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The Economic Model of $\overline{X}$ Control Chart Using Shewhart Method for Skewed Distributions

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Abstract

The purposes of this research were to present economic model and to compare the efficiency of $\bar{X}$ Control Chart using Shewhart Method for Skewed Distributions. The experiment data sets were Weibull Distribution, Lognormal Distribution, and Burr's Distribution using the expected value of all expenses per one single unit of time as standard. The coefficients of skewness (α_3) were 0.1, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, and 9.0. The variations of production level were 0.5σ, 1.0σ, 1.5σ, 2.0σ, 2.5σ, and 3.0σ, obtained by Monte Carlo Simulation Technique. Using an application program with PHP, a total number of 10,000 samples were repeatedly looped. The results indicated that the production level begin to vary from 3.0σ of Lognormal Distribution. The lowest expense was observed at the coefficient of skewness at (α_3) 6.

Keywords: control chart, economic model, skewness, variations of production**.**

1. Introduction

Statistical quality control chart is a population tool which widely uses to control manufacturing process in order to reduce any variation that may occur. Several advantages of this tool have be noted. For example, it is able to separate type of variation in process, and to analyze and indicate root cause of variation in process. Alert to control process person for stopping process for analysis defect of process and process improvement. The average controls chart by the shewhart control chart is likely to be used extensively, but it is convenient for only the data that has the normal distribution. Generally, even in a good process, sometimes it still has indication of root cause of variation or skewnees.

The design of shewhart controls chart requires the determination of three parameters: the sample size(n), the sampling interval(h), and the width of control limit(k) ($\overline{\overline{X}} \pm 3\sigma_{\overline{X}}$ when $k = 3$). Usually, in the practice of the shewhart controls chart designing, the important matter needed to concern is to have the least variation of the products, and to have the long of ARL, in order to get least economized expenses for inspection. For this reason, all expenses in the process will directly participate in the design of control of chart, especially selection of parameters; n, h, and k. The $\overline{X}$ Control Chart is the chart for controlling average value or controlling quality level of mean value. It is also a tool for controlling variance in production process. The effective quality control chart must be able to isolate production process variances in term of sources of nonconformity for further inspection and temporary production halting for maintenance. This is a preventive action in order to lower production cost and loss, and further inspection cost. Therefore, utilizing production control charts are to lower inspection cost and are the reason to study the economic model of quality control charts along with product quality control. Presently, several researchers have studied on the economic model of mean value control chart. Alexander [1] had studied the economic model of Taguchi Loss Function Control Charts under parameter definition. By that time, the outcome of the studies was still not suitable for product mean value control chart. Bai and Choi [2] had improved economic model of product $\overline{X}$ Control Chart using time period to random for variance. Later, Magalh$\tilde{a}$es and Epprecht [3] had studied the economic model of variable parameter mean value control chart by examining normally distributed data and using Shewhart's mean value control chart comparing with defined parameter mean value control chart. The entire expenses were utilized as comparison bottom line. The results showed that at the production process variances of 0.5σ , 0.75σ , 1.0σ , 1.25σ , 1.5 σ , 1.75 σ , and 2.0 σ , the variable parameter mean value control chart yields better effectiveness. However, this study is base on normally distributed data, which is contrast to the actual production situation. Chou et al. [4] introduced economic model of mean value control chart with Weibull distribution production data using warning cause as one of criteria. One year later, Al-Orainia, Rahim [5] studied the economic model of mean value control chart with Gamma control time period at parameter (λ ,2). Pongpullponsak [6] compared mean value and range control charts with weighed variance (using Nelson and Shewhart methods) under skewed data. The results indicated that the scale weighed variance (SWV) yielded the highest effectiveness when the data was Weibull's distribution (at the skewness of 0.1 to 3) and Nelson range control chart yielded the highest effectiveness at the coefficient of skewness at 0.1 to 9. Lin and Chou [7] introduced non-normal distribution and variable parameter mean value control chart (using t and Gamma distributions) to compare the varying defined production process from 0 to 3 in the form of ARL. Magalh $\tilde{a}$ es and Epprecht [3] had studied the economic model of variable parameter control chart with Shewhart's $\bar{X}$ Control Chart, which is suitable only for normal distributed data. But in real situation the data in production process are not in constantly normal distribution. In addition, the appropriate data for the average controls chart that uses in the qualilty control should have skewness. In case of non-normal distribution, Pongpullponsak [6] introduced $\bar{X}$ Chart using weighed variance method in which the efficiency is even higher than that of Shewhart.

The objective of this research is to introduce the economic model of Shewhart $\bar{X}$ Control Chart for the data that are transformed to be normally distributed. The data utilized in this study is formerly in Weibull, Lognormal, and Burr's distribution. Consequently, the expected value of expense per time unit is compared to Magalh $\tilde{a}$ es and Epprecht's [3] introductory.

2. Data distribution

2.1 Weibull Distribution

Cumulative distribution function :

$$F(y) = 1 - \frac{1}{e^{(y/\theta)^{\beta}}} \tag{1}$$

Population mean :

$$\mu = E(Y) = \frac{\theta}{\beta}\Gamma\left(\frac{1}{\beta}\right) \tag{2}$$

Population variance : $\sigma^2 = V(Y) = \frac{\theta^2}{\beta}\left\{2\Gamma\left(\frac{2}{\beta}\right) - \frac{1}{\beta}\left|\Gamma\left(\frac{1}{\beta}\right)\right|^2\right\}$ (3)

where θ is Scale Parameter, β is Shape Parameter

In this research, the defined values are as the following:

θ = 0.1,0.5,1,2,3,4,5,6,7,8 and 9

β = 3.2219,2.211,1.563,1.0,0.7686, ,0.6478, 0.5737, 0.5237,0.4873,0.4596, and 0.4376 (compatible with coefficient of skewness (α_3)=0.1,0.5,1,2,3,4,5,6,7,8,9)

2.2 Lognormal distribution

Cumulative distribution function : $F(y) = \phi\left(\frac{\ln(y)}{\sigma'}\right)$ (4)

Population Mean : $\mu = E(Y) = e^{\mu + \frac{\sigma^2}{2}}$ (5)

Population Variance : $\sigma^2 = V(Y) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$ (6)

given : $\omega = \exp\left(\sigma^2\right)$; Coefficient of Skewness(α_3) $\alpha_3 = (\omega+2)(\omega-1)^{1/2}$ (7)

when $\exp(\mu)$ is Scale Parameter

σ is Shape Parameter

2.3 Burr's distribution

Cumulative distribution function : $F(y) = 1 - \frac{1}{(1+y^c)^h}$ (8)

When c is dispersion parameter and h is family parameter.

$$\psi_j = \frac{\Gamma\left(\frac{j+1}{c}\right)\Gamma\left(h - \frac{j+1}{c}\right)}{c\Gamma(h)} \quad (9)$$

Population mean : $\mu = E(Y) = \psi_1$

Population Variance : $\sigma^2 = V(Y) = \psi_2$ (10)

3. Economic Model of Production Process Model

Magalhães and Epprecht [3] introduce expenditure function is existing in every single production hour. It bases on selection of optimum economic model value for parameters n_1, n_2, h_1, h_2, w_1, w_2, k_1, k_2 when developing expenditure function. The production process assumptions, utilized in expenditure function development, are as following:

3.1 Process Model

The following assumptions are the in-process product characteristic assumptions to be analyzed. The samples are assumed to be independent from each other and the initial production process will be under statistical control in which the $\bar{X}$ control chart equals to $\bar{\bar{X}}$ and standard deviation equals to σ'_x . Once a warning cause or nonconformity is existed, the mean value will shift from μ_0 to $\mu_0 + \delta\sigma'_x$ or $\mu_0 - \delta\sigma'_x$. While the process is still under control, the population is exponentially distributed with the mean value of $\frac{1}{\lambda}$ and not self reversible if any process change is existed. During process investigation, the probability of process continuation ability is an index variable δ_1 ($\delta_1 = 1$ if process is able to continue; $\delta_1 = 0$ if otherwise).The probability of process continuation ability during process repair or improvement is an index variable δ_2 ($\delta_2 = 1$ if process is able to continue; $\delta_1 = 0$ if otherwise). The μ , σ' and σ are assumed to be known in order to define parameters n_1, n_2, h_1, h_2, w_1, w_2, k_1, and k_2 of control chart.

The five production process expenditures caused by implementing economic model are as the following:

1. Expenditure caused by population control and sampling (C_{sam})
2. Expenditure caused by inspecting failure warning signal (C_{fa})
3. Expenditure caused by investigating for identifiable cause of nonconformity (C_r)
4. Expenditure caused by producing goods that is not conform to specifications while process is under control (C_{in})
5. Expenditure caused by producing goods that is not conform to specifications while process is not under control (C_{out})

3.2 Production Cycle

The production cycle is defined as production duration. Controlling of the production process is assumed to be constant at the beginning. Production cycle composes of two time periods which are under control period and not under control period as the details are described bellowing:

1. Time period where production process is still under control (T_{in}) : The time duration started from the beginning to the point where the warning cause is obviously identifiable

2. Time period where production process is not under control (T_{out}) : The time duration started from when the process starts changing until the failure warning is developed

3. Analyzing period (T_a) : Time period contributed to sample analysis and control chart result analysis

4. Inspecting period (T_{ass}) : Time period contributed to investigation of identifiable cause, once the production process is not under control

5. Repairing period (T_r) : Time period contributed to process repairing.

3.3 The burdened expenditure per one production cycle

The expenditure function is economically considered as per time unit expenditure function. $E(T)$ is expected value of production period duration and $E(C)$ is expected value of total expenses burdened in one production cycle. Hence, the expected value of total expenses per one time unit is

$$ECTU = \frac{E(C)}{E(T)} \qquad (11)$$

The expected value of total expenses per one production cycle composes of the summation of all existed expenses while production process is both under and out of control. Hence, $E(C)$ composes of

1. The expected value of expenditure per one production cycle due to the production of goods that is not conform to specification while the production process is under control ($E(C_{in})$) and out of control ($E(C_{out})$). Hence,

$$(E(C_{in})) + (E(C_{out})) = \frac{1}{\lambda}C_0 + C_1\left[AATS + E(T_a) + \delta_1 T_* + \delta_2 T_{**}\right] \qquad (12)$$

Given C_0 and C_1 are hourly expenditure due to the production of goods that is not conform to specification while the production process is under control and out of control respectively. The mean value of time period while production process is under control is $\frac{1}{\lambda}$. The Adjusted Average Time to Signal (AATS) is the expected value of time period since the production process starts changing until the failure warning signal $E(T_a)$ equals to $n'G$, where G is sampling time interval specified by control chart, n' is sample size while process is out of control, T_* is average time interval where the warning cause is detected and T_{**} is average time for process repairing.

2. The expected value of failure warning signal detection ($E(C_{fa})$)

$$E(C_{fa}) = \text{Y}\, E(F) \tag{13}$$

where Y is expense caused by failure warning signal detection.

$E(F)$ is average number of independent failure warning signal.

3. The expected value of expenses contributed to investing and repairing the cause of warning signal $\left(E(C_r)\right)$

$$E(C_r) = \text{w} \tag{14}$$

4. The expected value of expenses contributed to sampling and controlling $\left(E(C_{sam})\right)$

$$E(C_{sam}) = (a+bn)s+(a+bn')s' \tag{15}$$

where a is the fixed expense (direct) per one sample,

b is the variable expense (indirect) per one sample,

n is the average sample size while process is in control,

n' is the average sample size while process is out of control, and

s is the average number of sample specified by the control chart while process is in control.

s' is the average number of sample specified by the control chart while process is out of control. Hence, the expected value of total expenditure per one production cycle is obtained by integrating all equations from (12) to (15) which is :

$$E(C)=\frac{1}{\lambda}C_0+C_1\left[AATS+E(T_a)+\delta_1T_*+\delta_2T_{**}\right]+\text{Y}\,E(F)+\text{w}+(a+bn)s+(a+bn')s' \tag{16}$$

The Expected Cycle Time is the summation of average time periods of each sub cycle time, which is $E(T)=E(T_{in})+E(T_{out})+E(T_a)+E(T_{ass})+E(T_r)$

$$=\frac{1}{\lambda}+(1-\delta_1)E(T_{fa})+AATS+n'G+T_*+T_{**} \tag{17}$$

where $(1-\delta_1)E(T_{fa})$ is part of $E(T_{in})$

T_* and T_{**} are independent to each other or independent to process status (halting or continuing). Hence, from equation (11), we obtain:

$$ETCU = \frac{\frac{1}{\lambda}C_0 + C_1\left[AATS + E(T_a) + \delta_1 T_*\right] + TE(F) + w + (a + bn)s + (a + bn')s'}{\frac{1}{\lambda} + (1 - \delta_1)E(T_{fa}) + AATS + n'G + T_* + T_{**}} \qquad (18)$$

4. ECTU calculation (in case of skewed population)

Shewhart control chart is suitable only for normally distributed population. Therefore, in case of skewed population, the normal distribution conversion is required. Lin and Chou [7] converted population using the Central Tendency Limit Theorem which stated that if randomized sample, with size n, from any type of distribution with limited mean and standard deviation, the statistical value, obtained from mean value minus population mean and consequently divide by sample mean standard deviation, would yield tend-to-standard normal distribution [N (0,1)]. From the Central Tendency Limit Theorem, we obtain

Given Y_i is random variable of skewed population

M_i is population mean

S_i is standard deviation of Y_i when i = 1 , 2

Then $$\frac{Y_i - M_i}{S_i} \xrightarrow{d} N(0,1) \text{ or } \frac{\bar{X}_i - \mu_0}{\sigma/\sqrt{n_i}} = \frac{Y_i - M_i}{S_i} \qquad (19)$$

From equation (19) we obtain $$\bar{X}_i = \mu_0 + \frac{Y_i - M_i}{S_i}\,\sigma/\sqrt{n_i} \qquad (20)$$

When production process starts to change, process mean value will shift from μ_0 to $\mu_0 + \delta\sigma'$, thus from equation (19), it would yield

$$\frac{\bar{X}_i - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n_i}} = \frac{Y_i - M_i}{S_i} \qquad (21)$$

From equation (21) we obtain $$\bar{X}_i = \mu_0 + \delta\sigma + (Y_i - M_i)\frac{\sigma/\sqrt{n_i}}{S_i} \qquad (22)$$

In case of normal distributed population, Magalhães and Epprecht [3] introduced the calculation of expected variables using Shewhart control chart. For skewed population, variables could be calculated as the following:

4.1. Expected failure warning $E(F)$

Let F as the number of failure warning existed in a production cycle. F is a random variable depending on sample size (N) before process change.

Hence, expected number of failure warning ($E(F)$) is

$$E(F) = (\alpha_1 p_0 + \alpha_2 (1-p_0))Q \quad \text{when} \quad \alpha_i = P(\bar{X}_i < LCL_i) + P(\bar{X}_i > UCL_i) \tag{23}$$

Substituting equation (1), (4) and (8) in (23) would yield

$$\alpha_i = 1 - F(M_i + k_i S_i) + F(M_i - k_i S_i)$$

If Y~Weibull Distribution $\alpha_i = 1 - F(M_i + k_i S_i) + F(M_i - k_i S_i)$

$$= 1 + \frac{1}{e^{((M_i + k_i S_i)/\theta)^\beta}} + \frac{1}{e^{((M_i - k_i S_i)/\theta)^\beta}}$$

If Y~Lognormal Distribution $\alpha_i = 1 - F(M_i + k_i S_i) + F(M_i - k_i S_i)$

$$= 1 - \phi\left(\frac{\ln(M_i + k_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)}\right) - \phi\left(\frac{\ln(M_i - k_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)}\right)$$

If Y~Burr's Distribution $\alpha_i = 1 - F(M_i + k_i S_i) + F(M_i - k_i S_i)$

$$= 1 + \frac{1}{(1+(M_i + k_i S_i)^c)^k} + \frac{1}{(1+(M_i - k_i S_i)^c)^k}$$

When α_i is Type 1 failure probability

$F(x)$ is cumulative distribution function of normal distribution

p_0 is small sample size probability while process is under control

$1-p_0$ is large sample size probability while process is out of control

Hence, the probability determination of p_0, which is a conditional probability could be as the following : $p_0 = P(LWL_i < \bar{X}_i < UWL_i \,/\, LCL_i < \bar{X}_i < UCL_i)$ (24)

Substitute equation (1), (4) and (8) in (24) we obtain

$$p_0 = \frac{F(M_i + w_i S_i) - F(M_i - w_i S_i)}{F(M_i + k_i S_i) - F(M_i - k_i S_i)}$$

If Y~Weibull Distribution

$$p_0 = \frac{-\frac{1}{e^{((M_i + w_i S_i)/\theta)^\beta}} + \frac{1}{e^{((M_i - w_i S_i)/\theta)^\beta}}}{-\frac{1}{e^{((M_i + k_i S_i)/\theta)^\beta}} + \frac{1}{e^{((M_i + w_i S_i)/\theta)^\beta}}}$$

If Y~Lognormal Distribution
$$p_0 = \frac{\phi\left(\frac{\ln(M_i + w_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2}-1)}\right) - \phi\left(\frac{\ln(M_i + w_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2}-1)}\right)}{\phi\left(\frac{\ln(M_i + k_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2}-1)}\right) - \phi\left(\frac{\ln(M_i - k_i S_i)}{e^{2\mu+\sigma^2}(e^{\sigma^2}-1)}\right)}$$

If Y~Burr's Distribution
$$p_0 = \frac{-\frac{1}{(1+(M_i + w_i S_i)^c)^k} + \frac{1}{(1+(M_i - w_i S_i)^c)^k}}{\frac{1}{(1+(M_i + k_i S_i)^c)^k} + \frac{1}{(1+(M_i - k_i S_i)^c)^k}}$$

4.2. Average sampling size while process is under control (Q).

Given $Q = E(N)$ as average sampling point while process is under control.

N as number of sample before process starts to change.

Hence, warning cause existed between sample j and $j+1$ means the process average shifting from μ_0 to $\delta\sigma'$. When j is utilized prior to process change, which means $N = j$, similar to the existence of warning cause which is identifiable during the sampling interval T_j and T_{j+1}.

Hence

$$E(N) = Q = \frac{e^{-\lambda h_1} p_0 + e^{-\lambda h_2}(1-p_0)}{1 - e^{-\lambda h_1} p_0 - e^{-\lambda h_2}(1-p_0)} \tag{25}$$

4.3. Adjusted Average Time to Signal ($AATS$) calculation

Given $AATS$ as average time interval from the existence of warning cause to the existence of actual warning if

T_{out} is time interval from the existence of warning cause to the existence of actual warning,

A is random variable during the time of change,

R is post change time interval that from the last until the first post change samples,

Q is time interval from the first post change until the existence of warning signal, and

T' is time interval from the last sample prior to the change until the existence of change.

4.3. 1 $E(R)$ calculation

Reynolds[8] assumed $P(A=h_i)$ to be proportional to the time interval change A. So the probability of this interval length is:

$$P(A=h_1)=\frac{p_0h_1}{p_0h_1+(1-p_0)h_2} \tag{26}$$

$$P(A=h_2)=\frac{(1-p_0)h_2}{p_0h_1+(1-p_0)h_2} \tag{27}$$

$E(R)$ is reformulated as $E(R)=E(E(R/A))$ and $E(E(R/A))=E(E((h_i-T')/A))$

When T' is expected time interval of warning cause in the process which exists during the sample j and $j+1$. Therefore:

$$E(R)=\left\{h_1-\frac{1-e^{-\lambda h_1}(1+\lambda h_1)}{\lambda(1-e^{-\lambda h_1})}\right\}P(A=h_1)+\left\{h_2-\frac{1-e^{-\lambda h_2}(1+\lambda h_2)}{\lambda(1-e^{-\lambda h_2})}\right\}P(A=h_2) \tag{28}$$

4.3.2 Decision about $E(Q)$

$E(Q)$ depends on the position of the first sample point $(j+1)$ after the change (B). The probability that the point will be at the center $(B=B_1)$ of the warning zone $(B=B_2)$ or the actuation zone $(B=B_3)$ will depend on the length of time interval during the change which is:

$$P(B=B_1)=p_{11}P(A=h_1)+p_{22}P(A=h_2) \tag{29}$$

$$P(B=B_2)=p_{12}P(A=h_1)+p_{22}P(A=h_2) \tag{30}$$

$$P(B=B_3)=1-P(B=B_1)-P(B=B_2) \text{ where } p_{i1}=P(LWL_i<\overline{X}_i<UWL_i) \tag{31}$$

substitute equation (22) in (31) then we obtain

$$p_{i1}=\phi(M_i+w_iS_i-\delta S_i\sqrt{n_i})-\phi(M_i-w_iS_i-\delta S_i\sqrt{n_i}) \tag{32}$$

and $$p_{i2}=P(LCL_i<\bar{X}_i<LWL_i)+P(UWL_i<\bar{X}_i<UCL_i) \tag{33}$$

substitute equation (22) in (33) then we obtain:

$$p_{i2}=\phi(M_i+k_iS_i-\delta S_i\sqrt{n_i})-\phi(M_i+w_iS_i-\delta S_i\sqrt{n_i})+$$
$$\phi(M_i-w_iS_i-\delta S_i\sqrt{n_i})-\phi(M_i+k_iS_i-\delta S_i\sqrt{n_i}) \tag{34}$$

Given $(Q/B=B_1)=T_1$ when the first post-change sample point is in the middle and $(Q/B=B_2)=T_2$ when the first post-change sample point is in the warning zone.

Hence

$$E(Q)=E(T_1)P(B=B_1)+E(T_2)P(B=B_2) \tag{35}$$

When T_1 is time interval last from the first post change sample (the first post process change sample is in the middle: warning zone) until the existence of warning zone. If the first post process change is at the middle, time interval from the first post process change sample to the existence of warning signal, will be defined as

$$T_1 = \sum_{i=1}^{M_1} Y_i$$

The expected T_1 is $E(T_1) = E\left(\sum_{i=1}^{M_1} Y_i\right) = E\left(E\left(\sum_{i=1}^{M_1} Y_i \middle| D_1\right)\right) = E(D_1)E(V)$ (36)

When D_1 is random variable of sample point projected at the middle until the existence of warning signal and D_1 is geometrically distributed random variable with parameter $(1-p_1)$, when p_1 is the probability that the sample point will project at the middle. Therefore:

$$p_1 = p_{11} + p_{12} \sum_{i=1}^{\infty} p_{22}^{i-1} p_{21} \qquad (37)$$

when substitute p'_{ij} in pair

$$E(D_1) = \frac{1}{1-p_1} \qquad (38)$$

The random variables $Y_i'S$ are independent to each other and have unique distribution V; the time interval, while sample points are projecting outside the warning zone since the last sample point projected at the middle. The probability function V is defined as:

$$P(V = h_1) = p_{11} + p_{13} = 1 - p_{12}$$

$$P(V = h_1 + ih_2) = p_{12} p_{22}^{i-1} p_{21} + p_{12} p_{22}^{i-1} p_{23} = p_{12} p_{22}^{i-1}(1 - p_{22}) \text{ when } i = 1, 2, \ldots \qquad (39)$$

The calculation of expected value V :

$$E(V) = h_1 + h_2 \frac{p_{12}}{1-p_{22}} \qquad (40)$$

substitute equation $E(D_1)$ and $E(V)$ in $E(T_1)$ we obtain

$$E(T_1) = \frac{[h_1(1-p_{22}) + (h_{22} p_{12})]}{1 - p_{11} - p_{22} + p_{11}p_{22} - p_{12}p_{21}} \qquad (41)$$

In the same token, if the first post process change sample projects in the warning zone, we obtain:

$$E(T_2) = \frac{[h_2(1-p_{11}) + (h_1 p_{21})]}{1 - p_{11} - p_{22} + p_{11}p_{22} - p_{12}p_{21}} \qquad (42)$$

The determination of n, n', h' and S' are as the following:

4. 4 Determination of n

$$n = n_1 p_0 + n_2(1 - p_0) \qquad (43)$$

4.5 Determination of n'

$$n' = n_1 p_0(\delta) + n_2(1 - p_0(\delta)) \qquad (44)$$

4. 6 Determination of h'

$$h' = h_1 p_0(\delta) + h_2(1 - p_0(\delta)) \qquad (45)$$

4.7 Determination of

$$p_0(d) = P(-w_i - d\sqrt{n_i} < Z < w_i - d\sqrt{n_i} \mid -k_i - d\sqrt{n_i} < Z < k_i - d\sqrt{n_i}) \text{ when } i = 1,2 \qquad (46)$$

substitute equation (22) in equation (46) we obtain

$$p_0(\delta') = \frac{\phi(M_i + w_i S_i - \delta S_i \sqrt{n_i}) - \phi(M_i + w_i S_i - \delta S_i \sqrt{n_i})}{\phi(M_i + k_i S_i - \delta S_i \sqrt{n_i}) - \phi(M_i + k_i S_i - \delta S_i \sqrt{n_i})} \qquad (47)$$

4. 8 Determination of Q'

$$Q' = \frac{\text{expected cycle length while the process is off target}}{\text{average time between samples while the process is off target}}$$

$$Q' = \frac{AATS + n'G + \delta_1 T_* + T_{**}}{h'} \qquad (48)$$

4.9 Determination of

$$ECTU = \frac{E(C)}{E(T)} \qquad (49)$$

5. Numerical examples

The objective of this research is to introduce the economic model of Shewhart Control Chart when the data is transformed to be normally distributed. The data utilized in this research is formerly Weibull, Lognormal, and Burr’s distributed. Consequently, the expected value of expense per time unit obtained from the model will be compared with Magalh $\tilde{a}$ es and Epprecht’s [3] introductory.

To achieve the objective mentioned above, the research plans are as followed:

1. Simulating population data by Monte Carlo simulation technique. In this case, data simulation will be on the Weibull distribution, Lognormal distribution, and Burr's distribution, in order to use for transforming data to normal distribution. The number of sample for the simulated data will be used at 10,000 samples which is 100 times repeatedly looped using an application program with PHP.

2. Determining mean and variance of the Weibull distribution, lognormal distribution, and Burr's distribution.

3. Creating $\bar{X}$ control chart from the simulating population data.

4. Computing a_j from the the Weibull distribution, Lognormal distribution, and Burr's distribution.

5. Computing

$$p_0, p_{i1}, p_{i2}, P(A=h_1), P(A=h_2), p_0(d), p_{0i}(d'), p_{11,} p_{12}, p_{13}, p_{21}, p_{22}, p_{23}$$

from the Weibull distribution, Lognormal distribution, and Burr's distribution.

6. Computing ECTU from the coefficients of skewness under the Weibull distribution, Lognormal distribution, and Burr's distribution simulating, and then transforming into the normal distribution.

7. Supposing variable data that use to calculate the value ECTU are

$$G = T_* = T_0 = \frac{5}{60}\text{hours}, T_{**} = \frac{45}{60}\text{ hours}, \frac{1}{i} = 50 \quad , \quad ,$$

$$C_0 = \text{b}114.24/\text{hours}, \quad C_1 = \text{b}949.20/\text{hours}$$

$$Y = W = \text{b}977.40; \; a = 0, b = \text{b}4.22; d_2 = 0$$

8. In order to accomplish the optimization of the unit cost function, the following constrains were considered

$$n_1 \le n_2; n_1 \ge 1; 0.1 \le h_2 \le h_1; h_1 \ge 1; w_2 \le w_1 \quad and \quad w_2 \ge 0.1; k_2 \le k_1; k_1 \quad and \quad k_2 \ge 1$$

With an application program with PHP.

9. The results of calculation are as the following.

9.1 From Table 1, Table 2 and Figure 1 , it was observed that the ECTU of the average variables parameter controls chart (V_P)should not shift to be valuable a little more average variables fixes parameter controls chart(V_F). In other words, if process shifts from 0.5-3.0 the ECTU by V_P and V_F have decreasing cost when the process increasing shifts. When compared to economics V_P controls chart with economics V_F controls chart, we found that the ECTU of V_P controls chart have a little more V_F controls chart.

Table 1. Performance of Shewhart control chart for V_P

Δ	SH for V_P								ECTU
	n_1	n_2	h_1	h_2	w_1	w_2	k_1	k_2	
0	12	17	2.21	0.1	1.19	1.11	2.71	1.9	243.75
0.5	11	15	2.15	0.1	1.15	1.08	2.62	2	236.55

Table 1. (continute)

Δ	SH for V_P								ECTU
	n_1	n_2	h_1	h_2	w_1	w_2	k_1	k_2	
1	5	6	1.43	0.1	1.3	1.26	3.26	2.4	186.25
1.5	3	5	1.11	0.1	1.71	1.6	3.62	2.4	169.75
2	2	5	1	0.1	2.06	1.9	3.56	2.4	163.21
2.5	2	5	1	0.1	1.13	1.94	3.43	2.5	162.45
3	2	5	1	0.1	1.12	1.93	3.4	2.4	160.75

Table 2. Performance of Shewhart control chart for V_F

D	SH for V_F			ECTU
	n_0	h_0	k_0	
0	19	2.84	1.83	253.2
0.5	17	2.77	1.87	248.8
1	9	1.98	2.48	197.7
1.5	5	1.45	2.73	176.9
2	5	1.59	3.06	168.7
2.5	5	1.48	3.15	164.3
3	3	1.32	3.36	162.9

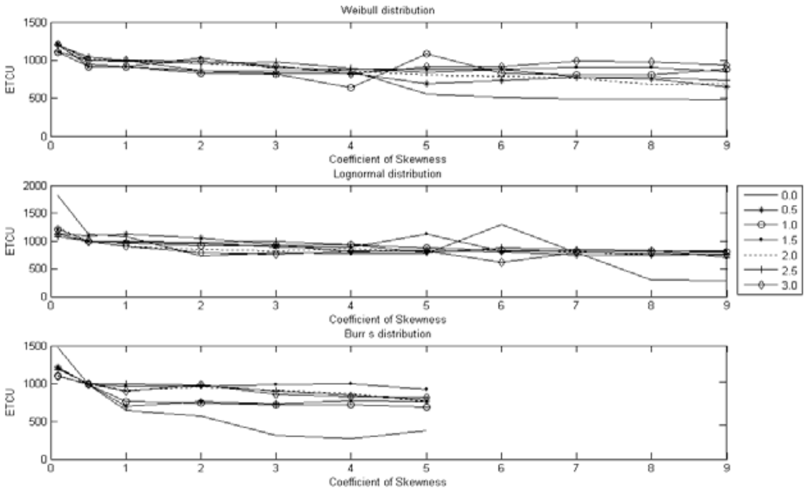
Figure 1. The comparative economic V_P control chart with economic control chart

9.2 From Table 3 and Figure 2, the ECTU values calculated from various distributions when the process control did not change are shown. In case of the Weibull distribution at the coefficient of skewness 0.1 – 9.0, the values of ECTU were continuingly decrease. At the coefficient of skewness 9.0, the value of ECTU was minimum. For the Lognormal distribution at the coefficient of skewness 0.1 - 2.0, the values of ECTU were continuingly decrease, and at the coefficient of skewness 3.0 - 9.0 the ECTU values were similar. The minimum of ECTU value was observed at the coefficient of skewness 2 and 3. For the Burr's distribution, at the coefficient of skewness 0.1 - 5.0 the ECTU values were continuingly decrease, where the ECTU value was minimum at the coefficient of skewness 5.0. When compared the value ECTU of 3 distributions, it was found that the minimum ECTU value was observed in the Weibull distribution at the coefficient of skewness 9.0.

Table 3. Comparison between distribution of ECTU

	Δ =0			Δ =0.5			Δ =1		
	Weibull	**lognormal**	**burr**	**Weibull**	**lognormal**	**burr**	**Weibull**	**lognormal**	**burr**
☆	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**
0.1	1203.43	1821.28	1471.05	0.1	1203.43	1119.2	1101.43	1214.2	1103.9
0.5	945.67	1124.77	981.32	0.5	1045.67	1088.7	908.76	986.16	986.16
1	909.89	1076.88	639.75	1	1003.42	1125.22	908.76	957.33	757.33
2	858.06	735.11	564.75	2	856.76	1056.64	825.30	925.30	735.30
3	860.8	783.98	313.53	3	830.75	932.82	822.97	922.97	722.97
4	860.8	768.22	267.37	4	832.59	808.67	634.51	934.51	713.51
5	559.4	773.1	372.67	5	688.67	833.29	1082.45	882.45	682.45
6	509.7	1303.03	-	6	733.29	802.62	832.70	832.70	-
7	486.57	788.63	-	7	772.62	753.16	810.89	821.89	-
8	496.26	295.68	-	8	753.16	753.08	805.58	812.58	-
9	480.32	280.48	-	9	653.08	753.69	879.25	797.25	-
	Δ =1.5			Δ =2			Δ =2.5		
	Weibull	**lognormal**	**burr**	**Weibull**	**lognormal**	**burr**	**Weibull**	**lognormal**	**burr**
☆	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**	**ECTU**
0.1	1211.76	1203.2	1207.1	1212.3	1246.3	1186.5	1113.56	1086.3	1186.3
0.5	918.76	988.17	988.17	1011.4	990.21	990.21	1002.12	992.30	992.30
1	918.76	977.24	967.24	998.07	906.55	906.55	987.7	997.29	992.29
2	1025.30	959.51	959.51	952.85	852.85	952.85	952.87	973.13	973.13
3	889.51	886.43	986.43	913.22	813.22	913.22	973.13	996.67	896.67
4	886.43	894.96	994.96	864.80	864.80	864.80	896.67	941.05	841.05
5	885.96	1123.11	923.11	807.32	811.32	757.32	841.05	786.48	786.48
6	885.11	815.22	-	791.51	831.51	-	886.48	872.63	-
7	905.22	822.1	-	769.75	809.75	-	772.63	852.39	-
8	905.11	823.80	-	676.23	739.23	-	772.39	839.31	-
9	853.80	823.51	-	694.81	764.81	-	739.31	703.44	-
	Δ =3								
	Weibull	**lognormal**	**burr**						
☆	**ECTU**	**ECTU**	**ECTU**						
0.1	1211.3	1126.3	1091.3						
0.5	998.45	994.43	994.43						
1	998.44	902.14	902.14						
2	986.55	786.55	986.55						
3	921.45	755.47	855.45						

Figure 2. The Comparative ETCU under various shifts and coefficients of Skewness from the Weibull distribution, the Lognormal distribution, and the Burr's distribution



When the process control changed, the Weibull distribution at the coefficient of skewness 0.1- 1.0 had the ECTU values continuingly decreased, while at the coefficient of skewness 2.0 - 9.0 the ECTU values fluctuated. At the coefficient of skewness 4.0 where the level of process control shifts was 1.0, the ECTU value was minimum. Similar to the Weibull distribution, the ECTU values for the Lognormal distribution fluctuated at the coefficient of skewness 0.1 - 5.0, whereas at the coefficient of skewness 6.0 - 9.0 the ECTU values were similar. The minimum ECTU value was observed at the coefficient of skewness 6.0 where the level of process control shifts was 1.5. In case of the Burr's distribution at the coefficient of skewness 5.0, the ECTU value was stable, while in every coefficient of skewness 1.0 - 5.0 when the level of process control shifts and the level procedure coefficient produces changed, the ECTU value was increase. When compared the ECTU values of 3 distributions it was found that the Lognormal distribution at the coefficient of skewness 6.0 where the procedure level shifts was 3.0, the ECTU value was minimum.

5. Conclusion and Discussion

5.1 Conclusion

1. Using the economic model of Shewhart control chart, the lowest per-unit expense was observed in the lognormal distribution at stable process with the coefficient of skewness at 2.

2. Once the process began to change, the ECTU value in the Weibull distribution at the coefficient of skewness of 2.0, 3.0, and 4.0 started to decrease. On the other hand, the ECTU value began to increase at the coefficient of skewness from 4.0.-9.0, except at the level where the process ~~is~~ was changed at 5.0.and 3.0. At the coefficient of skewness of 8.0 and at the level where the process was changed at 5.0, the ECTU value was the lowest. For the Lognormal distribution at the coefficient of skewness of 0.1 - 4.0, the ECTU value fluctuated and the ECTU began to increase at the coefficient of skewness of 5.0 - 6.0. At the coefficient of skewness of 2.0, when every level of process was changed, it gave the lowest ECTU value except at the level of the process change of 0.5 and 3.0. In case of the Burr's distribution at the coefficient of skewness from 0.1 to 5.0, almost ECTU values were similar. Conclusively, from three types of distribution, the lognormal distribution at the coefficient of skewness of 6.0 with the level of the process change at 3 had the lowest ECTU value.

5.2 Discussion

The aim of this research was to introduce the economic model of control chart using weighted variance method. Based on expected value of gross per-unit expense, the variance, estimated by proportional value, and weighing, from skewed distribution, was compared to the variance weighed by using Shewhart's control chart, from normal distribution. From the comparison, the lowest expense per time unit will be considered as the highest effectiveness. The results obtained from this research, indicated that in case of the normally distributed data, the variable parameter control chart yielded lower expenses than that of fixed parameter. For the skewed data, data analysis was classified into two cases. First, in case of Shewhart's control chart which is suitable for normally distributed data, the skewed data was converted into normal distribution. At stable process, Weibull's distribution, with coefficient of skewness at 9.0~~,~~ yielded the maximum effectiveness, which is in an agreement with Pongpullponsak [6]. At the process variation from $0.5s$ to $3.0s$, which yielded the higher right skewness, the sign of uncontrollable process existence in control chart was higher and faster. For Shewhart's control chart with the Weibull's distribution, at the coefficient of skewness 4.0 and process variation at 1.0, yielded the lowest per-time unit expense ($634.51 / unit).

6. Suggestion

The results from this study showed that at normally distributed population, the Shewhart variable parameter control chart yielded the lowest expense. In Weibull's distribution population with the level of process change of 5 and coefficient of skewness of 8.0, the expense was the lowest. In case of lognormal distribution population, the expense was the lowest at the level of process change of 3 and coefficient of skewness of 6.0. Finally, the lowest expense observed in Burr's distribution population was at the level of process change of 2.5 and coefficient of skewness of 1.0.

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