Inference Concerning the Conversion Efficiency for a Special Predator-prey System

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Abstract

In this communication, we consider the inference problem for the ratio of two interaction parameters, the so-called conversion efficiency of the Lotka-Volterra ordinary differential equations system (ODEs). The stochastic model under consideration views the actual population sizes as random perturbations of the solutions to these ODEs. Namely, we assume that the perturbations follow correlated Ornstein-Uhlenbeck processes and thus, no assumption is made that the random variables are independent. In this context, we establish the uniformly most powerful unbiased test for the conversion efficiency parameter. The asymptotic properties of the proposed test are derived. A simulation study is conducted and this provided strong evidence that corroborates with the usual asymptotic theory of optimal tests. To illustrate the procedure, the proposed method is applied to the Canadian mink-muskrat data set.

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Keywords: conversion efficiency, gaussian process, Lotka-Volterra ODEs, monte-carlo simulation, the uniformly most powerful unbiased test.

1. Introduction

We consider predator-prey system as described by the Lotka-Volterra system of differential equations [1,2]:

$$\frac{d\,x(t)}{d\,t} = (\eta - \beta\, y(t))\, x(t), \quad \frac{d\,y(t)}{d\,t} = (\gamma\, x(t) - \delta)\, y(t)$$

$$(x(0),\, y(0)) = (x_0, y_0) \quad fixed \tag{1}$$

where $\eta, \beta, \gamma, \delta$ are all positive quantities and the components of initial value (x_0, y_0) are the positive. We suppose that (x_0, y_0) is different from an equilibrium point and thus, the solution of the system (1) is not trivial. The system (1) does not admit an explicit analytical solution even if it admits a unique solution which belongs to a closed curve. Thus, the trajectory $(x(t),\, y(t))$ is a periodic function, whose period is denoted by $\rho(\gamma, \beta, \delta, \eta, x_0, y_0)$ and, is a function of $(\gamma, \beta, \delta, \eta, x_0, y_0)$.

Theoretically, $x(t)$ and $y(t)$ are the population sizes (at time t) of the prey and the predator, respectively. The parameter, η is the birth rate of the prey when the predator is absent, δ is the death rate of the predator when the prey is absent. In ecological modeling, the parameters δ and η, are usually considered as intrinsics at the species prey and predator. The parameters β and γ are the interaction parameters. The ratio γ / β is called "conversion efficiency" and represents a percentage of consumed prey that is converted into biomass predator. In the sequel, the conversion efficiency that is the parameter of interest is denoted by $\theta = \dfrac{\gamma}{\beta}$.

In practice, we have N pairs of observations $(X_i, Y_i)_{i=1,\,2,\,\ldots\ldots,\,N}$ collected at discrete times t_i, where $0 < t_i < t_{i+1}$; X_i and Y_i, represent respectively the sizes of the prey and the predator observed at time t_i, $i = 1, 2, \ldots\ldots, N$. Briefly, as in Froda and Nkurunziza [3], the logarithm of population sizes are considered as logarithm of the solution of ODE (1) plus error Markov processes. In this model, the error process is considered as mainly measurement (observation) error which does not interfere with the deterministic function. Interestingly, a such measurement type model has the advantage of preserving the periodic behaviour as well the irregularities of the trajectory that is commonly observed in practice (see Froda and Nkurunziza, [3]). In fact,

according to ecologists (see e.g. Kendall, et al., [4], Ginzburg and Taneyhill, [5] or Royama, [6], chapters 5-6), the trajectory of many animal population sizes has oscillatory behaviour. However, the trajectory of the observed predator-prey population sizes are not as smooth and regular as the solution of the ODEs (1).

Concerning the noise, we assume that each component of error process follows Ornstein-Uhlenbeck process that is the continuous version of a first-order autoregressive model AR(1) in discrete times. In fact, if $\left\{(e_t^X, t \geq 0\right\}$ is an Ornstein-Uhlenbeck process, then

$$\left\{(e_{t_i}^X, 0 < t_1 < t_2 < \ldots < t_N, \; with \; t_{i+1} - t_i \; cons\tan t \; for \; all \; i\right\}$$

a first-order autoregressive model AR(1). Indeed, the statistical model which is commonly used by ecologists with population cycles is the linear autoregressive (AR) model (see Kendall, et al., [4], Berryman, [7] or Royama,[6]), with an order less than or equal to 2. In our case, the periodicity is captured by the solution of the ODEs (1) and then, to simplify some computations, we can reduce the order by considering an AR(1) model. For other references on the study of cyclic behaviour in the ecological literature, we refer the reader to Haydon, et al. [8], Brillinger [10] and Boyce [11].

The point estimation problem of the parameters $\eta, \beta, \gamma, \delta$ is considered by Froda and Colavita [12] and Nkurunziza [4] proposed the likelihood ratio test for the interaction parameters $\beta \; and \; \gamma$

In the current investigation, the parameter of interest is the conversion efficiency and we would like to test

$$H_0: \gamma/\beta = \theta \leq \theta_0 \; against \; H_A: \theta > \theta_0.$$

For $\theta_0 = 1$, the above test allows us to test the homogeneity of the two interaction parameters $\gamma \; and \; \beta.$

As in Nkurunziza [4], the nuisance parameters $\delta \; and \; \eta$ are considered as constants with respect to the interaction parameters $\gamma \; and \; \beta$ (for more details, see Nkurunziza, [4]).

The rest of the paper is organized as follows. In Section 2, we present the statistical model and give some preliminary results. Section 3 presents the uniformly most powerful unbiased test, when the nuisance parameters are assumed to be known. In Section 4, we deal with the testing problem when these nuisance parameters are

unknown. Section 5 is Data Analysis and Simulations Studies. Finally, the conclusion is offered in Section 6.

2. Preliminaries and Statistical Model

In this section, we showcase the statistical model and set up some assumptions and notations used in this paper. To this end, let $(x(t),\ y(t))$ be the solution of ODEs (1) and let us assume that the N pairs of observations $(X_i,Y_i)_{i=1,\ 2,\ \dots\dots,N}$ are collected at discrete times $0 < t_1 < t_2 < \dots < t_N$, where $X_i \equiv X(t_i),\ Y_i \equiv Y(t_i)$. Further, these observations are generated by a process with continuous paths $\{(X(t),\ Y(t)),\ 0 \le t \le T\ \}$ satisfying

$$\log X_t = \log x(t) + e_t^X,\ \ \log Y_t = \log y(t) + e_t^Y, \tag{2}$$

here we assume that each noise component $\{(e_t^X,\ e_t^Y),\ 0 \le t \le T\}$ is Ornstein-Uhlenbeck process [13], with a particular dependence structure as described in Froda and Nkurunziza [3]. More precisely, we assume that

$$de_t^X = -c\,e_t^X\,dt + \tau\,dW_t^X,\ \ de_t^Y = -c\,e_t^Y\,dt + \tau\,dW_t^Y,\ \ c,\tau > 0, \tag{3}$$

Where $\{W_t^X,\ t \ge 0\}$ and $\{W_t^Y,\ t \ge 0\}$ are Wiener processes which satisfy Assumption (ℓ_1) as given in Froda and Nkurunziza [3]. Also, the initial random variables e_0^X and e_0^Y are assumed to satisfy assumption (ℓ_2) given in Nkurunziza [4].

As mentioned in the Introduction, the main objective is testing problem

$$H_0:\theta \le \theta_0\ \ against\ \ H_A:\theta > \theta_0. \tag{4}$$

for a given θ_0 Further, let μ_x and μ_y be the population means during a period and let $\tilde{\rho} = \rho(\gamma,\ \beta,\ \delta,\ \eta,\ x_0,\ y_0)$. We have,

$$\mu_x = \frac{1}{\tilde{\rho}}\int_s^{s+\tilde{\rho}} x(t)dt\ \ and\ \ \mu_y = \frac{1}{\tilde{\rho}}\int_s^{s+\tilde{\rho}} y(t)dt \tag{5}$$

From   (5),   we   get $(\mu_x, \mu_y) = (\delta/\gamma, \eta/\beta)$ and   therefore,

$\mu_y/\mu_x = \theta\eta/\delta$. Hence, when η, δ are fixed and known, the testing problem in (4) is equivalent to the following testing problem

$$H_0: \frac{\mu_y}{\mu_x} = \mu^\circ \leq \frac{\eta}{\delta}\theta_0 = \mu_0^\circ \ against \ H_A: \mu^\circ > \mu_0^\circ. \qquad (6)$$

In solving the testing problem in (4), we consider first the case when the nuisance parameters are known. In this case, we establish the *uniformly most powerful unbiased (UMPU)* test. Further, we consider the more realistic situation when the nuisance parameters need to be estimated. Hence, we replace these parameters by their strongly consistent estimators to obtain a test which is asymptotically as powerful as the uniformly most powerful unbiased test.

Given the testing problem (4) and the ODE (1), we note that the parameter of interest is an argument of an implicit function, which is the mean of the model (2). In establishing the UMPU we use the following theorem on a re-parametrization of the solution of ODE (1).

Theorem 1 (Nkurunziza, [4]) Let $\kappa_1 \ and \ \kappa_2$ be two positive real numbers, let (x_0, y_0) be fixed initial value and let

$(x(t; \gamma, \beta, \delta, \eta, x_0, y_0), \ y(t; \gamma, \beta, \delta, \eta, x_0, y_0))$ be a solution to the Lotka-Volterra ODE (1). Then,

$$x(t; \gamma, \beta, \delta, \eta, x_0, y_0) = \kappa_1 x\left(t; \kappa_1\gamma, \kappa_2\beta, \delta, \eta, \frac{x_0}{\kappa_1}, \frac{y_0}{\kappa_2}\right)$$

$$y(t; \gamma, \beta, \delta, \eta, x_0, y_0) = \kappa_2 y\left(t; \kappa_1\gamma, \kappa_2\beta, \delta, \eta, \frac{x_0}{\kappa_1}, \frac{y_0}{\kappa_2}\right),$$

$for \ t \geq 0, \ \eta, \ \beta, \ \gamma, \ \delta > 0.$

From Theorem (1), Nkurunziza [4] proves that, under some assumptions, the period of the populations is an intrinsic characteristic of the interacting species (see Nkurunziza, [4], Corollary 1), i.e. the period of the ODEs (1) $\rho(\gamma, \beta, \delta, \eta, x_0, y_0)$ is constant with respect to the interaction parameters (see Nkurunziza, [4]). Namely, that

holds if "there exists $(u_0, v_0) \in \Re_+^2$, fixed and that (x_0, y_0) is chosen such that,

for all $\gamma > 0,\ \beta > 0,\ x_0 = u_0 / \gamma\ and\ y_0 = v_0 / \beta$".

For the simplicity sake, we assume that (u_0, v_0) is known. In fact, (u_0, v_0)

can be replaced by its strongly consistent estimator $(\hat{u}_0, \hat{v}_0)$ (see Nkurunziza, [4]).

Further, let

$$\Gamma = (\gamma, \beta, \delta, \eta),\quad (x(t; \Gamma; x_0, y_0),\ y(t; \Gamma; x_0, y_0)) = (x(t),\ y(t)),$$
$$\xi(t) = x(t; 1, 1, \delta, \eta; u_0, v_0)\ and\ v(t) = y(t; 1, 1, \delta, \eta; u_0, v_0). \tag{7}$$

Following Theorem 1, $(x(t),\ y(t)) = \left(\dfrac{1}{\gamma} \xi(t), \dfrac{1}{\beta} v(t) \right),\ \forall t \ge 0.$ In passing,

note that Theorem 1 is a key result of this paper. Indeed, it permits to overcome the implicit parametrization inherent to the testing problem (4).

3. The uniformly most powerful unbiased test

In this section, we develop the UMPU test for the conversion efficiency parameter, θ. The composite null and alternative hypotheses are given as follows:

$$H_0: \theta \le \theta_0\ against\ H_A: \theta > \theta_0, \tag{8}$$

where θ_0 is positive and known.

In order to solve the testing problem (8), let us consider the differences

$$\Delta_x(j) = \log(X_j) - \log(\xi(j)),\quad \Delta_y(j) = \log(Y_j) - \log(v(j)),$$

where, for all $j = 1, 2, \ldots, N$, the pair (X_j, Y_j) are generated by the

stochastic model (2) and $(\xi(j), v(j))$ is given by (7). Also let $\phi = \exp(-c)$ and the

$N \times N$ matrix $\Omega = \sigma^2 (\phi^{|i-j|})_{i,\,j=1,\,2,\,\ldots,\,N}$. Further, let the vectors

$$\Delta_x = (\Delta_x(1),\, \Delta_x(2),\, \ldots,\, \Delta_x(N))',\quad \Delta_y = (\Delta_y(1),\, \Delta_y(2),\, \ldots,\, \Delta_y(N))',$$

and let the real quantities

$$\|e_N\|_\Omega^2 = e_N'\Omega^{-1}e_N, \quad \overline{\Delta}_x = (e_N'\Omega^{-1}e_N)^{-1}e_N'\Omega^{-1}\Delta_x,$$
$$\overline{\Delta}_y = (e_N'\Omega^{-1}e_N)^{-1}e_N'\Omega^{-1}\Delta_y, \tag{9}$$

where e_m is the column vector of dimension m with all entries equal to 1. Further, we have

$$\sigma^2\|e_N\|_\Omega^2 = [2 + (1-\phi)(N-2)](1+\phi)^{-1},$$
$$\overline{\Delta}_x = T_1\sigma^{-2}\|e_N\|_\Omega^{-2}, \quad \overline{\Delta}_y = T_2\sigma^{-2}\|e_N\|_\Omega^{-2}, \tag{10}$$

where

$$T_1 = \Delta_x(1) + \frac{1}{1+\phi}\sum_{j=2}^{N}(\Delta_x(j) - \phi\Delta_x(j-1))$$

$$T_2 = \Delta_y(1) + \frac{1}{1+\phi}\sum_{j=2}^{N}(\Delta_y(j) - \phi\Delta_y(j-1)). \tag{11}$$

The statistics (T_1, T_2) is complete and sufficient for the interaction parameters (γ, β) (see Nkurunziza, [4]).

Finally, let $\Phi(x)$ be the cumulative distribution function of a standard normal random variable and let I_A denote the indicator function of an event A. The following proposition gives a solution to the testing problem (4).

Proposition 1 Consider the testing problem defined in (8), at level $0 < \alpha < 1$ and suppose that $\rho = 0$, the UMPU α level test, is given by

$$\Psi = I_{\left\{(1/\sqrt{2})\|e_N\|_\Omega[\overline{\Delta}_x - \overline{\Delta}_y + \log(\theta_0)] < z_{1-\alpha}\right\}} \tag{12}$$

where $\Phi(z_\alpha) = 1 - \alpha$. The statistic $(\overline{\Delta}_x, \overline{\Delta}_y)$ is given by (9).   □

The proof follows from the application of Theorem 1 in Lehmann ([14], Chap. 5, p. 190).

In passing, note that the maximum likelihood estimator of θ is $\hat{\theta}$, such that

$$\log(\hat{\theta}) = -\overline{\Delta}_x + \overline{\Delta}_y = \frac{1+\phi}{2+(1-\phi)(N-2)}(T_1 - T_2).$$ (13)

Moreover, the statistic $(-\overline{\Delta}_x, \overline{\Delta}_y)$ is the maximum likelihood estimator of $(\log(\gamma), \log(\beta))$.

4. Asymptotic test and practical aspects

In the preceding section, we established the test function, Ψ, which is useful if the parameters, $\delta, \eta, \phi, \sigma$ are known. However, in practice, these parameters are usually unknown. Accordingly, we modify the test Ψ by replacing the parameters $\delta, \eta, \phi, \sigma$ with their strongly consistent estimators, $\hat{\delta}, \hat{\eta}, \hat{\phi}, \hat{\sigma}$. The estimators and their properties are given in Froda and Nkurunziza [3].

Let $\prod_{\Psi}(\kappa_1, \kappa_2)$ be the power of the test Ψ evaluated at the point (κ_1, κ_2) and let

$$\zeta(\phi) = \frac{1-\phi}{1+\phi}, \quad \zeta_N(\phi) = \frac{2+(1-\phi)(N-2)}{1+\phi} \quad and$$

$$\zeta_N(\hat{\phi}) = \frac{2+(1-\hat{\phi})(N-2)}{1+\hat{\phi}}.$$ (14)

Also, let a sequence of local alternatives defined as

$$H_{A;N} : \log(\theta) = \log(\theta_0) + \frac{\lambda}{\sqrt{N}}, \ \lambda \neq 0, \ N = 1, \ 2, \ldots$$

As a preliminary step, in the following proposition we prove that Ψ is consistent test.

Proposition 2 Let λ be fixed and positive real number and suppose that Proposition 1 holds. Then, uniformly in λ belonging to a fixed compact of $\mathfrak{R}^+$,

$$\lim_{N \to \infty} \prod_{\Psi}\left(\log(\theta_0) + \frac{\lambda}{\sqrt{N}}\right) = \Phi\left(z_{1-\alpha} + \frac{\sqrt{\zeta(\phi)}\lambda}{\sigma}\right).$$ □ (15)

By Proposition 2, the asymptotic power of Ψ is the right side of (15). For practical reasons, we modify the test Ψ by replacing the parameters $\delta, \eta, \phi, \sigma$ with theirs strongly consistent estimators, $\hat{\delta}, \hat{\eta}, \hat{\phi}, \hat{\sigma}$. Hence, the new test is

$$\Psi_a = I_{\left\{ \sqrt{\zeta_N(\hat{\phi})/2\hat{\sigma}^2} \left[(T_{a1} - T_{a2})/\zeta_N(\hat{\phi}) \right] + \log(\theta_0) \, < \, z_{1-\alpha} \right\}}, \tag{16}$$

where $\zeta_N(\hat{\phi})$ is given by (14). Further (T_{a1}, T_{a2}) is obtained from (11), by replacing $\delta, \eta, \phi, \sigma$ with $\hat{\delta}, \hat{\eta}, \hat{\phi}, \hat{\sigma}$. In a similar way, a plug-in estimator of θ is derived from (13), by replacing $\delta, \eta, \phi, \sigma$ with $\hat{\delta}, \hat{\eta}, \hat{\phi}, \hat{\sigma}$. Again, to simplify some computations, we assume that (u_0, v_0) is known. Nevertheless, by replacing (u_0, v_0) by its strongly consistent estimator $(\hat{u}_0, \hat{v}_0)$, we preserve the asymptotic properties of the test (see Nkurunziza, [4]).

Noting that, the proposed new test-statistic Ψ_a, is no longer the UMPU test. However, the following proposition proves that Ψ_a is indeed asymptotically as powerful as the uniformly most powerful unbiased test.

Proposition 3 Let λ be any fixed and positive real number and assume that Proposition 2 holds. Then, the asymptotic power of Ψ_a is the right side of relation (15).

5. Data Analysis and Simulations studies

In this section, we illustrate how to apply the test presented in this paper. To this end, we apply the test to the Canadian mink-muskrat data as well as to some of the generated data according to the statistical model (2). Also, we present some graphics which illustrate the consistency of the UMPU test.

5.1 Test on the Canadian Mink-Muskrat data set

This subsection illustrates the use of the test Ψ_a, on a real data set. The data set considered here is the mink-muskrat data as given in Brockwell and Davis ([15], p. 557-558). These data correspond to fur sales of the Bay of Hudson Company, in the years 1848-1912. Thus, we have 64 pairs of observations which represent the number of prey and predators recorded over 64 consecutive years.

In this data set, the prey is muskrat, whereas the predator is mink. In Bulmer [16], the author comments on the predator-prey relationship between the species listed in the Bay of Hudson Company records, and point out the fact that the muskrat cycle is due to predation by mink. Thus, the mink-muskrat is a predator-prey couple which seems to satisfy the requirement that the muskrat is the main food supply for the predator. Also, the same data has been used in Froda and Colavita [12] as well as in Froda and Nkurunziza [3] to illustrate their estimation methods.

Interestingly, Figure 1 highlights some periodicity in the data, and this is in agreement with the modeling of the Lotka-Volterra system of ODE.
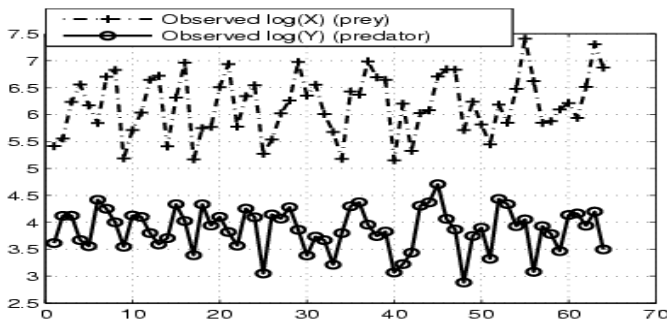

Figure 1. Observed log-populations sizes

5.1.1 Preliminary estimation and test result

In accordance with the test presented, we need to estimate firstly the parameters $(\delta, \alpha, \sigma^2, \phi, u_0, v_0)$ by using the method in Froda and Colavita [12] or in Froda and Nkurunziza [3]. Let $(\hat{\delta}, \hat{\alpha}, \hat{\sigma}^2, \hat{\phi}, \hat{u}_0, \hat{v}_0)$ be this preliminary estimate of the parameters $(\delta, \alpha, \sigma^2, \phi, u_0, v_0)$. We get

$$(\hat{\delta}, \hat{\alpha}, \hat{\sigma}^2, \hat{\phi}) = (0.538147, 0.917363, 0.3966, 0.7046),$$

and

$$(u_0, v_0) = (0.486203, 1.920740).$$

Consider testing problem $H_0: \theta \leq 1 \; against \; H_A: \theta > 1$. We have

$$\sqrt{\zeta_N(\hat{\phi})/2\hat{\sigma}^2} \left[(T_{a1} - T_{a2})/\zeta_N(\hat{\phi}) \right] + \log(\theta_0) = 11.60418$$

which gives the p-value approximately equals to 1. Therefore, the null hypothesis H_0: $\theta \leq 1$ is not rejected, at level 5%. It should be noted that, from (13), a consistency estimator of $\log(\theta)$ is given by $\log(\theta) = (-T_{a1} + T_{a2})/\zeta_N(\hat{\phi})$.

Thus, by some computations, we get -3.01572 as an estimate of $\log(\theta)$. Therefore, as an estimate of θ, we take $\exp(-3.01572) = 0.0490104 < 1$. Thus, the test result seems in agreement with the point estimation results. The following Table (1) gives some test results when θ_0 takes some values less than or equal to 0.0490104. The same Table (1) shows that when θ_0 is less than or equal to 0.03, we reject the null hypothesis H_0: $\theta \leq \theta_0$ at level 5%.

Table 1. Test results for some values of θ_0

θ_0	0.01	0.02	0.03	0.04
Z-statistic	-6.116	-3.4489	-1.8887	-0.7817
P-value	4.79681e-10	0.000281466	0.029466933	0.21719055

5.2 Test on the simulated data set

The main purpose of this subsection is to study the behavior of the power test for different sample sizes. To this end, we generate 1,000 samples of data sets of sizes 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 100. Also, in simulation studies, we take $(\gamma, \beta, \delta, \alpha, x_0, y_0) = (0.0008, 0.0206, 0.4417, 0.9760, 565.45, 78.34)$. For the nuisance parameters, σ, ρ, ϕ, we choose $(\sigma, \rho, \phi) = (0.82, 0.85, 0)$.

Figure (2) illustrates how the behavior of the power for the UMPU test is in agreement with the fact that the UMPU test is consistent.

Further, Figure (2) illustrates the behavior of the power test when the nuisance parameters σ, ρ are estimated. In particular, the consistence property is preserved, even if the sample sizes have to be relatively larger than the previous case when the nuisance parameters are known.
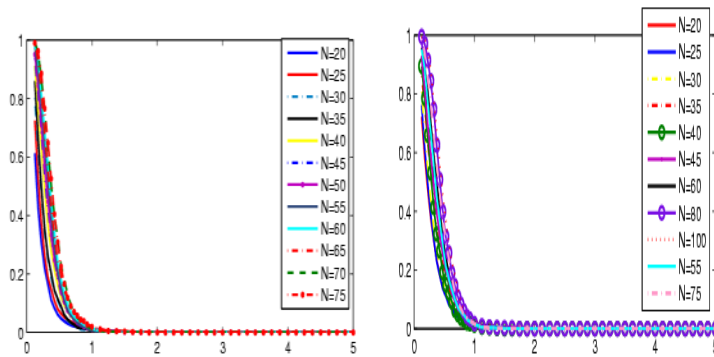
Figure 2. Simulated power by sample sizes

Our simulation experiments and numerical examples have provided strong evidence to corroborate the usual asymptotic theory related to the suggested inference method.

6. Summary and discussion

This paper deals with the inference problem concerning the variation of the conversion efficiency parameter. Methodologically, we use the stochastic model suggested in Froda and Nkurunziza [3]. As mentioned in the quoted paper, a such stochastic model has the advantage preserving both periodic behaviour and irregularities that are commonly observed in practice. In fact, many animal populations exhibit periodic behavior~(see e.g. Kendall, et al., [5], Ginzburg and Taneyhill, [6] or Royama, [7], chapters 5-6), that the trajectory of observed population sizes is not as smooth as the solution of the Lotka-Volterra ODE.

In testing problems, we derived the one-sided UMPU test for testing the conversion efficiency parameter when the nuisance parameters $(\delta, \eta, \sigma, \phi)$ are to be known. In particular, the test presented allows the ecologists to compare the interaction parameters γ and β. Furthermore, the suggested test is also useful for testing the one-sided hypothesis regarding the ratio, mean of the predator by the mean of the prey during a period of the ODEs (1).

Secondly, we extended this test to a more realistic situation. That is the case when the interaction parameters $(\delta, \eta, \sigma, \phi)$ are unknown. In this case, by replacing these parameters by their strongly consistent estimators $(\hat{\delta}, \hat{\eta}, \hat{\sigma}, \hat{\phi})$, we derive a

test which is asymptotically as powerful as the UMPU test for the known nuisance parameters case.

Finally, the most original contribution of this paper consists in the use of Theorem 1 to transform the hypothesis testing problem into some familiar problems. Basically, after this re-parametrization, we applied classical techniques in testing hypotheses, in particular, for the Gaussian case. Also, with Corollary 1, we provide a sufficient condition for the period of the ODEs (1) to be constant with respect to the interaction parameters γ and β as well as to the conversion efficiency.

Acknowledgements

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