs with intra-class correlation structure of errors using SUBAs with two unequal block sizes is studied. . Let ρ ($-\frac{1}{N-1} < \rho < 1$) be the correlation between errors of any two observations, each having the same variance σ^2**.**

Let (v, b, r, k_1, k_2, b_1, b_2, λ) denote a SUBA with two unequal block sizes, where k = sup(k_1, k_2) and $b = b_1 + b_2$, $2^{t(k)}$ denote a fractional replicate of 2^k in +1 and -1 levels, in which no interaction with less than five factors is confounded and n_0 be the number of centre points in the design.

Theorem (3.1): The design points, [1-(v, b, r, k_1, k_2, b_1, b_2, λ)]$2^{t(k)}$ ∪ (a, 0, 0, …, 0) 2^1∪ (n_0) will give a v-dimensional SOSRDs with intra-class correlation structure of errors using SUBAs with two unequal block sizes in N design points, where a^2 is positive real root of the fourth degree polynomial equation,

$$
\begin{aligned}
&[(8v-4N)]A^2a^8+\\
&\left[8vr2^{t(k)}A^2\right]a^6+\Big[2vr^2 2^{2t(k)}\\
&\qquad+\left\{\left((12-2v)\lambda-4r\right)N2^{t(k)}+(16\lambda-20v\lambda+4vr)\right\}2^{t(k)}\Big]A^2a^4\\
&\qquad+[4vr^2+(16v-20v)r\lambda]A^2 2^{2t(k)}a^2\\
&\qquad+[(5v-9)\lambda^2+(6-v)r\lambda-r^2]A^2N2^{2t(k)}\\
&\qquad+(vr+4\lambda-5v\lambda)A^2r^2 2^{3t(k)}=0.
\end{aligned}
\tag{3.1}
$$

Proof: For the design points generated from the SUBAs with two unequal block sizes, simple symmetry conditions are true. Further we have,

$$\sum_{u=1}^{N} x_{iu}^2 = r2^{t(k)} + 2a^2 = \text{ constant} = N\lambda_2 \quad \text{for all i} \tag{3.2}$$

$$\sum_{u=1}^{N} x_{iu}^4 = r2^{t(k)} + 2a^4 = \text{constant} = \ c\,N\lambda_4, \text{for all i} \tag{3.3}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \lambda 2^{t(k)} = \ \text{constant} = N\lambda_4, \text{for all } i \neq j \tag{3.4}$$

Substituting λ_2, λ_4 and c in condition 7 of (2.2) and on simplification, we get (3.1). The design exists only if at least one positive real root exits for equation (3.1). Solving equation (3.1) we get the SOSRDs with intra-class correlation structure of errors using SUBAs with two unequal block sizes with different ‘a’ values for the ‘v’ different factors. The variances of estimated slopes of these SOSRDs for $0 \leq \rho \leq 0.9$ and for $6 \leq v \leq 12$ are given in Table 1.

Note: It is observed that the SOSRDs preserve the property of slope rotatability under the intra-class structure of errors.

Example: We illustrate the above method with construction of SOSRD with intra-class correlation structure of errors for 12- factors with the help of a SUBA with two unequal block sizes with parameters (v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1).

The design points, **[**1- (v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1)] 2^4 U (a, 0, 0,…, 0) 2^1 U(n_0=1) will give a SOSRD with intra-class structure of errors using SUBA with two unequal block sizes in N = 233 design points for twelve factors. We have

$$\sum_{u=1}^{N} x_{iu}^2 = 64 + 2a^2 = N\lambda_2 \qquad (3.5)$$

$$\sum_{u=1}^{N} x_{iu}^4 = 64 + 2a^4 = c\,N\lambda_4 \qquad (3.6)$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 16 = N\lambda_4 \qquad (3.7)$$

From (3.6) and (3.7) we get $c = \frac{64+2a^4}{16}$. Substituting for λ_2, λ_4 and c in (3.7) and on simplification, we get the following biquadratic equation in a^2 (viz.)

$$(1 + 232\rho)^2(209a^8 - 1536a^6 + 1648a^4 + 8192a^2 - 32960) = 0. \qquad (3.8)$$

Equation (3.8) has only one positive real root a^2 = 5.586391472 ($\forall\rho$ $(-\frac{1}{N-1} < \rho < 1)$). It can be verified that condition 6 of (2.2) is also satisfied. This can be alternatively written directly from equation (3.1). Solving (3.8), we get a=2.363554838, $\forall\rho$. Substituting this value of 'a' in (3.5), (3.6) and (3.7) we obtain $\lambda_2 = 0.3226$, $\lambda_4 = 0.0687$ and c= 7.9010. From (2.3), we can obtain the variances and covariances. Further from (2.4), we have,

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right) = (0.0133 + 0.0625d^2)(1-\rho)\sigma^2.$$

We may point out here that this SOSRD with intra-class correlation structure of errors using SUBA with two unequal block sizes with parameters (v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1) has only 233 design points for 12 factors, where as the corresponding SOSRD with intra-class correlation structure of errors using central composite design (CCD) obtained by [11] needs 281 design points for 12 factors. Thus the method leads to a 12-factor SOSRD with intra-class structure of errors in less number of design points than the corresponding SOSRD with intra-class structure of errors using CCD.

4. A study of dependence of the variance function of the response at different design points

In this section we study the dependence of variance function of response at different design points of SOSRDs with intra-class correlated structure of errors using SUBAs with two unequal block sizes. Given ‘v’ factors and different values of the intra-class correlation coefficient ‘ρ’ and distance‘d’ between 0 and 1, the variances are tabulated.

From (2.4) the variance of the estimated derivative is obtained by,

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right) = V\left(\hat{b}_i\right) + 4x_i^2 V\left(\hat{b}_{ii}\right) + \sum_{j=1;j\neq i}^{v} x_j^2\, V\left(\hat{b}_{ij}\right) \qquad (4.1)$$

$$= \frac{\sigma^2(1-\rho)}{N\lambda_2} + \frac{\sigma^2(1-\rho)}{N\lambda_4} d^2 \qquad (4.2)$$

The numerical calculations are appended in the Appendix in Table 2.

Conclusion: From the Table 2, we conclude that,

(i) For given v and ρ, $V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$ increases as d increases.

(ii) For given v and d, $V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$ decreases as ρ increases.

Graphical representation of slope variances for SOSRD with intra-class correlation structure of errors using a SUBA with two unequal block sizes for (v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1) factors is given in Figure 1 in the Appendix.

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Appendix

Table 1. The variance of estimated derivatives (slopes) for the factors $6 \le v \le 12$.

ρ	v=6, b=7, r=3, k_1=2, k_2=3, b_1=3, b_2=4, λ=1,N=69, a=2.1287	v=8, b=12, r=4, k_1=2, k_2=3, b_1=4, b_2=8, λ=1, N=113, a=2.0444	v=10, b=11, r=5, k_1=4, k_2=5, b_1=5, b_2=6, λ=2, N=197, a=2.8928	v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1, N=233, a=2.3636
	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$
0	$0.0302\sigma^2+0.1250\,\sigma^2d^2$	$0.0248\sigma^2+0.1250\sigma^2d^2$	$0.0103\sigma^2+0.0313\sigma^2d^2$	$0.0133\sigma^2+0.0625\sigma^2d^2$
0.1	$0.0272\sigma^2+0.1125\,\sigma^2d^2$	$0.0223\sigma^2+0.1125\sigma^2d^2$	$0.0093\sigma^2+0.0281\sigma^2d^2$	$0.0120\sigma^2+0.0562\sigma^2d^2$
0.2	$0.0242\,\sigma^2+0.1000\,\sigma^2d^2$	$0.0198\sigma^2+0.1000\sigma^2d^2$	$0.0083\sigma^2+0.0250\sigma^2d^2$	$0.0106\sigma^2+0.0500\sigma^2d^2$
0.3	$0.0212\,\sigma^2+0.0875\,\sigma^2d^2$	$0.0173\sigma^2+0.0875\sigma^2d^2$	$0.0072\sigma^2+0.0219\sigma^2d^2$	$0.0093\sigma^2+0.0437\sigma^2d^2$
0.4	$0.0181\,\sigma^2+0.0750\,\sigma^2d^2$	$0.0149\sigma^2+0.0750\sigma^2d^2$	$0.0062\sigma^2+0.0188\sigma^2d^2$	$0.0080\sigma^2+0.0375\sigma^2d^2$
0.5	$0.0151\sigma^2+0.0625\,\sigma^2d^2$	$0.0124\sigma^2+0.0625\sigma^2d^2$	$0.0052\sigma^2+0.0156\sigma^2d^2$	$0.0067\sigma^2+0.0312\sigma^2d^2$
0.6	$0.0121\,\sigma^2+0.0500\,\sigma^2d^2$	$0.0099\sigma^2+0.0500\sigma^2d^2$	$0.0041\sigma^2+0.0125\sigma^2d^2$	$0.0053\sigma^2+0.0250\sigma^2d^2$
0.7	$0.0091\sigma^2+0.0375\,\sigma^2d^2$	$0.0074\sigma^2+0.0375\sigma^2d^2$	$0.0031\sigma^2+0.0094\sigma^2d^2$	$0.0040\sigma^2+0.0187\sigma^2d^2$
0.8	$0.0060\sigma^2+0.0250\,\sigma^2d^2$	$0.0050\sigma^2+0.0250\sigma^2d^2$	$0.0021\sigma^2+0.0063\sigma^2d^2$	$0.0027\sigma^2+0.0125\sigma^2d^2$
0.9	$0.0030\,\sigma^2+0.0125\,\sigma^2d^2$	$0.0025\sigma^2+0.0125\sigma^2d^2$	$0.0010\sigma^2+0.0031\sigma^2d^2$	$0.0013\sigma^2+0.0062\sigma^2d^2$

NB: $V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$ is obtained by using the equation (4.2).

Table 2. Study of dependence of estimated slope of SOSRDs with intraclass correlated structure of errors using SUBAs with two unequal block sizes at different design points for 6 ≤ v ≤ 12 for different values of ‘ρ’ and d.

v=6, b=7, r=3, k_1=2, k_2=3, b_1=3, b_2=4, λ=1 (σ^2=1)

ρ	d=0.1	d=0.2	d=0.3	d=0.4	d=0.5	d=0.6	d=0.7	d=0.8	d=0.9	d=1
0	0.0315	0.0352	0.0415	0.0502	0.0615	0.0752	0.0915	0.1102	0.1315	0.1552
0.1	0.0283	0.0317	0.0373	0.0452	0.0553	0.0677	0.0823	0.0992	0.1183	0.1397
0.2	0.0252	0.0282	0.0332	0.0402	0.0492	0.0602	0.0732	0.0882	0.1052	0.1242
0.3	0.0221	0.0247	0.0291	0.0352	0.0431	0.0527	0.0641	0.0772	0.0928	0.1087
0.4	0.0189	0.0211	0.0249	0.0301	0.0369	0.0451	0.0549	0.0661	0.0789	0.0931
0.5	0.0157	0.0176	0.0207	0.0251	0.0307	0.0376	0.0457	0.0551	0.0657	0.0776
0.6	0.0126	0.0141	0.0166	0.0201	0.0246	0.0301	0.0366	0.0441	0.0526	0.0621
0.7	0.0095	0.0106	0.0125	0.0151	0.0185	0.0226	0.0275	0.0331	0.0395	0.0466
0.8	0.0063	0.0070	0.0083	0.0100	0.0123	0.0150	0.0183	0.0220	0.0263	0.0310
0.9	0.0031	0.0035	0.0041	0.0050	0.0061	0.0075	0.0091	0.0110	0.0131	0.0155

v=8, b=12, r=4, k_1=2, k_2=3, b_1=4, b_2=8, λ=1 (σ^2=1)

ρ	d=0.1	d=0.2	d=0.3	d=0.4	d=0.5	d=0.6	d=0.7	d=0.8	d=0.9	d=1
0	0.0261	0.0298	0.0361	0.0448	0.0561	0.0698	0.0861	0.1048	0.1261	0.1498
0.1	0.0234	0.0268	0.0324	0.0403	0.0504	0.0628	0.0774	0.0943	0.1134	0.1348
0.2	0.0208	0.0238	0.0288	0.0358	0.0448	0.0558	0.0688	0.0838	0.1008	0.1198
0.3	0.0182	0.0208	0.0252	0.0313	0.0392	0.0488	0.0602	0.0733	0.0882	0.1048
0.4	0.0157	0.0179	0.0217	0.0269	0.0337	0.0419	0.0517	0.0629	0.0757	0.0899
0.5	0.0130	0.0149	0.0180	0.0224	0.0280	0.0349	0.0430	0.0524	0.0630	0.0749
0.6	0.0104	0.0119	0.0144	0.0179	0.0224	0.0279	0.0344	0.0419	0.0504	0.0599
0.7	0.0078	0.0089	0.0108	0.0134	0.0168	0.0209	0.0258	0.0314	0.0378	0.0449
0.8	0.0053	0.0060	0.0073	0.0090	0.0113	0.0140	0.0173	0.0210	0.0252	0.0300
0.9	0.0026	0.0030	0.0036	0.0045	0.0056	0.0070	0.0086	0.0105	0.0126	0.0150

v=10, b=11, r=5, k_1=4, k_2=5, b_1=5, b_2=6, λ=2 (σ^2=1)

ρ	d=0.1	d=0.2	d=0.3	d=0.4	d=0.5	d=0.6	d=0.7	d=0.8	d=0.9	d=1
0	0.0106	0.0116	0.0131	0.0153	0.0181	0.0216	0.0256	0.0303	0.0357	0.0416
0.1	0.0096	0.0104	0.0118	0.0138	0.0163	0.0194	0.0230	0.0273	0.0321	0.0374
0.2	0.0086	0.0093	0.0106	0.0123	0.0146	0.0173	0.0206	0.0243	0.0286	0.0333
0.3	0.0074	0.0081	0.0092	0.0107	0.0127	0.0151	0.0179	0.0212	0.0249	0.0291
0.4	0.0064	0.0070	0.0079	0.0092	0.0109	0.0130	0.0154	0.0182	0.0214	0.0250
0.5	0.0054	0.0058	0.0066	0.0077	0.0091	0.0108	0.0128	0.0152	0.0178	0.0208
0.6	0.0042	0.0046	0.0052	0.0061	0.0072	0.0086	0.0102	0.0121	0.0142	0.0166
0.7	0.0032	0.0035	0.0039	0.0046	0.0055	0.0065	0.0077	0.0091	0.0107	0.0125
0.8	0.0022	0.0024	0.0027	0.0031	0.0037	0.0044	0.0052	0.0061	0.0070	0.0084
0.9	0.0010	0.0011	0.0013	0.0015	0.0018	0.0021	0.0025	0.0030	0.0035	0.0041

v=12, b=13, r=4, k_1=3, k_2=4, b_1=4, b_2=9, λ=1 (σ^2=1)

ρ	d=0.1	d=0.2	d=0.3	d=0.4	d=0.5	d=0.6	d=0.7	d=0.8	d=0.9	d=1
0	0.0139	0.0158	0.0189	0.0233	0.0289	0.0358	0.0439	0.0533	0.0639	0.0758
0.1	0.0126	0.0142	0.0171	0.0210	0.0261	0.0322	0.0395	0.0480	0.0575	0.0682
0.2	0.0111	0.0126	0.0151	0.0186	0.0231	0.0286	0.0351	0.0426	0.0511	0.0606
0.3	0.0097	0.0110	0.0132	0.0163	0.0202	0.0250	0.0307	0.0373	0.0447	0.0530
0.4	0.0084	0.0095	0.0114	0.0140	0.0174	0.0215	0.0264	0.0320	0.0384	0.0455
0.5	0.0070	0.0079	0.0095	0.0117	0.0145	0.0179	0.0220	0.0267	0.0320	0.0379
0.6	0.0056	0.0063	0.0076	0.0093	0.0116	0.0143	0.0176	0.0213	0.0256	0.0303
0.7	0.0042	0.0047	0.0057	0.0070	0.0087	0.0107	0.0132	0.0156	0.0191	0.0227
0.8	0.0028	0.0032	0.0038	0.0047	0.0058	0.0072	0.0088	0.0107	0.0128	0.0152
0.9	0.0014	0.0015	0.0019	0.0023	0.0029	0.0035	0.0043	0.0053	0.0063	0.0075

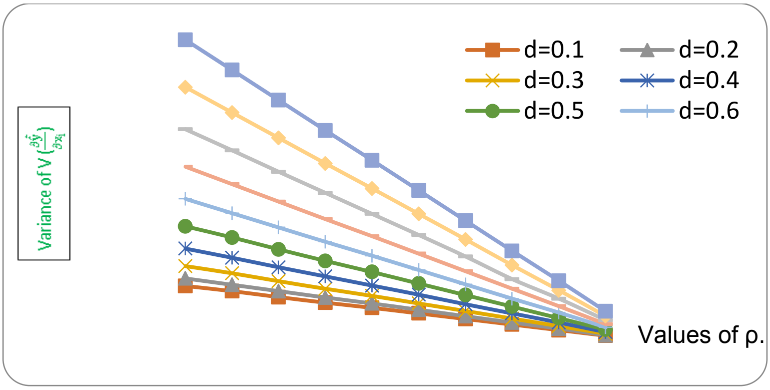

Figure 1. Graphical representation of slope variances.

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