Thailand Statistician
July 2008; 6(2) : 163-172
http://statassoc.or.th
Contributed paper

Estimation of Prosperity and Depression Periods in a Labour Market

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Received: 5 February 2008

Accepted: 9 May 2008.

Abstract

Economical growth and fall causes corresponding processes in the labour market. Even a brief analysis of these processes indicates their cyclic feature; period of prosperity follows up recession, depression, and recovery periods. Governmental institutions may influence on these processes due the economical and social programs to reduce the length of negative periods. In this point of view it is important to estimate the duration of the periods in advance. In this paper we derived formulas to evaluate the prosperity and depression periods in the labour market, and confirmed them in an example of the province of Alberta (Canada).

Keywords: employment rate, mathematical modeling, nonlinear dynamics, population dynamics, sociodynamics.

1. Introduction

For last three decades a new interdisciplinary concept, Sociodynamics aims at providing a frame of theoretical concepts for designing mathematical models for a broad class of dynamical phenomena within human society [5]. The methods developed so far integrate on the one side the theory of probabilistic systems and of nonlinear dynamics, and on the other side overarching concepts from the social sciences.

In the social systems at the level of the group, the organization, and the society, certain processes approach either a stationary state or a quasicyclic motion. In [4] the simple forms of interactions between quantified socioeconomic macrovariables are introduced, including in particular “cooperative” or “antagonistic” interactions; then a dynamic model is set up implying these kinds of interactions between its variables. The dynamics of the social processes are described by the logistic differential equations. Solving the equations one can imply the quantitative (time dependent) as well as qualitative (phase) expressions for chosen macrovariables.

In this study we present the solutions of the equations for the chosen social relation between the macrovariables. Then we analyze the dependences of the time intervals of certain social periods on the influence functions.

2. Theory: The Equations of Motion

The detailed analysis of simple forms of interactions between quantified socioeconomic macrovariables is introduced in [4]. In this paper we will refer to this methodology. This is why the same notation is held below. We will analyze interaction of two time dependent socioeconomic macrovariables *x* and *y*. Then we will derive the equations of motion for the macrovariables within four distinguished periods of dynamics. The time used in the equations below is dimensionless. *x* and *y* can be found as solutions of the logistic equations

$$\frac{dx}{d\tau} = x \cdot (a(y)s - x); \quad 0 < x < \infty \tag{1}$$

$$\frac{dy}{d\tau} = y \cdot (b(x)s - y); \quad 0 < y < \infty \tag{2}$$

Here, τ is the dimensionless time; $a(y)$ and $b(x)$ are the influence functions; *s* is adjustable saturation parameter. In [4] *a*(*y*) and *b*(*x*) are chosen as step functions. Depending on the influential configuration of macrovariables *a*(*y*) and *b*(*x*) to be equal to

a_-, a_+ and b_-, b_+ respectively. Consequently equations (1) and (2) become separable. Solutions of this type of differential equations can be found in any textbook on differential equations (for instance, [1]). Solving the equations one can obtain the following expressions for $x(\tau)$ and $y(\tau)$,

(a) *x* supports *y*, and *y* supports *x*,

$$x(\tau) = \frac{x_0 a_+ s \cdot \exp(a_+ s \tau)}{x_0 \exp(a_+ s \tau) + (a_+ s - x_0)} \tag{3}$$

$$y(\tau) = \frac{y_0 b_+ s \cdot \exp(b_+ s \tau)}{y_0 \exp(b_+ s \tau) + (b_+ s - y_0)} \tag{4}$$

(b) *x* suppresses *y*, but *y* supports *x*,

$$x(\tau) = \frac{x_0 a_+ s \cdot \exp(a_+ s \tau)}{x_0 \exp(a_+ s \tau) + (a_+ s - x_0)} \tag{5}$$

$$y(\tau) = \frac{y_0 |b_-| s}{-y_0 + (|b_-| s + y_0) \exp(|b_-| s \tau)} \tag{6}$$

(c) *x* suppresses *y*, and *y* suppresses *x*,

$$x(\tau) = \frac{x_0 |a_-| s}{-x_0 + (|a_-| s + x_0) \exp(|a_-| s \tau)} \tag{7}$$

$$y(\tau) = \frac{y_0 |b_-| s}{-y_0 + (|b_-| s + y_0) \exp(|b_-| s \tau)} \tag{8}$$

(d) *x* supports *y*, but *y* suppresses *x*

$$x(\tau) = \frac{x_0 |a_-| s}{-x_0 + (|a_-| s + x_0) \exp(|a_-| s \tau)} \tag{9}$$

$$y(\tau) = \frac{y_0 b_+ s \cdot \exp(b_+ s \tau)}{y_0 \exp(b_+ s \tau) + (b_+ s - y_0)} \tag{10}$$

Here, x_0 and y_0 are initial values of *x* and *y* respectively.

3. Employment Rate and Job Vacancies

A brief review of booming economies indicates that the high rate of progress in industry, agriculture and service sector causes a labour shortage. Some countries try to solve this problem by attracting more immigrants. However the following economical fall may cause a dramatic increase of unemployment rate. In this point of view the mathematical modeling of job vacancies (JV) – employment rate (ER) correlation becomes interesting and actual.

We have chosen the volume of JV and ER in the labour market as interacting macrovariables and denoted them by *x* and *y* respectively. It is logical to assume that *x* supports *y*, but *y* suppresses *x*. The qualitative phase diagram of this case is presented on the Fig. 1. In [4] this case is denoted as γ. Without losing generality it has been assumed that $x_s = y_s = z$, where (x_s, y_s) is a centre of quadrant system.

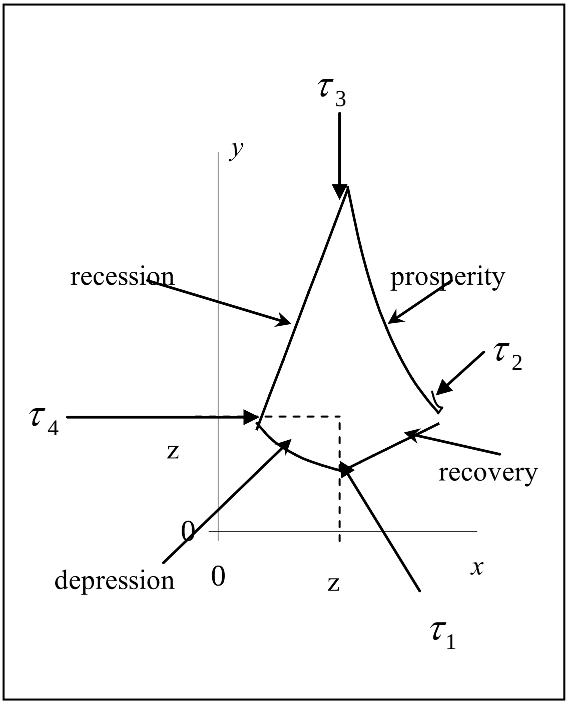

Figure 1: Case $\gamma : a_- = b_- = -1,\ a_+ = b_+ = 2,\ s = 10^{10}$

The Fig. 1 indicates a cyclical dynamics of the chosen macrovariables. We will interpret the periods of the cycle in manner presented in [4].

Prosperity period: Increase of JV causes the increase of ER, whereas the increase of ER works against the further growth of JV. This period is described by equations (5) and (6).

Recession period: Further growth of ER causes deterioration of the JV increasing process and leads to recession. During this period JV suppresses ER, and ER suppresses JV, i.e. cancellation of JV stimulate the further decrease of ER. The recession period is described by equations (7) and (8).

Depression period: ER dramatically goes down, and causes a depression in society. The observed slight increase of JV occurs only due the compelled reduction of jobs. This period is described by equations (9) and (10).

Recovery period: After ER reaches its lowest value; a recovery process begins in the labour force market due the progress of the economy. During this period JV supports ER, and ER supports JV. This period is described by equations (3) and (4).

These processes could be controlled by the governmental programs to make the prosperity period longer, and the depression period shorter. This is why mathematical prediction of these periods in time scale becomes important.

4. Depression and Prosperity Time Periods

Let τ_1 and τ_2 be beginning and ending of the depression period; τ_3 and τ_4 be beginning and ending of the prosperity period respectively. One can see from the Fig. 1 that,

$$x(\tau_1) = \frac{x_0 a_+ s \cdot \exp(a_+ s \tau)}{x_0 \exp(a_+ s \tau) + (a_+ s - x_0)} = z \qquad (3')$$

$$y(\tau_2) = \frac{y_0 b_+ s \cdot \exp(b_+ s \tau)}{y_0 \exp(b_+ s \tau) + (b_+ s - y_0)} = z \qquad (4')$$

$$x(\tau_3) = \frac{x_0 |a_-| s}{-x_0 + (|a_-| s + x_0)\exp(|a_-| s \tau)} = z \qquad (7')$$

$$y(\tau_4) = \frac{y_0 |b_-| s}{-y_0 + (|b_-| s + y_0)\exp(|b_-| s \tau)} = z \qquad (8')$$

Solving these algebraic equations, $(3')$, $(4')$, $(7')$ and $(8')$ we find expressions for the dimensionless phase change times, $\tau_1, \tau_2, \tau_3, \text{and}\ \tau_4$ respectively:

$$\tau_1 = \frac{1}{a_+ s} \ln \frac{z(a_+ s - x_0)}{x_0 (a_+ s - z)}$$

$$\tau_2 = \frac{1}{b_+ s} \ln \frac{z(b_+ s - y_0)}{y_0 (b_+ s - z)}$$

$$\tau_3 = \frac{1}{|a_-| s} \ln \frac{x_0 (|a_-| s + z)}{z(|a_-| s + x_0)}$$

$$\tau_4 = \frac{1}{|b_-| s} \ln \frac{y_0 (|b_-| s + z)}{z(|b_-| s + y_0)}$$

Consequently, the depression period,

$$\Delta\tau_{depr} = \left|\tau_1 - \tau_4\right| = \left|\frac{1}{a_+ s}\ln\frac{z(a_+ s - x_0)}{x_0(a_+ s - z)} - \frac{1}{|b_-|s}\ln\frac{y_0(|b_-|s + z)}{z(|b_-|s + y_0)}\right| \qquad (11)$$

and the prosperity period,

$$\Delta\tau_{pros} = \left|\tau_3 - \tau_2\right| = \left|\frac{1}{|a_-|s}\ln\frac{x_0(|a_-|s + z)}{z(|a_-|s + x_0)} - \frac{1}{b_+ s}\ln\frac{z(b_+ s - y_0)}{y_0(b_+ s - z)}\right| \qquad (12)$$

In the formulae (11) and (12) $\Delta\tau_{depr}$ and $\Delta\tau_{pros}$ are determined as two-variable functions of (a_+, b_-) and (a_-, b_+) respectively.

5. Analysis

The brief inspection of (11) and (12) shows that both quantities, $\Delta\tau_{depr}(a_+, b_-)$ and $\Delta\tau_{pros}(a_-, b_+)$ have similar functional forms with respect to a_+, b_-, a_-, and b_+. The graph of $\Delta\tau_{pros}(a_-, b_+)$ is given on the Fig. 2. One can see from the Fig. 2 that the depression time period as a two-variable function (similarly, the prosperity time period) does not have any local maximum nor minimum with respect to the variables. This means the even theoretically it is not possible to minimize the depression period or to maximize the prosperity period. However it is possible to evaluate the duration of these time periods for the given initial parameters, such as x_0, y_0, x_s, y_s, and s.

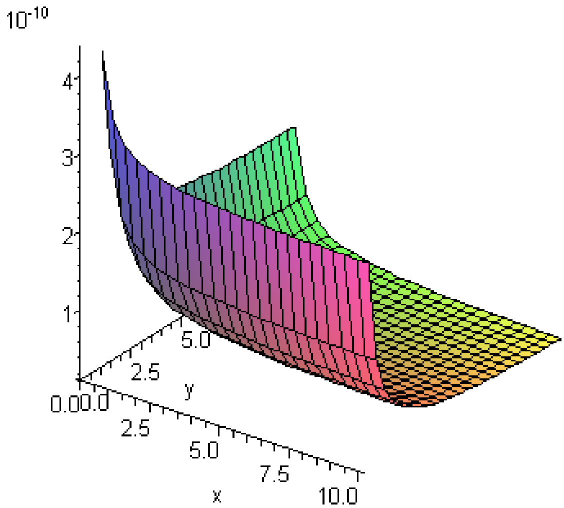

Figure 2: Prosperity period as a function of a_- and b_+

6. Application

In this section we are applying the discussed concept to the labour market dynamics of the province of Alberta (Canada). Whereas ER is available from different public resources [2, 3], the information about the JV is not registered. However it can be reasonably assumed that there is a direct correlation between JV and GDP, hence we can anticipate that the ER vs. GDP phase diagram has also cyclic character.

In Fig. 3 the temporal dynamics of ER and GDP are presented. The data for the graph have been taken from the E-Stat and the Pembina Institute reports [2, 3]. In [3] GDP is given in 1998$. Both, GDP and ER are normalized to the values for 1976.

The graph indicates periodicity of temporal dynamics for both, ER and GDP. A real data based ER vs. GDP phase diagram has a cyclic character (Fig. 4), which is typical for ER vs. JV phase diagram (Fig. 1), as it was shown above. The phase diagram on Figure 4 is built for year a time period from 1976 to 1991, which we suggest can be considered as one sociodynamical cycle.

One can see that the cycles on Figures 1 and 4 are different. This difference can be explained by the fact, that ER vs. GDP dependence is not identical to ER vs. JV dependence.

To achieve an agreement between the real data based phase diagram and theoretically estimated one we have to add more terms to the initial differential equations; i.e. to make our model more robust we have to consider more factors. This is a goal of our future studies and will not be discussed in this paper.

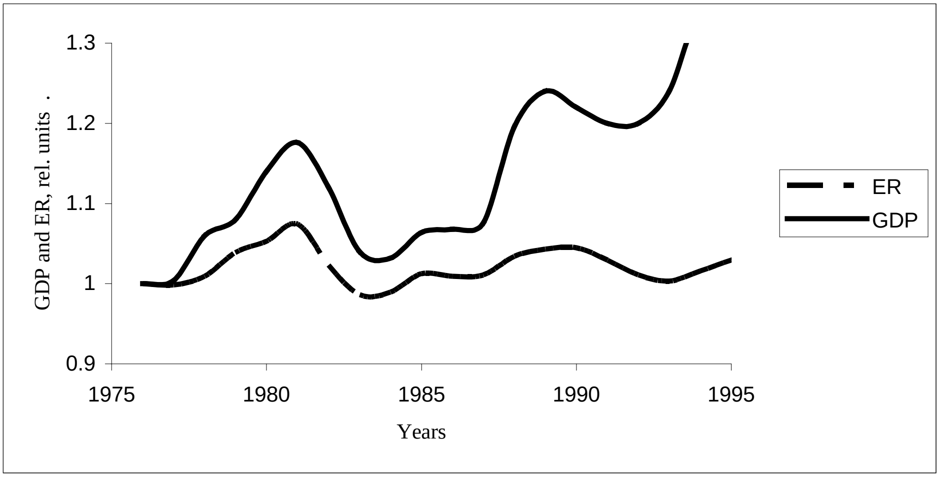

Figure 3: Temporal dynamics of GDP and ER for the province of Alberta (Canada)

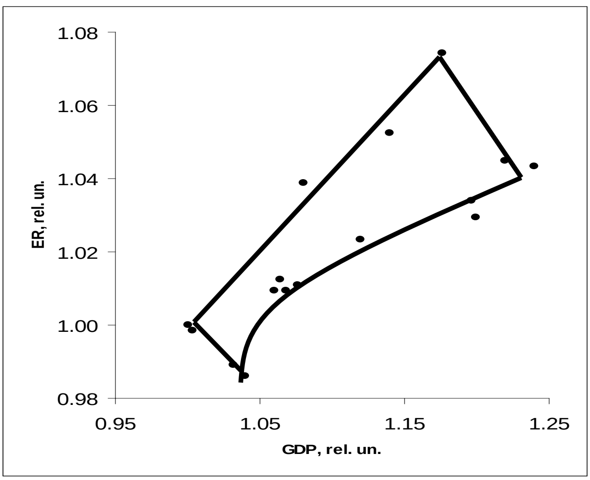

Figure 4: ER vs. GDP phase diagram for time period 1976 – 1991

7. Conclusion

In this paper using the concept developed by Weidlich [4] we analyzed the dynamics of the interaction of two macrovariables, JV and ER. It has been shown that the interaction of these macrovariables causes the cyclic processes. Each cycle consists of four distinct periods. We have driven formulae to evaluate so called depression and prosperity periods. The theoretical evaluations were compared with the real statistical data for the province of Alberta (Canada). In example of real statistical data we have shown that the relation JV vs. GDP also has cyclic character.

In our point of view the methodology proposed on this paper can be used to make prognosis regarding the social processes described above. We plan to consider the influence of more factors to the dynamics of these processes in our future works.

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