Thailand Statistician
July 2009; 7(2) : 161-177
www.statassoc.or.th
Contributed paper

Confidence intervals for the difference of two normal population means with one variance unknown

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Received: 13 March 2009
Accepted: 17 June 2009.

Abstract

Motivated by the recent work of Maity and Sherman (2006). The two-sample t-test with one variance unknown. The American Statistician 60, 163-166], we investigate, in this paper, confidence intervals for the difference between the means of two normal populations with one variance unknown. Asymptotic coverage probabilities and expected lengths of confidence intervals are derived. Monte Carlo simulations results indicate that the new confidence interval, based on Maity and Sherman, for the difference between two normal means with one variance unknown gives a shorter expected length than that of the well-known Welch-Satterthewaite confidence interval when the ratio of their population variances is large; otherwise, the Welch-Satterthewaite confidence interval is preferable.

Keywords: confidence interval, coverage probability, expected length, normal means.

1. Introduction

Confidence intervals for the difference between two normal population means have been studied intensively. When population variances are equal, a confidence interval based on the *t*-distribution with pooled sample variances is preferable; otherwise, the Welch-Satterthwaite (WS, hereafter) confidence interval is preferable, see e.g.

Satterthwaite [1, 2], Welch [3], Miao and Chiou [4], Moser et al. [5], Schechtman and Sherman [6] and references therein. Miao and Chiou [4] mentioned that the WS, Satterthwaite [1, 2], confidence interval also performs well, based on its coverage probability, in the case of two equal population variances. A confidence interval following the resulting of a preliminary *F*-test, that the population variances are equal, has been also examined recently, see e.g. Gans [7] and Kabaila [8] and references therein. This confidence interval is constructed under the preliminary *F*-test that is calculated first to motivate the prior belief (thought not certainty) that the population variances are equal, followed by the choice between a confidence interval based on the pooled estimate of the common variance and the WS confidence interval. Confidence intervals for the difference of two means when both normality and equal variances assumptions may be violated are also considered in Miao and Chiou [4] and references therein. These authors considered three confidence intervals; WS confidence interval and two adaptive confidence intervals. They used two pretests; Shapiro-Wilk test (Shapiro and Wilk [9]), the test for normality and the *t*-test for symmetry of distributions based on Miao et al. [10]. For normality pretest they suggested using the WS confidence interval for the difference of two means when data are from a normal distribution; otherwise, they suggested transforming the data onto the scale of logarithm, then applying the WS confidence interval to the log-transformed data and finally transform the interval back to the original scale. A confidence interval following the result of the symmetric pretest statistic *t*-test is constructed similarly. In addition, they suggested that the confidence interval following the resulting of the preliminary *t*-test outperforms other confidence intervals when data are from non-normal distributions. Niwitpong and Niwitpong [11] also derived coverage probabilities and expected lengths of confidence intervals for the difference between two normal means with a known ratio of variances. They reveal that the confidence interval based on *t*-statistic of Schechtman and Sherman [6] has a shorter expected length than that of the WS confidence interval when the ratio of variances is large. In this paper, we re-examine confidence intervals for the difference of two normal population means with one variance unknown. Maity and Sherman [12] mentioned that this situation arises, for example, when one is interested in comparing a standard treatment with a new treatment. A known variance comes from the standard treatment while an unknown variance comes from the new treatment. Mighy and Sherman found that their proposed *t*-test has more power than an existing Satterthwaite's test. However, they did not investigate the coverage probability and the expected length of the confidence interval for the difference of two normal population means when one variance is unknown.

Our aim in this short paper is therefore to derive the coverage probability and its expected length of the new confidence interval compared to the well known WS confidence interval. Typically, a confidence interval with a minimum coverage probability 1-α, when α is a level of significance, and a shorter expected length is preferable.

The paper is organized as follows. Section 2 presents confidence intervals for the difference of two normal population means with one variance unknown. Coverage probabilities and expected lengths of confidence intervals in Section 2 are derived in Section 3. Section 4 gives simulation results of coverage probabilities and the ratio of expected lengths of confidence intervals for the difference between two normal population means with one variance unknown for selected samples sizes with a range of values of known ratio of variances. Section 5 contains a discussion of the results and conclusions.

2. Confidence intervals for the difference of two normal population means

Let $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_m$ be random samples from two independent normal distributions with means with means μ_x, μ_y and standard deviations σ_x and σ_y, respectively The sample means and variances for X and Y are also denoted as $\overline{X}, \overline{Y}, S_X^2$ and S_Y^2, respectively, where

$$\overline{X} = \sum_{i=1}^{n} X_i / n, \overline{Y} = \sum_{j=1}^{m} Y_j / m \quad ,$$

$$S_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (X_i - \overline{X})^2, \; S_y^2 = (m-1)^{-1} \sum_{j=1}^{m} (Y_j - \overline{Y})^2 .$$

We are interested in 100(1-α)% confidence interval for $\theta = \mu_x - \mu_y$ when one of the variance is unknown.

2.1. The Confidence interval for θ based on Welch-Satterthwaite's method

When it is assumed that $\sigma_x^2 = \sigma_y^2$, it is well-known that, by using the pivotal quantity T_1 which is

$$T_1 = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}}$$

the 100(1-α) % confidence interval for θ is

$$CI_1 = \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2,n+m-2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}, (\bar{X} - \bar{Y}) + t_{1-\alpha/2,n+m-2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}} \right]$$

where S_p^2 is known as the pooled estimate of the sample variance and is given by

$$\frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}$$

Where $t_{1-\alpha/2}$ is the $(1-\alpha/2)th$ percentile of the *t*-distribution with n+m-2 degrees of freedom. Note that T_1 has an exact *t*-distribution with n+m-2 degrees of freedom.

In the case where the two variances differ, i.e. $\sigma_x^2 \neq \sigma_y^2$, the confidence interval for θ is constructed using the pivotal quantity T_2,

$$T_2 = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}}.$$

It is well-known that T_2 is *approximately* distributed as a *t*-distribution with degrees of freedom equal

$$\nu = \frac{(A+B)}{\frac{A^2}{n-1} + \frac{B^2}{m-1}}, A = \frac{S_X^2}{n}, B = \frac{S_Y^2}{m}$$

The 100(1-α)% confidence interval for θ is therefore

$$CI_2 = \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2,\nu} \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}, (\bar{X} - \bar{Y}) + t_{1-\alpha/2,\nu} \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \right]$$

Where $t_{1-\alpha/2,\nu}$ is the (1-α /2)th percentile of T_2 distribution with ν degrees of freedom.

The confidence interval CI_2 is known as the WS confidence interval.

2.2. Confidence interval for θ with one variance unknown

Following Maity and Sherman [12], Maity and Sherman proposed the test statistic T_3,

$$T_3 = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}}},$$

of the hypothesis $H_0 : \mu_X - \mu_Y = \Delta_0$ against the alternative hypothesis $H_1 : \mu_X - \mu_Y \neq (<,>)\Delta_0$. By means of Monte Carlo simulation, they found that the test statistic T_3 has better power than the test statistic T_2. However, they did not study the confidence interval for θ using the pivotal test statistic T_3. Although the correspondence between hypothesis testing, $H_0 : \mu_X - \mu_Y = \Delta_0$ against the alternative hypothesis $H_1 : \mu_X - \mu_Y \neq (<,>)\Delta_0$, and the confidence interval estimation had been well documented, the coverage probability and the expected length of the confidence interval for θ, when one variance unknown, have not been investigated.

As a result, it is of interest to construct the confidence interval for θ when one variance is unknown. The proposed confidence interval is constructed using the pivotal quantity T_3. Maity and Sherman [12] pointed out that the test statistic T_3 has an approximate *t*-distribution with ν_1 degrees of freedom where

$$\nu_1 = \frac{\left(\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}\right)^2}{\frac{(S_Y^2/m)^2}{m-1}}.$$

We choose the $t_{1-\alpha/2,\nu_1}$, which is the $(1-\alpha/2)th$ percentile of the *t*-distribution with ν_1 degrees of freedom, such that

$$\Pr\left[(\bar{X}-\bar{Y})-t_{1-\alpha/2,\nu_1}\sqrt{\frac{\sigma_x^2}{n}+\frac{S_y^2}{m}}<\theta<(\bar{X}-\bar{Y})+t_{1-\alpha/2,\nu_1}\sqrt{\frac{\sigma_x^2}{n}+\frac{S_y^2}{m}}\right]=1-\alpha.$$

Therefore, 100(1-α)% confidence interval for θ is

$$CI_3=\left[(\bar{X}-\bar{Y})-t_{1-\alpha/2,\nu_1}\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}},(\bar{X}-\bar{Y})+t_{1-\alpha/2,\nu_1}\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}\right].$$

We evaluate these confidence intervals i.e. CI_2 and CI_3 using their coverage probabilities and expected lengths which are derived in the next section. Generally, we prefer a confidence interval with minimum coverage probability equal to a pre-specified value 1-α and with a shorter expected length.

3. Coverage probabilities and expected lengths of confidence intervals for θ with one variance unknown

In this section, the coverage probabilities and the expected lengths of confidence intervals CI_2 and CI_3 are derived.

Theorem 1. The coverage probability and the expected length of CI_2 when one variance is unknown are respectively

$E\left[\Phi(w_1)-\Phi(-w_1)\right]$ and

$$\begin{cases} 2d\zeta\sqrt{s_1}F\left[\dfrac{-1}{2},\dfrac{m-1}{2},\dfrac{m+n-2}{2},\dfrac{s_1-s_2}{s_1}\right]; if\ s_2<2s_1, \\ 2d\zeta\sqrt{s_2}F\left[\dfrac{-1}{2},\dfrac{n-1}{2},\dfrac{m+n-2}{2},\dfrac{s_2-s_1}{s_2}\right]; if\ 2s_1\le s_2 \end{cases}$$

where $w_1 = t_{1-\frac{\alpha}{2},\nu} \dfrac{\left(\frac{S_X^2}{n}+\frac{S_Y^2}{m}\right)^{1/2}}{\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}\right)^{1/2}}$, $\Phi[.]$ is the cumulative distribution function of

$N(0,1)$, $s_1 = \dfrac{\sigma_X^2}{n(n-1)}$, $s_2 = \dfrac{\sigma_Y^2}{m(m-1)}$, $\varsigma = \dfrac{\sqrt{2}\Gamma\left(\frac{m+n-1}{2}\right)}{\Gamma\left(\frac{m+n-2}{2}\right)}$,

$F(a,b,c;k)$ is the hypergeometric function defined by

$F(a,b,c;k) = 1+\dfrac{ab}{c}\dfrac{k}{1!}+\dfrac{a(a+1)b(b+1)}{c(c+1)}\dfrac{k^2}{2!}+\ldots$ and $\Gamma[.]$ is the gamma function.

Proof.

From CI_2 we have

$$1-\alpha = \Pr\left[(\bar{X}-\bar{Y})-d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}} < \mu_x-\mu_y < (\bar{X}-\bar{Y})+d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}\right],$$

$d = t_{1-\alpha/2,\nu}$

$$= \Pr\left[-\frac{d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}} < \frac{\theta-(\bar{X}-\bar{Y})}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}} < \frac{d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}}\right]$$

$$= \Pr\left[-\frac{d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}} < Z < \frac{d\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}}\right], \quad z\sim N(0,1)$$

$$= E\left[I_{\{-w_1<Z<w_1\}}(\tau) \right], I_{\{-w_1<Z<w_1\}}(\tau) = \begin{cases} 1, \tau \in \{-w_1 < Z < w_1\} \\ 0, \ otherwise \end{cases}$$

$$= E\left[E\left[I_{\{-w_1<Z<w_1\}}(\tau) \right] | S \right], S = \left(S_X^2, S_Y^2\right)'$$

$$= E\left[\Phi(w_1) - \Phi(-w_1)\right].$$

The expected length of CI_2, is

$$\left[2dE\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \right] = 2d\left(E\left(\frac{S_X^2}{n} + \frac{S_Y^2}{m} \right)^{1/2} \right)$$

$$= 2dE\left(\frac{\sigma_X^2}{n(n-1)}\left(\frac{(n-1)S_X^2}{\sigma_X^2} \right) + \frac{\sigma_Y^2}{m(m-1)}\left(\frac{(m-1)S_Y^2}{\sigma_Y^2} \right) \right)^{1/2}$$

$$= 2dE\left(\frac{\sigma_X^2}{n(n-1)}(U_1) + \frac{\sigma_Y^2}{m(m-1)}(U_2) \right)^{1/2}, \ U_1 \sim \chi_{n-1}^2 \text{ and } U_2 \sim \chi_{m-1}^2$$

$$= 2dE\left(s_1 U_1 + s_2 U_2 \right)^{1/2}$$

$$= \begin{cases} 2d\zeta\sqrt{s_1} F\left[\frac{-1}{2}, \frac{m-1}{2}, \frac{m+n-2}{2}, \frac{s_1 - s_2}{s_1} \right]; if \ s_2 < 2s_1, \\ 2d\zeta\sqrt{s_2} F\left[\frac{-1}{2}, \frac{n-1}{2}, \frac{m+n-2}{2}, \frac{s_2 - s_1}{s_2} \right]; if \ 2s_1 \le s_2. \end{cases}$$

For more details of the proof of $E\left(\frac{\sigma_X^2}{n(n-1)}(U_1) + \frac{\sigma_Y^2}{m(m-1)}(U_2) \right)^{1/2}$, see Press [13]. This ends the proof.

The following Lemma 1 is needed for the proof of Theorem 2.

Lemma 1. Define

$$f(r)=\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m(m-1)}r\right)^{1/2} \text{ for } r>0.$$

Then $f(r)$ is a concave function in r.

Proof. It is easy to see that $f''(r)=-\frac{1}{4}\left[\frac{\sigma_Y^2}{m(m-1)}\right]^2\left[\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m(m-1)}r\right]^{-3/2}<0.$

Hence $f(r)$ is a concave function in r.

Theorem 2. The coverage probability and the expected length of CI_3 with one variance unknown are respectively

$$E\left[\Phi(w_2)-\Phi(-w_2)\right] \text{ and } 2t_{1-\alpha/2,\nu_1}\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}\right)^{1/2} \text{ where}$$

$$w_2=t_{1-\alpha/2,\nu_1}\frac{\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}}$$

Proof. It is straightforward to see that the coverage probability of CI_3 is

$$1-\alpha=\Pr\left((\bar{X}-\bar{Y})-d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}\le\theta\le(\bar{X}-\bar{Y})+d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}\right),$$

$$d_1=t_{1-\alpha/2,\nu_1}$$

$$=\Pr\left(-d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}\le\theta-(\bar{X}-\bar{Y})\le d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}\right)$$

$$= \Pr\left(-d_1 \frac{\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}} \le Z \le d_1 \frac{\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}} \right)$$

$$= E\left[I_{\{-w_2<Z<w_2\}}(\tau) \right], I_{\{-w_2<Z<w_2\}}(\tau) = \begin{cases} 1, \tau \in \{-w_2 < Z < w_2\} \\ 0, \ otherwise \end{cases}$$

$$= E\left[E\left[I_{\{-w_2<Z<w_2\}}(\tau) \right] | S_Y^2 \right]$$

$$= E\left[\Phi(w_2) - \Phi(-w_2)\right].$$

The length of CI_3 is $2d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}}$ and the expected length of CI_3 is

$$E\left(2d_1\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}} \right) = 2d_1 E\left(\sqrt{\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m}} \right)$$

$$\le 2d_1\left(E\left(\frac{\sigma_X^2}{n}+\frac{S_Y^2}{m} \right) \right)^{1/2}$$, using Jensen's inequality and Lemma 1, and $E_r\left(f(r)\right) \le f(E(r))$

$$= 2d_1\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m(m-1)} E\left(\frac{(m-1)S_Y^2}{\sigma_Y^2} \right) \right)^{1/2}$$

$$= 2d_1\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m(m-1)} E\left(U_2\right) \right)^{1/2}$$, since $U_2 \sim \chi^2_{m-1}$

$$= 2d_1\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m(m-1)}(m-1) \right)^{1/2}$$, $E\left(U_2\right) = m-1$

$$= \quad 2d_1\left(\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}\right)^{1/2}.$$

This ends the proof.

4. Simulation results

In this section, we compare confidence intervals for θ via Monte Carlo simulation, using functions written in **R** in a variety of situations to see how coverage probabilities and expected lengths of confidence intervals CI_2 and CI_3, see Theorems 1-2, may depend on sample sizes and on the ratio of the variances where we fixed $\sigma_X^2 = 1$. Our simulation experiments are as follows:

Setup 1. n=5 and m=5,

Setup 2. n=10 and m=5,

Setup 3. n=10 and m=10,

Setup 4. n=20 and m=10,

Setup 5. n=20 and m=20.

Setup 6. n=40 and m=20,

Setup 7. n=40 and m=40.

We chose σ_X^2/σ_Y^2 = 0.01, 0.25, 0.5, 0.8, 1, 2, 4, 6, 8, 10, 20, 40, 50, 80, 100, $\mu_X - \mu_Y = \Delta$, and without loss of generality, we can assume $\Delta = 0$. We compare confidence intervals of CI_2 and CI_3 based on their coverage probabilities and expected lengths, with a nominal value of 0.95 throughout. Comparison of coverage probabilities of the above confidence intervals, using Theorems 1-2, for Setup 1 - Setup 7, based on 10,000 simulations, are given in Table 1. The ratio of expected lengths for each interval, using Theorems 1-2, are also given in Table 1. From Table 1, coverage probabilities of

confidence intervals, CI_2 and CI_3 are about the nominal value of 0.95. Results of ratio of expected lengths of each intervals in Table 1 show that the confidence interval CI_3 has a shorter expected length than that of the WS confidence interval CI_2 for almost cases, except when n=m=5, n=m=10 and σ_X^2 / σ_Y^2=0.01. Moreover, the ratio of expected lengths is increasing when σ_X^2 / σ_Y^2 is increasing.

5. Discussion and Conclusion

We proposed, in this paper, the confidence interval for the difference between two normal population means with one variance unknown. As in Maity and Sherman [12], our proposed confidence interval, constructed using the pivotal quantity T_3 which has different ν_1 degrees of freedom from the WS confidence interval which is constructed using the pivotal quantity T_2 with ν degrees of freedom. The results show in Table 1 against the results of a real example of Maity and Sherman [12] showed that CI_3 is wider than CI_2, they also did not show other results of a variety of situations of the ratio of σ_X^2 / σ_Y^2. In addition, Table 1 shows that, for almost cases, coverage probabilities of CI_2 and CI_3 are very close to the nominal level of 0.95 while the confidence interval CI_3 is shorter than the confidence interval CI_2 for almost cases except for case n=m=5, n=m=10 and σ_X^2 / σ_Y^2=0.01. In other words, if the unknown variance, estimated by sample variance, from the second group is large, i.e. $\sigma_Y^2 = 100$, for known fixed value of $\sigma_X^2 = 1$ confidence interval CI_2 is slightly preferable for n=m=5 and n=m=10. The ratio of expected lengths is also greater than one when σ_X^2 / σ_Y^2 is increasing for other cases of small sample sizes, i.e. n=5, 10 and m=5 and 10. Obviously from Table 1, the length of the CI_3 confidence interval is shorter than that of the WS confidence interval. For a fixed value of σ_X^2 / σ_Y^2=1 and increasing the sample size of a known group from n=5 to n=40, we found that there is a slight decrease

in the ratio of expected lengths from 1.09968 to 1.00514. In other words, the CI_3 confidence interval performs slightly better than CI_2 confidence interval for small sample sizes only.

Table 1. The coverage probabilities of CI_2 , CI_3 and $E(CI_2)/E(CI_3)$ with the number of simulation runs = 10,000.

n	m	σ_Y^2	σ_X^2/σ_Y^2	CI_2	CI_3	$E(CI_2)/E(CI_3)$
5	5	100	0.01	0.95047	0.95062	0.97577
		4	0.25	0.94792	0.93988	1.02004
		2	0.5	0.95450	0.94444	1.05159
		1.25	0.8	0.95492	0.94574	1.08228
		1	1	0.95576	0.94674	1.09968
		0.5	2	0.95387	0.94908	1.15366
		0.25	4	0.94933	0.95007	1.21057
		0.67	6	0.94864	0.95028	1.24297
		0.1	10	0.94707	0.95008	1.28328
		0.05	20	0.94738	0.95006	1.32837
		0.025	40	0.94883	0.94999	1.35932
		0.02	50	0.94890	0.95003	1.36492
		0.0125	80	0.94937	0.95000	1.37428
		0.01	100	0.94993	0.94999	1.37738
10	5	100	0.01	0.94930	0.94934	1.04328
		4	0.25	0.94466	0.94194	1.04195
		2	0.5	0.94614	0.94183	1.03809
		1.25	0.8	0.94843	0.94319	1.03703
		1	1	0.94840	0.94314	1.03758
		0.5	2	0.95286	0.94754	1.04315
		0.25	4	0.95243	0.94911	1.06014
		0.67	6	0.95241	0.94982	1.07217
		0.1	10	0.95125	0.95015	1.08591

Table 1. (continued)

n	m	σ_Y^2	σ_X^2 / σ_Y^2	CI_2	CI_3	$E(CI_2) / E(CI_3)$
		0.05	20	0.94962	0.95011	1.10175
		0.025	40	0.95027	0.94997	1.11413
		0.02	50	0.94918	0.95008	1.11695
		0.0125	80	0.95027	0.95002	1.12628
		0.01	100	0.94953	0.94999	1.12476
10	10	100	0.01	0.95114	0.95124	0.99723
		4	0.25	0.94969	0.94760	1.00104
		2	0.5	0.95115	0.94882	1.01111
		1.25	0.8	0.95225	0.94943	1.02284
		1	1	0.95142	0.94908	1.03051
		0.5	2	0.95118	0.94985	1.05524
		0.25	4	0.94941	0.95011	1.08270
		0.67	6	0.94936	0.95004	1.09824
		0.1	10	0.94877	0.95004	1.11436
		0.05	20	0.94903	0.95003	1.13050
		0.025	40	0.94899	0.94997	1.14206
		0.02	50	0.94072	0.95000	1.14488
		0.0125	80	0.94908	0.95000	1.14882
		0.01	100	0.95083	0.95000	1.15004
20	10	100	0.01	0.94998	0.94999	1.05048
		4	0.25	0.94842	0.94787	1.03760
		2	0.5	0.94943	0.94855	1.02936
		1.25	0.8	0.94980	0.94859	1.02316
		1	1	0.94931	0.94808	1.02051
		0.5	2	0.95054	0.94916	1.01628
		0.25	4	0.95077	0.94955	1.02355
		0.67	6	0.95049	0.94986	1.03005
		0.1	10	0.95029	0.94996	1.03765
		0.05	20	0.94991	0.94997	1.04611

Table 1. (continued)

n	m	σ_Y^2	σ_X^2 / σ_Y^2	CI_2	CI_3	$E(CI_2)/E(CI_3)$
20	10	0.025	40	0.94990	0.94998	1.05191
		0.02	50	0.94995	0.95000	1.05325
		0.0125	80	0.95025	0.95000	1.05535
		0.01	100	0.95010	0.94999	1.05614
20	20	100	0.01	0.94944	0.94943	1.00692
		4	0.25	0.94988	0.94950	1.00166
		2	0.5	0.94995	0.94950	1.00361
		1.25	0.8	0.95038	0.94949	1.00829
		1	1	0.95024	0.94990	1.01174
		0.5	2	0.94996	0.94986	1.02458
		0.25	4	0.95038	0.95006	1.04019
		0.67	6	0.94932	0.94995	1.04876
		0.1	10	0.95014	0.94999	1.05763
		0.05	20	0.94968	0.95005	1.06578
		0.025	40	0.95037	0.94999	1.07052
		0.02	50	0.95032	0.94999	1.07154
		0.0125	80	0.94999	0.95000	1.07353
		0.01	100	0.95070	0.94999	1.07438
40	20	100	0.01	0.94979	0.94979	1.05411
		4	0.25	0.94977	0.94970	1.03991
		2	0.5	0.94963	0.94939	1.02954
		1.25	0.8	0.94960	0.94927	1.02111
		1	1	0.95040	0.95004	1.01710
		0.5	2	0.95028	0.94982	1.00847
		0.25	4	0.95006	0.95007	1.01040
		0.67	6	0.94992	0.94998	1.01384
		0.1	10	0.95021	0.95001	1.01800
		0.05	20	0.94991	0.94998	1.02273
		0.025	40	0.94996	0.94998	1.02593

Table 1. (continued)

n	m	σ_Y^2	σ_X^2 / σ_Y^2	CI_2	CI_3	$E(CI_2) / E(CI_3)$
40	20	0.02	50	0.94999	0.94998	1.02665
		0.0125	80	0.94999	0.95001	1.02779
		0.01	100	0.94980	0.95000	1.02819
40	40	100	0.01	0.94995	0.94994	1.01153
		4	0.25	0.94994	0.94978	1.00326
		2	0.5	0.95018	0.94991	1.00187
		1.25	0.8	0.94988	0.94997	1.00338
		1	1	0.94967	0.94992	1.00514
		0.5	2	0.95017	0.95003	1.01214
		0.25	4	0.95020	0.94993	1.02206
		0.67	6	0.94944	0.94999	1.02748
		0.1	10	0.94990	0.94997	1.03315
		0.05	20	0.94967	0.94998	1.03831
		0.025	40	0.95041	0.94998	1.04127
		0.02	50	0.94984	0.95000	1.04190
		0.0125	80	0.94974	0.94999	1.04280
		0.01	100	0.95042	0.94999	1.04308

Acknowledgements

The author thanks the three anonymous referees for their constructive suggestions and comments that resulted in an improved present of the paper.

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