



Thailand Statistician  
July 2009; 7(2) : 131-145  
www.statassoc.or.th  
Contributed paper

## **An application of multiple criteria decision making in combining environmental indices of five air pollution indicators**

**Satinee Lertprapai [a] and Montip Tiensuwan\* [b]**

[a] Department of Mathematics, Faculty of Science, Burapha University, Chon Buri 20131, Thailand.

[b] Department of Mathematics, Faculty of Science, Mahidol University, Bangkok 10400, Thailand.

**\*Author for correspondence;** e-mail: scmts@mahidol.ac.th

Received: 13 January 2009

Accepted: 12 June 2009.

### **Abstract**

In this paper we focus on environmental assessment based on the air pollution data measured on carbon monoxide, nitrogen dioxide, sulfur dioxide, ozone and particulate matter from seven monitoring stations in Bangkok Thailand for each season, covering the period 2002-2004 and apply Multiple Criteria Decision Making (MCDM) method and its modification to compute an overall air pollution index for making an overall judgment among these stations. Therefore, comparing and ranking of regions are feasible in terms of air pollution for environmental protection. We also study robustness of these overall indices.

---

**Keywords:** air pollution indicators, electre method, multiple criteria decision making.

### **1. Introduction**

Air pollution is one of the important problems in protecting healthy environment of this world. Environmental assessment is a key to any environmental protection policy and is now among the highest priorities in socioeconomic development around the world.

One of these cities interested in environment protection is Bangkok, the capital of Thailand. It has an air pollution problem like other large cities in the world. It is well known that land, air and water are the three sources for determination of the extent of pollution of different regions. There are always multiple indicators for environmental assessment. Due to these multiple indicators, it is not easy to determine whether the whole environment in question improves or whether the environmental policies under consideration achieve their desired goals. A single indicator representative of the whole set of indicators is often needed to evaluate the environment comprehensively and easily.

Multiple Criteria Decision Making (MCDM) method is a data integration technique used to describe situations where there is a need for integration of the results of different studies to make an overall judgement. MCDM method has been advocated by Hwang and Yoon [1], Zeleny [2], and Yoon and Hwang [3]. In 2004, Lertprapai et al. [4] focused on the application of MCDM to four air pollution data during years 1998-2001. Nowadays in protecting healthy environment of people in Bangkok not only the four air pollution but also particulate matter.

Therefore, in this paper we extend our study MCDM method and apply this technique and its modification to the air pollution data from Bangkok, Thailand in order to rank the regions with respect to all sources taken together which can then be ranked as the regions from the best to the worst in terms of the five air pollution measured on carbon monoxide, nitrogen dioxide, sulfur dioxide, ozone and particulate matter of environment protection for each season.

## 2. The MCDM procedure and its modification

### 2.1 The MCDM procedure

The MCDM method is a procedure to integrate multiple indicators into a single meaningful and overall index by combining  $(x_{i1}, \dots, x_{iN})$  for any row  $i$ ,  $i = 1, 2, \dots, K$  across all indicators  $j = 1, 2, \dots, N$ . This is based on the premises that in the absence of a natural ideal location, a best alternative would be the one which has the shortest distance from the hypothetical ideal location.

We begin with the description of the problem. We are given a data matrix  $X = (x_{ij}) : K \times N$  where the rows represent facilities which need to be compared or ranked with respect to the element  $x_{ij}$ 's, the columns represent various sources of the elements  $x_{ij}$ 's and  $x_{ij}$ 's themselves represent some quantitative information about the facilities. MCDM method provides a statistical method to combine the elements in any

row into a single value which can then be used to compare the rows on a linear scale. We can define an Ideal Row as one with the smallest observed value for each column

$$IDR = (\min_i x_{i1}, \dots, \min_i x_{iN}) = (u_1, \dots, u_N) \quad (2.1)$$

and a Negative-ideal Row (NIDR) as one with the largest observed value for each column

$$NIDR = (\max_i x_{i1}, \dots, \max_i x_{iN}) = (v_1, \dots, v_N). \quad (2.2)$$

For any given row  $i$ , we now compute the distance of each row from Ideal row and from Negative Ideal row based on the  $L_2$ -norm by using the formulae:

$$L_2(i, IDR) = \left[ \frac{\sum_{j=1}^N (x_{ij} - u_j)^2 w_j}{\sum_{i=1}^K x_{ij}^2} \right]^{1/2} \quad (2.3)$$

$$L_2(i, NIDR) = \left[ \frac{\sum_{j=1}^N (x_{ij} - v_j)^2 w_j}{\sum_{i=1}^K x_{ij}^2} \right]^{1/2} \quad (2.4)$$

where  $w_1, w_2, \dots, w_N$  are suitably chosen nonnegative weights between 0 and 1. An objective way to select the weights is to use Shannon's entropy [5] measure  $\phi$  based on the proportion  $p_{1j}, \dots, p_{Kj}$  for the  $j$ th column [6] where

$$p_{ij} = x_{ij} / \sum_{i=1}^K x_{ij}. \quad (2.5)$$

For the  $j$ th column,  $\phi_j$  is computed as

$$\phi_j = - \sum_{i=1}^K p_{ij} \log(p_{ij}) / [\log(K)]. \quad (2.6)$$

Obviously, it is assumed here that  $x_{ij}$ 's are positive.

The quantity  $\phi$  essentially provides a measure of closeness of the different proportions.

The smaller value of  $\phi$ , larger is the variation among the proportions for classifying the rows. So we can select the weights as

$$w_j = (1 - \phi_j) / \left[ \sum_{j=1}^N (1 - \phi_j) \right], \quad j = 1, \dots, N. \quad (2.7)$$

In addition to Shannon's entropy measure, we can also use the sample variance of these proportions, given by

$$s_{j/prop}^2 = \sum_{i=1}^K (p_{ij} - \bar{p}_j)^2 / (K - 1). \quad (2.8)$$

If  $\bar{x}_j$  and  $s_j^2$  denote the mean and variance of  $x_{ij}$  in the  $j$ th column,  $s_{j/prop}^2$  is directly proportional to  $s_j^2 / \bar{x}_j^2$ , which is the square of the sample coefficient of variation  $cv_j$  for the  $j$ th column. Therefore we propose to use  $w_j = cv_j$  for all  $j$ .

The various rows are now ranked based on an overall index  $I$  computed as

$$I_i = \frac{L_2(i, IDR)}{L_2(i, IDR) + L_2(i, NIDR)}, \quad i = 1, \dots, K. \quad (2.9)$$

In addition to  $L_2$ -norm we can also use the  $L_1$ -norm as a distance measure and rank the rows once again. The  $L_1$ -norm distance is defined as

$$L_1(i, IDR) = \frac{\sum_{j=1}^N |x_{ij} - u_j| w_j}{\sum_{i=1}^K x_{ij}} \quad (2.10)$$

$$L_1(i, NIDR) = \frac{\sum_{j=1}^N |x_{ij} - v_j| w_j}{\sum_{i=1}^K x_{ij}}, \quad (2.11)$$

where  $w_j$ 's are appropriate weights.

## 2.2 Modification of MCDM

We describe modification of MCDM [7]. Let  $d_i = [d_{i1}, d_{i2}, \dots, d_{iN}]$  represent the row-vector of  $d_{ij}$ 's, distance of  $x_{ij}$  from  $\min_i x_{i,j}$ ,  $1 \leq i \leq K, 1 \leq j \leq N$ , for  $i$ th row involving  $N$  columns, and  $d_i^- = [d_{i1}^-, d_{i2}^-, \dots, d_{iN}^-]$  represent the row-vector of  $d_{ij}^-$ 's, distance of  $x_{ij}$  from  $\max_i x_{i,j}$ ,  $1 \leq i \leq K, 1 \leq j \leq N$ , for  $i$ th row involving  $N$  columns. The modification is

$$L_i(d, d^-) = \left[ \sum_j \left\{ w_j (d_{ij}^k / d_{ij}^{-k}) / \left[ \sum_i x_{ij}^2 \right]^{k/2} \right\} \right]^{1/2} + \left[ \sum_j' \left\{ w_j R_j^k / \left[ \sum_i x_{ij}^2 \right]^{k/2} \right\} \right]^{1/2},$$

where  $\Sigma$  refers to all  $j$  for which  $d_{ij}^- > 0$  while  $\Sigma'$  refers to all  $j$  for which  $d_{ij}^- = 0$  and  $R_j$  is a finite quantity of our choice  $R_j = d_{ij} / d_{ij}^-$  subject to  $R_j \geq \max [d_{ij} / d_{ij}^-]$  taken over all  $i$  for which  $d_{ij}^- > 0$ . In the above equation  $k$  is a positive number.

To check the robustness of various sets of ranks produced by different methods, we will compute Spearman's rank correlation coefficient:

$$r = 1 - \frac{6 \sum_{i=1}^K \Delta_i^2}{K(K^2 - 1)}, \quad -1 \leq r \leq 1,$$

where  $\Delta_i$  = difference between ranks. It is obvious that a large value of  $r$  signifies good agreement.

### 2.3 Electre Method

The Electre method requires more extensive computations than the MCDM method [7]. It is used for comparing the status of two locations rather than ranking all of them together. We start with the  $K \times N$  data matrix  $X$  of observations and proceed as follows:

**Step 1:** Transform  $X = [X_1, X_2, \dots, X_N]$  to  $R = [R_1, R_2, \dots, R_N]$  where  $R_i = \frac{X_i}{\|X_i\|_2}$ .

**Step 2:** Transform  $R$  to  $V = RW$  where  $W = \text{diag}[w_1, w_2, \dots, w_N]$ .

**Step 3:** Construct two matrices **C** and **D**,

$$\text{where } c_{ij} = \sum_{k: V_{ik} \geq V_{jk}} w_k, \quad \text{and} \quad d_{ij} = \frac{\max_{k: V_{ik} < V_{jk}} |V_{ik} - V_{jk}|}{\max_k |V_{ik} - V_{jk}|}.$$

$$\text{Compute } \bar{c} = \frac{\sum \sum_{i \neq j} c_{ij}}{K(K-1)}, \quad \text{and} \quad \bar{d} = \frac{\sum \sum_{i \neq j} d_{ij}}{K(K-1)}.$$

**Step 4:** Construct matrices **F** and **G** such that  $f_{ij} = \begin{cases} 1 & ; c_{ij} \geq \bar{c} \\ 0 & ; \text{otherwise} \end{cases}$  and

$$g_{ij} = \begin{cases} 1 & ; d_{ij} \leq \bar{d} \\ 0 & ; \text{otherwise} \end{cases}.$$

**Step 5:** Define matrix **E** where  $e_{ij} = f_{ij} \cdot g_{ij}$ .

It should be noted that the weights  $w_i$ 's are obtained as discussed before, and that  $e_{ij} = 0$  means that row  $i$  is better than row  $j$ .

### 3. Application to air pollution

In this section we apply the previously described MCDM, modification of MCDM, and electre method to the air pollution data from Bangkok, Thailand. The main cause of air pollution in this city is the increasing number of motor vehicles. Carbon monoxide (CO), nitrogen dioxide (NO<sub>2</sub>) and sulfur dioxide (SO<sub>2</sub>), which are released directly from motor vehicles, are main air pollutants in Bangkok. The photochemical reaction on the

oxide of nitrogen is ozone ( $O_3$ ) which is a secondary pollutant. The last pollutant is particulate matter (PM-10). The data sets were provided by the Pollution Control Department of Thailand and were recorded by seven monitoring stations in Bangkok during 2002-2004. The monitoring stations are National Housing Authority, Huai Khwang, Nonsee Vitaya School, Singharatpitayakom School, Thonburi, Traffic Police Residence and Dindaeng. According to the Thai Meteorological Department summer, rainy, and winter is a period of March 16-May 15, May 16-October 15, and October 16-March 15, respectively. We averaged data in a whole year and each season for each indicator and station. The smaller the value of the observation the better the facility is.

We are interested in ranking monitoring stations in Bangkok with respect to all such indicators to determine which station is the best or the worst in air pollution by using MCDM method. We have  $K \times N$  data matrix  $X$  with  $K$  rows representing the monitoring stations and  $N$  columns representing the air pollutant indicators. We denote the elements of this matrix by  $x_{ij}$  corresponding to the monitoring station  $i$  and air pollutant indicator  $j$ . The fact is that the lower the value of an environmental indicator the better is the rank of the row for that specific indicator. Thus, rank one would normally correspond to the best row that is the best station. The air pollution data for each station during years 2002-2004 are shown in Table 1.

**Table 1.** Air pollution data during years 2002-2004.

Monitoring station	$O_3^*$	$CO^*$	$NO_2^*$	$SO_2^*$	$PM-10^*$
Year 2002					
(1) National Housing Authority	16.3400	0.8123	20.7797	6.2774	42.8524
(2) Huai Khwang	11.5492	0.9882	34.5680	5.9218	50.2723
(3) Nonsee Vitaya school	11.2682	0.7967	30.9848	7.1081	55.5594
(4) Singharatpitayakom school	17.5509	1.0079	18.9575	6.5709	50.3496
(5) Thonburi	11.7036	1.1101	28.3894	7.6350	60.0667
(6) Traffic Police Residence	9.0695	1.3523	33.9666	6.7971	35.0743
(7) Dindaeng	5.6088	2.3191	47.2247	8.7173	47.0584
Year 2003					
(1) National Housing Authority	19.6275	0.7766	21.2695	5.6317	45.7837
(2) Huai Khwang	15.7576	0.8439	32.4307	4.7384	45.6933
(3) Nonsee Vitaya school	13.0607	0.6187	30.2024	6.1187	62.4625
(4) Singharatpitayakom school	18.5210	0.7648	20.3156	4.4983	62.0479

**Table 1.** (continued)

Monitoring station	O3*	CO*	NO2*	SO2*	PM-10*
(5) Thonburi	16.1648	1.1520	27.7445	7.3807	53.9217
(6) Traffic Police Residence	12.3560	1.2147	33.3878	7.1525	34.1713
(7) Dindaeng	6.9589	1.5695	44.7820	6.6628	56.1434
Year 2004					
(1) National Housing Authority	21.5447	0.6682	19.6492	4.9977	47.0252
(2) Huai Khwang	13.6099	0.8666	33.5167	5.6818	56.3142
(3) Nonsee Vitaya school	13.2196	0.6204	26.1518	5.8421	67.4887
(4) Singharatpitayakom school	17.9264	0.7870	21.3065	5.2940	66.2020
(5) Thonburi	16.0654	1.0854	25.9813	6.3620	52.2094
(6) Traffic Police Residence	15.0625	1.0388	32.3761	7.4442	71.8597
(7) Dindaeng	6.2992	1.3459	45.8121	7.0369	65.0315

\* O<sub>3</sub>, SO<sub>2</sub>, NO<sub>2</sub> unit: part per billion (ppb, 1/1,000,000,000)

CO unit: part per million (ppm, 1/1,000,000)

PM-10 unit: microgramme per cubic meter ( $\mu\text{g}/\text{m}^3$ )

### 3.1 Results of MCDM method

To apply the MCDM method, we use both the distance measures  $L_2$  norm as in equations (2.3)-(2.4) and  $L_1$  norm as in equations (2.10)-(2.11) as well as the two choices of weights based on  $\phi$  as in equation (2.7) and coefficient of variation (cv) as in equation (2.8). We show below the results in four sets of the values of combined indices for each year and also for each season separately, i.e. summer, rainy and winter. The final ranks of the rows are then based on the average index. We also compute the standard deviation to show the closeness of the five indices in a row. Tables 2-5 present the results for whole year, summer, rainy, and winter, respectively.

From Table 2 we observe that most often the first rank is Nonsee Vitaya school station which means that this station is expected to be good in terms of air pollution. On the other hand, Dindaeng station performed poorly. When we compute separately for each season in each year, from Table 3 we observe that most of the first rank in summer season is Singharatpitayakom school station and the last rank is Dindaeng station. Tables 4-5 also show that most of the first rank in rainy season and winter season are Nonsee Vitaya school station and Huai Khwang station, respectively. The last ranks of both rainy and winter seasons are Thonburi, Traffic Police Residence and Dindaeng stations.

**Table 2.** Results of MCDM method on air pollution data during years 2002-2004.

Monitoring station	$L_1$		$L_2$		Mean	SD	rank
	$w_1$	$w_2$	$w_1$	$w_2$			
Year 2002							
(1) National Housing Authority	0.28750	0.25282	0.36315	0.33928	0.31069	0.04985	2
(2) Huai Khwang	0.33941	0.31017	0.34609	0.32146	0.32928	0.01645	3
(3) Nonsee Vitaya school	0.26842	0.22827	0.29399	0.26729	0.26449	0.02711	1
(4) Singharatpitayakom school	0.37429	0.34375	0.41525	0.39317	0.38162	0.03029	6
(5) Thonburi	0.36182	0.33579	0.35257	0.33123	0.34535	0.01431	4
(6) Traffic Police Residence	0.36214	0.36498	0.37279	0.37246	0.36809	0.00536	5
(7) Dindaeng	0.69577	0.73467	0.61556	0.63996	0.67149	0.05386	7
Year 2003							
(1) National Housing Authority	0.40824	0.38898	0.44214	0.42869	0.41701	0.02331	3
(2) Huai Khwang	0.42545	0.41721	0.44011	0.43061	0.42834	0.00959	4
(3) Nonsee Vitaya school	0.35241	0.32440	0.36281	0.34411	0.34593	0.01626	1
(4) Singharatpitayakom school	0.39570	0.37338	0.43198	0.41700	0.40451	0.02554	2
(5) Thonburi	0.59384	0.58466	0.57117	0.56547	0.57879	0.01286	6
(6) Traffic Police Residence	0.51452	0.52355	0.51927	0.52760	0.52123	0.00562	5
(7) Dindaeng	0.68417	0.70607	0.59989	0.61366	0.65095	0.05209	7
Year 2004							
(1) National Housing Authority	0.40650	0.40408	0.48290	0.47930	0.44320	0.04381	3
(2) Huai Khwang	0.44871	0.45241	0.45943	0.46188	0.45561	0.00610	4
(3) Nonsee Vitaya school	0.30012	0.29324	0.33156	0.32707	0.31300	0.01914	1
(4) Singharatpitayakom school	0.40914	0.40006	0.45164	0.44554	0.42659	0.02579	2
(5) Thonburi	0.50245	0.49920	0.51091	0.50681	0.50484	0.00511	5
(6) Traffic Police Residence	0.58562	0.57619	0.56418	0.55935	0.57133	0.01187	7
(7) Dindaeng	0.58997	0.59642	0.51930	0.52376	0.55736	0.04150	6



**Table 3.** Results of MCDM method on air pollution data in summer season for each year.

Monitoring station	$L_1$		$L_2$		Mean	SD	rank
	$w_1$	$w_2$	$w_1$	$w_2$			
Year 2002							
(1) National Housing Authority	0.35703	0.36233	0.40060	0.40226	0.38055	0.02421	2
(2) Huai Khwang	0.40655	0.38749	0.42055	0.40891	0.40588	0.01370	5
(3) Nonsee Vitaya school	0.32774	0.31059	0.35432	0.34380	0.33411	0.01912	1
(4) Singharatpitayakom school	0.37500	0.38146	0.41724	0.42025	0.39849	0.02357	4
(5) Thonburi	0.40233	0.39573	0.39347	0.38898	0.39513	0.00556	3
(6) Traffic Police Residence	0.39409	0.39762	0.42056	0.42087	0.40829	0.01443	6
(7) Dindaeng	0.67349	0.67674	0.60932	0.60962	0.64229	0.03793	7
Year 2003							
(1) National Housing Authority	0.28119	0.26309	0.31130	0.29834	0.28848	0.02094	2
(2) Huai Khwang	0.56476	0.56295	0.55863	0.55692	0.56082	0.00366	5
(3) Nonsee Vitaya school	0.44992	0.43666	0.45407	0.44543	0.44652	0.00746	4
(4) Singharatpitayakom school	0.23236	0.22019	0.28190	0.27130	0.25144	0.02979	1
(5) Thonburi	0.43604	0.42967	0.42117	0.41632	0.42580	0.00878	3
(6) Traffic Police Residence	0.57752	0.58239	0.57825	0.58164	0.57995	0.00242	6
(7) Dindaeng	0.80995	0.83250	0.73331	0.74835	0.78103	0.04772	7
Year 2004							
(1) National Housing Authority	0.32074	0.32307	0.40661	0.40722	0.36441	0.04909	2
(2) Huai Khwang	0.57905	0.57903	0.56618	0.56672	0.57275	0.00727	5
(3) Nonsee Vitaya school	0.36440	0.36095	0.38231	0.38028	0.37198	0.01087	3
(4) Singharatpitayakom school	0.25577	0.25629	0.34728	0.34751	0.30171	0.05275	1
(5) Thonburi	0.53298	0.53819	0.53637	0.54021	0.53694	0.00307	4
(6) Traffic Police Residence	0.77227	0.77140	0.67412	0.67384	0.72290	0.05650	7
(7) Dindaeng	0.65459	0.65661	0.58783	0.58879	0.62196	0.03886	6

**Table 4.** Results of MCDM method on air pollution data in rainy season for each year.

Monitoring station	$L_1$		$L_2$		Mean	SD	rank
	$w_1$	$w_2$	$w_1$	$w_2$			
Year 2002							
(1) National Housing Authority	0.24154	0.19522	0.30246	0.27125	0.25262	0.04564	1
(2) Huai Khwang	0.46592	0.43957	0.44752	0.42442	0.44436	0.01728	6
(3) Nonsee Vitaya school	0.26496	0.21737	0.28406	0.25260	0.25475	0.02808	2
(4) Singharatpitayakom school	0.29799	0.26517	0.32672	0.30053	0.29760	0.02523	3
(5) Thonburi	0.32682	0.29651	0.30913	0.28499	0.30437	0.01792	4
(6) Traffic Police Residence	0.39292	0.40221	0.41785	0.42075	0.40843	0.01317	5
(7) Dindaeng	0.79942	0.83958	0.70942	0.73852	0.77173	0.05875	7
Year 2003							
(1) National Housing Authority	0.48025	0.47601	0.50720	0.50372	0.49179	0.01594	4
(2) Huai Khwang	0.44538	0.44346	0.46742	0.46534	0.45540	0.01273	3
(3) Nonsee Vitaya school	0.34978	0.33049	0.38343	0.37017	0.35847	0.02323	1
(4) Singharatpitayakom school	0.36057	0.34729	0.41167	0.40309	0.38066	0.03153	2
(5) Thonburi	0.61122	0.60975	0.57923	0.57903	0.59481	0.01811	7
(6) Traffic Police Residence	0.57642	0.59581	0.55368	0.56624	0.57304	0.01781	5
(7) Dindaeng	0.61892	0.63029	0.54799	0.55427	0.58787	0.04275	6
Year 2004							
(1) National Housing Authority	0.41077	0.40253	0.48518	0.47839	0.44422	0.04360	3
(2) Huai Khwang	0.46154	0.46882	0.47393	0.47885	0.47079	0.00740	4
(3) Nonsee Vitaya school	0.26220	0.25101	0.30800	0.30004	0.28031	0.02795	1
(4) Singharatpitayakom school	0.29464	0.28049	0.35791	0.34854	0.32040	0.03854	2
(5) Thonburi	0.48411	0.47758	0.47933	0.47463	0.47891	0.00397	5
(6) Traffic Police Residence	0.59055	0.58222	0.57689	0.57192	0.58039	0.00797	7
(7) Dindaeng	0.59684	0.61350	0.52580	0.53439	0.56764	0.04402	6

**Table 5.** Results of MCDM method on air pollution data in winter season for each year.

Monitoring station	$L_1$		$L_2$		Mean	SD	rank
	$w_1$	$w_2$	$w_1$	$w_2$			
Year 2002							
(1) National Housing Authority	0.31163	0.27378	0.38805	0.36213	0.33390	0.05112	3
(2) Huai Khwang	0.23524	0.20352	0.28492	0.26192	0.24640	0.03506	1
(3) Nonsee Vitaya school	0.28678	0.24766	0.31226	0.28470	0.28285	0.02659	2
(4) Singharatpitayakom school	0.43426	0.39165	0.46539	0.43759	0.43222	0.03044	6
(5) Thonburi	0.39170	0.36333	0.38891	0.36506	0.37725	0.01513	5
(6) Traffic Police Residence	0.35889	0.35186	0.36237	0.35349	0.35665	0.00485	4
(7) Dindaeng	0.64369	0.68483	0.56687	0.59293	0.62208	0.05260	7
Year 2003							
(1) National Housing Authority	0.47225	0.45414	0.48571	0.47204	0.47104	0.01295	3
(2) Huai Khwang	0.33111	0.31424	0.37245	0.35801	0.34395	0.02619	1
(3) Nonsee Vitaya school	0.37854	0.35032	0.39492	0.37511	0.37472	0.01842	2
(4) Singharatpitayakom school	0.50069	0.47544	0.50176	0.48506	0.49074	0.01274	4
(5) Thonburi	0.67211	0.66277	0.64993	0.64218	0.65674	0.01330	7
(6) Traffic Police Residence	0.48880	0.49021	0.49822	0.49870	0.49398	0.00521	5
(7) Dindaeng	0.64980	0.67437	0.57226	0.58789	0.62108	0.04882	6
Year 2004							
(1) National Housing Authority	0.43123	0.42894	0.49672	0.49345	0.46258	0.03756	3
(2) Huai Khwang	0.39953	0.40304	0.42885	0.42962	0.41526	0.01620	2
(3) Nonsee Vitaya school	0.37256	0.36760	0.40789	0.40454	0.38815	0.02101	1
(4) Singharatpitayakom school	0.57281	0.56503	0.56529	0.55990	0.56576	0.00532	6
(5) Thonburi	0.48669	0.48764	0.51783	0.51568	0.50196	0.01711	4
(6) Traffic Police Residence	0.57887	0.56870	0.56322	0.55634	0.56678	0.00951	7
(7) Dindaeng	0.56618	0.57036	0.50481	0.50847	0.53745	0.03565	5

### 3.2 Results of MCDM Modification

Returning to the air pollution data set, we have applied the Modification of MCDM using only weights based on Shannon's entropy and using various values of  $k$ , these are reported in Table 6. The values of Spearman's rank correlation coefficients of two sets of ranks between MCDM and Modification are shown in Table 7. Table 8 shows these values for Modification within themselves for different values of  $k$ .

According to Modification of MCDM using various values of  $k$ , these are reported in Table 6. We observe that in 2002 most of the first rank is the Traffic Police

Residence station for  $k = 1.5, 2, 2.5$  and  $3$ . On the other hand, The Singharatpitayakom School and National Housing Authority station are the last rank. In 2003, the first rank is Huai Khwang station for all values of  $k$  and most of the last rank is Singharatpitayakom School ( $k = 1.5, 2, 2.5, 3$ ). In 2004, most of the first rank belong to Huai Khwang station ( $k = 1.5, 2, 2.5, 3$ ) and the last rank belongs to Traffic Police Residence station for all values of  $k$ .

**Table 6.** Results of Modification for each year.

Monitoring station	$k=1$	rank	$k=1.5$	rank	$k=2$	rank	$k=2.5$	rank	$k=3$	rank
Year 2002										
(1) National Housing Authority	0.2256	6	0.1443	6	0.0931	7	0.0602	7	0.0389	7
(2) Huai Khwang	0.1099	1	0.0399	2	0.0148	2	0.0055	2	0.0021	3
(3) Nonsee Vitaya school	0.1150	3	0.0490	4	0.0226	4	0.0110	4	0.0055	4
(4) Singharatpitayakom school	0.2706	7	0.1536	7	0.0912	6	0.0554	6	0.0340	6
(5) Thonburi	0.1690	5	0.0696	5	0.0303	5	0.0137	5	0.0064	5
(6) Traffic Police Residence	0.1101	2	0.0385	1	0.0137	1	0.0050	1	0.0018	1
(7) Dindaeng	0.1266	4	0.0441	3	0.0155	3	0.0055	3	0.0020	2
Year 2003										
(1) National Housing Authority	0.2923	4	0.1700	5	0.1055	5	0.0676	5	0.0440	5
(2) Huai Khwang	0.1321	1	0.0531	1	0.0222	1	0.0095	1	0.0041	1
(3) Nonsee Vitaya school	0.4479	7	0.3849	6	0.3605	6	0.3492	6	0.3423	6
(4) Singharatpitayakom school	0.4199	6	0.3872	7	0.3734	7	0.3672	7	0.3641	7
(5) Thonburi	0.3031	5	0.1627	4	0.0920	4	0.0541	3	0.0328	3
(6) Traffic Police Residence	0.2173	3	0.1293	3	0.0842	3	0.0571	4	0.0394	4
(7) Dindaeng	0.2094	2	0.0881	2	0.0375	2	0.0161	2	0.0070	2
Year 2004										
(1) National Housing Authority	0.1601	5	0.0729	5	0.0347	5	0.0168	6	0.0082	6
(2) Huai Khwang	0.1149	2	0.0388	1	0.0132	1	0.0046	1	0.0016	1
(3) Nonsee Vitaya school	0.1101	1	0.0461	2	0.0212	2	0.0102	3	0.0051	3
(4) Singharatpitayakom school	0.1472	3	0.0660	4	0.0302	4	0.0139	4	0.0064	4
(5) Thonburi	0.1482	4	0.0592	3	0.0241	3	0.0099	2	0.0040	2
(6) Traffic Police Residence	0.2248	7	0.0945	7	0.0409	7	0.0182	7	0.0083	7
(7) Dindaeng	0.1959	6	0.0824	6	0.0355	6	0.0157	5	0.0071	5

**Table 7.** Spearman's rank correlation coefficients between MCDM and Modification.

Year	Modification				
	$k=1$	$k=1.5$	$k=2$	$k=2.5$	$k=3$
2002	0.2143	-0.0714	-0.2143	-0.2143	-0.3571
2003	-0.6429	-0.7143	-0.7143	-0.7500	-0.7500
2004	0.8214	0.6071	0.6071	0.3571	0.3571

The result of Spearman's rank correlation coefficients between MCDM and Modification of MCDM in Table 7 shows that the robustness of the ranks is obvious in 2004 for all values of  $k$ . For  $k = 1$ , the Spearman's rank correlation coefficient is the largest positive for year 2004. Also the choice of  $k = 1$  performs better than other choices though value is small positive for year 2002 and large negative for year 2003. The values of Spearman's rank correlation coefficients for 2002 are negative which show that the correlation between MCDM and Modification is very low. For 2003, the values of Spearman's rank correlation coefficients are quite highly negative correlated thus a fairly strong negative relationship between MCDM and Modification. In Table 8, the results show that the ranks are fairly robust in view of the large values of Spearman's rank correlation coefficients uniformly in all cases.

**Table 8.** Spearman's rank correlation coefficients for Modification for different values of  $k$ 

Year	$k=1, k=1.5$	$k=1, k=2$	$k=1, k=2.5$	$k=1, k=3$	$k=1.5, k=2$	$k=1.5, k=2.5$	$k=1.5, k=3$	$k=2, k=2.5$	$k=2, k=3$	$k=2.5, k=3$
2002	0.9286	0.8929	0.8929	0.7857	0.9643	0.9643	0.9286	1.0000	0.9643	0.9643
2003	0.9286	0.9286	0.8571	0.8571	1.0000	0.9643	0.9643	0.9643	0.9643	1.0000
2004	0.9286	0.9286	0.7857	0.7857	1.0000	0.9286	0.9286	0.9286	0.9286	1.0000

### 3.3 Result of Electre method

We begin with the data matrix  $X$ , and follow Steps 1-5 of Electre method to eventually obtain the matrix  $E$ . The three  $E$ -matrices are shown in Table 9.

**Table 9 E-matrices**

station	1	2	3	4	5	6	7	station	1	2	3	4	5	6	7	station	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
2	0	0	1	0	0	0	0	2	0	0	1	1	0	0	0	2	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0
4	1	0	1	0	0	0	0	4	0	0	1	0	0	0	0	4	0	0	1	0	0	0	0
5	0	0	1	0	0	0	0	5	1	1	1	1	0	1	0	5	0	0	1	1	0	0	0
6	0	0	1	0	0	0	0	6	0	1	1	0	0	0	0	6	0	1	1	1	0	0	0
7	1	1	1	1	1	1	0	7	0	1	1	0	0	0	0	7	0	0	1	0	0	0	0
Year 2002								Year 2003								Year 2004							

Station 1:National Housing Authority 2:Huai Khwang 3:Nonsee Vitaya

4:Singharatpitayakom 5:Thonburi 6:Traffic Police Residence 7:Dindaeng

From Table 9, we conclude that in 2002, 2003 and 2004, the Nonsee Vitaya School station is the best and National Housing Authority station is the second. In addition, the worst station is Dindaeng station for 2002. Thonburi station and Traffic Police Residence station are the worst for 2003 and 2004 respectively.

#### 4. Conclusion

This paper presents a study on combination of five air pollution indicators, that is, carbon monoxide, nitrogen dioxide, sulfur dioxide, ozone and particulate matter using MCDM method. MCDM is a method to integrate multiple indicators for making an overall judgement, thus making a ranking of the rows and hence their comparison is feasible. This study used MCDM to evaluate and analyze the relative importance of various air pollution indicators based on the observations to compare and rank seven monitoring stations in Bangkok, Thailand in three-year period from 2002-2004. Modifications of MCDM and Electre methods are also used to rank the stations.

Our study reveals that, during the three year period, the performance in rank order of seven monitoring stations changed possibly depending on the seasons in each year. However, for the whole year the first rank is attributed to Nonsee Vitaya School station, while most of the last rank is attributed to Dindaeng station. The Dindaeng station is located in central of Bangkok and is very crowded of traffic particularly motorcycle and car. Further, in summer, rainy and winter seasons most of the first rank is attributed to Singharatpitayakom School, Nonsee Vitaya School, and Huai Khwang stations, respectively and most of the last rank are attributed to Dindaeng, Traffic Police Residence, and Thonburi stations, respectively. The Government needs to take care of these air pollution indicators for environmental protection, particularly the last ranked station in each season and year to improve health and life of Thai people since environmental protection is such an important issue in the world nowadays.

Our study is only a preliminary one and many things are left for further investigation, other effective and efficient methods may also be needed for further investigation such as the difference rank for each location are significant difference in term of mean air pollution indicator or rank base on maximum level for these air pollution indicators over some span of time, i.e. day, week or month.

### **Acknowledgements**

We would like to thank the Thailand Research Fund for the financial support and the Pollution Control Department of Thailand for providing the air pollution indicators data.

### **References**

- [1] Hwang, C.L. and Yoon K., Multiple Attribute Decision Making: Methods and Applications, A State-of-the-Art-Survey, Berlin: Springer-Verlag, 1981.
- [2] Zeleny, M., Multiple Criteria Decision Making, New York: McGraw-Hill, 1982.
- [3] Yoon, K. and Hwang, C.L., Multiple Attribute Decision Making: An Introduction, Sage, California, 1995.
- [4] Lertprapai, S., Tiensuwan, M., and Sinha, B.K., A statistical approach to combining environmental indices with an application to air pollution data from Bangkok, Thailand, *Pak. J. Statist.* 2004; 20:245-261.
- [5] Shannon, C.E., and Weaver, W., The Mathematical Theory of Communication, The University of Illinois Press, Urbana, Illinois, 1947.
- [6] Maitra, R., Ross, N.P., and Sinha Bimal K., On Some Aspects of Data Integration Techniques with Applications, Technical Report, Department of Mathematics and Statistics, UMBC, USA, 2002.
- [7] Sinha B.K., and Shah K.R., On some aspects of data integration techniques with environmental applications, *Environmetrics*, 2003; 14:409-416.