Thailand Statistician
July 2012; 10(2): 199-223
http://statassoc.or.th
Contributed paper

Bayesian Analysis of Random Coefficient Dynamic AutoRegressive Model

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Received: 7 December 2011

Accepted: 18 September 2012

Abstract

The goal of this work is to develop Random Coefficient AutoRegressive (RCAR) model and AutoRegressive (AR) model to Random Coefficient Dynamic AutoRegressive (RCDAR) model. The RCDAR model is considered by adding exogenous variables in RCAR model, so there are two variables in RCDAR model. This paper proposes the Bayesian analysis to estimate parameter of the first order RCDAR model. The non-informative prior is used to the Bayesian estimation procedure that works well for the AR model. Monte Carlo simulations was repeated for each situations in comparison of the coefficient from RCDAR model which is stationary, weakly stationary, and closed to non-stationary data. The results of coefficient estimator are satisfied weakly stationary data which is performed to fit possibly for large sample sizes.

Keywords: Bayesian analysis, Monte Carlo simulation, nonstationary, stationary .

1. Introduction

In economics it is very common to see that the data are often collected in the forms of time series. Researchers have been interested for modeling the time series model especially the model in form of conditional variance or volatility that can help to forecast the financial values in future time. At this point not only the mean but also some measure of the conditional variance of distribution is a primary importance.

A model that is widely used to the volatility model, is known as the AutoRegressive Conditional Heteroscedastic (ARCH) model introduced by Engle [1]. In this model, condition variance function is expressed in terms of mean and corrected return of the past time, that is similar in spirit to an AutoRegressive (AR) model. An extension is to consider Genalized AutoRegressive Conditional Heteroscedastic (GARCH) model studied by Bollerslev [2]. The GARCH model is given the past observations and the past volatility in deterministic.

An alternative way to model unobserved volatility is to use the Conditional Heteroscadastic Autoregressive Moving Average (CHARMA) model by Tsay [3]. Note that the CHARMA model is allowed the coefficients of an Autoregressive Moving Average (ARMA) model to be random. The special case of CHARMA model reduced the number of parameter to the Random Coefficient AutoRegressive (RCAR) model which was studied by Nicholls and Quinn [4] that obtained parameters by using the least squares estimates and maximum likelihood estimates. Wang [5] studied the Bayesian approach to estimate the first order of RCAR model.

The above models are considered in the case of univariate time series data. However the RCAR model can be improved by changing the constant parameter to exogenous variables in form of AR model called Random Coefficient Dynamic AutoRegressive (RCDAR) model which has some direct effect on the stationary and non-stationary data.

The Bayesian approach can be viewed as hierarchical model to estimate the parameter of the first order in RCDAR model. Bayesian data analysis is becoming more and more appealing because of its flexibility in handing complex models. We also illustrate how our proposed Bayesian analysis method that can be implemented using the software WinBUGS [6] available free on the web.

The rest of this paper is organized as follows: Section 2 describes the RCAR model and RCDAR model. Section 3 presents the parameter estimation procedure and how to implement the Gibbs sampling algorithm via WinBUGS. We apply our proposed model to simulation study in Section 4, and we discuss the results in Section 5.

2. Random Coefficient Dynamic AutoRegressive (RCDAR) model

For economics time series model, it is widely used univariate data and related the conditional variance that can be described in RCAR model. The RCAR model in general form is denoted RCAR(p) [7] written as

$$x_t = \alpha + \sum_{i=1}^{p} \beta_{ti} x_{t-i} + \sigma\, \varepsilon_t \quad , \quad t = 1, 2, \ldots$$

$$\underline{\beta}_t = \underline{\mu}_\beta + \sum\nolimits_{\beta}^{1/2} \underline{u}_t \tag{1}$$

where $\underline{\beta}_t = (\beta_{t1}, \ldots, \beta_{tp})^T$

$\underline{\mu}_\beta = (\mu_{\beta 1}, \ldots, \mu_{\beta p})^T$

It is assumed that ε_t 's and $\underline{u}_t$'s are iid from a distribution with zero mean and unit variance and $\underline{u}_t$ are called the white noise. Empirically, we can see that the constant term of RCAR model can be applied by adding the exogenous variables, y_t , as the AR model which is called Random Coefficient Dynamic AutoRegressive (RCDAR) model or RCDAR(p,q) model following

$$x_t = y_t + \sum_{t=1}^{p} \beta_{ti} x_{t-i} + \sigma\, \varepsilon_t \quad , \quad t = 1, 2, \ldots$$

$$\underline{\beta}_t = \underline{\mu}_\beta + \sum\nolimits_{\beta}^{1/2} \underline{u}_t$$

$$y_t = \sum_{j=1}^{q} \eta_j y_{t-j} + \delta_t \quad . \tag{2}$$

In this case, we will consider the first order of RCDAR model with p=q=1 and $\sigma = 1$: denoted by RCDAR(1,1) that can be written as

$$x_t = y_t + \beta_t x_{t-1} + \varepsilon_t \quad , \quad t = 1, 2, \ldots$$

$$\beta_t = \mu_\beta + \sigma_\beta\, u_t$$

$$y_t = \eta\, y_{t-1} + \delta_t \tag{3}$$

where β_t's are iid random variables with mean μ_β and variance σ^2_β, ε_t 's are iid random variables with mean 0 and variance σ^2_ε, δ_t 's are iid random variables with mean 0 and variance σ^2_δ, β_t's , ε_t 's, and δ_t 's are independent.

For the parameter estimation of RCDAR(1,1), it is seen from (3) that consisted of the intercept term η, the mean μ_β, variance σ^2_β of the coefficient β_t, the variance σ^2_ε of the ε_t , and the variance σ^2_δ of the δ_t , or defined as $\theta = (\eta, \mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon, \sigma^2_\delta)^T$.

3. Parameter Estimation for RCDAR(1,1)

To perform Bayesian data analysis for the RCDAR(1,1) model, we propose the three-level hierarchical model. The first level is the conditional distribution of the data x_t's given the observable x_{t-1} and y_t , coefficient η, β_t , and σ^2_ε. The second level consists of the conditional distribution β_t given the parameter μ_β and σ^2_β. Finally the last level shows the conditional distribution y_t given the observable y_{t-1} and parameter η and σ^2_δ . Consequently, given the sample variables $x_1, x_2, \ldots, x_n$ and the exogenous variables $y_1, y_2, \ldots, y_n$, we are able to express the RCDAR(1,1) model in the following hierarchical structure,

$$\begin{aligned} x_t \mid x_{t-1}, y_t, \eta, \beta_t, \sigma^2_\varepsilon &\sim N(y_t + \beta_t x_{t-1}, \sigma^2_\varepsilon) \\ \beta_t \mid \mu_\beta, \sigma^2_\beta &\sim N(\mu_\beta, \sigma^2_\beta) \\ y_t \mid y_{t-1}, \eta &\sim N(\eta\, y_{t-1}, \sigma^2_\delta) \\ (\eta, \mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon, \sigma^2_\delta) &\sim p(\eta, \mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon, \sigma^2_\delta) \end{aligned} \qquad (4)$$

where p(.) is the prior density of θ which unknown parameter. Following (4) , we can express the likelihood function of θ as,

$$\begin{aligned} &L(\eta, \mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon, \sigma^2_\delta \mid x_1, \ldots, x_n, y_1, \ldots, y_n) \\ &\quad = L(\mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon \mid x_1, \ldots, x_n, y_1, \ldots, y_n) L(\eta, \sigma^2_\delta \mid y_1, \ldots, y_n) \end{aligned}$$

where

$$L(\mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2 \mid x_1, \ldots, x_n, y_1, \ldots, y_n) = \phi(x_1; y_1, \sigma_\varepsilon) \prod_{t=2}^{n} \phi\left(x_t ; y_t + \mu_\beta x_{t-1}, \sqrt{\sigma_\varepsilon^2 + \sigma_\beta^2 x_{t-1}^2}\right) \quad (5)$$

and

$$L(\eta, \sigma_\delta^2 \mid y_1, \ldots, y_n) = \phi(y_1; 0, \sigma_\delta) \prod_{t=2}^{n} \phi\left(y_t; \eta y_{t-1}, \sigma_\delta\right)$$

where $\phi(x; \mu, \sigma)$ denotes the density function of a normal distribution with mean μ and standard deviation σ. Therefore, the joint posterior density of the parameters is given by,

$$f(\theta \mid x_1, \ldots, x_n, y_1, ..., y_n) \propto L(\theta \mid x_1, \ldots, x_n, y_1, ..., y_n) p(\theta)$$

where $p(\theta)$ is a prior density of θ. From the hierarchical structure in (4), the posterior density of μ_β can be written as

$$f(\mu_\beta \mid x_1, \ldots, x_n, y_1, ..., y_n) \propto \int f\left(\mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2 \mid x_1, \ldots, x_n, y_1, ..., y_n\right) d\sigma_\beta^2 d\sigma_\varepsilon^2$$

As a possible remedy, it is hard to integrate out the above density, so this problem is used the Markov Chain Monte Carlo (MCMC) method that is combined the Monte Carlo techniques and Markov chains. We will carry out the Gibbs sampler see Gelfand and Smith [8], a widely used MCMC method, to obtain the parameter from the posterior distribution using the software WinBUGS.

3.1 Choice of Prior Densities

The kind of prior has “no information” or called “non-information” that is used in place of a subjective prior distribution when no information available. By using non-informative prior, all of the information are obtained from the joint posterior density.

For the parameter estimation of $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2)^T$ in RCDAR(1,1) model, we use the uniform distribution as non-informative prior of η. The normal distribution is supported the non-informative prior of μ_β, and the gamma distribution is

supported the non-informative prior of σ_β^2 , σ_ε^2, and σ_δ^2. Specifically, we used prior as

$$\eta \sim Uniform(-0.999, 0.999)$$

$$\mu_\beta \sim Normal(0, 0.01)$$

, and $$\sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2 \sim Inverse-Gamma(2.001, 10^3).$$

Notice that for η, y_t is the first-order autoregressive process, then we define $|\eta| < 1$. The variance of the normal distribution on μ_β is small. We employ the inverse Gamma for σ_β^2 , σ_ε^2, and σ_δ^2, then we choose the parameter that can make the variance to be large.

3.2 Posterior Inference via Gibbs Sampling

To perform Bayesian data analysis for RCDAR(1,1) model, we are interested in the properties of the density of $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2)^T$. Deriving the joint posterior density for θ amounts to integrating out the unobserved coefficients $\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2$, and σ_δ^2, which are difficult to perform both analytically and numerically that are made the posterior about the parameters very difficult.

At this points, Markov Chain Monte Carlo (MCMC) methods are being increasingly used for the cases where marginal distribution of the random variables can not be obtained either analytically or numerically. MCMC methods consist of algorithms to construct a Markov Chain of the parameters such that its stationary distribution is our distribution of interest, i.e. the posterior distribution of the parameter of interest.

We use the Gibbs sampler [9] which is the most popular MCMC method, to obtain dependent samples from the posterior distribution. Specifically, we derive the condition densities, $f(\eta \mid \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2, \underline{x}, \underline{y})$, $f(\mu_\beta \mid \eta, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2, \underline{x}, \underline{y})$, $f(\sigma_\beta^2 \mid \eta, \mu_\beta, \sigma_\varepsilon^2, \sigma_\delta^2, \underline{x}, \underline{y})$, $f(\sigma_\varepsilon^2 \mid \eta, \mu_\beta, \sigma_\beta^2, \sigma_\delta^2, \underline{x}, \underline{y})$, and

$f(\sigma_\delta^2 \mid \eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2, \underline{x}, \underline{y})$ as the full conditional densities of $\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2$,and σ_δ^2 , respectively, based on Model (3).

The Gibbs sampling algorithm is:

1. Initialize $\eta^{(0)}, \mu_\beta^{(0)}, \sigma_\beta^{2(0)}, \sigma_\varepsilon^{2(0)}$, and $\sigma_\delta^{2(0)}$, for k=1,2,…,m+M
2. Draw $\eta^{(k)}$ from $f(\eta \mid \mu_\beta^{(k-1)}, \sigma_\beta^{2(k-1)}, \sigma_\varepsilon^{2(k-1)}, \sigma_\delta^{2(k-1)}, \underline{x}, \underline{y})$

   Draw $\mu_\beta^{(k)}$ from $f(\mu_\beta \mid \eta^{(k)}, \sigma_\beta^{2(k-1)}, \sigma_\varepsilon^{2(k-1)}, \sigma_\delta^{2(k-1)}, \underline{x}, \underline{y})$

   Draw $\sigma_\beta^{2(k)}$ from $f(\sigma_\beta^2 \mid \eta^{(k)}, \mu_\beta^{(k)}, \sigma_\varepsilon^{2(k-1)}, \sigma_\delta^{2(k-1)}, \underline{x}, \underline{y})$

   Draw $\sigma_\varepsilon^{2(k)}$ from $f(\sigma_\varepsilon^2 \mid \eta^{(k)}, \mu_\beta^{(k)}, \sigma_\beta^{2(k)}, \sigma_\delta^{2(k-1)}, \underline{x}, \underline{y})$

   Draw $\sigma_\delta^{2(k)}$ from $f(\sigma_\delta^2 \mid \eta^{(k)}, \mu_\beta^{(k)}, \sigma_\beta^{2(k)}, \sigma_\varepsilon^{2(k)}, \underline{x}, \underline{y})$

where m is burn-in and M is the number of samples generated for 4,000 iterations after 1,000 burn-in. Repeating the above sampling steps, we obtain a discrete-time Markov chain $\left\{\left(\eta^{(k)}, \mu_\beta^{(k)}, \sigma_\beta^{2(k)}, \sigma_\varepsilon^{2(k)}, \sigma_\delta^{2(k)}\right), k = 1, 2, \ldots\right\}$ whose stationary distribution is the joint posterior density of the parameters.

We carry out the Gibbs sampling algorithm as proposed above by means of a software package known as WinBUGS. In WinBUGS, a Markov chain of parameters of $\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2$, and σ_δ^2 are constructed by direct sampling from the corresponding full conditional densities as they are standard distribution. We can perform this procedure using a freeware WinBUGS [6].

The MCMC samples of $\theta = (\eta, \mu_\beta, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\delta^2)^T$ obtain via WinBUGS compute approximate posterior summaries of the parameters as the posterior estimation of θ. In particular, we use the posterior mean as point estimates of θ.

The Gibbs sampling algorithm of RCDAR(1,1) is specified in WinBUGS's code following

```
model{
y[1] ~ dnorm(0, tau.y)
x[1] ~ dnorm(0,tau.x)
beta[1] ~ dnorm(mu.beta,tau.beta)
```

```
for (i in 2:n){
    x[i]~ dnorm(alpha[i], tau.x)
    y[i]~ dnorm(mu.y[i], tau.y)
 alpha[i] <-  y[i] + (beta[i]*x[i-1])
 mu.y[i] <- eta*y[i-1]
 beta[i]  ~ dnorm(mu.beta,tau.beta)}
  tau.y  ~ dgamma(2.001,0.001)
  tau.x  ~ dgamma(2.001,0.001)
  sigma2.y <-  1/tau.y
  sigma2.x <-  1/tau.x
  eta ~ dunif(-0.999,0.999)
  mu.beta ~ dnorm(0,0.01)
  tau.beta  ~ dgamma(2.001,0.001)
  sigma2.beta <-  1/tau.beta
}
```

4. Simulation Study

The simulation study is to estimate parameter $\theta = (\eta, \mu_\beta, \sigma^2_\beta, \sigma^2_\varepsilon, \sigma^2_\delta)^T$ for the performance of Bayesian analysis. At the first, we generate data y_t, t=1,2,…,n from AR(1) model by taking η = 0.1, 0.5, and 0.9 following;

$$y_t = \eta\, y_{t-1} + \delta_t \quad : \quad \text{AR(1)} \qquad (6)$$

Hence, when $|\eta| < 1$, the data is stationary, but $|\eta| \to 1$, the data is nearly non-stationary process. In this case, we consider the $0 < \eta < 1$, then all autocorrelations are positive.

To illustrate the implication of AR(1) model, Figure 1 shows the 100 sample sizes for each 3 coefficients (η = 0.1, 0.5, and 0.9). It should be noted that η = 0.1 is stationary, η = 0.5 is weakly stationary, and η = 0.9 is closed to non-stationary process.

Next, we consider the RCDAR(1,1) model where y_t are generated form AR(1) model. Therefore we obtain the x_t in term of RCDAR(1,1) written as

$$x_t = y_t + \beta_t x_{t-1} + \varepsilon_t$$

$$\beta_t = \mu_\beta + \sigma_\beta u_t \qquad (7)$$

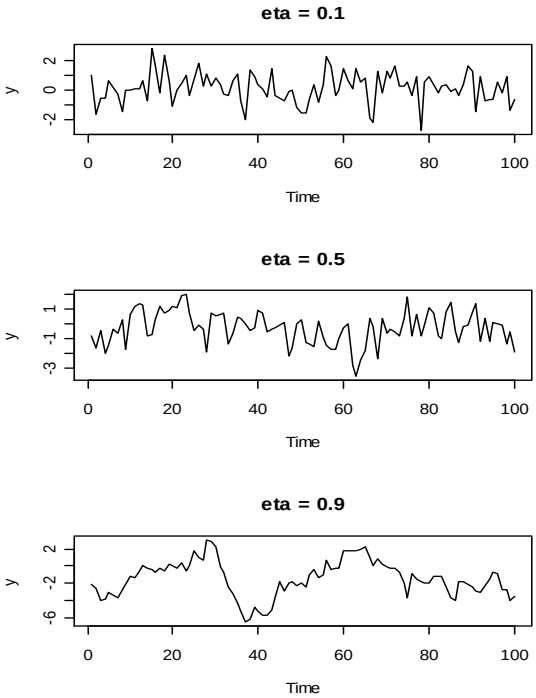


Figure 1. The time series plot of generated data from AR(1) for y_t (η = 0.1, 0.5, and 0.9).

In Figures 2-4, we present the data generating in 4 cases:

Case 1: σ^2 = 1, μ_β = 0.5, and σ_β^2 = 0.25

Case 2: σ^2 = 1, μ_β = 1, and σ_β^2 = 0

Case 3: σ^2 = 1, μ_β = - 0.995, and σ_β^2 = 0.01

Case 4: σ^2 = 1, μ_β = - 0.1, and σ_β^2 = 0.99

It should be noted that Case 1 is the stationary case, Nicholls and Quinn [4] derived the condition of μ_β and σ_β^2 if $\mu_\beta^2+\sigma_\beta^2<1$ [7]. Cases 2 and 4 show non-stationary process or called the random walk case that Nicholls and Quinn [4] defined $\mu_\beta^2+\sigma_\beta^2=1$ but Case 4 depended on η under y_t. However Case 3 tends to be around its mean values of zero as the stationary process but $\mu_\beta^2+\sigma_\beta^2>1$.

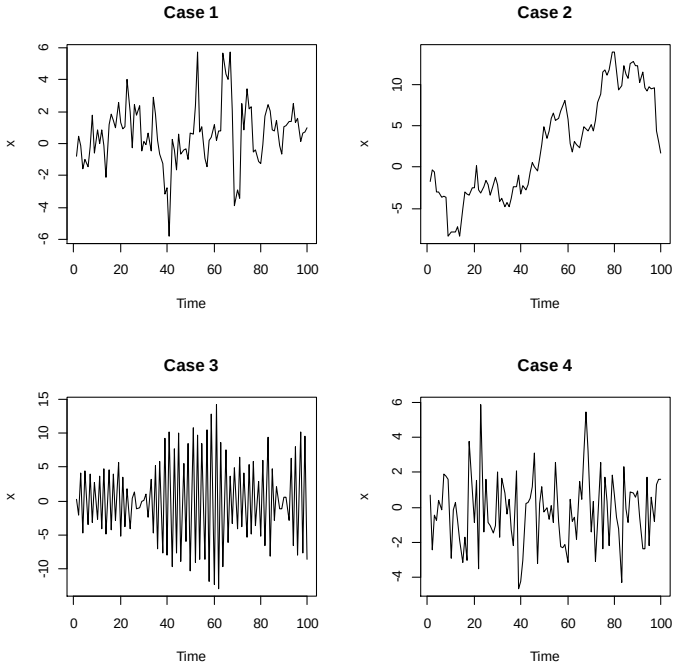


Figure 2. The time series plot of generated data from RCDAR(1,1) for x_t (η = 0.1).

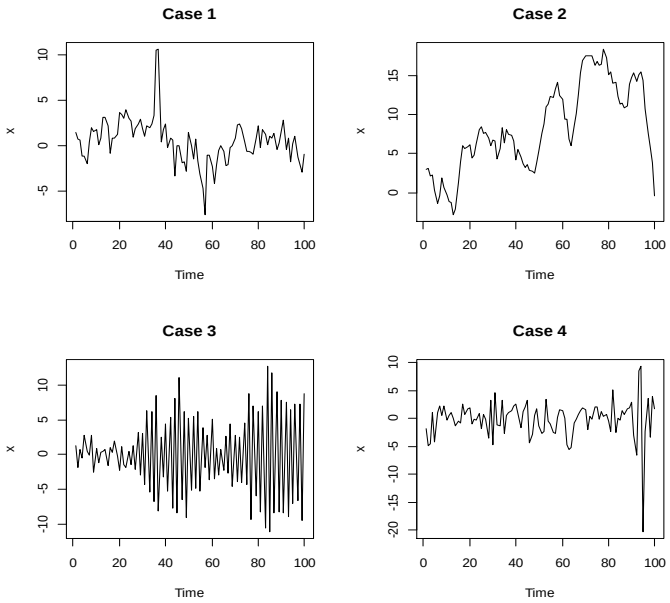


Figure 3. The time series plot of generated data from RCDAR(1,1) for x_t (η = 0.5).

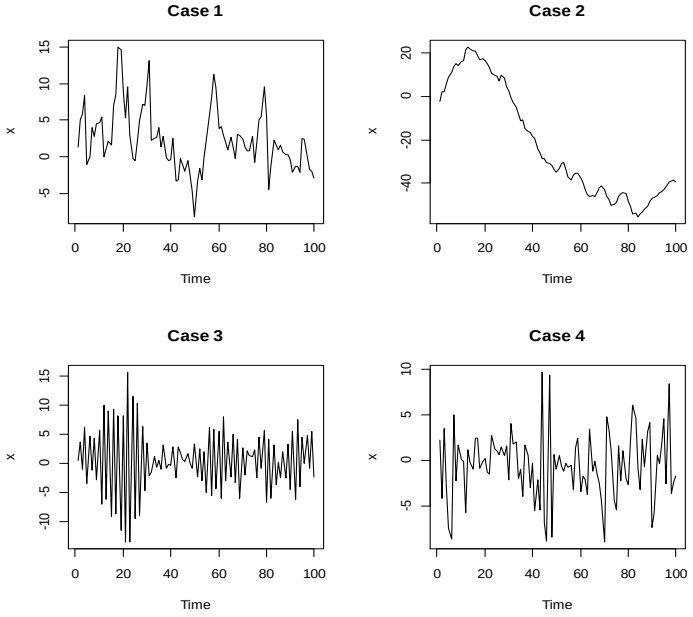


Figure 4. The time series plot of generated data from RCDAR(1,1) for x_t (η = 0.9).

For the Tables 1-6, the second column shows the true parameter of simulation, and the third and the forth columns represent the MC sample for average of posterior mean and standard deviation of posterior mean. For next two columns, the lower and upper bounds of 95% confidence interval captures the true parameters that computed from $mean \pm 1.96(s.d.)/\sqrt{500}$, and the last two columns list the t-statistics and p-value for hypothesis testing of true parameters.

Tables 1-2 provide the results of η = 0.1, and Figures 5-14 illustrate the histogram of parameter estimates at the sample sizes of 100 and 500. The results in both sample sizes are similar for all cases, especially Cases 1 and 4, most parameter estimates are non-significance on true parameters ($\eta, \mu_\beta, \sigma_\beta^2$). However the histograms of the most parameters are presented as the normal curve except Case 2 when the true parameters are μ_β and σ_β^2.

Tables 3-4 propose the results of η = 0.5, and Figures 15-24 illustrate the histogram of parameter estimates at the sample sizes of 100 and 500. When the sample sizes are 500, Cases 1 and 4 show p-value is larger than 0.01, so the average of posterior mean is not different for all true parameters. The histogram is similar at η = 0.1.

Tables 5-6 present the result of η = 0.9, and Figures 25-34 illustrate the histogram of parameter estimates at the sample sizes of 100 and 500. The results in both sample sizes show the statistically significant with true parameter except the μ_β and σ_β^2 under Case 4. For the small sample sizes, the histogram of η looks the skewness curve.

Table 1. The simulation of RCDAR(1,1) model for η = 0.1 and 100 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.1	0.094	0.103	0.085	0.103	-1.133	0.257
	μ_β = 0.5	0.490	0.091	0.481	0.498	-2.429	0.015
	σ_β^2 = 0.25	0.239	0.119	0.229	0.250	-1.941	0.052
	σ_ε^2 = 1	1.094	0.258	1.071	1.117	8.182*	0.000
	σ_δ^2 = 1	1.027	0.147	1.015	1.040	4.240*	0.000
Case 2	η = 0.1	0.106	0.045	0.097	0.115	1.353	0.176
	μ_β = 1	0.992	0.043	0.990	0.994	-8.294*	0.000
	σ_β^2 = 0	0.002	0.052	0.002	0.002	29.136*	0.000
	σ_ε^2 = 1	0.913	0.097	0.900	0.926	-12.867*	0.000
	σ_δ^2 = 1	1.025	0.059	1.013	1.038	4.059*	0.000
Case 3	η =0.1	0.090	0.094	0.081	0.098	-2.343	0.019
	μ_β =-.995	-0.987	0.024	-0.989	-0.985	6.999*	0.000
	σ_β^2 =0.01	0.010	0.005	0.010	0.011	2.223	0.026
	σ_ε^2 =1	1.059	0.257	1.036	1.082	5.156*	0.000
	σ_δ^2 =1	1.028	0.144	1.015	1.041	4.398*	0.000
Case 4	η =0.1	0.094	0.091	0.086	0.102	-1.270	0.204
	μ_β =-0.1	-0.098	0.129	-0.110	-0.087	0.224	0.822
	σ_β^2 =0.99	0.980	0.244	0.959	1.002	-0.824	0.409
	σ_ε^2 =1	1.109	0.338	1.079	1.138	7.207*	0.000
	σ_δ^2 =1	1.026	0.142	1.014	1.039	4.227*	0.000

* indicates significance at 1% level

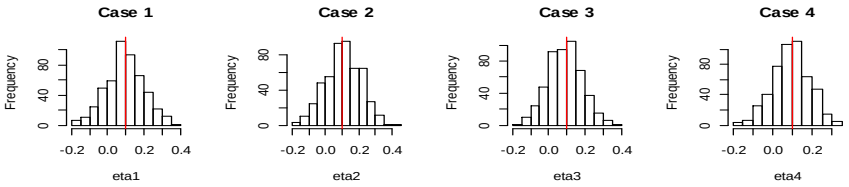


Figure 5. The histogram of RCDAR(1,1) model for η =0.1 .

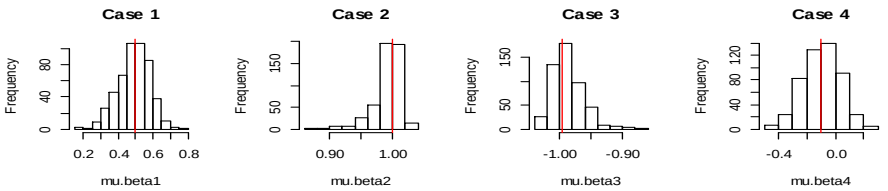


Figure 6. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

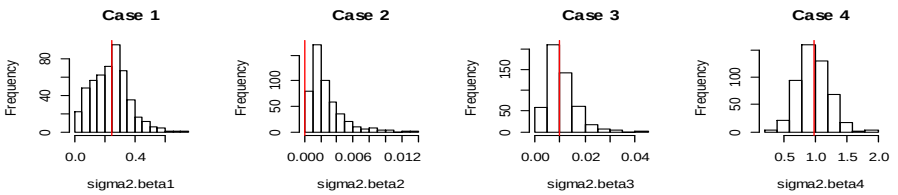


Figure 7. The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

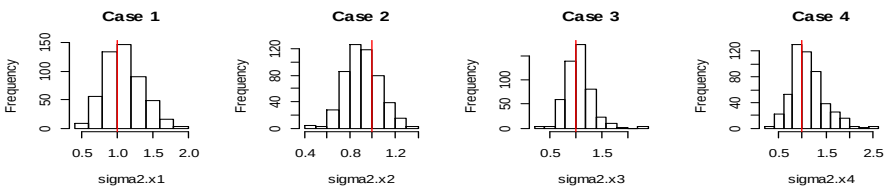


Figure 8. The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

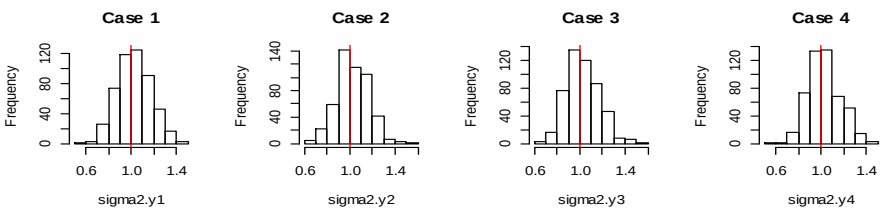


Figure 9. The histogram of RCDAR(1,1) model for σ_δ^2 = 1 .

Table 2. The simulation of RCDAR(1,1) model for η = 0.1 and 500 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.1	0.099	0.043	0.095	0.103	-0.091	0.927
	μ_β = 0.5	0.495	0.037	0.492	0.498	-2.598*	0.009
	σ_β^2 = 0.25	0.246	0.044	0.242	0.250	-1.638	0.102
	σ_ε^2 = 1	1.017	0.099	1.008	1.026	3.960*	0.000
	σ_δ^2 = 1	1.004	0.060	0.999	1.010	1.795	0.073
Case 2	η = 0.1	0.098	0.104	0.094	0.101	-0.991	0.322
	μ_β = 1	0.998	0.211	0.998	0.998	-8.983*	0.000
	σ_β^2 = 0	0.000	0.001	0.000	0.000	31.591*	0.000
	σ_ε^2 = 1	0.890	0.150	0.883	0.897	-30.447*	0.000
	σ_δ^2 = 1	1.007	0.141	1.001	1.013	2.638*	0.008
Case 3	η =0.1	0.100	0.046	0.096	0.104	0.148	0.881
	μ_β =-.995	-0.992	0.008	-0.993	-0.991	6.375*	0.000
	σ_β^2 =0.01	0.009	0.001	0.009	0.010	-0.830	0.406
	σ_ε^2 =1	1.011	0.116	1.001	1.021	2.197	0.028
	σ_δ^2 =1	1.004	0.063	0.998	1.009	1.454	0.146
Case 4	η =0.1	0.099	0.044	0.095	0.105	-0.491	0.623
	μ_β =-0.1	-0.102	0.059	-0.107	-0.097	-0.904	0.366
	σ_β^2 =0.99	0.994	0.099	0.985	1.002	0.905	0.365
	σ_ε^2 =1	1.017	0.139	1.005	1.029	2.781*	0.005
	σ_δ^2 =1	1.003	0.063	0.998	1.009	1.280	0.201

* indicates significance at 1% level

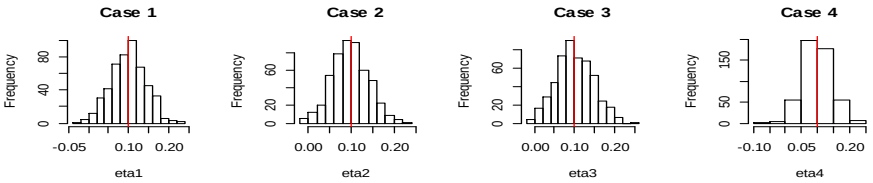


Figure 10. The histogram of RCDAR(1,1) model for η =0.1 .

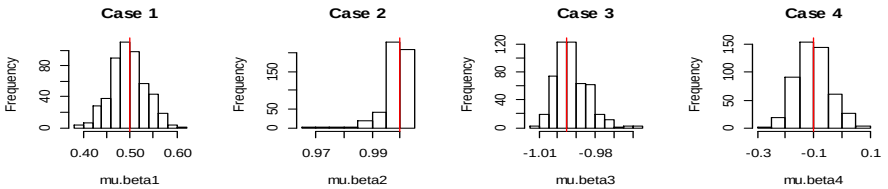


Figure 11. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

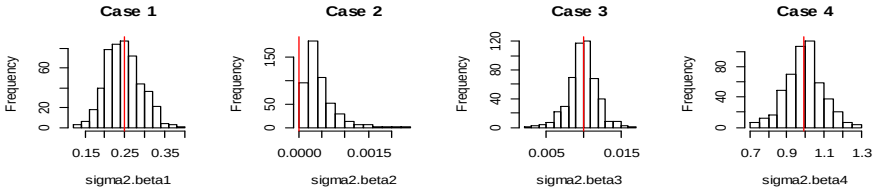


Figure 12. The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

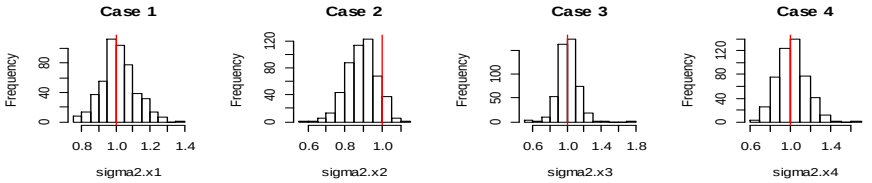


Figure 13. The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

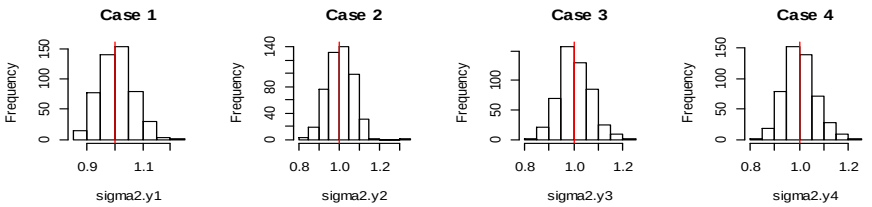


Figure 14. The histogram of RCDAR(1,1) model for σ_δ^2 = 1 .

Table 3. The simulation of RCDAR(1,1) model for η = 0.5 and 100 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.5	0.495	0.083	0.488	0.502	-1.222	0.222
	μ_β = 0.5	0.497	0.079	0.490	0.504	-0.791	0.428
	σ_β^2 = 0.25	0.240	0.098	0.231	0.248	-2.245	0.025
	σ_ε^2 = 1	1.075	0.262	1.052	1.098	6.411*	0.000
	σ_δ^2 = 1	1.017	0.138	1.005	1.029	2.834*	0.004
Case 2	η = 0.5	0.483	0.088	0.476	0.491	-4.103*	0.000
	μ_β = 1	0.996	0.012	0.995	0.997	-6.605*	0.000
	σ_β^2 = 0	0.001	0.001	0.001	0.001	26.742*	0.000
	σ_ε^2 = 1	0.885	0.157	0.871	0.899	-16.292*	0.000
	σ_δ^2 = 1	1.025	0.153	1.011	1.038	3.673*	0.000
Case 3	η =0.5	0.490	0.085	0.482	0.497	-2.496	0.012
	μ_β =-.995	-0.983	0.030	-0.985	-0.980	8.653*	0.000
	σ_β^2 =0.01	0.010	0.006	0.010	0.011	2.782*	0.005
	σ_ε^2 =1	1.034	0.226	1.015	1.054	3.454*	0.000
	σ_δ^2 =1	1.030	0.140	1.017	1.042	4.768*	0.000
Case 4	η =0.5	0.492	0.085	0.484	0.499	-2.059	0.039
	μ_β =-0.1	-0.097	0.128	-0.108	-0.085	0.511	0.609
	σ_β^2 =0.99	0.990	0.248	0.968	1.011	0.011	0.990
	σ_ε^2 =1	1.104	0.331	1.075	1.133	7.033*	0.000
	σ_δ^2 =1	1.013	0.147	1.000	1.026	2.016	0.044

* indicates significance at 1% level

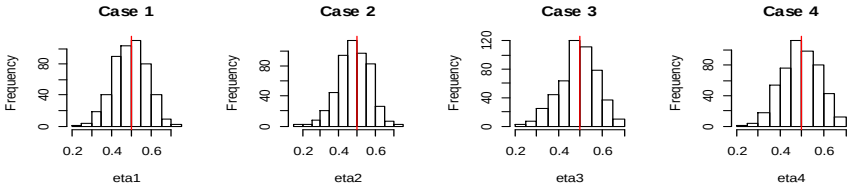


Figure 15. The histogram of RCDAR(1,1) model for η =0.5 .

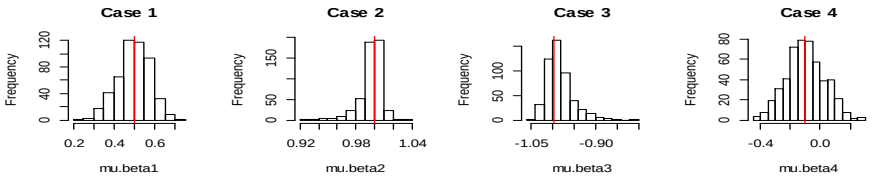


Figure 16. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

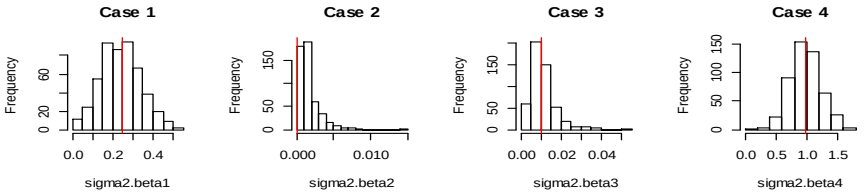


Figure 17. The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

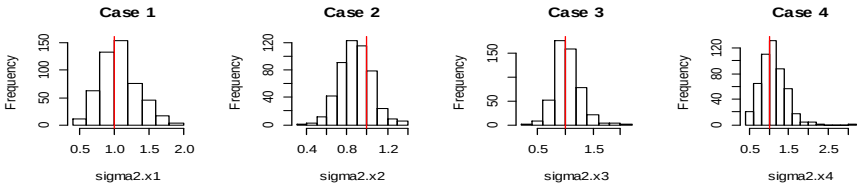


Figure 18. The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

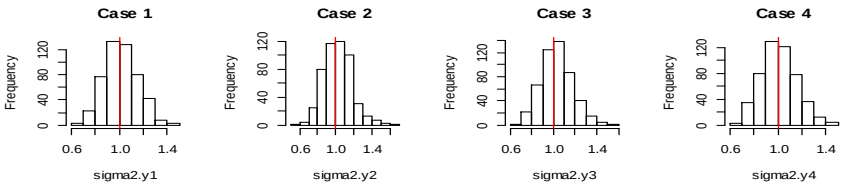


Figure 19. The histogram of RCDAR(1,1) model for σ_δ^2 = 1 .

Table 4. The simulation of RCDAR(1,1) model for η = 0.5 and 500 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.5	0.496	0.041	0.493	0.500	-1.717	0.086
	μ_β = 0.5	0.497	0.036	0.494	0.500	-1.457	0.145
	σ_β^2 = 0.25	0.252	0.036	0.249	0.255	1.742	0.082
	σ_ε^2 = 1	1.012	0.107	1.002	1.021	2.543	0.011
	σ_δ^2 = 1	1.004	0.064	0.998	1.009	1.504	0.133
Case 2	η = 0.5	0.496	0.039	0.493	0.500	-1.786	0.074
	μ_β = 1	0.999	0.002	0.999	0.999	-6.243*	0.000
	σ_β^2 = 0	0.000	0.000	0.000	0.000	32.121*	0.000
	σ_ε^2 = 1	0.873	0.082	0.865	0.880	-34.182*	0.000
	σ_δ^2 = 1	1.003	0.061	0.997	1.008	1.125	0.261
Case 3	η =0.5	0.496	0.037	0.492	0.499	-2.361	0.018
	μ_β =-.995	-0.992	0.008	-0.993	-0.991	6.247*	0.000
	σ_β^2 =0.01	0.010	0.002	0.009	0.010	0.495	0.620
	σ_ε^2 =1	1.013	0.105	1.004	1.023	2.958*	0.003
	σ_δ^2 =1	1.009	0.064	1.003	1.014	3.204*	0.001
Case 4	η =0.5	0.497	0.037	0.494	0.500	-1.390	0.164
	μ_β =-0.1	-0.105	0.056	-0.110	-0.100	-2.217	0.027
	σ_β^2 =0.99	0.993	0.097	0.984	1.001	0.729	0.466
	σ_ε^2 =1	1.014	0.131	1.002	1.025	2.409	0.016
	σ_δ^2 =1	1.000	0.060	1.000	1.011	2.265	0.023

* indicates significance at 1% level

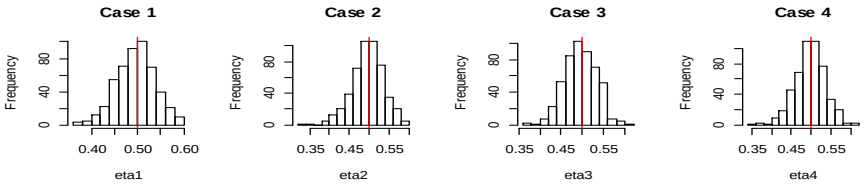


Figure 20. The histogram of RCDAR(1,1) model for η =0.5 .

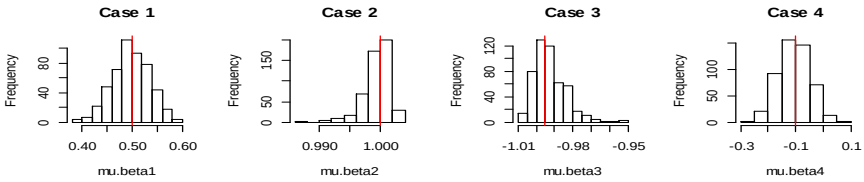


Figure 21. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

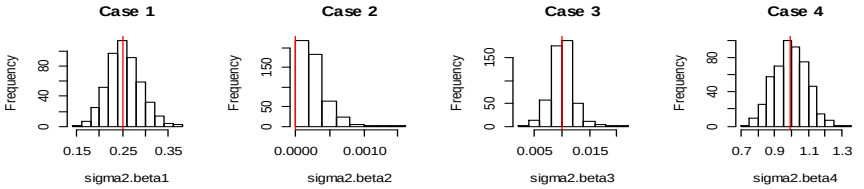


Figure 22. The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

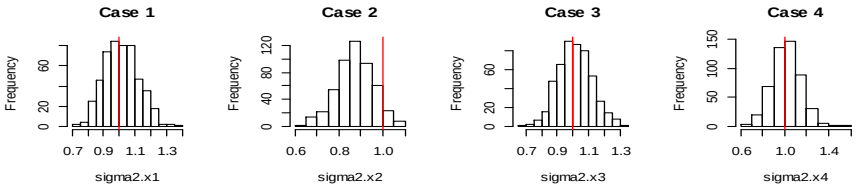


Figure 23. The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

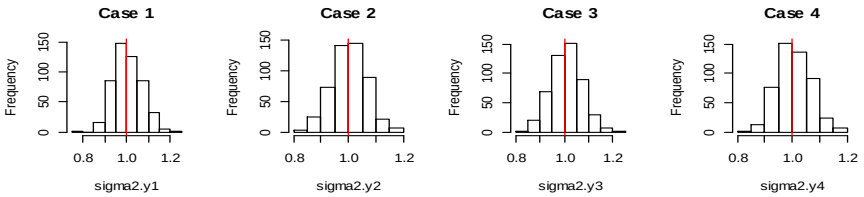


Figure 24. The histogram of of RCDAR(1,1) model for σ_δ^2 = 1 .

Table 5. The simulation of RCDAR(1,1) model for η = 0.9 and 100 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.9	0.880	0.050	0.876	0.885	-8.626*	0.000
	μ_β = 0.5	0.496	0.068	0.490	0.502	-0.995	0.320
	σ_β^2 = 0.25	0.260	0.068	0.254	0.266	3.324*	0.000
	σ_ε^2 = 1	1.054	0.299	1.028	1.081	4.101*	0.000
	σ_δ^2 = 1	1.027	0.147	1.014	1.040	4.173*	0.000
Case 2	η = 0.9	0.880	0.050	0.876	0.885	-8.626*	0.000
	μ_β = 1	0.999	0.003	0.999	1.000	-0.887	0.375
	σ_β^2 = 0	0.000	0.000	0.000	0.000	24.002*	0.000
	σ_ε^2 = 1	0.789	0.161	0.775	0.803	-29.161*	0.000
	σ_δ^2 = 1	1.027	0.147	1.014	1.040	4.173*	0.000
Case 3	η =0.9	0.879	0.050	0.874	0.883	-9.145*	0.000
	μ_β =-.995	-0.982	0.026	-0.984	-0.979	10.627*	0.000
	σ_β^2 =0.01	0.010	0.006	0.010	0.011	3.014*	0.002
	σ_ε^2 =1	1.031	0.229	1.011	1.051	3.070*	0.002
	σ_δ^2 =1	1.020	0.148	1.007	1.033	3.111*	0.001
Case 4	η =0.9	0.880	0.050	0.875	0.884	-8.711*	0.000
	μ_β =-0.1	-0.092	0.117	-0.102	-0.082	1.465	0.143
	σ_β^2 =0.99	1.003	0.204	0.985	1.021	1.465	0.143
	σ_ε^2 =1	1.028	0.385	1.053	1.121	5.076*	0.000
	σ_δ^2 =1	1.017	0.155	1.004	1.031	2.567*	0.010

* indicates significance at 1% level

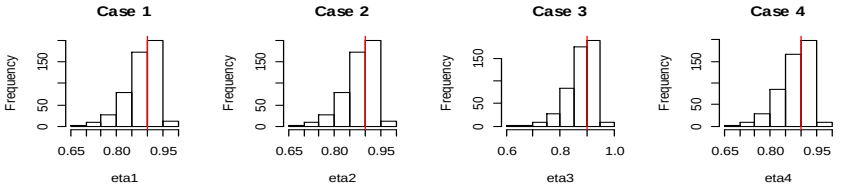


Figure 25. The histogram of RCDAR(1,1) model for η =0.9 .

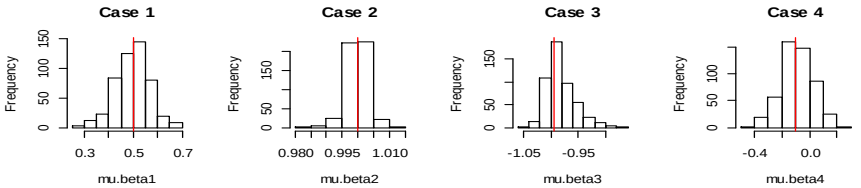


Figure 26. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

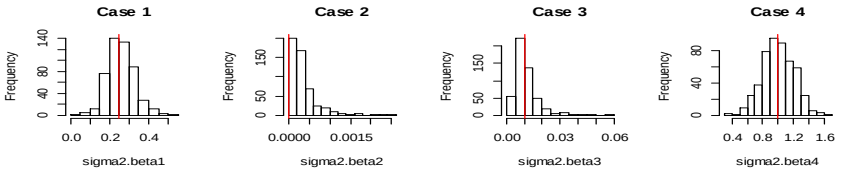


Figure 27. The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

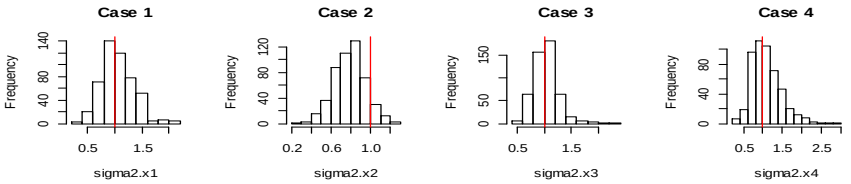


Figure 28. The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

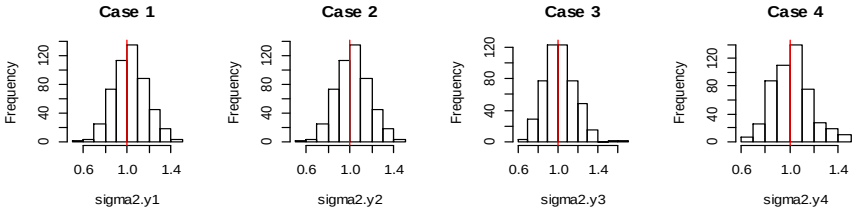


Figure 29. The histogram of of RCDAR(1,1) model for σ_δ^2 = 1 .

Table 6. The simulation of RCDAR(1,1) model for η = 0.9 and 500 sample sizes (500 replications) .

Case	true	mean	s.d.	lci	uci	t-value	p-value
Case 1	η = 0.9	0.895	0.020	0.894	0.897	-4.406*	0.000
	μ_β = 0.5	0.497	0.030	0.494	0.500	-1.763	0.078
	σ_β^2 = 0.25	0.250	0.026	0.247	0.252	0.101	0.919
	σ_ε^2 = 1	1.017	0.121	1.007	1.028	3.268*	0.001
	σ_δ^2 = 1	1.004	0.060	0.999	1.010	1.823	0.068
Case 2	η = 0.9	0.895	0.020	0.894	0.897	-4.406*	0.000
	μ_β = 1	0.999	0.000	0.999	0.999	-4.921*	0.000
	σ_β^2 = 0	0.000	0.000	0.000	0.000	34.656*	0.000
	σ_ε^2 = 1	0.753	0.113	0.743	0.763	-48.487*	0.000
	σ_δ^2 = 1	1.004	0.060	0.999	1.010	1.823	0.068
Case 3	η =0.9	0.897	0.019	0.895	0.898	-3.351*	0.000
	μ_β =-.995	-0.992	0.008	-0.993	-0.992	5.051*	0.000
	σ_β^2 =0.01	0.010	0.002	0.009	0.010	0.738	0.460
	σ_ε^2 =1	1.012	0.111	1.002	1.022	2.481	0.013
	σ_δ^2 =1	1.005	0.064	1.000	1.011	1.981	0.048
Case 4	η =0.9	0.896	0.020	0.894	0.898	-4.010*	0.000
	μ_β =-0.1	-0.101	0.050	-0.106	-0.097	-0.751	0.452
	σ_β^2 =0.99	0.998	0.089	0.990	1.005	2.001	0.045
	σ_ε^2 =1	1.016	0.168	1.001	1.031	2.184	0.029
	σ_δ^2 =1	1.005	0.061	1.000	1.011	2.163	0.030

* indicates significance at 1% level

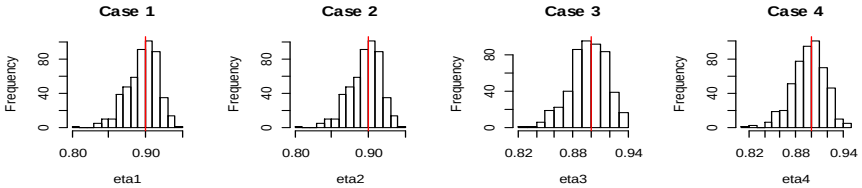


Figure 30. The histogram of RCDAR(1,1) model for η =0.9 .

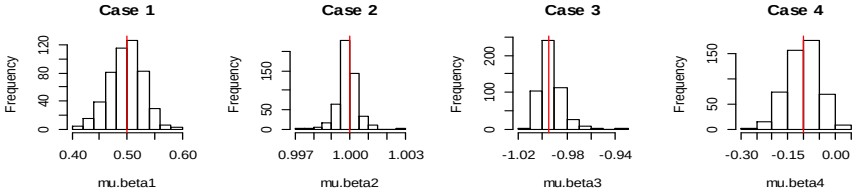


Figure 31. The histogram of RCDAR(1,1) model for μ_β =0.5, 1,-0.995 , and -0.1 .

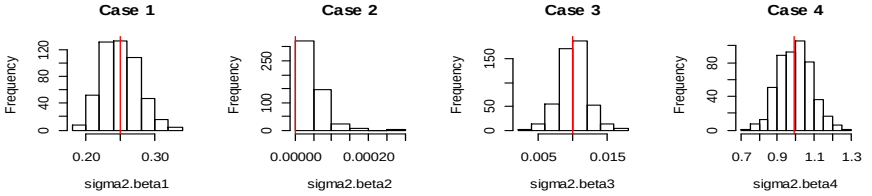


Figure 3: The histogram of RCDAR(1,1) model for σ_β^2 = 0.25, 0, 0.01 ,and 0.99 .

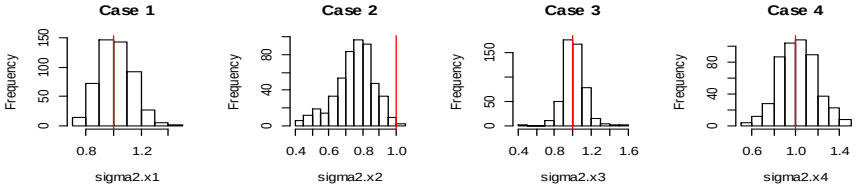


Figure 33: The histogram of RCDAR(1,1) model for σ_ε^2 = 1 .

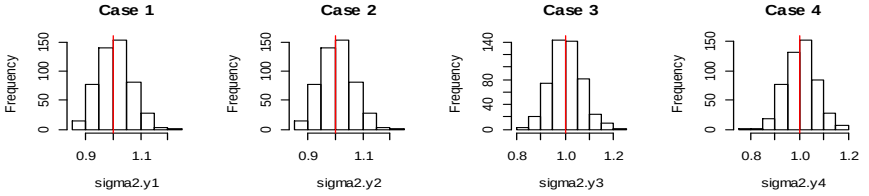


Figure 34: The histogram of of RCDAR(1,1) model for σ_δ^2 = 1 .

5. Discussion

In this paper, we have developed the Bayesian estimation for RCDAR(1,1) model based on stationary , weakly stationary, closed to non-stationry data. Extensive simulation experiments showed that the Bayesian estimation procedure using non-informative prior works well for the large sample sizes. In general, we find that RCDAR(1,1) model is robust to model when the data are weakly stationary and loss in efficiency due to the data are strong stationary and closed to non-stationary. We would recommended user to fit the RCDAR model to time series data where the sample size is large at weakly stationary for 2 variables.

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