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# Prediction Intervals for an Unknown Mean Gaussian Autoregressive Process Using the Residual Model

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**Abstract**

This paper presents a new one-step-ahead prediction interval for an unknown mean Gaussian autoregressive process (AR(p)) using the residual model. The proposed prediction interval is constructed by adding the multiplication of the correction factor and the percentile of sample errors in the estimated point forecast for an AR(p) process. The coverage probabilities of a new prediction interval and a standard prediction interval are also derived to be functionally independent of the population mean and the variance of the innovation process. The Monte Carlo simulation is used to investigate the behavior of this new prediction interval compared to the existing prediction interval based on their coverage probabilities and expected lengths. Simulation results have shown that almost cases of the new prediction interval have desired minimum coverage probabilities of 0.95 and 0.90. Moreover, this new one is better than a standard prediction interval for all the autoregressive parameter values and all sample sizes considered in this paper.

**Keywords:** AR(p), coverage probability, expected length, prediction interval, residual model.

## 1. Introduction

Recently, there has been increasing interest in constructing prediction intervals for an autoregressive process, see for example the standard textbooks of Box et al. [1] and Wei [2]. The conditional prediction interval for an autoregressive moving average model was proposed by de Luna [3]. However, this prediction interval does not have minimum coverage probability at the nominal confidence level for small and moderate sample sizes. Several authors also used the bootstrap methods for calculating prediction intervals, see for example Kim [4-6], Alonso, et al. [7] and Clements and Kim [8] and references therein. Although prediction intervals based on bootstrap methods have more coverage probability than $1-\alpha$, these methods are taken much time to calculate. Halkos and Kevork [9] proposed a prediction interval for a stationary AR(1) process with an almost unit root by considering the random walk as the true model. They found that their proposed prediction interval has less coverage probability than the nominal confidence level when $\phi_1$ is close to one. In this paper, we emphasis constructing simple prediction intervals, based on the residual model described by Olive [10], for an unknown mean Gaussian autoregressive process (AR(p)). In addition, we have derived coverage probabilities of this new prediction intervals and a standard prediction interval for an AR(p) process.

The plan of the paper is as follows. In Section 2, a standard prediction interval for an unknown mean Gaussian AR(p) process is presented. Section 3 proposes a new prediction interval for an unknown mean Gaussian AR(p) process using the residual model. The Monte Carlo simulation results are given in Section 4. The conclusion is presented in Section 5.

## 2. Standard Prediction Interval for an Unknown Mean Gaussian AR(p) Process

Suppose $\{Y_t; t=1,2,3,...,T\}$ is an unknown mean Gaussian AR(p) process satisfying

$$Y_t - \mu = \phi_1(Y_{t-1} - \mu) + ... + \phi_p(Y_{t-p} - \mu) + e_t \tag{1}$$

where $\mu$ is the population mean, $\phi_1,...,\phi_p$ are the autoregressive parameters and $e_t$ are unobservable independent errors having zero mean and finite variance, $e_t \sim N(0,\sigma_e^2)$. The equation (1) can be written symbolically in a compact form

$$\phi(B)(Y_t - \mu) = e_t \tag{2}$$

where $B$ is the back-shift operator defined by $B^j Y_t = Y_{t-j}$ and $\phi(\cdot)$ is the $p^{th}$ degree polynomials in $B$ such that $\phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p$ . The process in (2) is stationary when the roots of $\phi(z) = 0$ is outside the unit circle.

Now we present the standard prediction interval for $Y_{T+1}$ based on data $Y_1, Y_2, ..., Y_T$. It is well-known that, for known $(\mu, \phi_1, ..., \phi_p, \sigma_e)$, the optimal predictor in the mean square sense, is $\mu + (\phi_1 B + ... + \phi_p B^p)(Y_{T+1} - \mu)$ . Replacing the unknown $(\mu, \phi_1, ..., \phi_p, \sigma_e)$ by the estimators $(\hat{\mu}, \hat{\phi}_1, ..., \hat{\phi}_p, \hat{\sigma}_e)$ . The approximate $1-\alpha$ standard prediction interval for $Y_{T+1}$ is

$$PI_S \quad = \quad \left[ \hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) \pm Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e \right] \tag{3}$$

where $Z_{1-\frac{\alpha}{2}}$ is a $\left(1-\dfrac{\alpha}{2}\right)th$ quantile of the standard normal distribution, $\hat{\mu} = \overline{Y}$ , $\hat{\phi}_1, ..., \hat{\phi}_p$ are the ordinary least squares (OLS) estimators and

$$\hat{\sigma}_e^2 = \frac{\sum_{t=p+1}^{T} (Y_t - \overline{Y} - \hat{\phi}_1(Y_{t-1} - \overline{Y}) - \cdots - \hat{\phi}_p(Y_{t-p} - \overline{Y}))^2}{T - p - 1} . \tag{4}$$

In Theorem 1 below, we show the coverage probability of $PI_S$ .

**Theorem 1.** Suppose, from (1), $e_{T+1} \sim N(0, \sigma_e^2)$ , the coverage probability of $PI_S$ in (3), $P(Y_{T+1} \in PI_S)$ , is $\Delta_1 - \Delta_2$ where

$$\Delta_1 = \Phi\left( \frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left( \frac{Y_{T+1} - \hat{\mu}}{\sigma_e} \right) + Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left( \frac{Y_{T+1} - \mu}{\sigma_e} \right) \right),$$

$$\Delta_2 = \Phi\left( \frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left( \frac{Y_{T+1} - \hat{\mu}}{\sigma_e} \right) - Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left( \frac{Y_{T+1} - \mu}{\sigma_e} \right) \right)$$

and $\Phi(\cdot)$ is the standard normal distribution function.

**Proof of Theorem 1.**

$P(Y_{T+1} \in PI_S)$

$$= P\left[ \hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) - Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e \le Y_{T+1} \le \hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e \right]$$

$$= P\left[\begin{array}{l}\hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) - Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e \le \mu + \phi_1(Y_T - \mu) + \ldots + \phi_p(Y_{T+1-p} - \mu) + e_{T+1} \le \\ \hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e\end{array}\right]$$

$$= P\left[\begin{array}{l}\hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) - Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e \le \mu + (\phi_1 B + \ldots + \phi_p B^p)(Y_{T+1} - \mu) + e_{T+1} \le \\ \hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e\end{array}\right]$$

$$= P\left[\begin{array}{l}\hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) - Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e - \mu - (\phi_1 B + \ldots + \phi_p B^p)(Y_{T+1} - \mu) \le e_{T+1} \le \\ \hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + Z_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_e - \mu - (\phi_1 B + \ldots + \phi_p B^p)(Y_{T+1} - \mu)\end{array}\right]$$

$$= P\left[\begin{array}{l}\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) - Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + \ldots + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right) \le \frac{e_{T+1}}{\sigma_e} \le \\ \frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + \ldots + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\end{array}\right]$$

$$\begin{aligned}&= \Phi\left(\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + \ldots + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\right) \\ &\quad - \Phi\left(\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) - Z_{1-\frac{\alpha}{2}} \cdot \frac{\hat{\sigma}_e}{\sigma_e} - \frac{\mu}{\sigma_e} - (\phi_1 B + \ldots + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\right)\end{aligned}$$

## 3. Prediction Interval for an Unknown Mean Gaussian AR(p) Process Using the Residual Model

Our aim for model (1) is to construct the prediction interval for $Y_{T+1}$ based on data $Y_1, Y_2, ..., Y_T$. It is well-known that, for known $(\mu, \phi_1, \phi_2, ..., \phi_p)$, the optimal predictor, in the mean square sense, for model (1) is $\mu + (\phi_1 B + ... + \phi_p B^p)(Y_{T+1} - \mu)$ Replacing the unknown $(\mu, \phi_1, \phi_2, ..., \phi_p)$ in this expression by estimators $(\hat{\mu}, \hat{\phi}_1, \hat{\phi}_2, ..., \hat{\phi}_p)$, we obtain the predictor $\hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu})$. The approximate $(1-\alpha)100\%$ prediction interval for $Y_{T+1}$ of model (1) based on the residual model, described by Olive [10], is given by

$$PI_R = \left[\ \hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + a_T \cdot \hat{\xi}_{\frac{\alpha}{2}}\ ,\ \hat{\mu} + (\hat{\phi}_1 B + \ldots + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + a_T \cdot \hat{\xi}_{1-\frac{\alpha}{2}}\ \right] \qquad (5)$$

where $\hat{\mu}=\overline{Y}$, $\hat{\phi}_1,...,\hat{\phi}_p$ are the ordinary least squares (OLS) estimators ,the residual of model (1) is $Y_t-\hat{Y}_t$ and $\hat{\xi}_{\frac{\alpha}{2}}$ is the sample $\alpha/2$ percentile of the residuals from model (1) and $a_T$ is the correction factor. Define $a_T=\left(1+\frac{k}{T}\right)\sqrt{\frac{T}{T-p}}\sqrt{1+h_f}$ where

$$h_f=x_f^T(X^TX)^{-1}x_f\ ,\quad x_f=\begin{bmatrix}1\\ Y_T\\ Y_{T-1}\\ \vdots\\ Y_{T-p+1}\end{bmatrix},\quad X=\begin{bmatrix}1 & Y_p & Y_{p-1} & \cdots & Y_1\\ 1 & Y_{p+1} & Y_p & \cdots & Y_2\\ \vdots & \vdots & \vdots & & \vdots\\ 1 & Y_{T-1} & Y_{T-2} & \cdots & Y_{T-p}\end{bmatrix}$$

and $k$ is a constant. The coverage probability of $PI_R$ is found in Theorem 2.

**Theorem 2.** Suppose, from (1), $e_{T+1}\sim N(0,\sigma_e^2)$, the coverage probability of $PI_R$ in (5), $P(Y_{T+1}\in PI_R)$, is $\Delta_3-\Delta_4$ where

$$\Delta_3=\Phi\left(\frac{\hat{\mu}}{\sigma_e}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)\left(\frac{Y_{T+1}-\hat{\mu}}{\sigma_e}\right)+\frac{a_T}{\sigma_e}\cdot\hat{\xi}_{1-\frac{\alpha}{2}}-\frac{\mu}{\sigma_e}-(\phi_1B+...+\phi_pB^p)\left(\frac{Y_{T+1}-\mu}{\sigma_e}\right)\right),$$

$$\Delta_4=\Phi\left(\frac{\hat{\mu}}{\sigma_e}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)\left(\frac{Y_{T+1}-\hat{\mu}}{\sigma_e}\right)+\frac{a_T}{\sigma_e}\cdot\hat{\xi}_{\frac{\alpha}{2}}-\frac{\mu}{\sigma_e}-(\phi_1B+...+\phi_pB^p)\left(\frac{Y_{T+1}-\mu}{\sigma_e}\right)\right)$$

**Proof of Theorem 2.**

$P(Y_{T+1}\in PI_R)$

$$=P\left[\hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{\frac{\alpha}{2}}\le Y_{T+1}\le\hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{1-\frac{\alpha}{2}}\right]$$

$$=P\begin{bmatrix}\hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{\frac{\alpha}{2}}\le\mu+\phi_1(Y_T-\mu)+...+\phi_p(Y_{T+1-p}-\mu)+e_{T+1}\le\\ \hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{1-\frac{\alpha}{2}}\end{bmatrix}$$

$$=P\begin{bmatrix}\hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{\frac{\alpha}{2}}\le\mu+(\phi_1B+...+\phi_pB^p)(Y_{T+1}-\mu)+e_{T+1}\le\\ \hat{\mu}+(\hat{\phi}_1B+...+\hat{\phi}_pB^p)(Y_{T+1}-\hat{\mu})+a_T\cdot\hat{\xi}_{1-\frac{\alpha}{2}}\end{bmatrix}$$

$$= P\left[\begin{array}{l}\hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + a_T \cdot \hat{\xi}_{\frac{\alpha}{2}} - \mu - (\phi_1 B + ... + \phi_p B^p)(Y_{T+1} - \mu) \le e_{T+1} \le \\ \hat{\mu} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)(Y_{T+1} - \hat{\mu}) + a_T \cdot \hat{\xi}_{1-\frac{\alpha}{2}} - \mu - (\phi_1 B + ... + \phi_p B^p)(Y_{T+1} - \mu)\end{array}\right]$$

$$= P\left[\begin{array}{l}\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + \frac{a_T}{\sigma_e} \cdot \hat{\xi}_{\frac{\alpha}{2}} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right) \le \frac{e_{T+1}}{\sigma_e} \le \\ \frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + \frac{a_T}{\sigma_e} \cdot \hat{\xi}_{1-\frac{\alpha}{2}} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\end{array}\right]$$

$$= \Phi\left(\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + \frac{a_T}{\sigma_e} \cdot \hat{\xi}_{1-\frac{\alpha}{2}} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\right)$$
$$- \Phi\left(\frac{\hat{\mu}}{\sigma_e} + (\hat{\phi}_1 B + ... + \hat{\phi}_p B^p)\left(\frac{Y_{T+1} - \hat{\mu}}{\sigma_e}\right) + \frac{a_T}{\sigma_e} \cdot \hat{\xi}_{\frac{\alpha}{2}} - \frac{\mu}{\sigma_e} - (\phi_1 B + ... + \phi_p B^p)\left(\frac{Y_{T+1} - \mu}{\sigma_e}\right)\right)$$

In the next section, we present the simulation results, using Monte Carlo simulation, to estimate coverage probabilities and expected lengths of the prediction intervals (3) and (5) using the results of Theorems 1-2.

## 4. Monte Carlo Simulation

In this section, we report the results of using the Monte Carlo simulation to investigate the estimated coverage probabilities of the prediction intervals (3), (5) and their expected lengths. We used R program to generate the data from an AR(3) process in model (1) with parameters $(\mu, \sigma_e^2)$ = (0, 1), sample sizes; T = 25, 50, 100 and 250, The parameter values of the autoregressive process are chosen to include a wide range of real roots of the equation $1 - \phi_1 B - \cdots - \phi_p B^p = 0$. The number of simulation runs, M = 10,000 at level of significance $\alpha$ = 0.05 and 0.10. The constant number $k$ in $a_T$ has been selected so that coverage probabilities of the prediction intervals in (5) are at least $1-\alpha$. We found, using the Monte Carlo simulation, that $k$ = 7 and 9 are good choices for $\alpha$ = 0.10 and 0.05 respectively. Tables 1-2 show estimated coverage probabilities of the prediction intervals (3) and (5), $PI_S$ and $PI_R$, and their expected lengths for an AR(3) process at $\alpha$ = 0.05 and 0.10, respectively. As can be seen from Tables 1-2 and Figures 1-2, almost cases of the new prediction interval, $PI_R$, has a minimum estimated coverage probability $1-\alpha$, for all sample sizes and values of $(\phi_1, \phi_2, \phi_3)$ considered here. Consequently, the expected lengths of $PI_R$ are longer than that of $PI_S$ for all sample sizes since the prediction interval $PI_R$ has more coverage probabilities than $PI_S$.

## 5. Conclusion

We have proposed a new one-step-ahead prediction interval for an unknown mean Gaussian autoregressive process (AR(p)). The standard prediction Interval and the prediction interval based on the residual model are proposed in this study. The prediction interval based on the residual model performs better than the standard prediction Interval in terms of the coverage probability. Therefore, if we prefer a prediction interval with minimum coverage probability equal to a pre-specified value $1-\alpha$, the prediction interval based on the residual model is preferable to the standard prediction Interval.

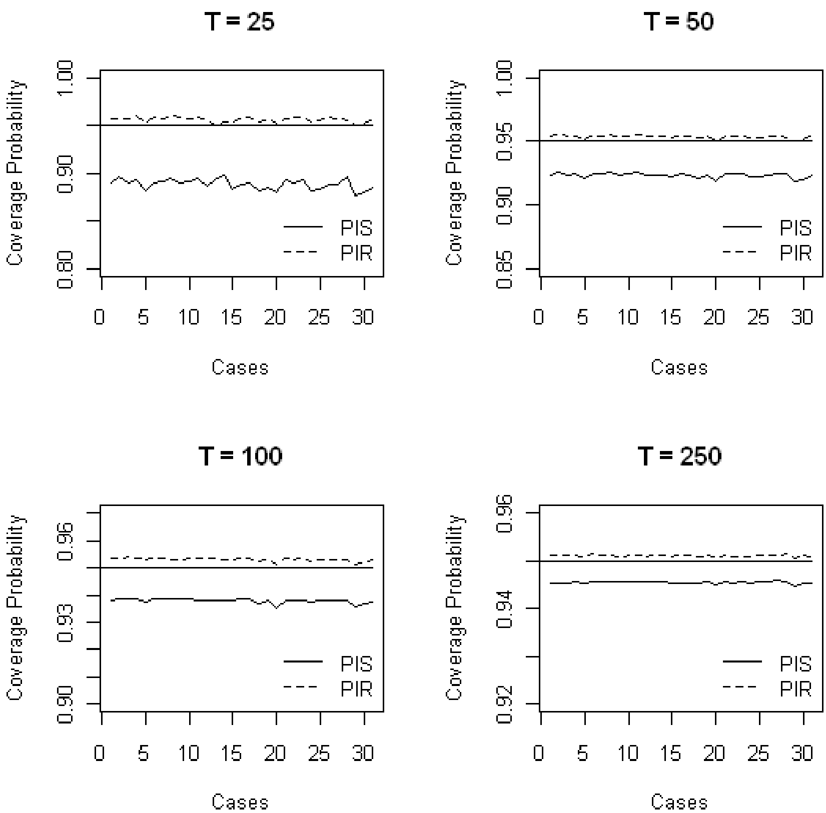


**Figure 1.** Comparison of the coverage probabilities of the prediction interval $PI_S$ and $PI_R$ where $\alpha$ = 0.05.

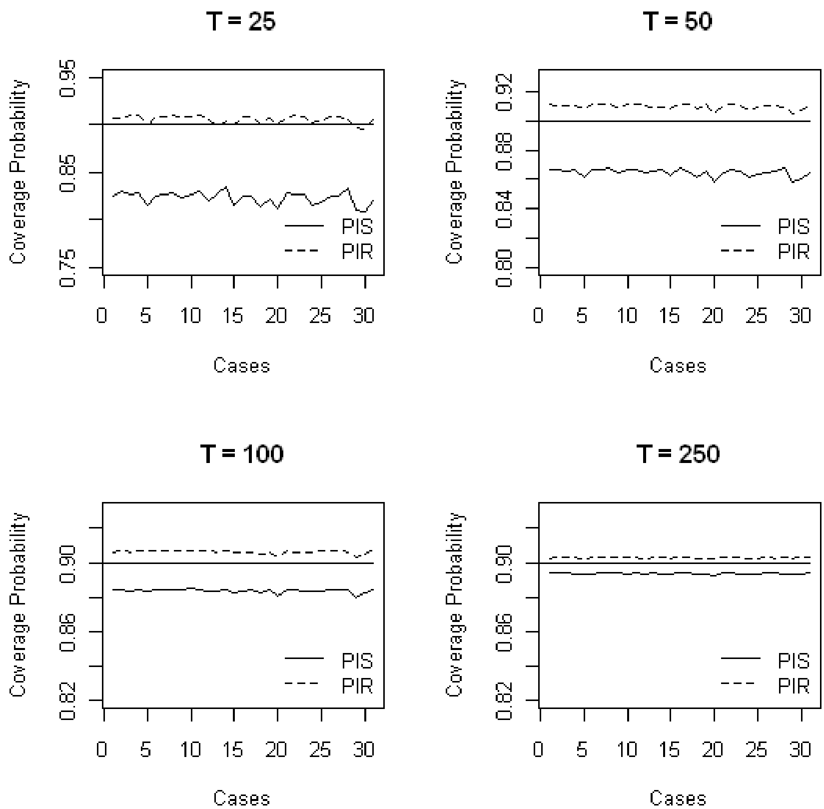


**Figure 2.** Comparison of the coverage probabilities of the prediction interval $PI_S$ and $PI_R$ where $\alpha$ = 0.10.

**Table1.** The estimated coverage probabilities and expected lengths of a 95% one-step-ahead prediction interval for AR(3) when $M = 10,000$, $\mu$ = 0 and $\sigma_e^2$ = 1.

| T | $(\phi_1, \phi_2, \phi_3)$ | Coverage probability | | Expected length | |
|---|---|---|---|---|---|
| | | $PI_S$ | $PI_R$ | $PI_S$ | $PI_R$ |
| 25 | (0, -0.65, 0.5) | 0.8890 | 0.9562 | 3.6422 | 4.9908 |
| | (0, -0.65, -0.5) | 0.8965 | 0.9571 | 3.7070 | 5.0063 |
| | (-0.4, -0.15, 0.3) | 0.8892 | 0.9574 | 3.6183 | 4.9836 |
| | (-0.4, -0.15, -0.3) | 0.8929 | 0.9597 | 3.6390 | 5.0117 |
| | (0.4, 0.15, 0.3) | 0.8817 | 0.9527 | 3.5686 | 4.9409 |
| | (0.4, 0.15, -0.3) | 0.8905 | 0.9586 | 3.6288 | 5.0097 |
| | (-0.8, -0.65, 0.2) | 0.8921 | 0.9577 | 3.6970 | 5.0166 |
| | (-0.8, -0.65, -0.2) | 0.8941 | 0.9596 | 3.6631 | 5.0141 |
| | (0.8, -0.65, 0.2) | 0.8890 | 0.9582 | 3.6137 | 4.9990 |
| | (0.8, -0.65, -0.2) | 0.8914 | 0.9572 | 3.6479 | 5.0087 |
| | (-0.95,-0.9,-0.5) | 0.8952 | 0.9586 | 3.6996 | 5.0207 |
| | (0.95, -0.9, 0.5) | 0.8864 | 0.9564 | 3.6067 | 5.0003 |
| | (1.75, -0.76, -0.1) | 0.8923 | 0.9510 | 3.7043 | 4.9718 |
| | (1.2, -0.3, -0.4) | 0.8972 | 0.9531 | 3.8029 | 4.9482 |
| | (0.8, -0.16, 0.2) | 0.8828 | 0.9535 | 3.5975 | 4.9905 |
| | (0.8, -0.16, -0.2) | 0.8886 | 0.9579 | 3.6153 | 4.9979 |
| | (0.55, 0.4, -0.5) | 0.8902 | 0.9580 | 3.6370 | 5.0270 |
| | (0.4, 0.3, 0.15) | 0.8813 | 0.9531 | 3.5846 | 4.9768 |
| | (0.4, 0.3, -0.15) | 0.8856 | 0.9566 | 3.6010 | 4.9961 |
| | (0, 0.15, 0.8) | 0.8802 | 0.9523 | 3.5719 | 4.9313 |
| | (0, 0.15, -0.8) | 0.8929 | 0.9568 | 3.6604 | 4.9994 |
| | (-0.8, -0.16, 0.2) | 0.8900 | 0.9579 | 3.6271 | 4.9988 |
| | (-0.8, -0.16, -0.2) | 0.8935 | 0.9586 | 3.6559 | 5.0068 |
| | (0, 0.65, 0.1) | 0.8821 | 0.9531 | 3.5834 | 4.9711 |
| | (0, 0.65, -0.1) | 0.8832 | 0.9546 | 3.5832 | 4.9765 |
| | (-0.4, 0.3, 0.15) | 0.8886 | 0.9583 | 3.6174 | 5.0103 |
| | (-0.4, 0.3, -0.15) | 0.8887 | 0.9575 | 3.6177 | 4.9951 |
| | (-1.2, -0.3, 0.4) | 0.8966 | 0.9558 | 3.7219 | 4.9765 |
| | (0, 0.9, 0.05) | 0.8767 | 0.9508 | 3.5635 | 4.9328 |
| | (0, 0.9, -0.05) | 0.8795 | 0.9515 | 3.5780 | 4.9580 |
| | (-0.55, 0.4, 0.5) | 0.8856 | 0.9561 | 3.5963 | 4.9837 |
| 50 | (0, -0.65, 0.5) | 0.9239 | 0.9542 | 3.7780 | 4.4235 |
| | (0, -0.65, -0.5) | 0.9258 | 0.9549 | 3.8095 | 4.4270 |
| | (-0.4, -0.15, 0.3) | 0.9237 | 0.9540 | 3.7700 | 4.4076 |
| | (-0.4, -0.15, -0.3) | 0.9244 | 0.9544 | 3.7792 | 4.4209 |
| | (0.4, 0.15, 0.3) | 0.9208 | 0.9521 | 3.7570 | 4.4041 |
| | (0.4, 0.15, -0.3) | 0.9248 | 0.9545 | 3.7778 | 4.4139 |
| | (-0.8, -0.65, 0.2) | 0.9248 | 0.9545 | 3.7907 | 4.4194 |
| | (-0.8, -0.65, -0.2) | 0.9255 | 0.9551 | 3.7911 | 4.4282 |
| | (0.8, -0.65, 0.2) | 0.9239 | 0.9543 | 3.7685 | 4.4119 |
| | (0.8, -0.65, -0.2) | 0.9251 | 0.9547 | 3.7921 | 4.4285 |
| | (-0.95,-0.9,-0.5) | 0.9257 | 0.9554 | 3.7989 | 4.4278 |
| | (0.95, -0.9, 0.5) | 0.9234 | 0.9538 | 3.7728 | 4.4113 |
| | (1.75, -0.76, -0.1) | 0.9239 | 0.9538 | 3.8001 | 4.4258 |
| | (1.2, -0.3, -0.4) | 0.9241 | 0.9539 | 3.7894 | 4.4183 |
| | (0.8, -0.16, 0.2) | 0.9226 | 0.9531 | 3.7714 | 4.4230 |
| | (0.8, -0.16, -0.2) | 0.9243 | 0.9546 | 3.7790 | 4.4241 |
| | (0.55, 0.4, -0.5) | 0.9235 | 0.9542 | 3.7672 | 4.4141 |
| | (0.4, 0.3, 0.15) | 0.9210 | 0.9526 | 3.7609 | 4.4085 |
| | (0.4, 0.3, -0.15) | 0.9233 | 0.9540 | 3.7712 | 4.4197 |
| | (0, 0.15, 0.8) | 0.9183 | 0.9508 | 3.7477 | 4.3900 |
| | (0, 0.15, -0.8) | 0.9242 | 0.9543 | 3.7818 | 4.4237 |
| | (-0.8, -0.16, 0.2) | 0.9249 | 0.9548 | 3.7784 | 4.4196 |
| | (-0.8, -0.16, -0.2) | 0.9249 | 0.9548 | 3.7815 | 4.4228 |
| | (0, 0.65, 0.1) | 0.9220 | 0.9531 | 3.7642 | 4.4110 |
| | (0, 0.65, -0.1) | 0.9228 | 0.9535 | 3.7650 | 4.4122 |
| | (-0.4, 0.3, 0.15) | 0.9235 | 0.9540 | 3.7712 | 4.4130 |
| | (-0.4, 0.3, -0.15) | 0.9242 | 0.9544 | 3.7729 | 4.4151 |
| | (-1.2, -0.3, 0.4) | 0.9248 | 0.9535 | 3.8154 | 4.4159 |
| | (0, 0.9, 0.05) | 0.9184 | 0.9504 | 3.7494 | 4.3939 |
| | (0, 0.9, -0.05) | 0.9198 | 0.9515 | 3.7600 | 4.4153 |
| | (-0.55, 0.4, 0.5) | 0.9240 | 0.9542 | 3.7758 | 4.4177 |

**Table1.** (continued)

| T | $(\phi_1, \phi_2, \phi_3)$ | Coverage probability | | Expected length | |
|---|---|---|---|---|---|
| | | $PI_S$ | $PI_R$ | $PI_S$ | $PI_R$ |
| 100 | (0, -0.65, 0.5) | 0.9382 | 0.9533 | 3.8515 | 4.1789 |
| | (0, -0.65, -0.5) | 0.9384 | 0.9533 | 3.8595 | 4.1789 |
| | (-0.4, -0.15, 0.3) | 0.9388 | 0.9539 | 3.8557 | 4.1837 |
| | (-0.4, -0.15, -0.3) | 0.9384 | 0.9534 | 3.8510 | 4.1736 |
| | (0.4, 0.15, 0.3) | 0.9375 | 0.9531 | 3.8518 | 4.1815 |
| | (0.4, 0.15, -0.3) | 0.9385 | 0.9534 | 3.8511 | 4.1743 |
| | (-0.8, -0.65, 0.2) | 0.9384 | 0.9534 | 3.8563 | 4.1790 |
| | (-0.8, -0.65, -0.2) | 0.9384 | 0.9532 | 3.8504 | 4.1692 |
| | (0.8, -0.65, 0.2) | 0.9384 | 0.9531 | 3.8525 | 4.1725 |
| | (0.8, -0.65, -0.2) | 0.9385 | 0.9536 | 3.8541 | 4.1794 |
| | (-0.95,-0.9,-0.5) | 0.9383 | 0.9533 | 3.8556 | 4.1760 |
| | (0.95, -0.9, 0.5) | 0.9379 | 0.9533 | 3.8474 | 4.1747 |
| | (1.75, -0.76, -0.1) | 0.9382 | 0.9534 | 3.8570 | 4.1853 |
| | (1.2, -0.3, -0.4) | 0.9381 | 0.9529 | 3.8540 | 4.1711 |
| | (0.8, -0.16, 0.2) | 0.9380 | 0.9532 | 3.8515 | 4.1797 |
| | (0.8, -0.16, -0.2) | 0.9386 | 0.9535 | 3.8537 | 4.1745 |
| | (0.55, 0.4, -0.5) | 0.9384 | 0.9533 | 3.8533 | 4.1783 |
| | (0.4, 0.3, 0.15) | 0.9371 | 0.9524 | 3.8438 | 4.1709 |
| | (0.4, 0.3, -0.15) | 0.9382 | 0.9530 | 3.8526 | 4.1749 |
| | (0, 0.15, 0.8) | 0.9354 | 0.9513 | 3.8339 | 4.1625 |
| | (0, 0.15, -0.8) | 0.9381 | 0.9534 | 3.8523 | 4.1812 |
| | (-0.8, -0.16, 0.2) | 0.9380 | 0.9530 | 3.8464 | 4.1685 |
| | (-0.8, -0.16, -0.2) | 0.9381 | 0.9533 | 3.8489 | 4.1764 |
| | (0, 0.65, 0.1) | 0.9373 | 0.9526 | 3.8424 | 4.1676 |
| | (0, 0.65, -0.1) | 0.9378 | 0.9528 | 3.8465 | 4.1712 |
| | (-0.4, 0.3, 0.15) | 0.9380 | 0.9531 | 3.8479 | 4.1726 |
| | (-0.4, 0.3, -0.15) | 0.9381 | 0.9531 | 3.8485 | 4.1728 |
| | (-1.2, -0.3, 0.4) | 0.9383 | 0.9531 | 3.8649 | 4.1757 |
| | (0, 0.9, 0.05) | 0.9358 | 0.9513 | 3.8361 | 4.1628 |
| | (0, 0.9, -0.05) | 0.9369 | 0.9526 | 3.8450 | 4.1752 |
| | (-0.55, 0.4, 0.5) | 0.9377 | 0.9529 | 3.8467 | 4.1755 |
| 250 | (0, -0.65, 0.5) | 0.9454 | 0.9511 | 3.8910 | 4.0200 |
| | (0, -0.65, -0.5) | 0.9454 | 0.9511 | 3.8924 | 4.0195 |
| | (-0.4, -0.15, 0.3) | 0.9454 | 0.9511 | 3.8914 | 4.0193 |
| | (-0.4, -0.15, -0.3) | 0.9455 | 0.9512 | 3.8908 | 4.0187 |
| | (0.4, 0.15, 0.3) | 0.9454 | 0.9510 | 3.8913 | 4.0171 |
| | (0.4, 0.15, -0.3) | 0.9457 | 0.9514 | 3.8942 | 4.0218 |
| | (-0.8, -0.65, 0.2) | 0.9456 | 0.9512 | 3.8939 | 4.0189 |
| | (-0.8, -0.65, -0.2) | 0.9456 | 0.9511 | 3.8932 | 4.0181 |
| | (0.8, -0.65, 0.2) | 0.9455 | 0.9510 | 3.8911 | 4.0157 |
| | (0.8, -0.65, -0.2) | 0.9457 | 0.9513 | 3.8953 | 4.0222 |
| | (-0.95,-0.9,-0.5) | 0.9455 | 0.9511 | 3.8925 | 4.0200 |
| | (0.95, -0.9, 0.5) | 0.9455 | 0.9510 | 3.8920 | 4.0163 |
| | (1.75, -0.76, -0.1) | 0.9456 | 0.9512 | 3.8930 | 4.0223 |
| | (1.2, -0.3, -0.4) | 0.9456 | 0.9513 | 3.8968 | 4.0234 |
| | (0.8, -0.16, 0.2) | 0.9454 | 0.9510 | 3.8920 | 4.0180 |
| | (0.8, -0.16, -0.2) | 0.9454 | 0.9511 | 3.8901 | 4.0182 |
| | (0.55, 0.4, -0.5) | 0.9454 | 0.9511 | 3.8909 | 4.0194 |
| | (0.4, 0.3, 0.15) | 0.9454 | 0.9510 | 3.8909 | 4.0182 |
| | (0.4, 0.3, -0.15) | 0.9456 | 0.9511 | 3.8939 | 4.0186 |
| | (0, 0.15, 0.8) | 0.9451 | 0.9510 | 3.8912 | 4.0227 |
| | (0, 0.15, -0.8) | 0.9457 | 0.9513 | 3.8953 | 4.0224 |
| | (-0.8, -0.16, 0.2) | 0.9454 | 0.9510 | 3.8893 | 4.0164 |
| | (-0.8, -0.16, -0.2) | 0.9455 | 0.9510 | 3.8917 | 4.0166 |
| | (0, 0.65, 0.1) | 0.9453 | 0.9508 | 3.8909 | 4.0159 |
| | (0, 0.65, -0.1) | 0.9455 | 0.9513 | 3.8926 | 4.0215 |
| | (-0.4, 0.3, 0.15) | 0.9456 | 0.9512 | 3.8919 | 4.0196 |
| | (-0.4, 0.3, -0.15) | 0.9459 | 0.9513 | 3.8973 | 4.0228 |
| | (-1.2, -0.3, 0.4) | 0.9457 | 0.9514 | 3.8966 | 4.0238 |
| | (0, 0.9, 0.05) | 0.9447 | 0.9505 | 3.8872 | 4.0148 |
| | (0, 0.9, -0.05) | 0.9453 | 0.9511 | 3.8916 | 4.0217 |
| | (-0.55, 0.4, 0.5) | 0.9453 | 0.9509 | 3.8882 | 4.0155 |

**Table2.** The estimated coverage probabilities and expected lengths of a 90% one-step-ahead prediction interval for AR(3) when $M = 10,000$, $\mu$ = 0 and $\sigma_e^2$ = 1.

| T | $(\phi_1, \phi_2, \phi_3)$ | Coverage probability | | Expected length | |
|---|---|---|---|---|---|
| | | $PI_S$ | $PI_R$ | $PI_S$ | $PI_R$ |
| 25 | (0, -0.65, 0.5) | 0.8252 | 0.9075 | 3.0619 | 4.0640 |
| | (0, -0.65, -0.5) | 0.8306 | 0.9073 | 3.0997 | 4.0583 |
| | (-0.4, -0.15, 0.3) | 0.8266 | 0.9100 | 3.0469 | 4.0582 |
| | (-0.4, -0.15, -0.3) | 0.8278 | 0.9109 | 3.0525 | 4.0690 |
| | (0.4, 0.15, 0.3) | 0.8146 | 0.8993 | 3.0059 | 4.0232 |
| | (0.4, 0.15, -0.3) | 0.8250 | 0.9088 | 3.0364 | 4.0457 |
| | (-0.8, -0.65, 0.2) | 0.8274 | 0.9091 | 3.0910 | 4.0551 |
| | (-0.8, -0.65, -0.2) | 0.8285 | 0.9097 | 3.0646 | 4.0531 |
| | (0.8, -0.65, 0.2) | 0.8242 | 0.9092 | 3.0362 | 4.0531 |
| | (0.8, -0.65, -0.2) | 0.8265 | 0.9078 | 3.0607 | 4.0637 |
| | (-0.95,-0.9,-0.5) | 0.8301 | 0.9109 | 3.0958 | 4.0616 |
| | (0.95, -0.9, 0.5) | 0.8204 | 0.9056 | 3.0283 | 4.0461 |
| | (1.75, -0.76, -0.1) | 0.8271 | 0.8986 | 3.1182 | 4.0422 |
| | (1.2, -0.3, -0.4) | 0.8352 | 0.9033 | 3.2045 | 4.0245 |
| | (0.8, -0.16, 0.2) | 0.8150 | 0.9006 | 3.0118 | 4.0245 |
| | (0.8, -0.16, -0.2) | 0.8251 | 0.9092 | 3.0460 | 4.0676 |
| | (0.55, 0.4, -0.5) | 0.8244 | 0.9089 | 3.0440 | 4.0602 |
| | (0.4, 0.3, 0.15) | 0.8135 | 0.8997 | 2.9952 | 4.0077 |
| | (0.4, 0.3, -0.15) | 0.8218 | 0.9074 | 3.0272 | 4.0471 |
| | (0, 0.15, 0.8) | 0.8118 | 0.9001 | 2.9914 | 3.9927 |
| | (0, 0.15, -0.8) | 0.8290 | 0.9069 | 3.0730 | 4.0571 |
| | (-0.8, -0.16, 0.2) | 0.8269 | 0.9105 | 3.0574 | 4.0668 |
| | (-0.8, -0.16, -0.2) | 0.8270 | 0.9093 | 3.0583 | 4.0598 |
| | (0, 0.65, 0.1) | 0.8154 | 0.9024 | 3.0143 | 4.0489 |
| | (0, 0.65, -0.1) | 0.8184 | 0.9037 | 3.0224 | 4.0452 |
| | (-0.4, 0.3, 0.15) | 0.8249 | 0.9094 | 3.0381 | 4.0538 |
| | (-0.4, 0.3, -0.15) | 0.8251 | 0.9088 | 3.0476 | 4.0711 |
| | (-1.2, -0.3, 0.4) | 0.8329 | 0.9050 | 3.1327 | 4.0569 |
| | (0, 0.9, 0.05) | 0.8100 | 0.8977 | 2.9917 | 3.9933 |
| | (0, 0.9, -0.05) | 0.8089 | 0.8955 | 2.9883 | 4.0084 |
| | (-0.55, 0.4, 0.5) | 0.8207 | 0.9060 | 3.0272 | 4.0474 |
| 50 | (0, -0.65, 0.5) | 0.8665 | 0.9112 | 3.1753 | 3.6889 |
| | (0, -0.65, -0.5) | 0.8665 | 0.9107 | 3.1907 | 3.6868 |
| | (-0.4, -0.15, 0.3) | 0.8658 | 0.9110 | 3.1677 | 3.6847 |
| | (-0.4, -0.15, -0.3) | 0.8667 | 0.9110 | 3.1688 | 3.6788 |
| | (0.4, 0.15, 0.3) | 0.8619 | 0.9079 | 3.1571 | 3.6764 |
| | (0.4, 0.15, -0.3) | 0.8666 | 0.9111 | 3.1688 | 3.6821 |
| | (-0.8, -0.65, 0.2) | 0.8669 | 0.9115 | 3.1842 | 3.6925 |
| | (-0.8, -0.65, -0.2) | 0.8675 | 0.9118 | 3.1773 | 3.6881 |
| | (0.8, -0.65, 0.2) | 0.8649 | 0.9100 | 3.1629 | 3.6791 |
| | (0.8, -0.65, -0.2) | 0.8671 | 0.9116 | 3.1812 | 3.6896 |
| | (-0.95,-0.9,-0.5) | 0.8674 | 0.9121 | 3.1864 | 3.6883 |
| | (0.95, -0.9, 0.5) | 0.8649 | 0.9102 | 3.1633 | 3.6824 |
| | (1.75, -0.76, -0.1) | 0.8656 | 0.9097 | 3.1858 | 3.6824 |
| | (1.2, -0.3, -0.4) | 0.8669 | 0.9100 | 3.1853 | 3.6863 |
| | (0.8, -0.16, 0.2) | 0.8628 | 0.9089 | 3.1598 | 3.6814 |
| | (0.8, -0.16, -0.2) | 0.8675 | 0.9121 | 3.1754 | 3.6932 |
| | (0.55, 0.4, -0.5) | 0.8662 | 0.9112 | 3.1709 | 3.6877 |
| | (0.4, 0.3, 0.15) | 0.8617 | 0.9082 | 3.1556 | 3.6750 |
| | (0.4, 0.3, -0.15) | 0.8655 | 0.9112 | 3.1693 | 3.6888 |
| | (0, 0.15, 0.8) | 0.8580 | 0.9054 | 3.1397 | 3.6543 |
| | (0, 0.15, -0.8) | 0.8651 | 0.9104 | 3.1704 | 3.6877 |
| | (-0.8, -0.16, 0.2) | 0.8664 | 0.9111 | 3.1692 | 3.6823 |
| | (-0.8, -0.16, -0.2) | 0.8659 | 0.9104 | 3.1707 | 3.6770 |
| | (0, 0.65, 0.1) | 0.8620 | 0.9081 | 3.1531 | 3.6731 |
| | (0, 0.65, -0.1) | 0.8642 | 0.9097 | 3.1614 | 3.6798 |
| | (-0.4, 0.3, 0.15) | 0.8650 | 0.9105 | 3.1647 | 3.6823 |
| | (-0.4, 0.3, -0.15) | 0.8663 | 0.9109 | 3.1747 | 3.6929 |
| | (-1.2, -0.3, 0.4) | 0.8677 | 0.9099 | 3.2006 | 3.6826 |
| | (0, 0.9, 0.05) | 0.8582 | 0.9052 | 3.1436 | 3.6595 |
| | (0, 0.9, -0.05) | 0.8604 | 0.9074 | 3.1529 | 3.6810 |
| | (-0.55, 0.4, 0.5) | 0.8649 | 0.9104 | 3.1640 | 3.6813 |

**Table2.** (continued)

| T | $(\phi_1, \phi_2, \phi_3)$ | Coverage probability | | Expected length | |
|---|---|---|---|---|---|
| | | $PI_S$ | $PI_R$ | $PI_S$ | $PI_R$ |
| 100 | (0, -0.65, 0.5) | 0.8839 | 0.9063 | 3.2327 | 3.4867 |
| | (0, -0.65, -0.5) | 0.8844 | 0.9070 | 3.2384 | 3.4903 |
| | (-0.4, -0.15, 0.3) | 0.8837 | 0.9062 | 3.2290 | 3.4844 |
| | (-0.4, -0.15, -0.3) | 0.8843 | 0.9068 | 3.2333 | 3.4895 |
| | (0.4, 0.15, 0.3) | 0.8832 | 0.9064 | 3.2322 | 3.4928 |
| | (0.4, 0.15, -0.3) | 0.8839 | 0.9064 | 3.2287 | 3.4830 |
| | (-0.8, -0.65, 0.2) | 0.8842 | 0.9066 | 3.2347 | 3.4883 |
| | (-0.8, -0.65, -0.2) | 0.8840 | 0.9064 | 3.2289 | 3.4815 |
| | (0.8, -0.65, 0.2) | 0.8842 | 0.9067 | 3.2302 | 3.4848 |
| | (0.8, -0.65, -0.2) | 0.8848 | 0.9071 | 3.2369 | 3.4922 |
| | (-0.95,-0.9,-0.5) | 0.8842 | 0.9067 | 3.2347 | 3.4862 |
| | (0.95, -0.9, 0.5) | 0.8837 | 0.9064 | 3.2305 | 3.4869 |
| | (1.75, -0.76, -0.1) | 0.8836 | 0.9061 | 3.2320 | 3.4864 |
| | (1.2, -0.3, -0.4) | 0.8840 | 0.9064 | 3.2363 | 3.4879 |
| | (0.8, -0.16, 0.2) | 0.8829 | 0.9055 | 3.2281 | 3.4817 |
| | (0.8, -0.16, -0.2) | 0.8837 | 0.9062 | 3.2284 | 3.4822 |
| | (0.55, 0.4, -0.5) | 0.8839 | 0.9058 | 3.2285 | 3.4784 |
| | (0.4, 0.3, 0.15) | 0.8824 | 0.9050 | 3.2250 | 3.4787 |
| | (0.4, 0.3, -0.15) | 0.8840 | 0.9062 | 3.2311 | 3.4846 |
| | (0, 0.15, 0.8) | 0.8807 | 0.9042 | 3.2237 | 3.4837 |
| | (0, 0.15, -0.8) | 0.8842 | 0.9067 | 3.2374 | 3.4948 |
| | (-0.8, -0.16, 0.2) | 0.8840 | 0.9063 | 3.2295 | 3.4813 |
| | (-0.8, -0.16, -0.2) | 0.8837 | 0.9060 | 3.2298 | 3.4841 |
| | (0, 0.65, 0.1) | 0.8833 | 0.9060 | 3.2296 | 3.4874 |
| | (0, 0.65, -0.1) | 0.8836 | 0.9064 | 3.2269 | 3.4846 |
| | (-0.4, 0.3, 0.15) | 0.8843 | 0.9067 | 3.2317 | 3.4858 |
| | (-0.4, 0.3, -0.15) | 0.8840 | 0.9067 | 3.2318 | 3.4889 |
| | (-1.2, -0.3, 0.4) | 0.8839 | 0.9061 | 3.2394 | 3.4841 |
| | (0, 0.9, 0.05) | 0.8801 | 0.9033 | 3.2175 | 3.4735 |
| | (0, 0.9, -0.05) | 0.8822 | 0.9054 | 3.2266 | 3.4863 |
| | (-0.55, 0.4, 0.5) | 0.8839 | 0.9064 | 3.2301 | 3.4861 |
| 250 | (0, -0.65, 0.5) | 0.8938 | 0.9026 | 3.2672 | 3.3664 |
| | (0, -0.65, -0.5) | 0.8939 | 0.9030 | 3.2681 | 3.3700 |
| | (-0.4, -0.15, 0.3) | 0.8939 | 0.9028 | 3.2663 | 3.3664 |
| | (-0.4, -0.15, -0.3) | 0.8937 | 0.9028 | 3.2657 | 3.3672 |
| | (0.4, 0.15, 0.3) | 0.8936 | 0.9027 | 3.2665 | 3.3684 |
| | (0.4, 0.15, -0.3) | 0.8937 | 0.9028 | 3.2652 | 3.3673 |
| | (-0.8, -0.65, 0.2) | 0.8942 | 0.9031 | 3.2702 | 3.3713 |
| | (-0.8, -0.65, -0.2) | 0.8942 | 0.9033 | 3.2692 | 3.3729 |
| | (0.8, -0.65, 0.2) | 0.8940 | 0.9029 | 3.2667 | 3.3669 |
| | (0.8, -0.65, -0.2) | 0.8937 | 0.9030 | 3.2659 | 3.3688 |
| | (-0.95,-0.9,-0.5) | 0.8940 | 0.9031 | 3.2686 | 3.3722 |
| | (0.95, -0.9, 0.5) | 0.8934 | 0.9025 | 3.2627 | 3.3646 |
| | (1.75, -0.76, -0.1) | 0.8941 | 0.9031 | 3.2685 | 3.3693 |
| | (1.2, -0.3, -0.4) | 0.8937 | 0.9028 | 3.2669 | 3.3680 |
| | (0.8, -0.16, 0.2) | 0.8937 | 0.9027 | 3.2659 | 3.3677 |
| | (0.8, -0.16, -0.2) | 0.8940 | 0.9030 | 3.2676 | 3.3686 |
| | (0.55, 0.4, -0.5) | 0.8940 | 0.9030 | 3.2682 | 3.3688 |
| | (0.4, 0.3, 0.15) | 0.8932 | 0.9024 | 3.2618 | 3.3641 |
| | (0.4, 0.3, -0.15) | 0.8937 | 0.9026 | 3.2651 | 3.3657 |
| | (0, 0.15, 0.8) | 0.8927 | 0.9020 | 3.2613 | 3.3639 |
| | (0, 0.15, -0.8) | 0.8940 | 0.9030 | 3.2677 | 3.3694 |
| | (-0.8, -0.16, 0.2) | 0.8939 | 0.9029 | 3.2666 | 3.3686 |
| | (-0.8, -0.16, -0.2) | 0.8936 | 0.9029 | 3.2639 | 3.3671 |
| | (0, 0.65, 0.1) | 0.8936 | 0.9025 | 3.2652 | 3.3658 |
| | (0, 0.65, -0.1) | 0.8936 | 0.9025 | 3.2646 | 3.3658 |
| | (-0.4, 0.3, 0.15) | 0.8939 | 0.9031 | 3.2675 | 3.3711 |
| | (-0.4, 0.3, -0.15) | 0.8938 | 0.9027 | 3.2657 | 3.3667 |
| | (-1.2, -0.3, 0.4) | 0.8936 | 0.9028 | 3.2670 | 3.3683 |
| | (0, 0.9, 0.05) | 0.8930 | 0.9021 | 3.2645 | 3.3654 |
| | (0, 0.9, -0.05) | 0.8936 | 0.9029 | 3.2659 | 3.3694 |
| | (-0.55, 0.4, 0.5) | 0.8939 | 0.9031 | 3.2673 | 3.3709 |

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