Thailand Statistician
Jan 2013; 11(1): 67-76
http://statassoc.or.th
Contributed paper

A Decision-Making Framework for a Single Item Economic Order Quantity Model with Two Constraints

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Received: 12 November 2012

Accepted: 23 December 2012

Abstract

A Single item economic order quantity (EOQ) model is a stylized model using crisp arithmetic approach in decision making process with demand unit cost and dynamic ordering cost varies with the quantity produced/Purchased under two constraints. This paper considers the modification of objective function, limited capital investment and limited storage area in the presence of estimated parameters. The model is developed for the problem by employing Non Linear Programming modeling approaches over an infinite planning horizon. It incorporates all concepts of crisp arithmetic approach, the quantity ordered, the demand per unit and compares with other model that of the crisp would optimal ordering policy of the problem over an infinite time horizon is also suggested. Investigation of the properties of an optimal solution allows developing an algorithm for obtaining solution through LINGO 13.0 version whose validity is illustrated through an example problem. Furthermore, sensitivity analysis of the optimal solution is studied with respect to changes in different parameter values and to draw managerial insights of proposed model.

Keywords: Single item, EOQ, unit cost, dynamic ordering cost.

1. Introduction

Since its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners [3] and [8] have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers. Recently, for a single product with demand related to unit price Cheng [1] has solved the EOQ model. Urgeletti [2], Clark [3], Hardley and Whitin [4] and Taha [5] have introduced EOQ formula but their treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

During the Second World War, this operations research mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of economics, management sciences, public administration, behavioral science, social work, commerce, engineering and different branches of Mathematics etc. But various paradigmatic changes in science and mathematics concern the concept of fixed setup cost. In science, this change has been manifested by a gradual transition from the traditional view, which insists that static setup cost is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for static setup cost in all its manifestations; hence it is regarded as unscientific. According to the modern view, dynamic setup cost is considered essential to real market; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle dynamic setup cost, Roy and Maiti [6] give significant contributions in this direction which have been applied in many fields including production related areas.

But Roy and Maiti [6, 7] have considered the space constraint with the objective goal in fuzzy environment and attacked the fuzzy optimization problem directly using either fuzzy non-linear or fuzzy geometric programming techniques. Whitin [8] introduced different methods to control the inventory by using several assumptions. Recently two sophisticated models were developed by Tripathy and Pattnaik [9, 10] to deal with reliability constraint with deterministic demand, Tripathy and Pattnaik [10] considered the unit cost of production is inversely related to both process reliability and demand. Tripathy and Pattnaik [11] studied an entropic inventory model with two component demand allowing price discounts for perishable items to get maximum profit in a fuzzy model. Pattnaik [12] investigated entropic order quantity model with cash discounts and Tripathy et al. [13] discussed the fuzzy EOQ model with demand and reliability dependent unit cost of production.

The purpose of this paper is to investigate the effect of the approximation made by using the average cost when determining the optimal values of the policy variables. This paper focuses exclusively on the inventory holding cost with demand dependent unit cost, dynamic ordering cost and two constraints such as capital investment and storage capacity for demand dependent unit cost in crisp decision space. A policy iteration algorithm is designed for non linear programming (NLP) with the help of LINGO 13.0 versions software. Numerical experiment is carried out to analyze the magnitude of the approximation error. This model has encouraged researchers to look for a better model to optimize total costs.

Table 1. Summary of the Related Research.

Author	Model	Type of Model	Demand	Setup Cost	Holding Cost	Unit Cost is a function of	Constraint	Sensitivity Study
Tripathy et al. [10]	Crisp	NLP	Constant	Constant	$\frac{H\lambda q^2}{2r^2}$	Reliability and demand	Reliability	Yes
Tripathy et al. [11]	Crisp	NLP	Constant	Constant	$\frac{Hq^2}{2\lambda r^2}$	Reliability and demand	Reliability	Yes
Roy et al. [6]	Crisp	NLP	Constant	Variable	$\frac{1}{2}C_1 q$	Demand	Storage	No
Tripathy et al. (this paper)	Crisp	NLP	Constant	Variable	$\frac{1}{2\times 100}C_1 K D^{-\beta} q$	Demand	Capital Investment and Storage	Yes

In this paper a single item EOQ model is developed where unit price varies inversely with demand and ordering cost increases with the increase in production. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for crisp model are compared. The major assumptions used in the above research articles are summarized in Table 1. The remainder of this paper is organized as follows. In Section 2, assumptions and notations are provided for the development of the model and the mathematical formulation is developed. In Section 3, the numerical example is presented to illustrate the development of the model. The sensitivity analysis is carried out in Section 4 to observe the changes in the optimal solution. Finally Section 5 deals with the summary and the concluding remarks.

2. Mathematical Model

A single item inventory model with demand dependent unit price and variable setup cost under limited capital investment and storage constraints is formulated as

$$Min\ C(D,q) = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q$$

Such that $\frac{1}{2}\ uq \le U,$

and $Aq \le B,$

$\forall\ D, q\ \ge 0$ (1)

where, C = average total cost,

q = number of order quantity,

D = demand per unit time,

C_1 = holding cost per item per unit time.

$C_{03}q^{\nu}$ = Setup Cost, (C_{03} (> 0) and ν (0< ν < 1) are constants),

$KD^{-\beta}$ = Unit Production Cost, (K (> 0) and β (> 1) are constants. A, B, u and U are non negative real numbers. Here lead time is zero, no back order is permitted and replenishment rate is infinite. For this crisp NLP model the solution is obtained by through LINGO software with 13.0 versions.

3. Numerical Examples

For a particular EOQ problem, let C_{03} = \$4, K = 100, C_1 =\$2, ν = 0.5, β = 1.5, u =\$0.5, U= (\$3.5, \$5.5), A = (5, 10) units, and B = (90, 50, 105) units. For these values the optimal value of optimal demand rate D*, productions batch quantity q*, minimum average total cost C* (D*, q*), $\frac{1}{2}uq^*$ and Aq* obtained by NLP are given in Table 2.

Table 2. Optimal Values of the Proposed Inventory Model.

Examples	Iteration	u	U	A	B	D^{*}	q^{*}	$C^{*}(D^{*},q^{*})$	$\frac{1}{2}uq^{*}$	Aq^{*}
1	21	0.5	3.5	5	90	13.25404	14	41.92723	3.5	70
	53	0.5	12500*	5	250000*	500	50000	17.88854	12500	250000
	53	1.522034*	38051.49*	5	250000*	500	50000	17.88854	38050.85	250000
	52	1.522034*	38051.49*	0*	0*	500	50000	17.88854	38050.85	0
	21	0.5	3.5	1.234568*	90	13.25404	14	41.92723	3.5	17.283952
2	23	0.5	3.5	10	50	9.308755	5	49.60392	1.25	50
	23	1.234568*	3.558909*	10	50	9.308755	5	49.60392	3.08642	50
3	21	0.5	3.5	10	90	11.38003	9	45.05123	2.25	90
4	26	0.5	5.5	5	105	15.26587	21	39.27128	5.25	105

After 21 iterations Table 3 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of Roy and Maiti [6] are also compared with the results of present model. The optimum replenishment quantity q^{*} and Aq^{*} for both the models are equal but the optimum quantity demand D^{*} is 9.31 and 9.21 for comparing model, hence -1.06% less from present model. The minimum total average cost $C^{*}(D^{*},q^{*})$ is 49.60 and 54.43 for comparing model, hence 9.73% more from the present model. It permits better use of present model as compared to other related model. The results are justified and agree with the present model. It indicates the consistency of the crisp space of EOQ model from other comparing model [6].

Table 3. Comparative Analysis of the Proposed Inventory Model.

Model	Solution Method	Iteration	u	U	Demand D^*	Quantity Q^*	Total Average Cost $C^*(D^*, q^*)$	$\frac{1}{2}uq^*$	Aq^*
Crisp Model	NLP	23	1.2346	3.5589	9.308755	5	49.60392	3.0864	50
Crisp Model Roy et al. [6]	NLP	-	-	-	9.21	5	54.43	-	50
% Change	-	-	-	-	-1.060883007	0	9.729231077	-	0

4. Sensitivity Analysis

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q), $\frac{1}{2}uq$ and Aq when only one parameter changes and others remain unchanged the computational results are described in Table 4. As a result

- q^*,D^*, $C^*(D^*, q^*)$ and Aq^* are highly sensitive to the parameter 'u' but $\frac{1}{2}uq^*$ is insensitive to 'u'.
- $q^*, D^*, C^*(D^*, q^*)$, Aq* and $\frac{1}{2}uq^*$ are moderately sensitive to the parameter 'U'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$ and $\frac{1}{2}uq^*$ are highly sensitive to the parameter 'A'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$ and $\frac{1}{2}uq^*$ are insensitive to the parameter 'B'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$ and $\frac{1}{2}uq^*$ are insensitive to the parameter 'C_1 '.
- q^*,$\frac{1}{2}uq^*$and Aq^* are insensitive to the parameter 'C_{03}' but $D^*$and $C^*(D^*, q^*)$ are moderately sensitive to 'C_{03}'.
- q^*, $\frac{1}{2}uq^*$ and Aq^* are insensitive to the parameter 'K' but $D^*$and $C^*(D^*, q^*)$ are moderately sensitive to 'K'.

Table 4. Sensitivity Analysis of the Parameters u, U, A, B, C_1, C_{03} and K .

Parameter	Value	Iteration	D^*	q^*	$C^*(D^*, q^*)$	% Change in $C^*(D^*, q^*)$	$\frac{1}{2}uq^*$	Aq^*
u	1	23	10.44	7	46.94	11.957146	3.5	35
	2	22	8.25	3.5	52.60	25.4613529	3.5	17.5
	3	21	7.19	2.33	56.24	34.143300	3.5	11.65
	4	22	6.53	1.75	58.98	40.678862	3.5	8.75
	5	22	6.05	1.4	61.20	45.973369	3.5	7
	10	19	4.80	0.7	68.66	63.756323	3.5	3.5
U	4	25	13.88	16	41.03	-2.1380139	4	80
	5	24	14.46	18	40.26	-3.9813505	4.5	90
	10	24	14.46	18	40.26	-3.9813505	4.5	90
	20	24	14.46	18	40.26	-3.9813505	4.5	90
	30	24	14.46	18	40.26	-3.9813505	4.5	90
	50	24	14.46	18	40.26	-3.9813505	4.5	90
A	10	20	11.38	9	45.05	7.451005	2.25	90
	15	25	9.91	6	48.14	14.821156	1.5	90
	30	23	7.83	3	53.96	28.691116	0.75	90
	50	22	6.59	1.8	58.71	40.024633	0.45	90
	100	19	5.22	0.9	65.85	57.066708	0.225	90
	200	22	4.14	0.45	73.89	76.226572	0.1125	90
B	150	21	13.25	14	41.93	0	3.5	70
	200	21	13.25	14	41.93	0	3.5	70
	250	21	13.25	14	41.93	0	3.5	70
	300	21	13.25	14	41.93	0	3.5	70
	400	21	13.25	14	41.93	0	3.5	70
	1000	21	13.25	14	41.93	0	3.5	70

Table 4. (Continue).

Parameter	Value	Iteration	D^*	q^*	$C^*(D^*, q^*)$	% Change in $C^*(D^*, q^*)$	$\frac{1}{2}uq^*$	Aq^*
C_1	3	22	13.39	14	42.07	0.343428	3.5	70
	4	24	13.51	14	42.21	0.681920	3.5	70
	5	19	13.64	14	42.35	1.015665	3.5	70
	6	20	13.76	14	42.49	1.344878	3.5	70
	7	24	13.88	14	42.63	1.669774	3.5	70
	10	30	14.23	14	43.03	2.6202065	3.5	70
C_{03}	5	22	11.46	14	45.21	7.841014	3.5	70
	6	22	10.18	14	48.10	14.716140	3.5	70
	7	24	9.21	14	50.68	20.88168	3.5	70
	8	24	8.45	14	53.04	26.499795	3.5	70
	9	24	7.83	14	55.21	31.680734	3.5	70
	10	23	7.31	14	57.23	36.501934	3.5	70
K	110	22	14.11	14	44.66	6.514788	3.5	70
	120	21	14.93	14	47.31	12.834738	3.5	70
	130	25	15.74	14	49.86	18.981244	3.5	70
	140	26	16.52	14	52.40	24.971957	3.5	70
	150	25	17.28	14	54.85	30.821521	3.5	70
	200	26	20.88	14	66.38	58.331328	3.5	70

5. Conclusions

Inventory modelers have so far considered auto type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased may represent the tractable decision making procedure in crisp environment. In this paper the real life inventory model for single item with limited capital investment and limited storage capacity constraints is solved by NLP technique in crisp decision space. Also, numerical examples are given to discuss the effects of the demand dependent unit cost, dynamic setup cost and constraints on optimal solutions. The results of numerical examples show that the managers should try them best to reduce the cost by maintaining the production system in order to cut down the total cost, meanwhile, the capital investment and storage capacity should be considered to lower the total cost when the manufacturer makes the production plan and Some sensitivity analyses on the tolerance limits have been presented. The results of the crisp model are compounded with those of other crisp model which reveals that the present model obtains better result than the other crisp model. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

The current work can be extended in order to incorporate the allocation of more constraints and the consideration of the multi-item problem. A further issue that is worth exploring is that of changing demand. Finally, few additional aspects that it is intended to take into account in the near future are imposing promotion and pricing through a new optimization model and stochastically of the quality of the quality of the products.

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