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Inferential Estimates from the One-parameter Half-normal Model

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Abstract

In this paper, some inferential estimates are determined from a doubly censored sample, which follow the half normal model. A simulation study is conducted to illustrate the predictive results for a future response. A real life survival data study of 25 patients who were diagnosed with leukemia at a local hospital is utilized to illustrate the predictive results. The survival days of 19 patients with leukemia are used only for the statistical analysis. Six patients out of 25 patients were lost to follow-up, and they are the first three and the last three patients in the recorded sample. The survival days of the patients were recorded in order. The Bayesian sensitivity of the predictive survival days with respect to several values of the hyperparameter are reported. The summary results of the posterior parameter and hyperparameters are given by using of the Markov Chain Monte Carlo method.

1. Introduction

The data from an experiment may form a complete sample or a censored sample. In a life testing experiment, a censored sample is a sample, where a scientific decision is made after observing the lifetimes of a reasonable number of items. Recently, several statistical research work have been completed based on censored sample, for example, Ahmed and Saleh [1], Ahmed and Rahbar [2], Buhamra et al. [3], among

others. A number of studies have already been appeared based on a type II censored sample, for instance by Khan [4]. Buhamra et al. [5] described in details of the non-parametric inferences for the quantile under left truncation and right censoring. Ahmed [6] also discussed an asymptotic estimation of reliability in a life-testing model. Bakilzi and Ahmed [7] discussed the estimation of reliability function in a Weibull lifetime model. Ahmed et al. [8] discussed a parametric estimation for the Birnbaum-Saunders lifetime distribution under a new parameterization approach. Doubly censored sample is a sample, where the record of the first few observations and the last few observations are not available from a life testing experiment. In such a case, the remaining observations form a doubly censored sample, Sarhan [9], among others.

Predictive inference has been used in engineering and biomedical sciences to solve many challenging problems. Predictive inference has been studied by many authors, for example, Thabane [10], Thabane and Haq [11], among others. Predictive inference usually involves the derivation of probability model of future response(s) given the data. From the predictive model, one may infer about future characteristics of the model such as the mean, standard deviation, moments, skewness, kurtosis, and tolerance region.

There are several methods to predictive inference, which include (i) classical method, (ii) the maximum likelihood method, (iii) the fiducial method, (iv) Fisher's relative likelihood method, (v) the structural method, and (vi) the Bayesian method. To derive the predictive model of future responses, one may consider the Bayesian method, which has been widely used in recent years for solving many scientific problems from the engineering and biomedical fields. The Bayesian method considers prior distribution for the parameters. The prior is used to drive the posterior density for the parameters.

The posterior density carries updated information through prior knowledge for the purpose of deriving informative predictive density for future responses. More about the Bayesian method, the readers are referred to Ahsanullah and Ahmed [12], Bernardo and Smith [13], Berger [14], and Geisser [15], among others. The Bayesian method has been used by many authors for determining predictive models. Berger [14] considered a general Bayesian predictive problem. Geisser [16] discussed the inferential use of predictive distributions. Geisser [15] discussed various Bayesian predictive problems for future responses. Evans and Nigm [17] used the Bayesian method to derive future responses from the Weibull distribution. Additional applications of the Bayesian method to predictive inference have been discussed for instance by Thabane [10], Thabane and Haq [11], Gelman et al. [18], among others.

The doubly censored samples from a life testing experiment may follow several statistical probability models such as the exponential, gamma, Weibull, Rayleigh, normal, log-normal, and half-normal. A number of studies related to the half normal model have published in the refereed journals by several authors, for example, Wiper et al. [19], Olmos et al. [20], among others based on non-censored samples. These studies deal with the parameter from the half-normal model but from an application point of view, the inference for future response plays a significant role of statistical inference. In real life one may observe a doubly censored sample that is being used in modeling the half-normal distribution. Therefore, the predictive inference for future responses from such a model is very important. We consider one-parameter half-normal model due to some computation convenience (for example, HPD intervals) given a doubly censored sample. The goal of this paper is to obtain predictive inference on the basis of a doubly censored sample from the half-normal model whose density function is given by

$$p(x|\sigma) = \begin{cases} \frac{1}{\sigma}\sqrt{\frac{2}{\pi}}\exp\left\{-\frac{x^2}{2\sigma^2}\right\}, x \geq 0;\ \sigma > 0, \\ 0 \qquad\qquad \text{elsewhere,} \end{cases} \qquad (1)$$

where σ is a scale parameter.

The rest of the paper is organized as follows: Section 2 presents the likelihood function and predictive model for a single future response, given a doubly censored sample from the one-parameter half-normal model. The formulae for the predictive moments, skewness, and kurtosis are presented in Section 3. The highest predictive density interval is described in Section 4. To illustrate the results, a simulation study and a real data study are presented in Section 5. Finally, a conclusion is added in Section 6.

2. The likelihood function and predictive model

To derive the likelihood function, let $x_1, \ldots, x_v$ be an ordered random sample of size v from model (1), where $x_1 \leq \cdots \leq x_g$ is the g smallest ordered observations and $x_{p+1} \leq \cdots \leq x_v$ is the $(v - p)$ largest ordered observations from the sample. The smallest and the largest observations may be unavailable from an experiment. Only the remaining ordered observations $\mathbf{x} = (x_{g+1}, \ldots, x_p)$ may be used for statistical analysis. It is assumed that the sample data are modeled by the one-parameter half-normal model. Following Khan [21] and by setting the location parameter, $\mu = 0$, the likelihood function of σ for a given $\mathbf{x} = (x_{g+1}, \ldots, x_p)$ is given by

$$\mathrm{L}(\sigma|\mathbf{x}) \propto \sum_{\omega=0}^{\infty}\sum_{\gamma=0}^{g}(-1)^{(1+\omega)\gamma+\omega(v-p)}\binom{g}{\gamma}\frac{\left((2\omega-1)!!\right)^{\gamma+v-p}x_{g+1}^{-(2\omega+1)\gamma}}{2^{w(\gamma+v-p)}}x_p^{-(2\omega+1)(v-p)}$$
$$\times\sigma^{-(p-g)+(2\omega+1)(\gamma+v-p)}\exp\left\{-\frac{\sum_{\alpha=g+1}^{p}x_\alpha^2+(v-p)x_p^2+\gamma\,x_{g+1}^2}{2\sigma^2}\right\},$$

where (2ω-1)!! is the term of double factorial notation, which can be simplified by Mathematica Software, Wolfram Research [22]. We use L(σ|**x**) to derive predictive inference for future response. Khan [4] considered a predictive inference problem by making use of the Bayesian approach with an inverted gamma prior. In the present study, we consider the same inverted gamma prior for σ with hyperparameters (η and δ). For more information about hyperparameters, the reader is referred to Berger [14]. Thus, the predictive density for a single future response (z) is given by

$$p(z|\mathbf{x})$$
$$\propto\begin{cases}\displaystyle\sum_{\omega=0}^{\infty}\sum_{\gamma=0}^{g}(-1)^{(1+\omega)\gamma+\omega(v-p)}\binom{g}{\gamma}\frac{\left((2\omega-1)!!\right)^{\gamma+v-p}x_{g+1}^{-(2\omega+1)\gamma}\Gamma\left(\frac{p-g+2\delta-(2\omega+1)(\gamma+v-p)}{2}\right)}{2^{\delta-(\omega+1)(\gamma+v-p)-1}} & \\ \times x_p^{-(2\omega+1)(v-p)}\left[\eta+z^2+\displaystyle\sum_{\alpha=g+1}^{p}x_\alpha^2+(v-p)x_p^2+\gamma\,x_{g+1}^2\right]^{-\frac{p-g+2\delta-(2\omega+1)(\gamma+v-p)}{2}} & ;\text{for } z\geq 0,\\ 0 \qquad \text{elsewhere.} & \end{cases} \quad (2)$$

3. The predictive moments, skewness, and kurtosis

The predictive raw moments are defined as follows:

$$\mu'_1 = E(z) = \int_{z=0}^{+\infty} z\,p(z|\mathbf{x})\;dz;\ \mu'_2 = E(z^2) = \int_{z=0}^{+\infty} z^2\,p(z|\mathbf{x})\;dz;$$

$$\mu'_3 = E(z^3) = \int_{z=0}^{+\infty} z^3\,p(z|\mathbf{x})\;dz;$$

and $\mu'_4 = E(z^4) = \int_{z=0}^{+\infty} z^4\,p(z|\mathbf{x})\;dz;$

where $p(z|\mathbf{x})$ is defined in Eqn. (2).

The corrected predictive moments are defined as follows:

$$\mu_2 = \mu'_2 - {\mu'_1}^2;\ \mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2{\mu'_1}^3;$$

and $\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$.

The predictive skewness and kurtosis are defined by β_1 and β_2 respectively,
where $\beta_1 = \mu_3^2/\mu_2^3$,
and $\beta_2 = \mu_4/\mu_2^2$.

4. Highest predictive density interval

An HPD interval is the interval which allows the most probable values of a given distribution at a given significance level, subject to the condition that the density function has the same value at both end points. An HPD interval U is of the form $U = \{z : p(z|\mathbf{x}) \geq c_\alpha\}$, where c_α is the largest constant such that $\Pr(z \in U|\mathbf{x}) = 1-\alpha$, and α denotes the significance level, see for example, Box and Tiao [23]. In this case, one can utilize the predictive density, $p(z|\mathbf{x})$ derived in Eqn. (2). The HPD interval $[c_1, c_2]$ for z must simultaneously satisfy the following conditions:

$$\Pr(c_1 \leq z \leq c_2) = 1 - \alpha \text{ and } p(c_1|\mathbf{x}) = p(c_2|\mathbf{x}),$$

where c_1 and c_2 are to be chosen so that $p(c_1|\mathbf{x}) = p(c_2|\mathbf{x})$. For arbitrary c_1 and c_2:

$$\Pr(c_1 < z < c_2)$$
$$= \int_{z=c_1}^{c_2} \psi_1(\mathbf{x}) \sum_{\omega=0}^{\infty} \sum_{\gamma=0}^{g} (-1)^{(1+\omega)\gamma+\omega(v-p)} \binom{g}{\gamma} \frac{\left((2\omega-1)!!\right)^{\gamma+v-p} x_{g+1}^{-(2\omega+1)\gamma} \Gamma\left(\frac{p-g+2\delta-(2\omega+1)(\gamma+v-p)}{2}\right)}{2^{\delta-(\omega+1)(\gamma+v-p)-1}}$$
$$\times x_p^{-(2\omega+1)(v-p)} \left[\eta + z^2 + \sum_{\alpha=g+1}^{p} x_\alpha^2 + (v-p)x_p^2 + \gamma\, x_{g+1}^2\right]^{-\frac{p-g+2\delta-(2\omega+1)(\gamma+v-p)}{2}} dz.$$

A closed form solution for c_1 and c_2 is not tractable. However, for a specific set of data, a solution may be determined.

5. Numerical studies

We considered two numerical studies. The first study describes a simulated sample from the half-normal model, and the second study presented based on a real data sample set of survival days of leukemia patients. Eqn. (2) is utilized for both studies.

5.1 Simulation study

We have simulated a sample of size ν= 40 from the half normal model with scale parameter σ = 1. To make a doubly censored sample, data are arranged in order. The first two observations and the last three observations are discarded from the ordered sample. Therefore, the value for the x_p is the 35th position of the data in order. This sample is utilized to evaluate the predictive density for a single future response.

We used the numerical integration command 'NIntegrate' in conjunction with the computational software Mathematica version 7.0, Wolfram Research [24] to determine the normalizing constants, predictive raw moments, corrected moments, the measures of skewness and kurtosis, standard deviation, and to plot the predictive graphs. This package is also utilized to carry out all related calculations.

We also made an attempt to estimate the hyperparameters for η and δ. Unfortunately, the MLEs for η and δ do not yield closed form representations for η and δ, and the NewtonRaphs on iterative algorithm fails to achieve solutions for η and δ, simultaneously. Thus, we consider some arbitrary positive values for η and δ (since η >0, δ>0), and these values are substituted in Eqn. (2) to determine the predictive density for a single future response.

Comparing the estimated values of mean and standard deviation in Table 1, it is commented that the predictive density is less sensitive to the values of η and more sensitive to the values of δ. That is, for larger values of δ the predictive density carries smaller variance.

The effect of varying the hyperparameters on the predictive raw moments and corrected moments can be observed from Tables 1 & 2. When η is increased and δ is fixed, it is observed that both the predictive mean and standard deviation slightly increase. When η is fixed and δ is increased, then there is a decrease in the mean and standard deviation. In addition, the estimated values of the measures of skewness and kurtosis with respect to β_1 and $\beta_2$are given in Table 2. Based on the values of β_1 and β_2, it is commented that the predictive density for a single future response given a doubly censored sample from the one-parameter half-normal model is positively skewed.

The 95% HPD intervals are determined based on the discussion given in Section 4. The results of 95% HPD intervals with respect to certain combinations of the hyperparameters are reported in Table 3.

Table 1: Predictive raw moments and standard deviations (SD) for a single future response with several values of the hyperparameters given a simulated doubly censored sample.

Hyperparameters		μ'_1	μ'_2	μ'_3	μ'_4	SD (z)
η	δ					
1	1	5.24183	43.6155	465.807	5,887.99	4.01730
1	2	5.10983	41.4484	431.775	5,329.53	3.91638
1	3	4.98692	39.4759	401.454	4,841.66	3.82185
1	4	4.87218	37.6750	374.348	4,413.97	3.73321
1	5	4.76480	36.0260	350.031	4,037.69	3.64997
2	1	5.24341	43.6415	466.216	5,894.70	4.01847
3	1	5.24500	43.6675	466.625	5,901.42	4.01964
4	1	5.24658	43.6935	467.034	5,908.13	4.02081
5	1	5.24817	43.7196	467.443	5,914.85	4.02198
6	1	5.24975	43.7456	467.852	5,921.57	4.02315
2	2	5.11138	41.4733	432.158	5,335.69	3.91753
2	3	4.98844	39.4998	401.813	4,847.33	3.82299
2	4	4.87367	37.6979	374.685	4,419.18	3.73433
2	5	4.76626	36.0476	350.348	4,042.50	3.65107
2	6	4.66549	34.5316	328.442	3,709.90	3.57278
3	2	5.11293	41.4982	432.541	5,341.85	3.91869
4	2	5.11482	41.5123	432.967	5,348.59	3.91917
5	2	5.11603	41.5480	433.307	5,354.19	3.92099
6	2	5.11758	41.5729	433.690	5,360.36	3.92215
7	2	5.11913	41.5978	434.073	5,366.53	3.92330

Table 2: Corrected predictive moments, measures of skewness and kurtosis for a single future response with several values of the hyperparameters given a simulated doubly censored sample.

Hyperparameters		μ_2	μ_3	μ_4	β_1	β_2
η	δ					
1	1	16.1387	67.9899	1,046.820	1.0997	4.0192
1	2	15.3380	63.2311	952.486	1.1080	4.0488
1	3	14.6065	58.9073	868.575	1.1135	4.0711
1	4	13.9368	54.9824	793.922	1.1168	4.0874
1	5	13.3223	51.4200	727.461	1.1182	4.0988
2	1	16.1481	68.0433	1,047.90	1.0995	4.0186
3	1	16.1575	68.0966	1,048.99	1.0993	4.0181
4	1	16.1669	68.1500	1,050.07	1.0991	4.0176
5	1	16.1763	68.2033	1,051.16	1.0989	4.0171
6	1	16.1857	68.2567	1,052.24	1.0987	4.0165
2	2	15.3471	63.2823	953.502	1.1079	4.0483
2	3	14.6152	58.9562	869.525	1.1134	4.0707
2	4	13.9452	55.0290	794.809	1.1166	4.0871
2	5	13.3303	51.4644	728.288	1.1181	4.0985
2	6	12.7648	48.2274	668.995	1.1183	4.1058
3	2	15.3561	63.3335	954.518	1.1077	4.0478
4	2	15.3630	63.3827	955.439	1.1076	4.0481
5	2	15.3742	63.4359	956.551	1.1074	4.0469
6	2	15.3832	63.4870	957.568	1.1072	4.0465
7	2	15.3923	63.5382	958.585	1.1070	4.0459

Table 3: Sensitivity of 95% HPD intervals with respect to specific combinations of the hyperparameters given a simulated doubly censored sample.

Hyperparameters		95% HPD intervals
η	δ	
1	1	(2.7845, 14.4521)
1	2	(3.3241, 13.9043)
1	3	(3.3641, 12.9831)
1	4	(3.4331, 11.5320)
1	5	(3.5501, 10.7002)
2	1	(2.9052, 15.8664)
3	1	(1.7015, 16.1002)
4	1	(1.4502, 17.4528)
5	1	(1.0056, 18.2423)
6	1	(0.9953, 20.6214)

The method “Bayesian Markov chain Monte Carlo (MCMC)” is a useful alternative to obtain the posterior distributions of the parameters. The implementation of this method can be obtained by making use of the software WinBUGS. On the basis of the simulated doubly censored sample, the summary results of the parameters are reported in Table 4 based on 60,000 iterative samples. Figure 1 is the posterior kernel densities and trace plots for the parameters and hyperparameters.

Table 4: Summary results for the posterior parameters and hyperparameters given a simulated doubly censored sample by making use of the software WinBUGS.

Node	Mean	SD	MC Error	2.5%	Median	97.5%	Start	Sample
δ	19.99	14.09	0.06018	2.46	16.85	55.77	1,001	60,000
η	99.81	71.25	0.3527	12.27	83.36	280.20	1,001	60,000
σ	0.06208	0.01493	$6.445E^{-5}$	0.03646	0.06082	0.09466	1,001	60,000

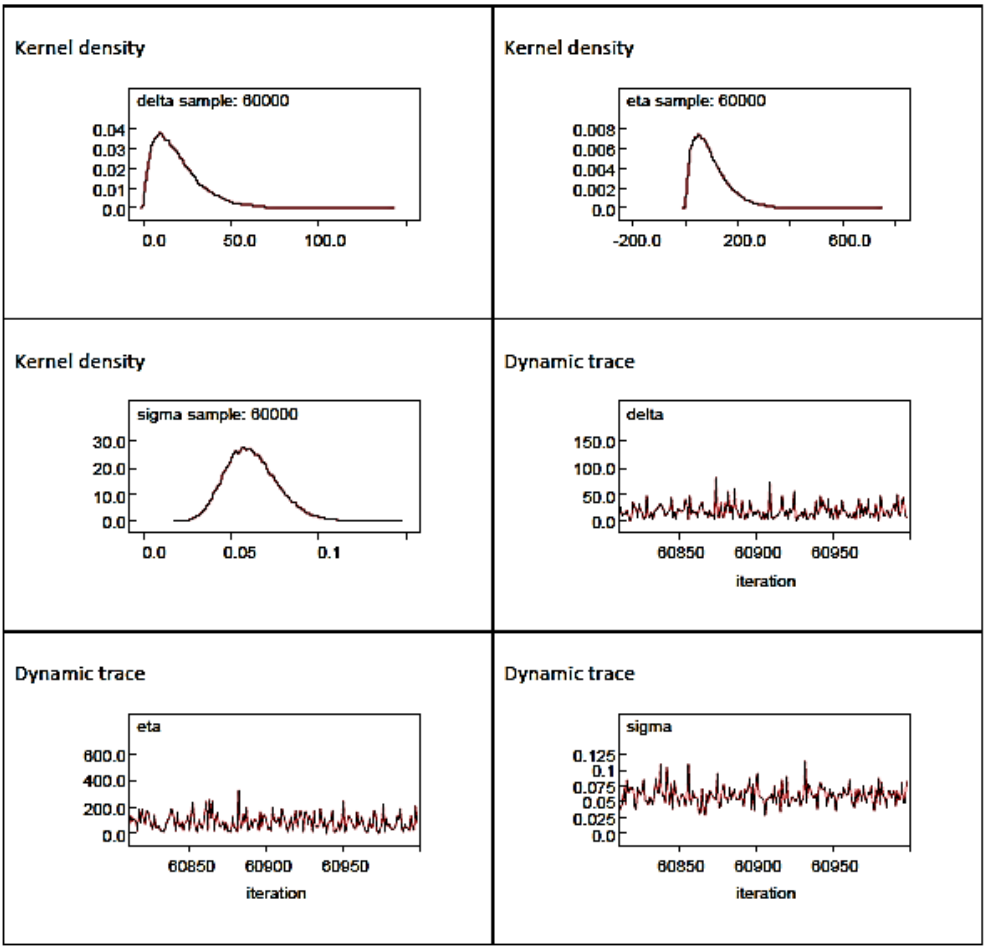

Figure 1: Posterior kernel densities and dynamic trace plots for the parameter and hyperparameters based on simulated censored sample.

5.2 Real data study

In this section we considered a real data study related to leukemia, which is a cancer of the blood and is usually characterized by an abnormal proliferation of white blood cells. These white blood cells help fight infection in our body. When cells are damaged early, old cells die, and new cells take their place. People with leukemia, the bone marrow produce a large number of abnormal white blood cells, that do not function properly, and make a problem for normal blood cells to do their regular activities. People with leukemia may have treatments such as chemotherapy, targeted therapy, biological therapy, radiation therapy, and stem cell transplant.

There were 25 patients diagnosed with leukemia in 2002 at a local hospital in Newark, New Jersey. The survival days of 19 patients out of 25 who were suffering from leukemia are considered for statistical analysis, and there were six patients out of 25 that lost to follow-up. They were the first three and the last three patients survival days from an ordered sample. These survival days of the patients may be ordered to form a doubly censored sample.

The empirical distribution function (EDF) is compared with the corresponding theoretical cumulative distribution function (CDF) given the sample of survival days. We choose a probability distribution for which the EDF and CDF are the closest. By making use of Mathematica v7.0 software (Wolfram Research [24]), or any advanced symbolic computation package, one can calculate the mean squared and absolute differences between the EDF and CDF, and select the model of best fit by visual inspection. The question may arise as to whether the selection of the probability model providing the best fit will be dependent on a given difference measure. Accordingly, two measures of discrepancy are being used to determine the best fit model. The mean squared differences i.e., $\sum_{i=1}^{n}(\mathrm{F}(\mathrm{x_i}) - \mathrm{G}(\mathrm{x_i}))^2/n$, and the absolute mean differences i.e.,$\sum_{i=1}^{n}|\mathrm{F}(\mathrm{x_i}) - \mathrm{G}(\mathrm{x_i})|/n$ are considered, where F isthe empirical distribution function, G is the functional representation of the cumulativedistribution function, and *n* is the observed sample size. Thus, it is obtained that the half-normal model best fits the data considering the lowest differences (mean squared differences= 0.0010135; absolute mean differences = 0.021602). The gamma model has the next lowest differences (mean squared differences = 0.0015963; absolute mean differences = 0.0352381).The exponential model has the highest differences (mean squared differences = 0.0094878;absolute mean differences = 0.086806). Furthermore, to confirm the best model half-normal, the calculated Anderson-Darling Test Statistics is obtained as 0.4964 with its corresponding p-value = 0.7834. Eqn. (2) is used to the data, and the predictive moments of a single future response are estimated with respect to some values of the hyperparameters. The results of95% HPD intervals with respect to certain combination of the hyperparameters are reported in Table 5.

Table 6 includes the estimates of the predictive raw moments and standard deviations with the combination of several values of the hyperparameters. Table 7 presents the estimates of the corrected moments and the shape characteristics with respect to several values of the hyperparameters. Figure 2 represents the posterior kernel densities and trace plots for the parameter and hyperparameters. The posterior

kernel density of each parameter gives their positive shape of the distribution and the summary results are reported in Table 8.

Table 5: Sensitivity of 95% HPD intervals with respect to specific combinations of the hyperparameters given a doubly censored survival sample of leukemia patients.

Hyperparameters		95% HPD intervals
η	δ	
1	1	(74.1662, 229.0321)
1	2	(74.9278, 228.7902)
1	3	(75.1011, 224.9136)
1	4	(76.9201, 220.4365)
1	5	(77.5120, 216.1783)
2	1	(74.1201, 230.1025)
3	1	(73.9206, 231.1287)
4	1	(73.0461, 232.2067)
5	1	(72.1008, 233.2506)
6	1	(70.5906, 234.1321)

Table 6: Predictive raw moments and standard deviations (SD) for a single future response with several values of the hyperparameters given a doubly censored survival sample of leukemia patients.

Hyperparameters		μ_1'	μ_2'	μ_3'	μ_4'	SD (z)
η	δ					
1	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8094
1	2	98.15	15,201.10	3.0039×10^{6}	6.9866×10^{8}	74.6126
1	3	95.41	14,370.90	2.7640×10^{6}	6.2661×10^{8}	72.5690
1	4	92.89	13,622.40	2.5528×10^{6}	5.6443×10^{8}	70.6660
1	5	90.54	12,944.90	2.3658×10^{6}	5.1059×10^{8}	68.8915
2	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8095
3	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8096
4	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8097
5	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8098
6	1	101.12	16,125.40	3.2772×10^{6}	7.8253×10^{8}	76.8099
2	2	98.15	15,201.10	3.0039×10^{6}	6.9866×10^{8}	74.6126
2	3	95.41	14,370.90	2.7641×10^{6}	6.2660×10^{8}	72.5690
2	4	92.89	13,622.40	2.5528×10^{6}	5.6444×10^{8}	70.6660
2	5	90.55	12,944.90	2.3658×10^{6}	5.1059×10^{8}	68.8915
2	6	88.37	12,329.40	2.1997×10^{6}	4.6373×10^{8}	67.2343
3	2	98.15	15,201.10	3.0039×10^{6}	6.9866×10^{8}	74.6127
4	2	98.15	15,201.10	3.0039×10^{6}	6.9867×10^{8}	74.6128
5	2	98.15	15,201.10	3.0039×10^{6}	6.9868×10^{8}	74.6129
6	2	98.15	15,201.10	3.0039×10^{6}	6.9869×10^{8}	74.6130
7	2	98.15	15,201.10	3.0039×10^{6}	6.9870×10^{8}	74.6131

Table 7: Corrected predictive moments, measures of skewness and kurtosis for a single future response with several values of the hyperparameters given a doubly censored survival sample of leukemia patients.

Hyperparameters		μ_2	μ_3	μ_4	β_1	β_2
η	δ					
1	1	5,899.69	453,390	1.32613×10^{8}	1.00105	3.81003
1	2	5,567.04	419,012	1.19539×10^{8}	1.01761	3.85710
1	3	5,266.26	387,835	1.07999×10^{8}	1.02988	3.89417
1	4	4,993.68	359,651	9.78222×10^{7}	1.03387	3.92280
1	5	4,746.04	334,215	8.88477×10^{7}	1.04486	3.94443
2	1	5,899.70	453,391	1.32613×10^{8}	1.00105	3.81003
3	1	5,899.70	453,391	1.32614×10^{8}	1.00105	3.81003
4	1	5,899.70	453,391	1.32613×10^{8}	1.00105	3.81003
5	1	5,899.70	453,391	1.32613×10^{8}	1.00105	3.81003
6	1	5,899.70	453,391	1.32613×10^{8}	1.00105	3.81002
2	2	5,567.04	419,012	1.19539×10^{8}	1.01761	3.85709
2	3	5,266.26	387,835	1.07999×10^{8}	1.02988	3.89417
2	4	4,993.68	359,652	9.78228×10^{8}	1.03873	3.92280
2	5	4,746.04	334,215	8.88479×10^{7}	1.04486	3.94443
2	6	4,520.45	311,271	8.09276×10^{7}	1.04889	3.96035
3	2	5,567.05	419,014	1.1954×10^{8}	1.01760	3.85709
4	2	5,567.07	419,014	1.1954×10^{8}	1.01760	3.85709
5	2	5,567.07	419,014	1.1954×10^{8}	1.01760	3.85709
6	2	5,567.07	419,014	1.1954×10^{8}	1.01760	3.85709
7	2	5,567.07	419,014	1.1954×10^{8}	1.01760	3.85709

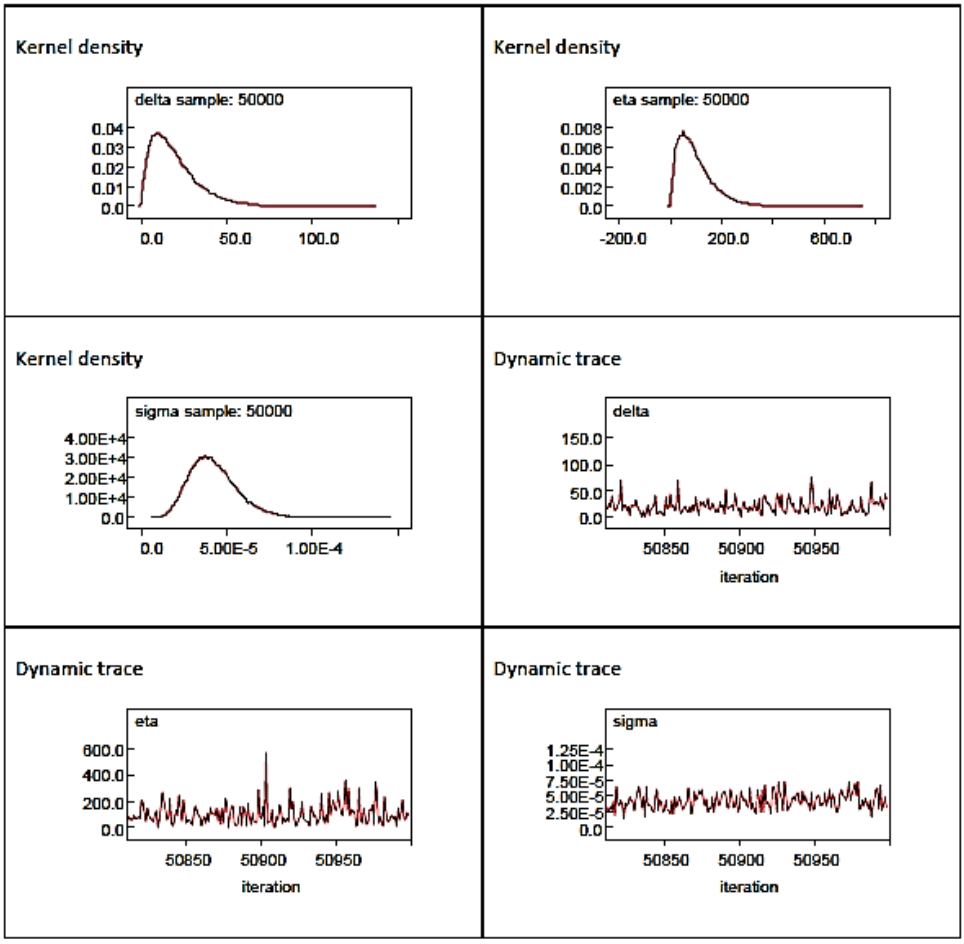

Figure 2: Posterior kernel densities and dynamic trace plots for the parameter and hyperparameters of the survival days of leukemia patients.

We used the WinBUGS software to obtain the posterior distributions of the parameters. In the survival data of 19 leukemia patients, there are 1,001 burn-in samples excluded, and the summary results of the parameters are reported in Table 8 based on the additional 50,000 iterative samples.

Table 8: Summary results for the posterior parameters and hyperparameters given survival days of the leukemia patients by making use of the software WinBUGS

Node	Mean	SD	MC Error	2.5%	Median	97.5%	Start	Sample
δ	20.04	14.14	0.05871	2.471	16.84	56.13	1,001	50,000
η	99.85	71.29	0.3489	12.27	83.56	280.40	1,001	50,000
σ	$4.237E^{-5}$	$1.375E^{-5}$	$5.991E^{-8}$	$1.998E^{-5}$	$4.084E^{-5}$	$7.315E^{-5}$	1,001	50,000

Figure 3 presents the predictive mean and standard deviation based on several values of the hyperparameters from both studies.

6. Conclusion

The first four raw and corrected moments of the predictive density for a single future response are reported. The measures of skewness and kurtosis of a single future response are given. The highest predictive density intervals for a single future response are obtained based on several combinations of the hyperparameters. We reported the summary results for the posterior model parameters and hyperparameters given the simulated data and also the survival days of the leukemia patients. Furthermore, the posterior kernel densities and dynamic trace for the parameters and hyperparameters are graphically displayed. An advanced computational software package, 'Mathematica version 7.0', was used to show the graphical representation of the predictive density for a single future response and also to carry out all related calculations.

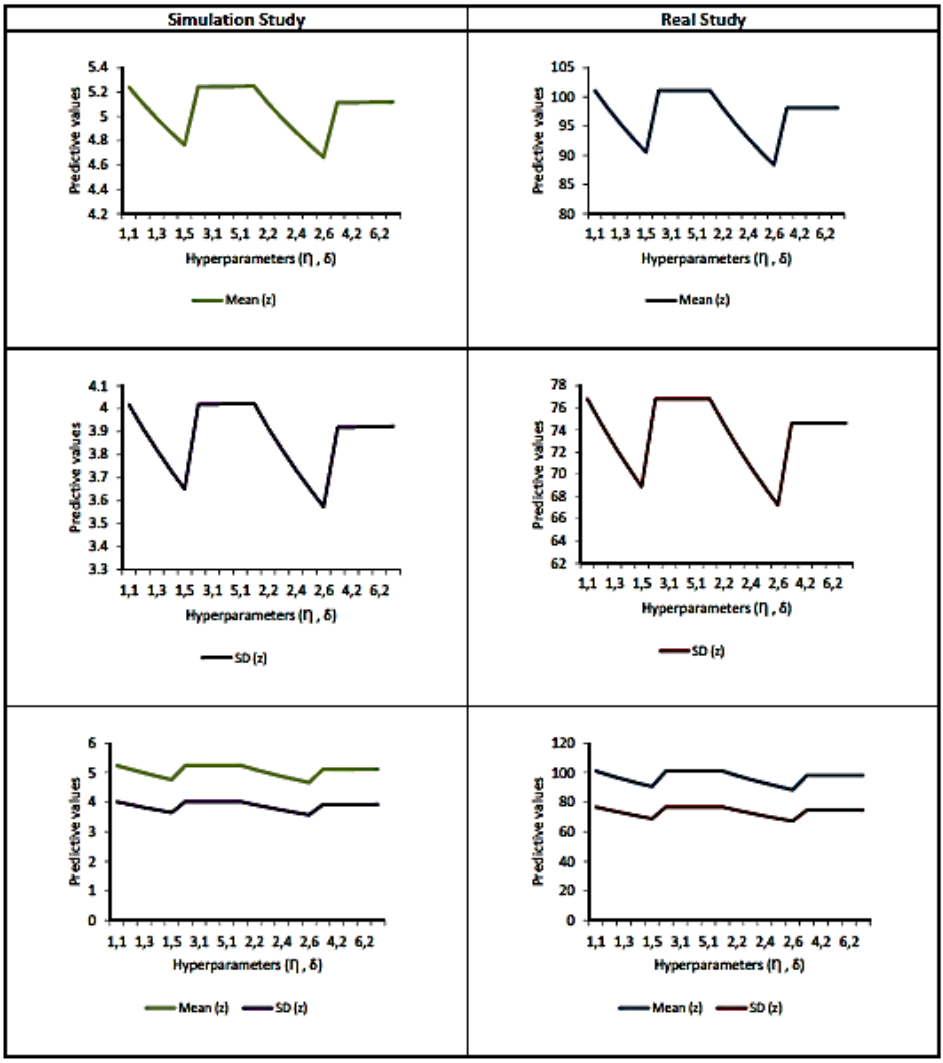

Figure 3: Predictive mean and standard deviation plots with respect to certain values of the hyperparameters from both studies.

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