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Deteriorated Economic Order Quantity (EOQ) Model with Variable Ordering Cost

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Abstract

The economic order quantity model is framed for analyzing the effect of variable ordering cost with deteriorated items. The objective of this model is to maximize the net profit so as to determine the order quantity. For any given number of replenishment cycles the existence of a unique optimal replenishment schedule can be obtained and further the concavity of the net profit function of the inventory system in the number of replenishments is established. The numerical analysis shows that an appropriate policy can benefit the retailer and that policy is important, especially for deteriorating items. Finally, sensitivity analyses of the optimal solution with respect to the major parameters are also studied to draw some decisions with managerial implications for competitive advantage.

Keywords: EOQ, deterioration, crisp, profit, variable ordering cost.

1. Introduction

Most of the literature on inventory control and production planning has dealt with the assumption that the demand for a product will continue infinitely in the future either in a deterministic or in a stochastic fashion. This assumption does not always hold true. Inventory management plays a significant role for production system in businesses since it can help companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth for achieving competitive advantage in the globe. The mathematical modeling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed.

The classical analysis builds a model of an inventory system and calculates the EOQ which minimize the costs so that satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory for long run. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks of the product.

The EOQ model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Comprehensive reviews of inventory models can be found in Osteryoung et al. [1], Tripathy et al. [2] and Salameh et al. [3]. In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain et al. [4] developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Mishra [5] explored the inventory model for time dependent holding cost and deterioration with salvage value where shortages are allowed. Rafaat [6] derived continuous deteriorated inventory models for the cost effective production process. Gupta et al. [7] examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Goyal et al. [8] considers the effect of different marketing policies, e.g. the price per unit product and the advertisement frequency on the demand of a perishable item. Bose et al. [9] considered an economic order quantity (EOQ) inventory model for deteriorating goods developed with a linear, positive trend in demand allowing inventory shortages and backlogging. Bose et al. [9] and Hariga [10] considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga [11] argued that the analysis of Bose et al. [9] contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Pattnaik [12] explained a single item EOQ model with demand dependent unit cost and variable setup cost. Padmanabhan et al. [13] presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. Unlike the work of Wee [14] who studied the case of partial backlogging. Padmanabhan et al. [13] considered the cases of no backlogging, and of partial and full backlogging. Waters et al. [15] and Pattnaik [16] discussed different types inventory models like deterministic, reliability, fuzzy, entropy cost, price sensitive demand etc.. Pattnaik [17] explained a non-linear profit-maximization entropic order quantity model for deteriorating items with stock dependent demand rate.

The most recent work found in the literature is that of Hariga [18] who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Pattnaik [19] presented an entropic order quantity (EnOQ) model under instant deterioration for perishable items with constant demand where discounts are allowed. Tsao et al. [20] studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked. Tripathy et al. [21] reviewed a single item EOQ model with two constraints. Tripathy et al. [2] investigated optimal EOQ model for deteriorating items with promotional effort cost. Tripathy et al. [22] explained continuous inventory replenishment and pricing models for deteriorating items with stock and price sensitive demand in finite horizon.

Salameh et al. [3] studied profit maximization economic order quantity model for deteriorated items with time and price dependent linear and decreasing demand for finite planning horizon under promotion factor for acquiring more profit. Pattnaik [23] investigated a single item EOQ model with demand-dependent unit cost and dynamic setup cost. All mentioned above inventory literatures with deterioration or the percentage of on-hand inventory due to deterioration is lost have the basic assumption that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. This paper postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with the percentage of on-hand inventory due to deterioration is not lost with dynamic ordering cost and then compares the optimal results with fixed ordering cost in traditional model. Replenishment decision under wasting the percentage of on-hand inventory due to deterioration are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the finite planning horizon. The major assumptions used in the above research articles are summarized in Table1.

Table 1. Summary of the related researches.

Author(s) and published Year	Structure of the model	Demand	Demand patterns	Deterioration	Ordering Cost	Planning	Model
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	Constant	Finite	Cost
Tsao et al. (2008)	Crisp (EOQ)	Time and Price	Linear and Decreasing	Yes	Constant	Finite	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Constant	Finite	Profit
Pattnaik (2011)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Constant	Finite	Profit
Present Paper (2013)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes	Variable	Finite	Profit

The remainder of the paper is organized as follows. In section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in section 3. The solution procedure is given in section 4. In section 5, numerical example is presented to illustrate the development of the model. The sensitivity analysis is carried out in section 6 to observe the changes in the optimal solution. Finally section 7 deals with the summary and the concluding remarks.

2. Notations and Assumptions

r	Consumption rate
t_c	Cycle length
h	Holding cost of one unit for one unit of time.
HC (q)	Holding cost per cycle
c	Purchasing cost per unit
P_s	Selling Price per unit
α	Percentage of on-hand inventory that is lost due to deterioration
q	Order quantity
$K \times (q^{\gamma-1})$	Ordering cost per cycle where, $0 < \gamma < 1$
q^*	Traditional economic ordering quantity (EOQ)
$\varphi(t)$	On-hand inventory level at time t
$\pi_1(q)$	Net profit per unit of producing q units per cycle
$\pi(q)$	Average profit per unit of producing q units per cycle

3. Mathematical Model

Denote $\varphi(t)$ as the on-hand inventory level at time t. During a change in time from point t to t+dt, where t + dt > t, the on-hand inventory drops from $\varphi(t)$ to $\varphi(t+dt)$. Then $\varphi(t+dt)$ is given as:

$$\varphi(t+dt) = \varphi(t) - r\,dt - \alpha\,\varphi(t)\,dt \qquad (1)$$

Equation (1) can be re-written as:

$$\frac{\varphi(t+dt) - \varphi(t)}{dt} = -r - \alpha\varphi(t) \qquad (2)$$

and $dt \to 0$, equation (2) reduces to:

$$\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r = 0 \qquad (3)$$

Equation (3) is a differential equation, solution is

$$\varphi(t) = \frac{-r}{\alpha} + \left(q + \frac{r}{\alpha}\right) \times e^{-\alpha t} \quad (4)$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. The cycle length, t_c, is determined by first substituting t_c into equation (4) and then setting it equal to zero to get:

$$t_c = \frac{1}{\alpha} \ln\left(\frac{\propto q + r}{r}\right) \quad (5)$$

Equation (4) and (5) are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r}$.

The total cost per cycle, TC(q), is the sum of the ordering cost per order, purchasing cost per cycle, cq and the holding cost per cycle, HC(q).

$$HC(q) = \int_0^{tc} h\varphi(t)dt = h \times \left[\frac{q}{\propto} - \frac{r}{\alpha^2} \ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (7)$$

Ordering cost per order is variable in nature and function of q. So, OC(q) = $K \times q^{(\gamma - 1)}$ it is negatively related to q.

$$TC(q) = \left(K \times q^{(\gamma-1)}\right) + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \times \ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (8)$$

The total cost per unit of time, TCU (q), is given by dividing equation (8) by equation (5) to give:

$$TCU(q) = \left[\left(K \times q^{(\gamma-1)}\right) + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln\left(\frac{\alpha q + r}{r}\right)\right]\right] \times \left[\frac{1}{\alpha} \ln\left(\frac{\propto q + r}{r}\right)\right]^{-1}$$

$$= \frac{K\propto + (c\propto + h)q}{\ln\left(1 + \frac{\propto q}{r}\right)} - \frac{hr}{\alpha} \quad (9)$$

As α approaches zero in equation (8) and (9) reduces to TC(q) =$\left(\mathrm{K}\times \mathrm{q}^{(\gamma-1)}\right)+\mathrm{cq}+\frac{\mathrm{hq}^2}{2\mathrm{r}}$ and TCU (q) = $\mathrm{Krq}^{(\gamma-2)}+\mathrm{cr}+\frac{\mathrm{hq}}{2}$ respectively, whose solution is given by the traditional EOQ formula, $\mathrm{q}^*=\left[\frac{\mathrm{h}}{2\mathrm{Kr}(2-\gamma)}\right]^{1/\gamma-3}$ since L is zero. The total profit per cycle is $\pi_1(\mathrm{q})$.

$$\pi_1(\mathrm{q})=\mathrm{q}\times \mathrm{P_s}-\mathrm{TC}(\mathrm{q})=(\mathrm{q}\times \mathrm{P_s})-(\mathrm{K}\times \mathrm{q}^{(\gamma-1)})-\mathrm{cq}-\frac{\mathrm{hq}^2}{2\mathrm{r}} \qquad (10)$$

Where L, the number of units lost per cycle due to deterioration, and TC (q) the total cost per cycle, are calculated from equations (6) and (8), respectively. The average profit π(q) per unit time is obtained by dividing t_c in π_1(q). Hence the profit maximization problem is

Maximize π_1 (q)

$\forall$ q≥ 0 (11)

4. Solution Procedure (Optimization)

The optimal ordering quantity q per cycle can be determined by differentiating equation (10) with respect to q, then setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is concave in terms of ordering quantity q. The second order derivates of equation (10) with respect to q are strictly negative. Consider the following propositions.

Result 1. The net profit π_1 (q) per cycle is concave in q.

Conditions for optimal q is

$$\frac{\mathrm{d}\pi_1(\mathrm{q})}{\mathrm{dq}}=(\mathrm{P_s})-\left(\mathrm{K}(\gamma-1)\mathrm{q}^{\gamma-2}+\frac{\mathrm{chq}}{\mathrm{r}}\right)=0 \qquad (12)$$

The second order derivative of the net profit per cycle with respect to q can be expressed as:

$$\frac{\mathrm{d}^2\pi_1(\mathrm{q})}{\mathrm{dq}^2}=-\,\mathrm{K}(\gamma-1)(\gamma-2)\mathrm{q}^{\gamma-3}-\frac{\mathrm{hc}}{\mathrm{r}} \qquad (13)$$

Since r > 0, $h > 0, K > 0, q > 0, c > 0$ and $0 < \gamma < 1$ so the equation (13) is negative. Result 1 shows that the second order derivative of equation (11) with respect to q are strictly negative.

The objective is to determine the optimal values of q to maximize the unit profit function of equation (11). It is very difficult to derive the optimal values of q, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variables.

5. Numerical Example

Consider an inventory situation where K is Rs. 200 per order, h is Rs. 5 per unit per unit of time, r is 1200 units per unit of time, c is Rs. 100 per unit, the selling price per unit P_s is Rs. 125, γ is 0.5 and α is 0%, . The optimal solution that maximizes equation (10) and q^* are determined by using LINGO 13.0 version software and the results are tabulated in Table 2.

Table 2. Optimal Values of the Proposed Model.

Model	Iteration	t^*	L^*	EOQ	Dynamic OC	$\pi_1(q)$	$\pi(q)$
Crisp	417	5.000043	-	$q^* =$ 6000.052	2.59	74997.42	14999.35501
Crisp	41	0.258	-	309.839	-	7345.9678	28450.81

6. Sensitivity Analyses

It is interesting to investigate the influence of the major parameters K, h, r, c, P_s and γ on retailer's behavior. The computational results are shown in Table 3 indicates the following managerial phenomena:

- t_c the replenishment cycle length , q the optimal replenishment quantity, π_1 the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are insensitive to the parameter K but OC variable setup cost is sensitive to the parameter K.
- t_c the replenishment cycle length , q the optimal replenishment quantity, OC variable setup cost and π_1 the optimal net profit per unit per cycle are sensitive

to the parameter h but π the optimal average profit per unit per cycle is insensitive to the parameter h.

- q the optimal replenishment quantity, OC variable setup cost, π_1 the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are sensitive to the parameter r but t_c the replenishment cycle length is insensitive to the parameter r.
- t_c the replenishment cycle length , q the optimal replenishment quantity, OC variable setup cost, π_1 the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are sensitive to both the parameters c and P_s.
- t_c the replenishment cycle length , q the optimal replenishment quantity and π the optimal average profit per unit per cycle are insensitive to the parameter γ but OC variable setup cost and π_1 the optimal net profit per unit per cycle are sensitive to the parameter γ.

Fig. 1 represents the relationship between the order quantity q and dynamic setup cost OC. and Fig. 3 represents the three dimensional mesh plot of order quantity q and net profit per cycle π_1.

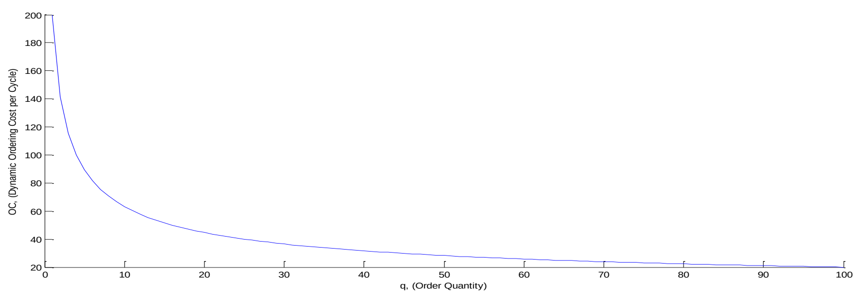

Figure 1. Two dimensional plot of Order Quantity, q and Dynamic Ordering Cost, OC.

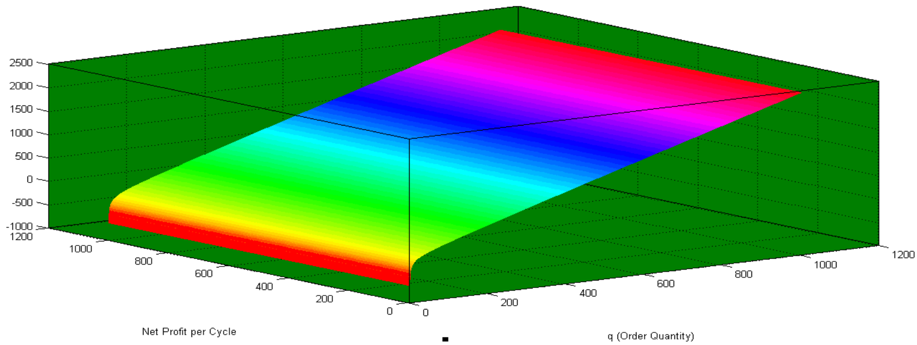

Figure 2. Three dimensional mesh plot of order quantity q and net profit per cycle $\pi_1(q)$.

Table 3. Sensitivity Analyses of the parameters K, h, r, c, P_s and γ.

Parameter	Value	Iteration	t^*	q^*	OC	$\pi_1(q)$	$\pi(q)$	% Change in $\pi_1(q)$
K	150	351	5.000032	6000.039	1.94	74998.06	14999.52	0.000853
	250	381	5.000054	6000.065	3.23	74998.06	14999.52	0.000853
	500	422	5.000108	6000.129	6.45	74993.55	14998.39	0.00516
h	3	348	8.333367	10000.04	1.10	124998.0	14999.70	66.6697
	8	372	3.125054	3750.065	3.27	46871.73	14998.69	37.5022
	10	420	2.500061	3000.073	3.65	37496.35	14998.17	50.0031
r	1100	380	5.000049	5500.054	2.70	68747.30	13749.33	8.3333
	1500	372	5.000031	7500.046	2.31	93747.69	18749.42	25.0012
	2000	370	5.000020	10000.04	1.10	124998.0	24999.50	66.6697
c	50	435	15.00001	18000.01	1.49	674998.5	44999.88	800.028
	80	348	9.000018	10800.02	1.92	242998.1	26999.73	224.009
	120	392	1.000481	1200.577	5.77	2994.227	2992.788	96.0076
P_s	120	379	4.00006	4800.072	2.89	47997.11	11999.10	36.0017
	150	324	10.00002	12000.02	1.83	299998.2	29999.7	300.011
	200	434	20.00001	24000.01	1.29	1199999	59999.92	1500.05
γ	0.3	372	5.000011	6000.013	0.45	74999.55	14999.88	0.00284
	0.7	370	5.000147	6000.177	14.71	74985.29	14996.62	0.0162
	0.9	370	5.000279	6000.335	83.79	74916.21	14982.40	0.1083

7. Conclusions

In this paper, a modified EOQ model is introduced which investigates the optimal order quantity assumes that a percentage of the on-hand inventory is not wasted due to deterioration for variable setup cost characteristic features and the inventory conditions govern the item stocked. This paper provides a useful property for finding the optimal profit and ordering quantity for deteriorated items. A new mathematical model with dynamic setup cost is developed and compared to the traditional EOQ model numerically. The economic order quantity, q^* and the net profit for the modified model, were found to be more than that of the traditional, q, i.e. $q^* > q$ and the net profit respectively. The modified average profit per unit per cycle is more than that of the traditional average profit per unit per cycle. Hence the utilization of variable setup cost makes the scope of the application broader. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analysis with respect to the major parameters. The model in this study is a general framework that considers variable setup cost without wasting the percentage of on-hand inventory due to deterioration simultaneously.

In the future study, it is hoped to further incorporate the proposed model into several situations such as shortages are allowed and the consideration of multi-item problem. Furthermore, it may also take partial backlogging into account when determining the optimal replenishment policy.

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