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Estimating Predictive Inference for Responses from the Generalized Rayleigh Model based on Complete Sample

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Abstract

In this paper, the likelihood function given a complete sample from the two-parameter generalized Rayleigh model is derived. By making use of the Bayesian framework, the posterior density function, the predictive density for a single future response, a bivariate future response, and several future responses are derived. A comparison of the predictive variability of the maximum likelihood estimates and some of its neighborhood estimates are provided. The predictive means, standard deviations, 95% highest predictive density intervals, and the shape characteristics for a single future response are determined. A real data set is utilized to illustrate the predictive results.

Keywords: Statistical inference, Generalized Rayleigh model, Likelihood function, Posterior density, Predictive inference.

1. Introduction

A large amount of life data has been collected and analyzed in connection with recent advances in engineering and the biomedical sciences. These data may form a complete sample. In order to make a scientific conclusion about the nature of the data, it is important to use statistical and computational techniques. The generalized Rayleigh model has been used in modeling the life data. Most of the studies on this model have focused on estimation of the parameters, or on the reliability and hazard functions. There are a number of studies related to the generalized Rayleigh model that have appeared in the refereed journals, but none of them discussed predictive model for responses.

Inference about the future responses is known as predictive inference. Many authors have studied the predictive inference to solve challenging issues in engineering and biomedical sciences in recent years, for example, Mahdi et al [1], Thabane [2], Thabane and Haq [3], Khan [4-6], Khan and Kabir [7], Khan et al. [8-10], Khan et al. [11], Khan and Provost [12], among others.

One may consider the Bayesian method to derive predictive inference. The Bayesian method considers prior distribution for the parameters and the prior is used to drive the posterior density for the parameters. More about the Bayesian method, the readers are referred to Ahsanullah and Ahmed [13], Bernardo and Smith [14], Berger [15], and Geisser [16], among others.

Additional applications of the Bayesian method to predictive inference have been discussed for instance by Khan et al. [8-10, 17], Khan et al. [18], Khan et al. [11], Khan and Provost [12], Khan and Provost [19], Khan et al. [20], Thabane [2], Thabane and Haq [3], Ali-Mousa and Al-Sagheer [21], and Raqab [22], among others.

Recorded complete sample from an experiment may follow several statistical probability models such as the exponential, gamma, Weibull, normal, half-normal, log-normal, Rayleigh, generalized Rayleigh, and inverse gaussian. The two-parameter generalized Rayleigh has been used in modeling the lifetime data. Several authors have studied this model to estimate the parameters, to name a few, Raqab and Kundu [23] described in details the generalized Rayleigh model with several properties. Kundu and Raqab [24] used different methods to estimate the parameters. Al-khedhairi et al. [25] proposed estimations for the two-parameter generalized Rayleigh model based on grouped and censored data. They used goodness of fit tests to justify the two-parameter generalized Rayleigh model fits the data set better than other models.

Inference about the future responses from the generalized Rayleigh is also important. Therefore, the aim of this paper is to derive the predictive inference from the two-parameter generalized Rayleigh model given an observed complete sample.

Following Kundu and Raqab [24], the probability density function (pdf) for the two-parameter generalized Rayleigh model is given by

$$p(x \mid \alpha,\ \lambda) = \begin{cases} 2\ \alpha\ \lambda^2\ x\ \exp\left\{-(\lambda x)^2\right\}\ \left(1 - \exp\left\{-(\lambda x)^2\right\}\right)^{\alpha-1},\ x \geq 0;\ \alpha, \lambda > 0, \\ 0 \quad \text{elsewhere,} \end{cases} \tag{1}$$

where α and λ are the shape and scale parameters respectively; and the distribution function is

$$F(x \mid \alpha,\ \lambda) = \left(1 - \exp\left\{-(\lambda x)^2\right\}\right)^{\alpha}.$$

The rest of the paper is organized as follows: Section 2 presents the predictive model, which includes the likelihood function, posterior density function, predictive densities for a single future response, bivariate future response, and several future responses given a complete sample of observations from the generalized Rayleigh model. To illustrate the results, a numerical example is presented in Section 3. Finally, a conclusion is added in Section 4.

2. The predictive model

Let z be a future response, then following Khan [26], the predictive density of z given the observed data x is

$$p(z \mid \mathbf{x}) = \int\int p(z \mid \alpha,\ \lambda)\ p(\alpha,\ \lambda \mid \mathbf{x})\ d\lambda\ d\alpha,$$

where $p(\alpha,\ \lambda \mid \mathbf{x})$ is the posterior density function, and $p(z \mid \alpha,\ \lambda)$ represents the probability density function of a future response (z) that may be defined from model (1). The posterior density is given by

$$p(\alpha,\ \lambda \mid \mathbf{x}) = \Psi(\mathbf{x})\ \mathrm{L}(\alpha, \lambda \mid \mathbf{x})\ p(\alpha,\ \lambda),$$

where $\mathrm{L}(\alpha,\ \lambda\,|\,\mathbf{x})$ is the likelihood function, $p(\alpha,\ \lambda)$ is the prior density, and the reciprocal of the normalizing constant is

$$\Psi(\mathbf{x})^{-1} = \int\!\!\int \mathrm{L}(\alpha, \lambda\,|\,\mathrm{x})\, p(\alpha,\ \lambda)\ d\lambda\ d\alpha.$$

To derive the likelihood function, let $x_1,\ .\ .\ .\ ,\ x_n$ be a random sample of size *n* from model (1). Thus, $\mathbf{x}\ =\ (x_1,\ .\ .\ .\ ,\ x_n)'$ forms an observed sample. Then given a set of data $\mathbf{x}\ =\ (x_1,\ .\ .\ .\ ,\ x_n)$ from (1), the likelihood function is given by

$$\mathrm{L}(\alpha, \lambda\,|\,\mathbf{x})\ \propto\ \alpha^n\ \lambda^{2n}\ \exp\left\{-\sum_{i=1}^{n}(\lambda x_i)^2\right\}\ \left[\prod_{i=1}^{n}(x_i)\right]\ \left[\prod_{i=1}^{n}\left(1 - \exp\left\{-(\lambda x_i)^2\right\}\right)^{\alpha-1}\right].$$

Ahmed [27] discussed an estimation theory under uncertain prior information. Ahsanullah and Ahmed [13] discussed in details on Bayes and empirical Bayes estimates of survival and hazard functions of a class of distribution. Ahmed and Tomkins [28] estimated lognormal mean by making use of an uncertain prior information. Khan [26] considered predictive inference problem for an independent future sample from a two-parameter exponential model given a type II censored sample by making use of the Bayesian approach. Khan et al. [17] derived the Bayesian predictive models given a doubly censored sample from the two-parameter exponential model by means of a conjugate prior for the scale parameter. Khan et al. [29] derived the Bayesian predictive inference from the one-parameter Rayleigh life model under type II censored sample considering a conjugate prior for the scale parameter. Khan and Provost [19] derived the Bayesian predictive inference from the two-parameter Rayleigh life model under type II censored sample considering a uniform prior for the location parameter and a conjugate prior for the scale parameter. Khan et al. [20] used the same prior to derive the predictive inference from the two-parameter Rayleigh life model given a doubly censored sample. Here, it is assumed that the prior density for the scale parameter (λ) is given by

$$p(\lambda)\ \propto\ \lambda\ \exp\{-\lambda\},\ \lambda > 0. \qquad (2)$$

Following Khan et al. [9], the shape parameter (α) has a uniform prior over the interval $(0,\ \alpha)$ which is given below:

$$p(\alpha) \propto \frac{1}{\alpha},\ \alpha > 0. \tag{3}$$

It is assumed that α and λ are independently distributed. Thus, the joint prior density of α and λ is

$$p(\alpha,\ \lambda) \propto \frac{\lambda \exp\{-\lambda\}}{\alpha},\ \alpha,\ \lambda > 0. \tag{4}$$

Considering the prior density in (4), the posterior density of α and λ is given by

$$\mathrm{p}(\alpha, \lambda \,|\, \mathbf{x}) = \Psi_0(\mathbf{x})\, \alpha^{n-1}\, \lambda^{2n+1} \exp\left\{-\sum_{i=1}^{n} (\lambda x_i)^2 - \lambda\right\} \left[\prod_{i=1}^{n} (x_i)\right] \left[\prod_{i=1}^{n} \left(1 - \exp\left\{-(\lambda x_i)^2\right\}\right)^{\alpha-1}\right],$$

where $\Psi_0(\mathbf{x})$ is a normalizing constant.

For the predictive density of *z* given a sample $\mathbf{x} = (x_1,\ .\ .\ .\ ,\ x_n)$ is given by

$$p(z \,|\, \mathbf{x}) = \int\int p(z \,|\, \alpha,\ \lambda)\ p(\alpha,\ \lambda \,|\, \mathbf{x})\ d\lambda\ d\alpha.$$

The above model may then be utilized to evaluate the predictive density for responses from the two-parameter generalized Rayleigh model.

2.1 Predictive density for a single future response

Let *z* be a single future response from the model specified by (1), where *z* is independent of the observed data. Then, the predictive density for a single future response (*z*) given $\mathbf{x} = (x_1,\ .\ .\ .\ ,\ x_n)$ is

$$p(z \,|\, \mathbf{x}) = \int_{\alpha=0}^{+\infty}\int_{\lambda=0}^{+\infty} p(z \,|\, \alpha,\ \lambda)\ p(\alpha,\ \lambda \,|\, \mathbf{x})\ d\lambda\ d\alpha,$$

where $p(z \,|\, \alpha,\ \lambda)$ may be defined from model (1), see Khan [26], Khan et al. [8, 17].

Thus, the predictive density for a single future response is given by

$$p(z\,|\,\mathbf{x}) = \begin{cases} \Psi_1(\mathbf{x}) \int_{\alpha=0}^{+\infty}\int_{\lambda=0}^{+\infty} \alpha^n\ \lambda^{2n+3}\ z\ \exp\left\{-(\lambda z)^2\right\}\ \left(1 - \exp\left\{-(\lambda z)^2\right\}\right)^{\alpha-1} \\ \times \exp\left\{-\sum_{i=1}^{n}(\lambda x_i)^2 - \lambda\right\} \left[\prod_{i=1}^{n}(x_i)\right] \left[\prod_{i=1}^{n}\left(1 - \exp\left\{-(\lambda x_i)^2\right\}\right)^{\alpha-1}\right] d\lambda\ \mathrm{d}\alpha, \\ \qquad \text{for } z > 0;\ \alpha, \lambda > 0, \\ 0 \qquad \text{elsewhere}, \end{cases} \quad (5)$$

where $\Psi_1(\mathbf{x})$ is a normalizing constant.

2.2 Predictive density for more than one future response

Statistical inference about more than one future response is important, as pointed out by Bain and Engelhardt [30], Khan et al. [9, 10]. Let z_1 and z_2 be two independent future responses from model (1). Then to derive the joint predictive model of z_1 and z_2, the posterior density of $p(\alpha,\ \lambda\,|\,\mathbf{x})$ obtained in Section 2 is utilized. Thus, the predictive density for a bivariate future response is given by

$$p(z_1,\ z_2\,|\,\mathbf{x}) = \begin{cases} \Psi_2(\mathbf{x}) \int_{\alpha=0}^{+\infty}\int_{\lambda=0}^{+\infty} \alpha^{n+1}\ \lambda^{2n+5}\ \prod_{i=1}^{2}(z_i)\ \exp\left\{-\sum_{i=1}^{2}(\lambda z_i)^2\right\} \\ \times \prod_{i=1}^{2}\left(1 - \exp\left\{-(\lambda z_i)^2\right\}\right)^{\alpha-1} \times\ \exp\left\{-\sum_{i=1}^{n}(\lambda x_i)^2 - \lambda\right\} \\ \times \left[\prod_{i=1}^{n}(x_i)\right] \left[\prod_{i=1}^{n}\left(1 - \exp\left\{-(\lambda x_i)^2\right\}\right)^{\alpha-1}\right] d\lambda\ \mathrm{d}\alpha, \\ \qquad \text{for } z_i > 0;\ \alpha, \lambda > 0, \\ 0 \qquad \text{elsewhere}, \end{cases} \quad (6)$$

where $\Psi_2(\mathbf{x})$ is a normalizing constant.

Similarly, let $z_1, \ . \ . \ . \ , z_m$ be the *m* ordered future responses from model (1). Thus,

$$p(z_1, \ . \ . \ . \ , z_m \mid \mathbf{x}) = \begin{cases} \Psi_m(\mathbf{x}) \int_{\alpha=0}^{+\infty} \int_{\lambda=0}^{+\infty} \alpha^{n+m-1} \ \lambda^{2n+2m+1} \prod_{i=1}^{m} (z_i) \ \exp\left\{-\sum_{i=1}^{m} (\lambda z_i)^2\right\} \\ \times \prod_{i=1}^{m} \left(1 - \exp\left\{-(\lambda z_i)^2\right\}\right)^{\alpha-1} \ \times \ \exp\left\{-\sum_{i=1}^{n} (\lambda x_i)^2 - \lambda\right\} \\ \times \left[\prod_{i=1}^{n} (x_i)\right] \left[\prod_{i=1}^{n} \left(1 - \exp\left\{-(\lambda x_i)^2\right\}\right)^{\alpha-1}\right] d\lambda \ \mathrm{d}\alpha, \\ \qquad \text{for } z_i > 0; \ \alpha, \lambda > 0, \\ 0 \qquad \text{elsewhere}, \end{cases} \tag{7}$$

where $\Psi_m(\mathbf{x})$ is a normalizing constant. For $m = 1$, the above predictive density reduces to the predictive density for a single future response obtained in equation (5); when $m = 2$, the above predictive density reduces to the predictive density for a bivariate future response obtained in equation (6); and so on.

3. Illustrative example

In this section a real data set that was originally given in Nelson [31] is used. Al-khedhairi et al. [25] used the same data set to perform the goodness of fit tests and to identify the best fit model. The lifetimes of eight unequally spaced inspections are given as follows:

6.12, 19.92, 29.64, 35.40, 39.72, 45.24, 52.32, 63.48.

These data were used for goodness of fit tests for the exponential distribution (ED), generalized exponential distribution (GED), and generalized Rayleigh distribution (GRD). Al-khedhairi et al. [25] estimated the maximum likelihood estimators for the parameters by using the data set. They used model testing criteria; the log-likelihood function and the Kolmogorov-Smirnov (K-S) test statistics. They concluded that the GRD $\left(\hat{\alpha} = 0.684, \ \hat{\lambda} = 1.425 \times 10^{-4}\right)$ fits the data much better than ED $\left(\hat{\alpha} = 1.2097 \times 10^{-2}\right)$ and GED $\left(\hat{\alpha} = 2.0285 \times 10^{-2}, \ \hat{\lambda} = 1.7839\right)$. The results of the Kolmogorov-Smirnov (K-S) test statistics of the empirical model and the fitted model given the data set were: K-S (ED) = 0.214; K-S (GED) = 0.144; and K-S (GRD) = 0.105. The final conclusion was the data set follows the two-parameter generalized Rayleigh model.

The present study deals with the same data set by assuming that the eight items' lifetimes were generated from a life testing experiment. Al-khedhairi et al. [25] considered the lifetimes for eight unequally spaced inspections with the number of failures recorded in each time interval and estimated the maximum likelihood estimators for the parameters α and λ. The estimated values are substituted in the predictive model and an attempt is made to display the predictive density graphically. Unfortunately, the estimated values did not display the predictive density. It may be the case that the data point for each item's lifetime is considered to form a complete sample. An iterative technique is used to display the predictive density. In this case, we considered some initial values of the estimates (since $\alpha, \lambda > 0$), and those values are substituted into the predictive density (5) to display it graphically.

The numerical integration command 'NIntegrate' in conjunction with the symbolic computational software Mathematica version 7.0, Wolfram Research [32] is applied to determine the normalizing constants and to plot the predictive graphs. The predictive means, standard deviations, highest predictive density (HPD) intervals, and the measures of skewness and kurtosis are obtained. The Mathematica package is also utilized to carry out all related calculations.

An HPD interval is the interval which includes the most probable values of a given predictive density at a given significance level, subject to the condition that the density function has the same value at the end points. Suppose a_1 and a_2 are to be arbitrarily chosen values. The HPD interval [a_1, a_2] for z must simultaneously satisfy the following two conditions:

$$\Pr(a_1 \le z \le a_2) = 1 - \alpha \quad \text{and} \quad p(a_1 \mid \mathbf{x}) = p(a_2 \mid \mathbf{x}).$$

For more about HPD intervals, the reader is referred to Box and Tiao [33], Khan [26]. Thus, in light of the expression derived for $p(z \mid \mathbf{x})$, in equation (5), one can obtain the numerical solutions for a_1 and a_2. In the present study, by setting $p(a_1 \mid \mathbf{x}) = p(a_2 \mid \mathbf{x})$ and aiming for $\Pr(a_1 \le z \le a_2) = 0.95$, 95% HPD intervals from equation (5) are obtained.

Figure 1 shows the graphical representation of the predictive densities with respect to certain values of the estimated parameters. The predictive density with respect to the estimated parameters ($\hat{\alpha}= 1.40,\ \hat{\lambda}= 0.018$) gives higher variability than that of the predictive density with ($\hat{\alpha}= 1.50,\ \hat{\lambda}= 0.02$). The predictive densities with $(\hat{\alpha}= 1.20,\ \hat{\lambda}= 0.014)$ gives higher variability than that of $(\hat{\alpha}= 1.30,\ \hat{\lambda}= 0.016)$. Similarly, in the case of Figure 2, the predictive density with the estimated parameters $(\hat{\alpha}= 1.02,\ \hat{\lambda}= 0.001)$ has the highest variability than that of the predictive densities with $(\hat{\alpha}= 1.10,\ \hat{\lambda}= 0.012)$; $(\hat{\alpha}= 1.05;\ \hat{\lambda}= 0.010)$; and $(\hat{\alpha}= 1.03,\ \hat{\lambda}= 0.005),$ respectively.

It is observed that when the estimated values (α) goes less than one with smaller scale parameter (λ), the predictive density does not produce its graph and therefore, does not yield any inference. For the estimate of the shape parameter, $\alpha \geq 0.50$, one would have a right skewed unimodal density function as proposed by Raqab and Kundu [23], which agrees the predictive results.

The predictive means, standard deviations, HPD intervals with the combinations of certain values of the parameters which are given in Table 1. The measures of skewness and kurtosis and normalizing constants are given in Table 2. Based on the iterative values of $\beta_1,\ \alpha,$ and λ in Table 2, it may be commented that the predictive density for a single future response given a complete sample from the two-parameter generalized Rayleigh model is positively skewed.

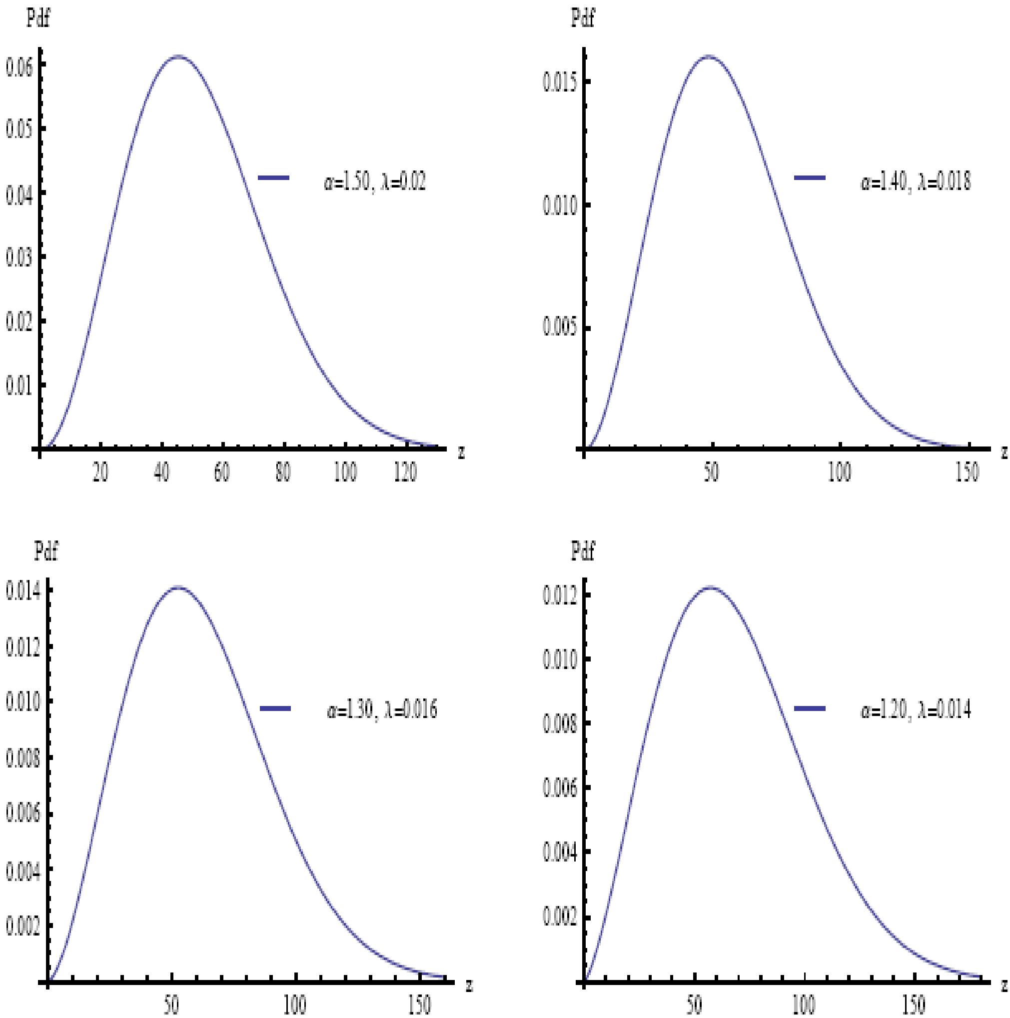

Figure 1. Comparison of variability of the predictive densities for a single future response with respect to certain iterative values of the parameters.

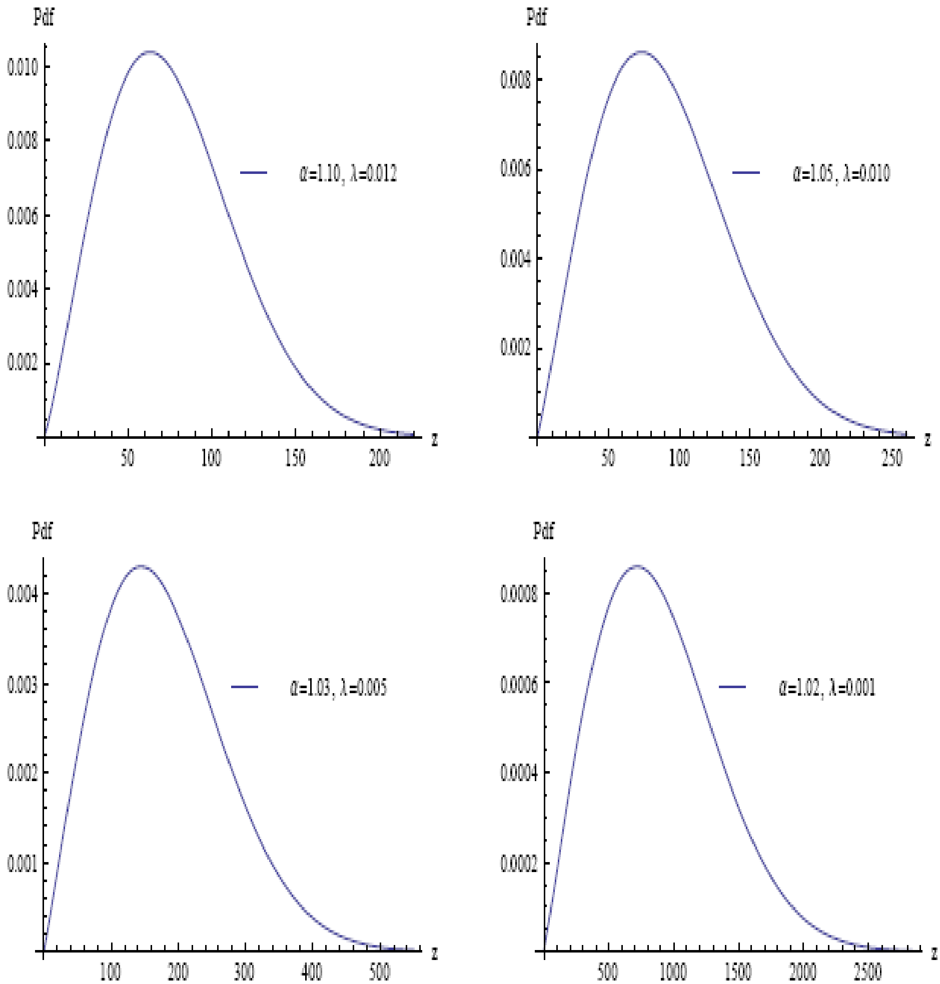

Figure 2. Comparison of variability of the predictive densities for a single future response with respect to certain iterative values of the parameters.

Table 1. Predictive mean, standard deviation, and HPD intervals for a single future response with some iterative values of the parameters given a complete sample.

Values of parameters		Means (z)	Standard Deviations (z)	95% HPD intervals
α	λ			
1.50	0.020	48.3471	22.5675	(7.8035, 92.4673)
1.40	0.018	56.2684	24.9361	(10.7331, 105.9830)
1.30	0.016	61.3983	27.9765	(11.0114, 115.5528)
1.20	0.014	67.9790	32.1577	(11.0331, 128.1190)
1.10	0.012	76.7034	38.0233	(11.0640, 143.9200)
1.05	0.010	90.2461	45.7288	(11.9380, 169.5650)
1.03	0.005	179.273	92.0130	(22.4144, 337.9660)
1.02	0.001	893.102	461.228	(278.943, 1914.3400)

Table 2. Predictive shape characteristics and normalizing constants for a single future response with some iterative values of the parameters given a complete sample.

Values of parameters		β_1	β_2	$\Psi_1(\mathbf{x})$
α	λ			
1.50	0.020	0.2572	2.9847	5.5153×10^{-18}
1.40	0.018	0.2816	3.1184	1.21396×10^{-17}
1.30	0.016	0.2473	2.9494	6.1851×10^{-18}
1.20	0.014	0.2591	2.9325	2.42265×10^{-18}
1.10	0.012	0.3113	3.0307	6.95083×10^{-19}
1.05	0.010	0.3191	3.0159	1.80454×10^{-19}
1.03	0.005	0.3514	3.1028	8.35731×10^{-22}
1.02	0.001	0.3667	3.1454	5.53648×10^{-28}

4. Conclusion

The predictive models for a single future response, a bivariate future response, and several future responses from the two-parameter generalized Rayleigh model by making use of the Bayesian method are derived. The normalizing constant for each of the predictive density is estimated to plot the predictive density accurately. The predictive means, standard deviations, and the highest predictive density intervals are obtained for the predictive density of a single future response. The measures of skewness and kurtosis of the predictive model are also given. Thus, one may infer that the predictive interval is narrower when the combinations of the iterative values, $\hat{\alpha} = 1.50$ and $\hat{\lambda} = 0.020$ and the predictive interval is wider with the combinations of the iterative values, $\hat{\alpha}$ = 1.40; 1.30; 1.20; 1.10; 1.05; 1.03, 1.02, and $\hat{\lambda}$ = 0.018; 0.016; 0.014; 0.012; 0.010; 0.005; 0.001, respectively. An advanced computational software package, ‘Mathematica version 7.0’, is used to show the graphical representation of the predictive density for a single future response and also to carry out all related calculations.

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