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On Measure of Second Order Slope Rotatable Designs Using Partially Balanced Incomplete Block Type Designs

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Abstract

In this paper, a new method of construction of measure of second order slope rotatable designs using partially balanced incomplete block type (PBIB) designs is suggested which enables us to assess the degree of slope-rotatability for a given response surface design.

Keywords: Second order response surface designs, second order slope rotatable designs (SOSRD), measure of SOSRD.

1. Introduction

Response surface methodology is a statistical technique that is very useful in design and analysis of scientific experiments. In many experimental situations the experimenter is concerned with explaining certain aspects of a functional relationship $Y = f(x_1, x_2, ..., x_v) + e$, where Y is the response and $x_1, x_2, ..., x_v$ are the levels of v-quantitative variables or factors and e is the random error. Response surface methods are useful where several independent variables influence a dependent variable. The independent variables are assumed to be continuous and controlled by the experimenter. The response is assumed to be as random variable. For example, if a chemical engineer wishes to find the temperature (x_1) and pressure (x_2) that maximizes the yield (response) of his process, the observed response Y may be written as a function of the levels of the temperature (x_1) and pressure (x_2) as $Y=f(x_1,x_2)+e$.

The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter [1]. The study of rotatable designs is mainly emphasized on the estimation of differences of yields and its precision. Estimation of differences in responses at two different points in the factor space will often be of great importance. If difference in responses at two points close together is of interest then estimation of local slope (rate of change) of the response is required. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in an animal etc. [2].

Hader and Park [3] introduced slope rotatable central composite designs (SRCCD). Victorbabu and Narasimham [4] studied in detail the conditions to be satisfied by a general second order slope rotatable designs (SOSRD) and constructed SOSRD using balanced incomplete block designs (BIBD). Victorbabu and Narasimham [5] constructed SOSRD using PBIB type designs. Victorbabu [6] suggested a review on SOSRD. Park and Kim [7] suggested a measure of slope rotatability for second order response surface designs. Jang and Park [8] suggested a measure and a graphical method for evaluating slope rotatability in response surface designs. Victorbabu and Surekha [9] studied measure of SOSRD using central composite designs. Victorbabu and Surekha [10] studied measure of SOSRD using BIBD. Victorbabu and Surekha [11] constructed measure of SOSRD using pairwise balanced designs (PBD). Victorbabu and Surekha [12] studied measure of SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. These measures are useful to enable us to assess the degree of slope rotatability for a given second order response surface designs.

2. Conditions for Second Order Slope Rotatable Designs

Suppose we want to use the second order response surface design $D=((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum\sum_{i<j} b_{ij} x_{iu} x_{ju} + e_u \qquad (1)$$

where x_{iu} denotes the level of the i^{th} factor $(i = 1,2,\ldots,v)$ in the u^{th} run $(u=1,2,\ldots,N)$ of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2, is said to be SOSRD if the variance of the estimate of first order partial derivative of $Y_u(x_1,x_2,\ldots,x_v)$ with respect to each of independent variables (x_i) is only a function of the distance $(d^2 = \sum_{i=1}^{v} x_i^2)$ of the point $(x_1,x_2,\ldots,x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions [3, 4].

1. $\sum x_{iu} = 0$, $\sum x_{iu}x_{ju} = 0$, $\sum x_{iu}x_{ju}^2 = 0$, $\sum x_{iu} x_{ju} x_{ku} = 0$,

   $\sum x_{iu}^3 = 0$, $\sum x_{iu}x_{ju}^3 = 0$, $\sum x_{iu}x_{ju} x_{ku}^2 = 0$, $\sum x_{iu}x_{ju}x_{ku}x_{lu} = 0$;

   for $i \neq j \neq k \neq l$;

2. (i) $\sum x_{iu}^2 =$ constant $= N\mu_2$; (ii) $\sum x_{iu}^4 =$ constant $= cN\mu_4$; for all i

3. $\sum x_{iu}^2 x_{ju}^2 =$ constant $= N\mu_4$; for $i \neq j$

4. $\dfrac{\mu_4}{\mu_2^2} > \dfrac{v}{(c+v-1)}$

5. $[v(5-c)-(c-3)^2]\,\mu_4 + [v(c-5)+4]\,\mu_2^2 = 0$ (2)

where c, μ_2 and μ_4 are constants.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\mu_4(c+v-1)\sigma^2}{N\left[\mu_4(c+v-1)-v\mu_2^2\right]}, \quad V(\hat{b}_i) = \frac{\sigma^2}{N\mu_2}, \quad V(\hat{b}_{ij}) = \frac{\sigma^2}{N\mu_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\mu_4}\left[\frac{\mu_4(c+v-2)-(v-1)\mu_2^2}{\mu_4(c+v-1)-v\mu_2^2}\right], \quad Cov(\hat{b}_0,\hat{b}_{ii}) = \frac{-\mu_2\sigma^2}{N[\mu_4(c+v-1)-v\mu_2^2]},$$

$$\mathrm{Cov}(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\mu_2^2-\mu_4)\sigma^2}{(c-1)N\mu_4[\mu_4(c+v-1)-v\mu_2^2]}$$ and other covariances vanish. (3)

3. SOSRD Using PBIB Type Designs

Take an incomplete block arrangement with constant block size and replication in which some pair of treatments occur λ_1 times each $(\lambda_1 \neq 0)$ and some other pairs do not occur at all $(\lambda_2 = 0)$ (the design need not be PBIBD). Take this as the first design. For the second design take the incomplete block design with all missing pairs (in the first design) once each with $k = 2, \lambda_1^{'} = 0, \lambda_2^{'} = 1$. Such pairs of PBIB type designs can be constructed in a straight forward manner in particular using existing two-associate PBIB designs with one of the $\lambda 's$ equal to zero.

Let $D_1 = (v, b_1, r_1, k_1, \lambda_1 \neq 0, \lambda_2 = 0)$ be an incomplete block design with constant replication in which only some pair of treatments occur a constant number of times $\lambda_1(\lambda_2 = 0)$. $[(1-(v,b,r_1,k_1,\lambda_1,\lambda_2 = 0)]$ denote the design points generated from the transpose of the incidence matrix of incomplete block design. 2^k denotes a suitable choice of fractional replication of 2^k factorial in which no interaction with less than five factors is confounded with levels +1 and -1. $[(1-(v,b,r_1,k_1,\lambda_1,\lambda_2 = 0)]2^{k_1}$ is the $b_1 2^{k_1}$ design points generate from D_1 by "multiplication" [13]. Let $D_2 = (v,b_2,r_2,k_2 = 2,\lambda_1^{'} = 0,\lambda_2^{'} = 1)$ be the associated second design containing only the missing pairs of treatments of above design D_1. Let $[a_1 - (v,b_2,r_2,k_2 = 2,\lambda_1^{'} = 0,\lambda_2^{'} = 1)]2^{k_2}$ are the $b_2 2^2$ design points generated from D_2 by multiplication. Let $(a,0,...,0)2^1$ denote the design points generated from $(a,0,...,0)$ point set and n_0 denote the number of central points. The method of construction of SOSRD using PBIB type designs is given in the following result [5].

Result: The design points,

$$\left[1-\left(v,b,r_1,k_1,\lambda_1,\lambda_2=0\right)\right]2^{k_1}\cup$$
$$\left[a_1-\left(v,b,r_2,k_2=2,\lambda_1^{'}=0,\lambda_2^{'}=1\right)\right]2^{2}\cup(a,0,...,0)2^{1}\cup n_0$$

will give a v-dimensional SOSRD in $N=b_1 2^{k_1}+b_2 2^2+2v+n_0$ design points, where a^2 is positive real root of the fourth degree polynomial equation.

$$\left[2^{k_1-1}4v-N2^{k_1}\right]a^8+2^{k_1-1}v\left[4r_2\left(\lambda_1 2^{k_1+2}\right)^{\frac{1}{2}}+r_1 2^{k_1+2}\right]a^6$$
$$+\left[\begin{array}{l}2^{k_1-1}\left[v\left(\begin{array}{l}\left[r_1^2 2^{2k_1}+r_2^2\lambda_1 2^{k_1+2}+r_1 r_2 2^{k_1+1}\left(\lambda_1 2^{k_1+2}\right)^{\frac{1}{2}}\right]+\\ 4\left(r_1 2^{k_1-1}-\lambda_1 2^{k_1}\right)+4\lambda_1 2^{k_1-1}(r_2-3)\end{array}\right)+4\lambda_1 2^{k_1+1}\right]-\\ N\left[\lambda_1 2^{2k_1}(r_2-3)+r_1 2^{2k_1}+v\lambda_1 2^{2k_1-1}\right]\end{array}\right]a^4$$
$$+2^{k_1-1}\left[\begin{array}{l}\left[v\left[\lambda_1 2^{k_1-1}(r_2-3)+\left(r_1 2^{k_1-1}-\lambda_1 2^{k_1}\right)\right]+\lambda_1 2^{k_1+1}\right]\\ \left[4r_2\left(\lambda_1 2^{k_1+2}\right)^{\frac{1}{2}}+r_1 2^{k_1+2}\right]\end{array}\right]a^2$$
$$+2^{k_1-1}\left[\begin{array}{l}\left[v\left[\lambda_1 2^{k_1-1}(r_2-3)+\left(r_1 2^{k_1-1}-\lambda_1 2^{k_1}\right)\right]+\lambda_1 2^{k_1+1}\right]\\ \left[r_1^2 2^{2k_1}+r_2^2\lambda_1 2^{k_1+2}+r_1 r_2 2^{k_1+1}\left(\lambda_1 2^{k_1+2}\right)^{\frac{1}{2}}\right]\end{array}\right]$$
$$-N\left[2^{3k_1-2}\left[\left(r_1+\lambda_1(r_2-3)\right)^2+v\lambda_1^2(r_2-3)\right]+v\lambda_1 2^{2k_1-1}\left(r_1 2^{k_1-1}-\lambda_1 2^{k_1}\right)\right]=0$$

Note: Values of SOSRD using PBIB type designs can be obtained by solving the above equation.

4. Conditions of Measure of SOSRD

Following Hader and Park [3], Victorbabu and Narasimham [4], Park and Kim [7], equations 1, 2, 3, 4, 5 of (2), and (3) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further we have,

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$Cov(b_i, b_{ii}) = Cov(b_i, b_{ij}) = Cov(b_{ii}, b_{ij}) = Cov(b_{ij}, b_{il}) = 0$ for all $i \neq j \neq l$.

The measure of slope rotatability for second order response surface design [7] can be obtained by using the following equation,

$$Q_v(D) = \frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[(v(b_i) - \frac{1}{v}\sum_{i=1}^{v} v(b_i)) + \frac{(4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij})) - \frac{1}{v}\sum_{i=1}^{v}(4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij}))}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}((4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij})) - \frac{1}{v}\sum_{i=1}^{v}(4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij})))^2$$

$$+2\sum_{i=1}^{v}\left[\left[4v(b_{ii}) - \frac{(4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij}))}{v}\right]^2 + \sum_{\substack{j=1\\ j\neq i}}^{v}\left[v(b_{ij}) - \frac{(4v(b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} v(b_{ij}))}{v}\right]^2\right]$$

$$\left.+4(v+4)\left[4cov^2(b_i, b_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{v} cov^2(b_i, b_{ij})\right] + 4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\ j\neq i}}^{v} cov^2(b_{ii}, b_{ij}) + \sum_{\substack{j<l\\ j,l\neq i}}^{v}\sum cov^2(b_{ij}, b_{il})\right]\right\}$$

where $Q_v(D)$ is the proposed measure of slope-rotatability. Further it is greatly simplified to $Q_v(D) = \frac{1}{\sigma^4}\left[4V(b_{ii}) - V(b_{ij})\right]^2$ [7].

5. Construction of Measure of SOSRD Using PBIB Type Designs

Take an incomplete block arrangement with constant block size and replication in which some pair of treatments occur λ_1 times each $(\lambda_1 \neq 0)$ and some other pairs do not occur at all $(\lambda_2 = 0)$ (the design need not be PBIBD). Take this as the first design. For the second design take the incomplete block design with all missing pairs (in the first design) once each with $k = 2, \lambda_1^{'} = 0, \lambda_2^{'} = 1$. Such pairs of PBIB type designs can be constructed in a straight forward manner in particular using existing two-associate PBIB designs with one of the $\lambda 's$ equal to zero.

Let $D_1 = (v, b_1, r_1, k_1, \lambda_1 \neq 0, \lambda = 0)$ be an incomplete block design with constant replication in which only some pair of treatments occur a constant number of times $\lambda_1 (\lambda_2 = 0)$. $[(1-(v, b, r_1, k_1, \lambda_1, \lambda_2 = 0)]$ denote the design points generated from the transpose of the incidence matrix of incomplete block design. $[(1-(v, b, r_1, k_1, \lambda_1, \lambda_2 = 0)]2^{k_1}$ are the $b_1 2^{k_1}$ design points generate from D_1 by "multiplication" [13].

Let $D_2 = (v, b_2, r_2, k_2 = 2, \lambda_1^{'} = 0, \lambda_2^{'} = 1)$ be the associated second design containing only the missing pairs of treatments of above design D_1. Let $[a_1 - (v, b_2, r_2, k_2 = 2, \lambda_1^{'} = 0, \lambda_2^{'} = 1)]2^{k_2}$ are the $b_2 2^2$ design points generated from D_2 by "multiplication". $(a, 0, ..., 0)2^1$ denote the design points generated from $(a, 0, ..., 0)$ point set and n_0 denote the number of central points. Thus, we have the total number of experimental points $N = b_1 2^{k_1} + b_2 2^2 + 2v + n_0$ and

$$\sum x_{iu}^2 = r_1 2^{k_1} + r_2 2^{k_2} a_1^2 + 2a^2 = N\mu_2 \quad (4)$$

$$\sum x_{iu}^4 = r_1 2^{k_1} + r_2 2^{k_2} a_1^4 + 2a^4 = cN\mu_4 \quad (5)$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda_1 2^{k_1} = \lambda_2^{'} 2^{k_2} a_1^4 = N\mu_4 \quad (6)$$

From (6) we get $a_1^4 = \dfrac{\lambda_1 (2^{k_1 - k_2})}{\lambda_2^{'}} = \lambda_1 2^{k_1 - 2} \quad \left(\text{since } \lambda_2^{'} = 1, k_2 = 2\right)$

Measure of SOSRD using PBIB type designs can be obtained by

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e-V(b_{ij})\right]^2$$

where

$$e=\frac{(v-1)\left[\begin{array}{l}2^{k_1}\lambda_1 n_0+2^{k_1+1}\lambda_1 v-2^{k_1+2}r_1 a^2-r_1^2 2^{2k_1}+\\ b_1\lambda_1 2^{2k_1}+b_2\lambda_1 2^{k_1+2}-16r_2^2\left(\lambda_1 2^{k_1-2}\right)-\\ \left(8r_2\left(\lambda_1 2^{k_1-2}\right)^{\frac{1}{2}}\left(r_1 2^{k_1}+2a^2\right)\right)\end{array}\right]+a^4\left[2^{k_1+1}b_1+2n_0+8b_2+4\right]+(r_1-\lambda_1+r_2\lambda_1)\left[2^{k_1+1}v+2^{k_1}n_0+b_1 2^{2k_1}+b_2 2^{k_1+2}\right]}{\left[2^{k_1}(r_1+(r_2-1)\lambda_1+2a^4\right]\left[\begin{array}{l}(r_1-\lambda_1)\left(b_1 2^{2k_1}+2^{k_1+1}v+2^{k_1}n_0+b_2 2^{k_1+2}\right)\\ +2^{k_1}v\left(\lambda_1 n_0-4r_1 a^2\right)+r_2\lambda_1\left(b_1 2^{2k_1}+2^{k_1}n_0\right)\\ +2^{k_1+1}\left(\lambda_1 v(r_2+v)+b_1 a^4\right)+\left(8b_2+2n_0\right)a^4\\ +2^{k_1+2}\left((r_2+v)b_2\lambda_1-vr_2^2\lambda_1\right)+2^{2k_1}v\left(b_1\lambda_1-r_1^2\right)\\ -vr_2\left(\lambda_1 2^{k_1-2}\right)^{\frac{1}{2}}\left(r_1 2^{k_1+3}+16a^2\right)\end{array}\right]}$$

The following table gives the values of $Q_v(D)$ for SOSRD using various parameters of PBIB type designs, n_0 and a, the value of 'a' which make SOSRD using PBIB type designs. It can be verified that $Q_v(D)$ is zero if and only if a design D is slope-rotatable. $Q_v(D)$ becomes larger as D deviates from a slope-rotatable design.

Tables. Values of measure of SOSRD using PBIB type designs.

Example 1

D_1=(v=6, b_1=4, r_1=2, k_1=3, λ_1=1, λ_2=0) ; D_2=(v=6, b_2=3, r_2=1, k_2=2, $\lambda_1^{'}$=0, $\lambda_2^{'}$=1)					
a	n_0=1,N=57	n_0=2,N=58	n_0=3,N=59	n_0=4,N=60	n_0=5,N=61
1.0	7.0565×10^{-4}	5.6149×10^{-4}	4.6102×10^{-4}	3.8744×10^{-4}	3.3148×10^{-4}
1.3	1.5999×10^{-3}	9.5129×10^{-4}	6.3812×10^{-4}	4.6157×10^{-4}	3.5145×10^{-4}
1.6	7.7003×10^{-3}	2.1328×10^{-3}	9.3627×10^{-4}	5.0482×10^{-4}	3.0609×10^{-4}
1.9	3.9735×10^{-3}	1.0323×10^{-3}	3.7557×10^{-4}	1.5698×10^{-4}	6.9206×10^{-5}
2.2	2.6796×10^{-5}	9.3936×10^{-12}	1.0138×10^{-5}	2.7628×10^{-5}	4.4291×10^{-5}
2.5	3.9108×10^{-4}	4.1137×10^{-4}	4.2061×10^{-4}	4.2213×10^{-4}	4.1826×10^{-4}
2.8	1.3120×10^{-3}	1.2525×10^{-3}	1.1930×10^{-3}	1.1345×10^{-3}	1.0776×10^{-3}
3.1	2.6047×10^{-3}	2.4476×10^{-3}	2.3009×10^{-3}	2.1622×10^{-3}	2.0362×10^{-3}
*	2.247524	2.199963	2.1540	2.109919	2.068321

* indicates exact SOSRD using PBIB type design values.

Here, we may point out that measure of SOSRD using PBIB type designs has 57 design points for 6-factors, where as the corresponding measure of obtained by Victorbabu and Surekha [10-12] using BIBD $(v=6,b=15,r=5,k=2,\lambda=1)$, PBD $(v=6,b=7,r=3,k_1=3,k_2=2,\lambda=1)$, SUBA with two unequal block sizes $(v=6,b=7,r=3,k_1=2,k_2=3,b_1=3,b_2=4,\lambda=1)$ need 73, 69 and 69 design points respectively.

Example: 2

D_1=(v=8, b_1=8, r_1=3, k_1=3, λ_1=1, λ_2=0) ; D_2=(v=8, b_2=4, r_2=1, k_2=2, λ_1'=0, λ_2'=1)					
a	n_0=1,N=97	n_0=2,N=98	n_0=3,N=99	n_0=4,N=100	n_0=5,N=101
1.0	6.8423×10^{-5}	5.4358×10^{-5}	4.4257×10^{-5}	4.6937×10^{-5}	3.1018×10^{-5}
1.3	2.2060×10^{-4}	1.3191×10^{-4}	8.6358×10^{-5}	5.9971×10^{-5}	4.3506×10^{-5}
1.6	1.8748×10^{-3}	5.1130×10^{-4}	2.1004×10^{-4}	1.0367×10^{-4}	5.6624×10^{-5}
1.9	8.9562×10^{-4}	2.3712×10^{-4}	8.0653×10^{-5}	7.8027×10^{-5}	9.9492×10^{-6}
2.2	1.1285×10^{-6}	7.5112×10^{-6}	1.5122×10^{-5}	2.2119×10^{-5}	1.4927×10^{-4}
2.5	1.5145×10^{-4}	1.5359×10^{-4}	1.5437×10^{-4}	1.5410×10^{-4}	1.5305×10^{-4}
2.8	3.7890×10^{-4}	3.6763×10^{-4}	3.5646×10^{-4}	3.4545×10^{-4}	3.3465×10^{-4}
3.1	6.6575×10^{-4}	6.4114×10^{-4}	6.1755×10^{-4}	5.9495×10^{-4}	5.7329×10^{-4}
*	2.181096	2.140030	2.098152	2.055566	2.012478

* indicates exact SOSRD using PBIB type design values.

For v=8 factors, this new method needs 97 design points, whereas the corresponding measure of SOSRD constructed using BIBD $(v=8, b=28, r=7, k=2, \lambda=1)$, PBD $(v=8, b=15, r=6, k_1=4, k_2=3, k_3=2, \lambda=2)$, SUBA with two unequal block sizes $(v=8, b=12, r=4, k_1=2, k_2=3, b_1=4, b_2=8, \lambda=1)$ need 129, 257 and 113 design points respectively.

Example: 3

D_1=(v=10, b_1=8, r_1=4, k_1=5, λ_1=2, λ_2=0) ; D_2=(v=10, b_2=5, r_2=1, k_2=2, $\lambda_1^{'}$=0, $\lambda_2^{'}$=1)					
a	n_0=1,N=169	n_0=2,N=170	n_0=3,N=171	n_0=4,N=172	n_0=5,N=173
1.0	5.5750×10^{-5}	5.2187×10^{-5}	5.0485×10^{-5}	4.6340×10^{-5}	4.3902×10^{-5}
1.3	6.6369×10^{-5}	5.9894×10^{-5}	5.4634×10^{-5}	5.0272×10^{-5}	4.6590×10^{-5}
1.6	9.6764×10^{-5}	7.8653×10^{-5}	6.6112×10^{-5}	5.6969×10^{-5}	5.0032×10^{-5}
1.9	2.4686×10^{-4}	1.4686×10^{-4}	9.9748×10^{-5}	7.3504×10^{-5}	5.7217×10^{-5}
2.2	9.0416×10^{-4}	2.9628×10^{-4}	1.4564×10^{-4}	8.6438×10^{-5}	5.7221×10^{-5}
2.5	2.8561×10^{-4}	1.1977×10^{-4}	6.2068×10^{-5}	3.6168×10^{-5}	2.2690×10^{-5}
2.8	1.1191×10^{-5}	5.4230×10^{-6}	2.5207×10^{-6}	1.0536×10^{-6}	3.4700×10^{-7}
3.1	4.4639×10^{-6}	5.3458×10^{-6}	6.1239×10^{-6}	6.8021×10^{-6}	7.3877×10^{-6}
*	2.956809	2.928641	2.900871	2.873608	2.846966

* indicates exact SOSRD using PBIB type design values.

In case of v=10 factors, this new method needs 169 design points, whereas the corresponding measure of SOSRD constructed using BIBD $(v=10,b=45,r=9,k=2,\lambda=1)$ and PBD $(v=10,b=11,r=5,k_1=5,k_2=4,\lambda=2)$ need 201, and 197 design points respectively

6. Conclusion

In this paper, measure of slope rotatability for second order response surface designs using partially balanced incomplete block type designs is suggested which enables us to assess the degree of slope-rotatability for a given response surface design. We note thus the new method sometimes leads to designs with less number of design points.

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