Thailand Statistician
July 2011; 9(2) : 129-141
http://statassoc.or.th
Contributed paper

Entropic Order Quantity (EnOQ) Model under Cash Discounts

Monalisha Pattnaik
Department of Business Administration, Utkal University, Bhubaneswar, 751004, India.
Email: monalisha1977@gmail.com

Received: 7 January 2011
Accepted: 14 May 2011

Abstract

A new type of replenishment policy is suggested in an entropic order quantity model for a perishable product with two component demand under cash discounts particularly over a finite time horizon. To handle this multiplicity of objectives in a pragmatic approach, entropic ordering quantity model with discounted selling price during pre and post deterioration of perishable items to optimize its payoff is proposed. Furthermore, numerical experiments are conducted to evaluate the difference between the crisp EnOQ and EOQ models separately. The proposed paper reveals itself as a pragmatic alternative to other approaches based on two component demand function with very sound theoretical underpinnings but with few possibilities of actually being put into practice. The results indicate that this can become a good model and can be replicated by researchers in neighbourhood of its possible extensions.

Keywords: cash discounts, deterioration, two component demand.

1. Introduction

This paper deals with an entropic order quantity model with cash discounts under pre and post deterioration of the perishable products. The predominant criterion in traditional inventory models is minimization of long run average cost per unit time. The costs considered are usually fixed and variable ordering cost and holding cost. Costs associated with disorder in a system tied up in inventory are accounted for by including an entropy cost in the total costs. Entropy is frequently defined as the amount of disorder in a system. Jaber, Bonney, Rosen and Moualek [1] proposed an analogy between the behaviour of production system and the behaviour of physical system. Pattnaik [2,3] introduced the use of entropy cost in the context of EOQ model. This paper suggested the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement process.

In deriving the economic order quantity model, the case of discounting on selling price is usually omitted. But in real world, it exists and is quite flexible in nature. On the other hand, in order to motivate customers to order more quantities, usually suppliers offer discount on selling prices both pre and post deterioration rate. Dave, Fitzapatrick and Baker [4] developed an inventory model under continuous discount pricing. Khouja [5] studied an inventory problem under the condition that multiple discounts can be used to sell excess inventory. Shah and Shah [6] mentioned that discount is considered temporarily for exponentially decaying inventory model. However, most of the studies except few, do not attempt to unify the two research streams: temporary price reductions and deterioration. This paper represents the issue in details.

Recently a number of research articles appeared which deal with the EOQ problem under constant demand. In last two decades the variability of inventory level dependent demand rate on the analysis of inventory system was described by researchers like Pal, Goswami and Choudhury [7] and Goswami and Choudhury [8]. They described the demand rate as the power function of on hand inventory. There is a vast literature on stock development inventory and its outline can be found in the review article by Urban [9] where he unified two types of inventory level dependent demand by considering a periodic review model. Researchers such as Chung, Tsu and Liang [10], Goyal and Giri [11], Raafat [12] and Skouri, Konstantaras, Papachristos and Ganas [13] discussed the EOQ model assuming time value of money, demand rate, deterioration rate, shortages and so on a constant or probabilistic number or an exponential function. This paper considers two component demand function which is initially depending on inventory up to deterioration and then becoming a constant function till stock is zero.

Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles are available in inventory literature considering deterioration. Interested readers may consult the survey paper of Weatherford and Bodily [14] and Panda, Saha and Basu [15].

The purpose of this paper is to investigate the effect of the approximation made by using the average payoff when determining the optimal values of the policy variables. In this paper it is focused exclusively on the cost of entropy. A policy iteration algorithm is designed with the help of Deb [16] and optimum solution is obtained through LINGO software. In order to make the comparisons equitable a particular evaluation function based on discount is suggested. Numerical experiments are carried out to analyse the magnitude of the approximation error. The remainder of this paper is organised as follows. In section 2 assumptions and notations are provided for the development of the model. Section 3 describes the model formulation. In section 4, an illustrative numerical experiment is given to illustrate the procedure of solving the model. Section 5 evaluates the sensitivity analysis to develop the critical discussion of the CEnOQ and CEOQ models separately. Finally section 6 summarizes the concluding remarks and provides some suggestions for future research.

Table 1. Major Characteristics of Inventory Models on Selected Researches.

Author(s) and published Year	Structure of the Model	Deterioration	Inventory Model Based on	Discount allowed	Demand	Back-logging allowed
Panda et al. (2009)	Crisp	Yes (Heaviside)	EOQ	Yes	Stock dependent	No
Jaber et al. (2008)	Crisp	Yes (on hand inventory)	EnOQ	No	Unit selling price	No
Chung et al. (2007)	Crisp	Yes (exponential)	EOQ	No	Selling price	Yes (partial)
Skouri et al. (2007)	Crisp	Yes (Weibull)	EOQ	No	Ramp	Yes (partial)
Pattnaik (2010)	Crisp	Yes (constant)	EnOQ	Yes (Instant deterioration)	Constant	No
Pattnaik (2011)	Crisp	Yes (Heaviside)	EnOQ	Yes (Post deterioration)	Stock dependent	No
Present paper (2011)	Crisp	Yes (Heaviside)	EnOQ	Yes (pre & post deterioration)	Stock dependent	No

2. Notations and Assumptions

Notations

C_0 : set up cost

c : per unit purchase cost of the product

s : constant selling price of the product per unit *(s>c)*

h : holding cost per unit per unit time

r_1 : discount offer per unit before deterioration.

r_2 : discount offer per unit after deterioration.

α_1 : the effect of pre-deterioration discount on demand.

α_2 : the effect of post deterioration discount on demand.

R : demand function.

S : entropy generation rate.

$(EC)_{WD}$: entropy cost with deterioration in the cycle.

$(EC)_{WOD}$:entropy cost without deterioration in the cycle.

Q_1 : order level for pre- and post deterioration discount on selling price.

T_1 : cycle length of the inventory model.

Assumptions

1. Replenishment rate is infinite.
2. The deterioration rate $\tilde{\theta} = \theta H(t-\tau), (0 < \theta \leq 1)$ constant

   Where *t* is the time measured from the instant arrivals of a fresh replenishment indicating that the deterioration of the items begins after a time τ from the instant of the arrival in stock.

   $H(t-\tau)$ is the well known Heaviside's function. $H(t-\tau) = \begin{cases} 1, t \geq \tau \\ 0, \text{otherwise} \end{cases}$
3. Demand depends on the on hand inventory up to time τ from time of fresh replenishment beyond which it is constant and defined as follows.

   $$R(I(t)) = \begin{cases} a + bI(t), & t < \tau \\ a, t \geq \tau \end{cases}$$

   Where *a>0* is the initial demand rate independent of stock level and condition of inventory and *b>0* is the stock sensitive demand parameter *I (t)* is the instantaneous inventory level at time *t*.
4 $r_1, (0 \leq r_1 \leq 1)$ is the percentage pre- deterioration discount offer on unit selling price. $\alpha_1 = (1-r_1)^{-n_1}$, (n_1, the set of real numbers), is the effect of pre-

deterioration discount on demand. r_2, $(0 \le r_2 \le 1)$ is the percentage discount offer on unit selling price during deterioration. $\alpha_2 = (1-r_2)^{-n_2}$ (n_2, the set of real numbers), is the effect of discounted selling price on demand during deterioration. α_2 is determined from priori knowledge of the seller such that the demand rate is influenced with the reduction rate of selling price.

5. Entropy generation rate must satisfy $S = \frac{d\sigma(t)}{dt}$ where, *σ(t)* is the total entropy generated by time *t* and *S* is the rate at which entropy is generated. The entropy cost is computed by dividing the total commodity flow in a cycle of duration T_i. The total entropy generated over time T_i as $\sigma(T_i) = \int_0^{T_i} S dt$,

$$S = \frac{R(I(t))}{s}$$

Entropy cost per cycle is

$$EC\ (T_i) = (EC)_{WOD} + (EC)_{WD} = \frac{D(\tau)}{\sigma(\tau)} + \frac{Q_{i\ WD}}{\sigma\ (T_i)}\ (i=1)$$

EC is measured in an appropriate price unit with no deterioration and with deterioration respectively.

3. Mathematical Model

At the beginning of the replenishment cycle the inventory level raises to Q_1. As time progresses it decreases due to instantaneous stock dependent demand up to the time *τ*. After τ deterioration starts and the inventory level decreases for deterioration and constant demand. Ultimately inventory reaches zero level at T_1. It is assumed that before the start of deterioration from time t_1, r_1 % discount on unit selling price of the product is given in order to boost the demand of fresh items. Clearly, this discount is continued for the period of time $(\tau - t_1)$. As deterioration starts from τ, r_2% discount on unit selling price is provided to enhance the demand of decreased quality items. This discount is continued for the rest of the replenishment cycle. Then the behaviour of inventory level is governed by the following system of linear differential equations and it is depicted in Figure 1.

$$d\ (I\ (t))\ /\ dt = -a - b\ I\ (t) \qquad 0 \le t \le t_1 \qquad (1)$$

$$= -\alpha_1 (a + b I(t)) \qquad t_1 \le t \le \tau \qquad (2)$$

$$= -\left[\alpha_2 a + \theta\, I(t)\right] \qquad \tau \le t \le T_1 \qquad (3)$$

With the initial boundary condition

$I(0) = Q_1,\ 0 \le t \le t_1$

$I(T_1) = 0,\ \tau \le t \le T_1$

Solving the equations,

$$I(t) = \frac{a}{b}\left[e^{-bt} - 1\right] + Q_1 e^{-bt}, 0 \le t \le t_1 \qquad (4)$$

$$I(t) = \frac{a}{b}\left[e^{-\alpha_1 b\,(t-t_1)-bt_1} - 1\right] + Q_1\, e^{\alpha_1 b\,(t_1-t)-bt_1} t_1 \le t \le \tau\,, \qquad (5)$$

$$I(t) = \frac{a\alpha_2}{\theta}\left[e^{\theta(T_1-t)} - 1\right], \quad \tau \le t \le T_1 \qquad (6)$$

Now, at the point t = τ we have from equations (5) and (6)

$$Q_1 = \left[\frac{a\alpha_2}{\theta}\left(e^{\theta(T_1-\tau)} - 1\right) + \frac{a}{b}\right] \times e^{\alpha_1 b\,(\tau-t_1)+b\,t_1} - \frac{a}{b} \qquad (7)$$

By using equations (4), (5) and (6) for different I (t) in the respective intervals the holding cost of inventories in the cycle is,

$$HC = h\int_{o}^{t_1} I\,(t)dt + h\int_{t_1}^{\tau} I(t)\,dt + h\int_{\tau}^{T_1} I\,(t)\,dt$$

$$= h\int_0^{t_1}\left[\frac{a}{b}\left[e^{-bt} - 1\right] + Q_1 e^{-bt}\right]dt +$$

$$h\int_{t_1}^{\tau}\left[\frac{a}{b}\left[e^{-\alpha_1 b\,(t-t_1)-bt_1} - 1\right] + Q_1\, e^{\alpha_1 b\,(t_1-t)-bt_1}\right]dt +$$

$$h\int_{\tau}^{T_1}\left[\frac{a\alpha_2}{\theta}\left[e^{\theta(T_1-t)} - 1\right]\right]dt,$$

Purchase cost in the cycle is given by $PC = cQ_1$.

Entropy cost in the cycle is $EC = (EC)_{WOD} + (EC)_{WD}$

$$= \frac{D(\tau)}{\sigma(\tau)} + \frac{Q_{i\ WD}}{\sigma\,(T_1)}$$

Where $D(\tau) = \int_0^{t_1} R(I(t))dt + \int_{t_1}^{\tau} R(I\,(t))dt$

$Q_{i\ WD} = Q_1$

$$\sigma(\tau) = \int_0^{\tau} S dt = \int_0^{t_1} \frac{R(I(t))}{s} dt + \int_{t_1}^{\tau} \frac{R(I(t))}{s} dt = \frac{1}{s} D(\tau)$$

$$\sigma(T_1) = \int_{\tau}^{T_1} \frac{R(I(t))}{s} dt = \int_{\tau}^{T_1} \frac{a}{s} dt = \frac{a}{s}(T_1 - \tau)$$

$$EC = s + \frac{sQ_1}{a(T_1 - \tau)}$$

Total sales revenue in the order cycle can be found as

$$SR = s\left[\int_0^{t_1} (a + bI(t))dt + \alpha_1(1 - r_1)\int_{t_1}^{\tau} [a + bI(t)]dt + \alpha_2(1 - r_2)\int_{\tau}^{T_1} a\ dt\right]$$

Thus total profit per unit time of the system is $\pi_1(r_1, r_2, t_1, T_1) = \frac{TP_1}{T_1}$

$$= \frac{1}{T_1}[SR - PC - HC - EC - OC]$$

On integration and simplification of the relevant costs, the total profit per unit time becomes

$$\pi_1 = \frac{1}{T_1}\Bigg[sat_1 + s\alpha_2(1 - r_2)a(T_1 - \tau) + (sb - h)\left(-\frac{at_1}{b} + \left(Q_1 + \frac{a}{b}\right)\frac{1 - e^{-bt_1}}{b}\right)$$
$$+ [s\alpha_1 b(1 - r_1) - h]\left[\frac{a(t_1 - \tau)}{b} + \left(\frac{a}{b} + Q_1\right)e^{-bt_1}\left(\frac{1 - e^{\alpha_1 b(t_1 - \tau)}}{b\alpha_1}\right)\right] + s\alpha_1(1 - r_1)a(\tau - t_1)$$
$$- h\frac{a\alpha_2}{\theta}\left[\frac{e^{\theta(T_1 - \tau)} - 1}{\theta} - (T_1 - \tau)\right] - s - \frac{sQ_1}{a(T_1 - \tau)} - cQ_1 - C_0\Bigg] \quad (8)$$

The pre deterioration discount on selling price is to be given in such a way that the discounted selling price is not less that the unit cost of the product, i.e. $s(1 - r_1) - c > 0$ similarly, $s(1 - r_2) - c > 0$ .Applying these constraints on unit total profit function, the maximization problem is

Maximize $\pi_1(r_1, r_2, t_1, T_1)$

Subject to $\{r_1, r_2\} < 1 - \frac{c}{s} \quad \forall\ r_1, r_2, t_1, T_1 \geq 0.$ (9)

The objective here is to determine the optimal values of r_1, r_2, t_1 and T_1 to maximize the unit profit function. It is very difficult to derive the results analytically. Thus some numerical methods must be applied to derive the optimal values of r_1, r_2, t_1 and

T_1, hence the unit profit function. There are several methods to cope with constraint optimization problem numerically. But here LINGO software is applied to derive the optimal values of the decision variables.

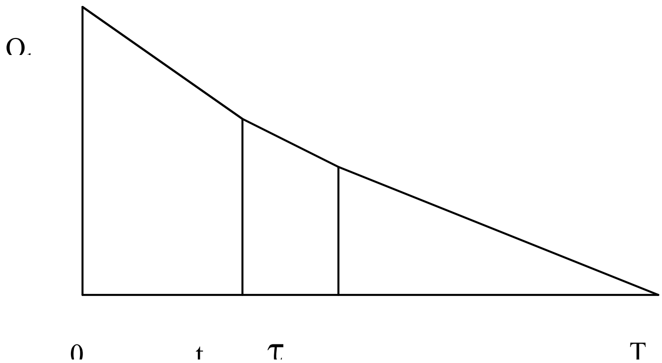

Figure 1. Inventory Model of both Pre and Post Deterioration Discounts.

4. Numerical Example

Let the parameter values are a=80, b=0.3, h=0.6, s=10.0, C_0=100.0, c=4.0, θ=0.03, τ =1.2, n_1=2.0 and n_2=2.0. With the help of LINGO software the optimum values of the decision variables are listed in Table 2.

For the entropic order quality model with both pre and post deterioration discounts, the pre-deterioration discount on unit selling price is 37.09% and discount starts at time 0.1770513 and continued to τ =1.2, then the product starts to deteriorate. During this time in order to enhance inventory depletion rate, 50.09% discount is provided for remaining time of replenishment cycle. Hence the profit per unit time is 692.7293 with the optimal order quantity, 1595.249 and the cycle length, 2.858826 respectively. The unit profit and order quantity for the case of crisp EOQ model are 749.6949 and 1652.540 less by 8.2233565% and 3.5913516% respectively from CEnOQ model. The cycle length is 2.418452 and 15.40462% more than that for CEnOQ model with both types of discounts.

The relative difference in Table 2 is marked larger than the ones found in the previous experiments when the ordering policy is optimal. This indicates that there exists case in which the relative difference is non-negligible.

Table 2. Optimal Values of the Decision Variables, Order Quantity and Profit per unit time of CEnOQ and CEOQ Models.

Nature of Discount	Decision variables	CEnOQ	CEOQ	% change
With pre and post deterioration discounts	r_1	0.3709945	0.3898235	-5.0752774
	r_2	0.5009602	0.5668512	-13.152911
	t_1	0.1770513	0.1709742	3.432395
	T_1	2.858826	2.418452	15.40402
	Q_1	1595.249	1652.540	-3.5913516
	EC	130.2092	-	-
	π_1	692.7293	749.6949	-8.2233565

Comparative Evaluation

From the Table 3 for pre and post deterioration discounted CEnOQ model 37.09**%** and 50.09% discounts are provided on unit selling price to earn more profit than that with other models. It also indicates the entropy cost is provided on the model to earn 20.84% more profits for the present CEnOQ model. This paper investigates a computing schema for the EOQ in introducing the entropy cost. The results are approximate i.e. it permits better use of CEnOQ with pre and post deterioration discounts as compare to CEOQ arising with only post deterioration discount. It indicates the consistency of the both aforesaid models. Though higher amount of percentage discounts on unit selling price in the form of pre and post deterioration discounts for larger periods of time results in lower unit sales revenue, still it is more profitable. Because the inventory depletion rate is much higher for discount enhanced demand resulting lower amount of inventory holding cost and deteriorated items. Therefore it is always profitable to apply both forms of discounts on unit selling price to earn more profit.

Table 3. Relative Error of Pre and Post Deterioration Discounted CEnOQ Model with other Models.

Model	r_1	r_2	t_1	T_1	Q_1	EC	π_1
CEnOQ	0.37099	0.50096	0.1770513	2.858826	1595.249	130.2092	692.7293
CEnOQ (Pattnaik, 2010)	-	0.04635	-	1.917616	173.6135	21.3170	362.0803
% Change	-	980.716	-	49.08230	818.8508	510.823	91.31925
CEOQ (Panda et al., 2009)	0.38982	-	0.1709739	1.20	376.47	573.2467	-
% Change	-4.83013	-	3.5545776	138.2355	323.73868	-	20.84314

Critical Discussions

In Table 4 some sensitivity analysis of the model with both types of discounts is performed by changing the parameter values +20% and +40%, taking one parameter at a time and keeping the remaining parameter values unchanged. It is found from Table 4 that the model is highly sensitive for the error in the estimation of the parameter value ‘a’ for both the models. It is moderately sensitive for the change in the parameter value ‘C_0’ for both the models. Low sensitivity it found for change in parameter value ‘c’.

Sensitivity analysis reveals some common characteristics, e.g., profit and EnOQ increase as unit purchase cost decreases. From Table 4 it is found that, the decrement of unit purchase cost and initial demand result the increment of the cycle length but the pre deterioration discount starting time is remaining same with τ . Similarly decrement of the ordering cost results little change in both the discounts and it is very approximate to all the results for CEnOQ model. But for the CEnOQ model the pre deterioration discount and pre deterioration discount starting time are insensitive to both ‘a’ and ‘C_0’. Conversely, if the unit cost is low than by providing more discounts and more time pre deterioration discount the inventory depletion rate can be increased, resulting lower amount of holding cost and deterioration cost, hence higher unit profit.

Table 4. Sensitivity Analysis of Pre and Post Deterioration Discounted CEnOQ and CEOQ Models.

Model	CEnOQ					CEOQ			
Parameter	C	a		C_0		A		C_0	
% Change	+40	+20	+40	+20	+40	+20	+40	+20	+40
r_1	-	-	-	0.3694	0.3698	0.3898	0.3898	0.3898	0.3898
% Change	-	-	-	0.4271	0.3138	0	0	0	0
r_2	0.84	0.2623	0.3502	0.5068	0.5054	-	-	-	0.5738
% Change	-40.362	90.9551	43.051	-1.153	-0.878	-	-	-	-1.228
t_1	1.2	1.2	1.2	0.1778	0.1778	0.1710	0.1710	0.1710	0.1710
% Change	-85.25	-85.25	-85.25	-0.375	-0.276	0.0002	0.0002	0.0003	0.0005
T_1	3.533	4.4968	3.488	2.7286	2.7610	1.2	1.2	1.2	2.3130
% Change	-19.08	-36.425	-18.04	4.7718	3.5426	101.54	101.54	101.54	4.5594
Q_1	10937.5	169.136	303.44	1514.8	1535.4	75.294	150.59	376.47	1579.1
% Change	-85.415	843.174	425.72	5.3141	3.8998	2094.78	997.39	338.96	4.6537
π_1	4401.307	83.1690	203.19	721.37	714.09	47.9827	179.30	639.91	775.06
% Change	-84.2608	732.918	240.93	-3.971	-2.991	146.243	318.13	17.155	-3.273

5. Conclusion

This paper provides an entropic order quantity model for perishable items with two component demand in which the criterion is to optimise the expected total discounted finite horizon payoff. To compute the optimal values of the policy parameters a simple and quite efficient policy model was designed. Finally, in numerical experiments the solution from the discounted model evaluated and compared to the solutions of EOQ policies.

However, it is observed that the performance differences among a set of different inventory policies in the existing literature. Although there are minor variations that do not appear significant in practical terms, at least when solving the single level, incapacitated version of the lot sizing problem. From the analysis it is demonstrated that the retailer’s profit is highly influenced by offering pre and post discount on selling price. The results of this study give managerial insights to decision maker developing an optimal replenishment decision for deteriorating product. Compensation mechanism should also be included to induce collaboration between retailer and dealer in a meaningful supply chain.

In general, for normal parameter values the relative payoff differences seem to be fairly small. Conventional wisdom suggests that workflow collaboration in an entropic model in a varying deteriorating product in market place are promising mechanism and achieving a cost effective replenishment policy.

The approach proposed in the paper based on EnOQ model seems to be a pragmatic way to approximate the optimum payoff of the unknown group of parameters in inventory management problems. The assumptions underlying the approach are not strong and the information obtained seems worthwhile. Its use may restrict the model's applicability in the real world. Incorporation of cash discounts structure and entropy cost into the analysis can be really challenging but they are more realistic and therefore can justifiably be used in actual situations. Given the inherent complexities of the real world, simple models lend themselves to the search for solutions, which are relatively robust over a wide range of situations rather than optimal for a narrow set of circumstances. So, this paper sets the stage to incorporate entropy cost into the analysis. The proposed model can be extended in several ways. For instance, it may be generalized the model to allow for shortages, partial backlogging, quantity discounts, time value of money and others.

References

[1] Jaber, M.Y., Bonney, M., Rosen, M.A., and Moualek, I., Entropic order quantity (EnOQ) model for deteriorating items. *Applied Mathematical modeling,* 2009; 33: 564-578.

[2] Pattnaik, M., An entropic order quantity model (EnOQ) under instant deterioration of perishable itpems with price discounts. *International Mathematical Forum*, 2010; 5: 49-52.

[3] Pattnaik, M., An entropic order quantity model with post deterioration cash discounts. *International Journal of Contemporary Mathematical Sciences,* 2011; 6: 931-939.

[4] Dave, D.S., Fitzapatrick, K.E., and Baker, J.R., An advertising inclusive production lot size model under continuous discount pricing. *Computational Industrial Engineering,* 1995; 30: 147-159.

[5] Khouja, M., Optimal ordering, discounting and pricing in the single period problem. *International Journal of Production Economics,* 2000; 65: 201-216.

[6] Shah, N.H., and Shah, Y.K., An EOQ model for exponentially decaying inventory under temporary price discounts. *Cahiers du CERO,* 1993; 35: 227-232.

[7] Pal, S., Goswami, K., and Chaudhuri, K.S., A deterministic inventory model for deteriorating items with stock dependent demand rate. *Journal of Production Economics,* 1993; 32: 291-299.

[8] Goswami, A., and Choudhury, K.S., An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. *Journal of Operational Research Society,* 1995; 46: 771-782.

[9] Urban, T.L., Inventory model with inventory level dependent demand a comprehensive review and unifying theory. *European Journal of Operational Research,* 2005; 162: 792-804.

[10] Chung, Y.D., Tsu, P. H., and Liang, Y., Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. *European Journal of Operational Research,* 2007; 181: 668-678.

[11] Goyal, S.K., and Giri, B.C., Recent trends in modelling of deteriorating inventory. *European Journal of Operational Research,* 2001; 134: 1-16.

[12] Raafat, E., Survey of literature on continuously deteriorating inventory model. *Journal of Operational Research Society,* 1991; 42: 27-37.

[13] Skouri, K., Konstantaras, I., Papachristos, S., and Ganas, I., Inventory models with ramp type demand rate, partial backlogging and weibull deterioration rate. *European journal of Operational Research,* 2009; 1: 79-92.

[14] Weatherford, L.R., and Bodily, S.E., A taxonomy and research overview of perishable asset revenue management: yield management, overbooking, and pricing. *Operations Research,* 1992; 40: 831-844.

[15] Panda, S., Saha, S., and Basu, M., An EOQ model for perishable products with discounted selling price and stock dependent demand. *CEJOR,* 2009; 17: 31-53.

[16] Deb, K., *Optimization for Engineering Design*. Prentice-Hall of India: New Delhi, 2000.