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An Improved Estimator for a Gaussian AR(1) Process with an Unknown Drift and Additive Outliers

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Abstract

This paper presents a new estimator for a Gaussian AR(1) process with an unknown drift and additive outliers. We apply the improved recursive median adjustment to the weighted symmetric estimator of Park and Fuller [1]. We consider the following estimators: the weighted symmetric estimator ($\hat{\rho}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_{R-W}$) proposed by Niwitpong [2], the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RMD-W}$) proposed by Panichkitkosolkul [3] and the improved recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{IRMD-W}$). Using Monte Carlo simulations, we compare the mean square error (MSE) of estimators. Simulation results have shown that the proposed estimator, $\hat{\rho}_{IRMD-W}$, provides a MSE lower than those of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$ and $\hat{\rho}_{RMD-W}$ for almost all situations.

Keywords: additive outliers, AR(1) model, parameter estimation, recursive median.

1. Introduction

Time series observations are sometimes influenced by interrupting phenomena, such as strikes, outbreaks of war, sudden political or economic crises, unexpected hot or cold waves, and even unnoticed errors of typing or recording. Such values are usually referred to as outliers. Because outliers are known to wreck havoc on the parameter estimation, it is therefore important to have procedures that will deal such outlier effects. Outliers in a time series were first studied by Fox [4], who introduced two statistical models for times series contaminated by outliers, namely, additive outliers (AO) and innovations outliers (IO). Additive outlier corresponds to the situation in which a gross error of observation or recording error affects a single observation [4]. An innovations outlier affects not only the particular observation but also subsequent observations [4]. A time series that does not contain any outliers is called an outlier-free series.

Suppose an outlier-free time series $\{X_t\,; t=2,3,...,n\}$ follows an AR(1) model:

$$X_t = \mu + \rho(X_{t-1} - \mu) + e_t \tag{1}$$

where μ is the population mean, ρ is an autoregressive parameter, $\rho \in (-1,1)$, e_t are unobservable independent errors and identically $N(0,\sigma_e^2)$ distributed. For $|\rho| = 1$, the model (1) is called the random walk model, otherwise it is called a stationary AR(1) process when $|\rho| < 1$. For ρ close to one or near a non-stationary process, the mean and variance of this model change over time. Let the observed time series be denoted by $\{Y_t\}$. In the simple case when $\{X_t\}$ has a single additive outlier at time point T $(1 < T < n)$, model (1) can be modified as to

$$Y_t = X_t + \delta I_t^{(T)} \tag{2}$$

where δ represents the magnitude of the additive outlier effect and $I_t^{(T)}$ is an indicator variable such that

$$I_t^{(T)} = \begin{cases} 1\ , & t = T \\ 0\ , & t \neq T. \end{cases}$$

It is known that the ordinary least squares estimator of ρ, which is denoted by $\hat{\rho}_{OLS}$, for (1) is biased (see e.g., Shaman and Stine [5]). Therefore, statisticians have suggested methods to reduce the bias. Park and Fuller [1] proposed the weighted symmetric estimator of ρ, which is denoted by $\hat{\rho}_W$. So and Shin [6] applied a recursive mean adjustment to the OLS estimator (abbreviated, R-OLS) and they concluded that the mean square error of the R-OLS estimator, which is denoted by $\hat{\rho}_{R-OLS}$, is smaller than the OLS estimator for $\rho \in (0,1)$. They also showed that the $\hat{\rho}_{R-OLS}$ estimator has a coverage probability which is close to the nominal value. Niwitpong [2] applied the recursive mean adjustment to the weighted symmetric estimator of Park and Fuller [1] (abbreviated, R-W). Panichkitkosolkul [3] proposed an estimator for an unknown mean

Gaussian AR(1) process with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (abbreviated, RMD-W). He found that the $\hat{\rho}_{RMD-W}$ estimator provides mean square error lower than those of $\hat{\rho}_W$ and $\hat{\rho}_{R-W}$ for almost all situations. We, therefore, apply the improved recursive median adjustment to the weighted symmetric estimator (abbreviated, IRMD-W) for model (1) when there are additive outliers in time series data. Because the outliers do not affect the median values, we replace the recursive mean adjustment with the improved recursive median adjustment to the weighted symmetric estimator. Our aim in this paper is to compare four estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$, in terms of mean square error (MSE) of estimators.

The remainder of this paper is organized as follows. Section 2 describes the $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ in detail. Simulation results obtained from Monte Carlo simulation are shown in Section 3. In Section 4, all the estimators are illustrated and compared through macro-economic real example. A discussion of the results and conclusions are presented in Section 5.

2. Detailed Description of the Estimators

Park and Fuller [1] proposed the weighted symmetric estimator of ρ given by

$$\hat{\rho}_W = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y})^2 + n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y})^2}. \qquad (3)$$

Niwitpong [2] replaces $\bar{Y}$ by $\bar{Y}_t = \frac{1}{t}\sum_{i=1}^{t} Y_i$ in (3). The estimator of ρ obtained as a result of this recursive mean adjustment is

$$\hat{\rho}_{R-W} = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y}_t)(Y_{t-1}-\bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y}_t)^2}. \qquad (4)$$

When there are outliers in time series data, it affects the recursive mean $\bar{Y}_t$ in (4). Panichkitkosolkul [3] replaced the recursive mean in (4) by the recursive median. The estimator of ρ obtained as a result of the recursive median adjustment is

$$\hat{\rho}_{RMD-W} = \frac{\sum_{t=2}^{n}(Y_t - \tilde{Y}_t)(Y_{t-1} - \tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{Y}_t)^2} \qquad (5)$$

where $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$.

We can also reduce the effect of outliers on an estimator of ρ in model (1) by using the improved recursive median adjustment. The improved recursive median values are derived from computing the double recursive median. There are two steps for computing the improved recursive median. First, we compute the recursive median ($\tilde{Y}_t$) by using time series data Y_t . Second, we calculate the recursive median again by using the recursive median obtained from the first step. Therefore, the recursive median in (5) is replaced by the improved recursive median. The proposed estimator of ρ obtained as a result of this improved recursive median adjustment is given by

$$\hat{\rho}_{IRMD-W} = \frac{\sum_{t=2}^{n}(Y_t - \ddot{Y}_t)(Y_{t-1} - \ddot{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \ddot{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \ddot{Y}_t)^2} \qquad (6)$$

where $\ddot{Y}_t = median(\tilde{Y}_1, \tilde{Y}_2, ..., \tilde{Y}_t)$ and $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$. An R function to compute the estimator in (6) is given in the Appendix.

In the next section, we present the Monte Carlo simulation results from estimating the mean square error (MSE) of these estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{RMD-W}$.

3. Simulation Results

In this section we examine−via Monte Carlo simulations−the performance of the proposed estimator for a Gaussian AR(1) process with unknown drift and additive outliers, with particular emphasis on comparisons between the new and existing approaches. Data are generated from an AR(1) process[1] with an unknown drift and additive outliers. The following parameter values were used; (μ, σ_e^2) = (0, 1); ρ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9; sample sizes n = 25, 50, 100 and 250; the magnitude of the AOs effect $\delta = 3\sigma_e$ and $5\sigma_e$; the percentage of additive outliers $p = 5\%$ and 10% .

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[1] We generate $Y_1 \sim N(0, \frac{\sigma_e^2}{1-\rho^2})$ and simulate the time series of length $n+50$ but the time series used in calculations are $\{Y_{51}, Y_{52}, ..., Y_{n+50}\}$.

All simulations were performed using programs written in the R statistical software [7, 8] with the number of simulation runs, M = 10,000. In addition, the additive outliers occurred randomly. Tables 1-2 and Figures 1-4 show the estimated MSEs of all estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$. As can be seen from Tables 1-2 and Figures 1-4, the MSE of $\hat{\rho}_W$ is larger than the MSEs of the other estimators especially when ρ is close to one and for small sample sizes. These values decrease as sample sizes get larger. $\hat{\rho}_W$ performs well for $n \geq 50$. On the other hand, the new estimator, $\hat{\rho}_{IRMD-W}$, provides the lowest MSE in all scenarios except when ρ = 0.1 and small sample sizes (n = 25 and 50). Additionally, the $\hat{\rho}_{IRMD-W}$ performs very well with respect to the other three estimators. The proposed estimator, $\hat{\rho}_{IRMD-W}$ in (6), dominates all estimators since the MSE of the proposed estimator is the lowest for almost all cases. For the rest, the MSE of $\hat{\rho}_{RMD-W}$ is less than that of $\hat{\rho}_{R-W}$ and $\hat{\rho}_W$ for almost all situations. The $\hat{\rho}_{RMD-W}$ often ranks the second best following the proposed estimator. Furthermore, the MSEs showed in Table 1 are less than those reported in Table 2 because time series data of Table 1 have less outliers.

4. An Example

In this section we applied a proposed estimator to a macro-economic time series. A real data set is the yearly real exchange rates between the USA and Sudan from ERS International Macroeconomic Data Set [9]. This series comes from 1970 to 2008 giving a total of 39 observations. The time series plot, the ACF and the PACF, as shown in Figures 5-6, suggest that an AR(1) model is suitable. We detect the additive outliers of this series by using an iterative detecting procedure proposed by Chang et al. [10] via the R statistical software (see i.e., Cryer and Chan [11], p.257-259 and p.455). We found that the time indices of potential AO are $t = 22$ and 23 (year 1991 and 1992) and we also construct all estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ and the standard errors of these estimators. The standard errors formulae are also shown in Figure 7.

As presented in Table 3, the proposed estimator, $\hat{\rho}_{IRMD-W}$, provides the standard error of the estimator less than those of the $\hat{\rho}_W$, $\hat{\rho}_{R-W}$ and $\hat{\rho}_{RMD-W}$ about 11.6%, 11.1% and 10.0%, respectively. The real example in this section confirms that the proposed estimator $\hat{\rho}_{IRMD-W}$ is much better than the other estimators.

Table1. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 5\%$.

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	R-W	RMD-W	IRMD-W	W	R-W	RMD-W	IRMD-W
25	0.1	0.0421	0.0374	0.0353	0.0423	0.0390	0.0343	0.0313	0.0329
	0.2	0.0499	0.0424	0.0400	0.0403	0.0545	0.0461	0.0423	0.0363
	0.3	0.0607	0.0509	0.0474	0.0408	0.0769	0.0650	0.0613	0.0463
	0.4	0.0716	0.0599	0.0570	0.0441	0.1036	0.0886	0.0855	0.0610
	0.5	0.0793	0.0664	0.0641	0.0444	0.1324	0.1141	0.1112	0.0761
	0.6	0.0934	0.0788	0.0771	0.0505	0.1623	0.1420	0.1391	0.0939
	0.7	0.1029	0.0882	0.0874	0.0549	0.1914	0.1688	0.1669	0.1115
	0.8	0.1121	0.0975	0.0979	0.0602	0.2101	0.1873	0.1872	0.1224
	0.9	0.1174	0.1045	0.1054	0.0634	0.2207	0.1984	0.1993	0.1296
50	0.1	0.0215	0.0200	0.0193	0.0235	0.0229	0.0210	0.0193	0.0196
	0.2	0.0260	0.0233	0.0216	0.0215	0.0341	0.0306	0.0281	0.0232
	0.3	0.0308	0.0275	0.0257	0.0217	0.0491	0.0442	0.0410	0.0304
	0.4	0.0367	0.0325	0.0307	0.0228	0.0683	0.0619	0.0585	0.0412
	0.5	0.0439	0.0391	0.0370	0.0250	0.0878	0.0802	0.0773	0.0529
	0.6	0.0485	0.0435	0.0423	0.0266	0.1062	0.0979	0.0950	0.0640
	0.7	0.0514	0.0468	0.0457	0.0280	0.1180	0.1097	0.1074	0.0701
	0.8	0.0493	0.0454	0.0448	0.0260	0.1170	0.1090	0.1073	0.0683
	0.9	0.0436	0.0412	0.0406	0.0227	0.1063	0.1001	0.0987	0.0606
100	0.1	0.0114	0.0107	0.0101	0.0112	0.0133	0.0125	0.0111	0.0107
	0.2	0.0149	0.0138	0.0126	0.0113	0.0238	0.0221	0.0191	0.0150
	0.3	0.0199	0.0183	0.0168	0.0128	0.0402	0.0378	0.0337	0.0253
	0.4	0.0258	0.0239	0.0222	0.0151	0.0594	0.0562	0.0516	0.0387
	0.5	0.0317	0.0295	0.0278	0.0181	0.0783	0.0746	0.0701	0.0519
	0.6	0.0353	0.0331	0.0317	0.0198	0.0957	0.0915	0.0875	0.0640
	0.7	0.0361	0.0342	0.0329	0.0203	0.1037	0.0996	0.0958	0.0678
	0.8	0.0320	0.0306	0.0297	0.0171	0.0983	0.0946	0.0920	0.0620
	0.9	0.0235	0.0228	0.0222	0.0123	0.0727	0.0704	0.0685	0.0426
250	0.1	0.0051	0.0049	0.0045	0.0045	0.0074	0.0071	0.0057	0.0050
	0.2	0.0079	0.0075	0.0067	0.0052	0.0164	0.0158	0.0130	0.0105
	0.3	0.0119	0.0113	0.0103	0.0074	0.0294	0.0285	0.0249	0.0201
	0.4	0.0163	0.0157	0.0146	0.0103	0.0462	0.0450	0.0409	0.0333
	0.5	0.0203	0.0196	0.0185	0.0128	0.0621	0.0608	0.0567	0.0461
	0.6	0.0228	0.0221	0.0212	0.0145	0.0755	0.0740	0.0704	0.0568
	0.7	0.0218	0.0212	0.0205	0.0135	0.0798	0.0784	0.0756	0.0591
	0.8	0.0179	0.0175	0.0169	0.0107	0.0687	0.0675	0.0657	0.0486
	0.9	0.0101	0.0100	0.0097	0.0056	0.0408	0.0402	0.0390	0.0259

Table2. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 10\%$.

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	R-W	RMD-W	IRMD-W	W	R-W	RMD-W	IRMD-W
25	0.1	0.0435	0.0387	0.0357	0.0408	0.0440	0.0386	0.0337	0.0336
	0.2	0.0552	0.0469	0.0424	0.0399	0.0656	0.0565	0.0489	0.0413
	0.3	0.0717	0.0603	0.0548	0.0436	0.0983	0.0852	0.0754	0.0590
	0.4	0.0928	0.0784	0.0726	0.0527	0.1396	0.1221	0.1118	0.0852
	0.5	0.1129	0.0961	0.0909	0.0623	0.1845	0.1631	0.1516	0.1139
	0.6	0.1362	0.1173	0.1122	0.0748	0.2334	0.2089	0.1970	0.1460
	0.7	0.1553	0.1355	0.1318	0.0864	0.2749	0.2477	0.2371	0.1726
	0.8	0.1672	0.1476	0.1449	0.0939	0.3156	0.2851	0.2748	0.1966
	0.9	0.1740	0.1558	0.1536	0.0982	0.3373	0.3065	0.2990	0.2090
50	0.1	0.0233	0.0215	0.0196	0.0219	0.0265	0.0244	0.0200	0.0202
	0.2	0.0333	0.0299	0.0265	0.0234	0.0457	0.0415	0.0325	0.0275
	0.3	0.0455	0.0406	0.0358	0.0268	0.0739	0.0678	0.0547	0.0436
	0.4	0.0630	0.0568	0.0514	0.0359	0.1102	0.1022	0.0868	0.0678
	0.5	0.0789	0.0717	0.0664	0.0445	0.1504	0.1402	0.1245	0.0973
	0.6	0.0944	0.0866	0.0810	0.0538	0.1892	0.1778	0.1618	0.1237
	0.7	0.1023	0.0945	0.0903	0.0583	0.2237	0.2110	0.1960	0.1462
	0.8	0.1017	0.0946	0.0910	0.0572	0.2370	0.2238	0.2119	0.1526
	0.9	0.0914	0.0859	0.0835	0.0506	0.2239	0.2120	0.2032	0.1386
100	0.1	0.0128	0.0120	0.0107	0.0114	0.0160	0.0150	0.0110	0.0110
	0.2	0.0207	0.0192	0.0162	0.0132	0.0322	0.0302	0.0214	0.0179
	0.3	0.0311	0.0289	0.0252	0.0184	0.0575	0.0547	0.0421	0.0344
	0.4	0.0434	0.0406	0.0364	0.0255	0.0892	0.0854	0.0703	0.0576
	0.5	0.0572	0.0541	0.0497	0.0345	0.1241	0.1195	0.1036	0.0843
	0.6	0.0657	0.0624	0.0586	0.0400	0.1567	0.1515	0.1361	0.1091
	0.7	0.0703	0.0671	0.0641	0.0427	0.1840	0.1781	0.1651	0.1291
	0.8	0.0643	0.0617	0.0594	0.0377	0.1823	0.1766	0.1671	0.1239
	0.9	0.0461	0.0446	0.0431	0.0257	0.1435	0.1389	0.1334	0.0899
250	0.1	0.0063	0.0060	0.0049	0.0047	0.0095	0.0091	0.0052	0.0051
	0.2	0.0127	0.0122	0.0098	0.0076	0.0245	0.0237	0.0154	0.0133
	0.3	0.0220	0.0212	0.0180	0.0138	0.0470	0.0459	0.0338	0.0295
	0.4	0.0333	0.0323	0.0289	0.0223	0.0766	0.0751	0.0605	0.0532
	0.5	0.0444	0.0432	0.0398	0.0308	0.1088	0.1070	0.0915	0.0804
	0.6	0.0515	0.0502	0.0474	0.0362	0.1378	0.1358	0.1214	0.1059
	0.7	0.0525	0.0514	0.0492	0.0365	0.1552	0.1530	0.1410	0.1195
	0.8	0.0451	0.0443	0.0428	0.0301	0.1501	0.1480	0.1402	0.1138
	0.9	0.0254	0.0250	0.0243	0.0153	0.1001	0.0986	0.0953	0.0693

Table3. The parameter estimates and the standard error of estimators of the US/Sudan of real exchange rates series.

Methods	Estimates	Standard Error (SE)
W	0.6766	0.11871
R-W	0.6799	0.11820
RMD-W	0.6860	0.11706
IRMD-W	0.7435	0.10641

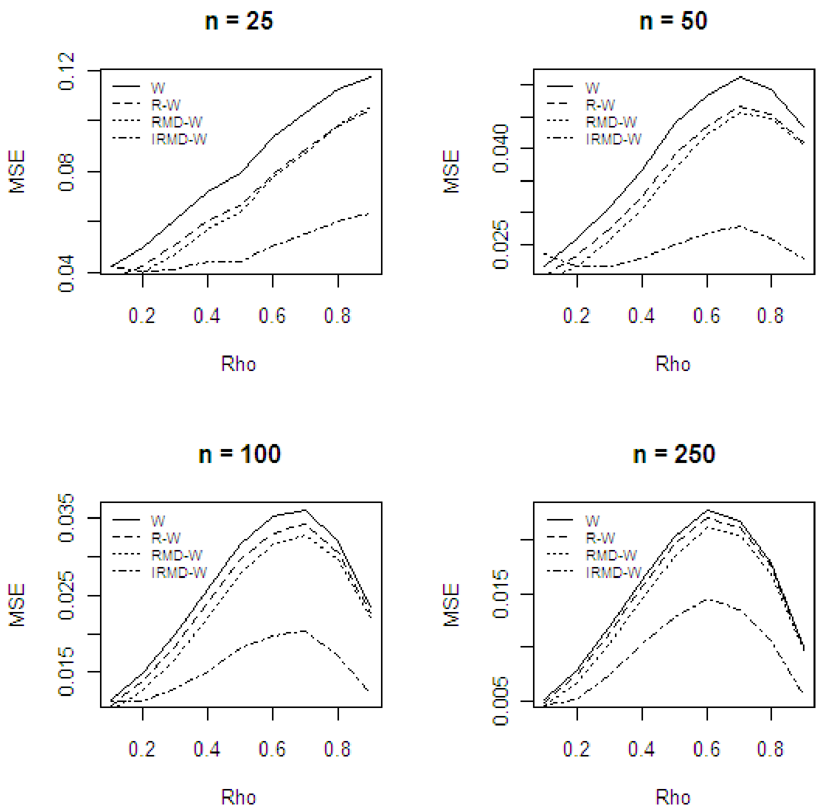

Figure 1. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 5\%$ and magnitude of the AOs effect; $\delta = 3\sigma_e$.

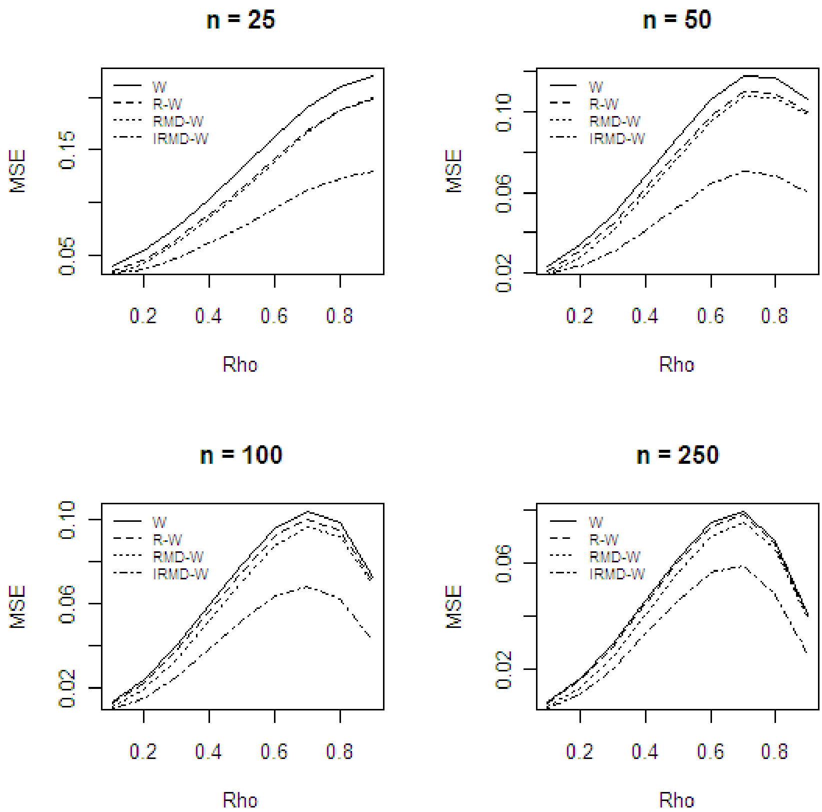

Figure 2. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 5\%$ and magnitude of the AOs effect; $\delta = 5\sigma_e$.

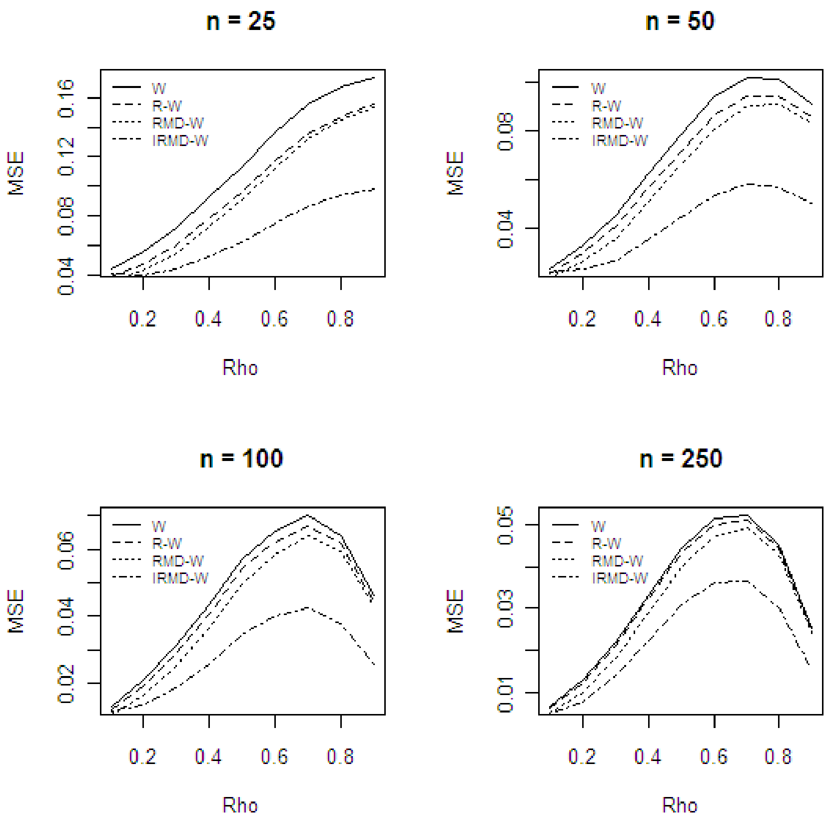

Figure 3. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 10\%$ and magnitude of the AOs effect; $\delta = 3\sigma_e$.

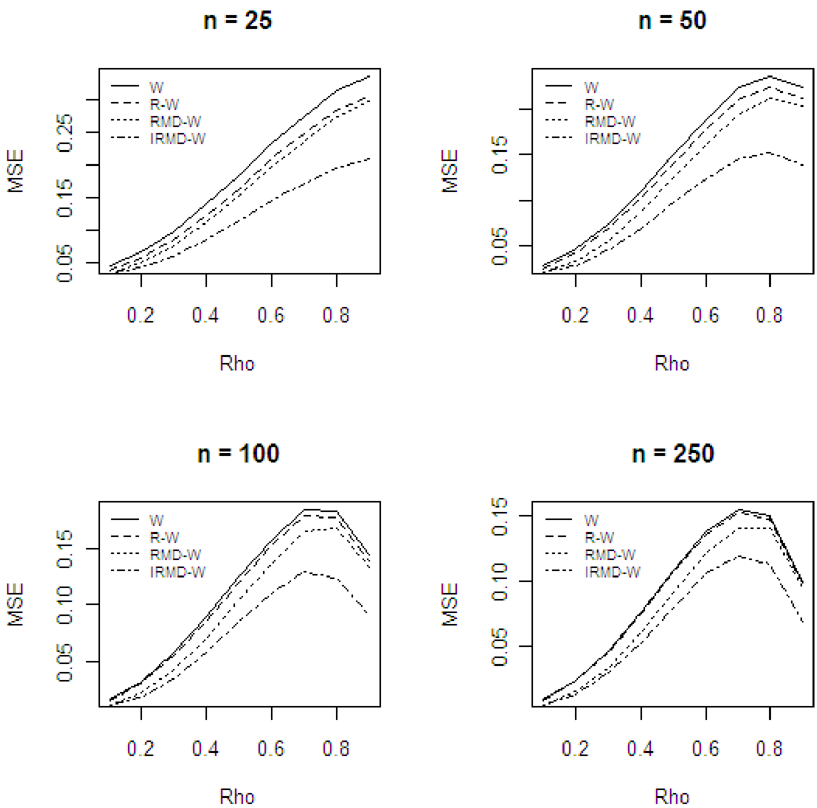

Figure 4. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{RMD-W}$ and $\hat{\rho}_{IRMD-W}$ when percentage of additive outliers; $p = 10\%$ and magnitude of the AOs effect; $\delta = 5\sigma_e$.

Figure 5. The US/Sudan of real exchange rates; annual from 1970 to 2008.

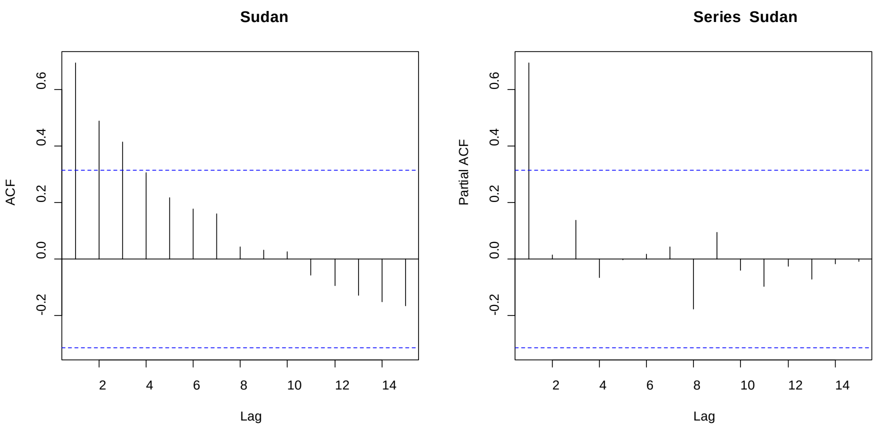

Figure 6. ACF and PACF of the US/Sudan of real exchange rates.

Standard Error of Estimators	
$SE(\hat{\rho}_W) = \dfrac{\hat{\sigma}_W}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\bar{Y})^2}}$,	$\hat{\sigma}_W^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\bar{Y}-\hat{\rho}_W(Y_{t-1}-\bar{Y}))^2}{n-2}$
$SE(\hat{\rho}_{R-W}) = \dfrac{\hat{\sigma}_{R-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\bar{Y}_{t-1})^2}}$,	$\hat{\sigma}_{R-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\bar{Y}_t-\hat{\rho}_{R-W}(Y_{t-1}-\bar{Y}_{t-1}))^2}{n-2}$
$SE(\hat{\rho}_{RMD-W}) = \dfrac{\hat{\sigma}_{RMD-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\tilde{Y}_{t-1})^2}}$,	$\hat{\sigma}_{RMD-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\tilde{Y}_t-\hat{\rho}_{RMD-W}(Y_{t-1}-\tilde{Y}_{t-1}))^2}{n-2}$
$SE(\hat{\rho}_{IRMD-W}) = \dfrac{\hat{\sigma}_{IRMD-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\ddot{Y}_{t-1})^2}}$,	$\hat{\sigma}_{IRMD-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\ddot{Y}_t-\hat{\rho}_{IRMD-W}(Y_{t-1}-\ddot{Y}_{t-1}))^2}{n-2}$

Figure 7. The standard error of all estimators.

5. Discussion and Conclusion

We have proposed a new estimator for a Gaussian AR(1) process with an unknown drift and additive outliers. This proposed estimator of ρ is obtained by applying the improved recursive median adjustment to the weighted symmetric estimator. The improved recursive median values are derived from computing the double recursive median. Furthermore, the weighted symmetric estimator ($\hat{\rho}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_{R-W}$), the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RMD-W}$) and the proposed estimator ($\hat{\rho}_{IRMD-W}$) are compared in this study. The new estimator $\hat{\rho}_{IRMD-W}$, performs better than $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, and $\hat{\rho}_{RMD-W}$ in terms of the MSE for almost all cases. One reason behind this is that the additive outliers do not affect the median values. Moreover, the improved recursive median values applied in the formula for $\hat{\rho}_{IRMD-W}$ in (6) can also reduce the mean square error (MSE) of the estimator. Therefore, the proposed estimator ($\hat{\rho}_{IRMD-W}$) which is based on the improved recursive median adjustment is superior to the existing estimators.

Finally, let us mention a problem for further research, which goes beyond the scope of the present paper but is of practical interest. In practice, a statistician or an econometrician has one time series set that is contaminated by various kinds of outliers (i.e., additive outliers (AO) and innovations outliers (IO)). Thus, it would be interesting to see whether, in this context, the proposed approach still maintains an edge over the other methodologies.

Appendix: R function for Proposed Estimator

```
rho.irmdw <- function (y)
 {
        T   <- length(y)
        ss1 <- rep(0,T)
        ss2 <- rep(0,T)
        ss3 <- rep(0,T)
        rmed <- rep(0,T)
        for (j in 1:T) {  rmed[j] <- median(y[1:j])  }
        for(i in 2:T) {  ss1[i] <- (y[i]-median(rmed[1:i]))*(y[i-1]-median(rmed[1:(i-1)]))  }
        for(j in 3:T) {  ss2[j] <- (y[j-1]- median(rmed[1:(j-1)]))^2  }
        for(k in 1:T) { ss3[k] <- (y[k]- median(rmed[1:k]))^2  }
        rho.hat.irmdw <- sum(ss1[2:T])/(sum(ss2[3:T])+(1/T)*sum(ss3))
        return(rho.hat.irmdw)
}
```

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