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On Coverage Probability of a Prediction Interval for an Unknown Mean AR(1) Process Using Combined Predictors

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Abstract

A new prediction interval for an unknown mean first-order autoregressive process (AR(1)) using combined predictors from a stationary process and a non stationary process is investigated in this paper. The coverage probabilities of a new prediction interval and a standard prediction interval are also derived to be functionally independent of the population mean and the variance of the innovation process. Monte Carlo simulation shows that a new prediction interval has a desired minimum coverage probability $1-\alpha$, which is better than a standard prediction interval for all the autoregressive parameter values used and for all sample sizes considered in this paper.

Keywords: AR(1), combined predictors, coverage probability, prediction interval.

1. Introduction

Sanchez [5] proposed a combined predictors to forecast the h-steps-ahead forecast for an AR(1) process near the non-stationary AR(1) process. Combined predictors consist of a predictor from a stationary AR(1) process and another predictor from a non- stationary AR(1) process. Sanchez compared these combined predictors with the differenced stationary predictor, a pretest differenced stationary predictor, a fractional differenced stationary predictor and classical combined predictors. Sanchez assessed these combined predictors using the prediction mean square error (PMSE). A combined predictors is superior to others predictors when the autoregressive parameter of this process approaches one. In other words, this predictor produces a smaller PMSE than others. Niwitpong [3] proposed a new prediction interval for an AR(1) process using combined predictors of Sanchez [5]. He found that a new prediction interval has a minimum coverage probability 0.95 which is better than a standard prediction interval.

Our aim in this paper is to construct a one-step-ahead prediction interval based on the combined predictors for an unknown mean AR(1) process of Sanchez [5]. As in Niwitpong [3], we have derived coverage probabilities of this new prediction interval and a standard prediction interval. The coverage probabilities of these two prediction intervals are shown to be functionally independent of the mean process and the variance of the innovation process. This important result allows us to set the mean process equals zero and the error variance of this process at equals one and this result is valid for all possible parameter values of the mean and the error variance of this process. This leads to a great reduction in computational effort. We have assessed these two prediction intervals based on a minimum coverage probability $1-\alpha$. In other words, we have compared these two prediction interval using a minimum coverage probability of $1-\alpha$ as a criterion, see e.g., Casella and Berger [1].

Section 2 describes the method to construct prediction intervals for an unknown mean AR(1) process based on a standard method and the combined predictors method. Section 3 gives the idea to compute the coverage probabilities of both prediction intervals. Section 4 presents Monte Carlo simulation results of the coverage probabilities of these two prediction intervals. The conclusion is in Section 5.

2. Prediction Intervals for an unknown mean AR(1) process

Suppose $\{Y_t\}$ is a non-zero mean AR(1) process satisfying

$$Y_t - \mu \;=\; \rho(Y_{t-1}-\mu)+e_t, \qquad \text{.. (1)}$$

where μ is a population mean, ρ is an autoregressive parameter, $\rho \in (-1,1)$, $t \in \{1,2,3,...,T\}$ and e_t are unobservable independent errors having zero mean and finite variance.

The ordinary least squares (OLS) estimator of ρ is denoted $\hat{\rho}$ and is given by

$$\hat{\rho} \;=\; \frac{\sum_{t=2}^{T}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=2}^{T}(Y_{t-1}-\bar{Y})^2}. \qquad \text{..(2)}$$

Also an unbiased estimator of μ is $\hat{\mu}=\bar{Y}$. For a stationary process, for known μ and ρ, the optimal predictor of Y_{T+1} is $\rho(Y_T-\mu)$. Replacing the unknown μ and ρ by the estimators $\hat{\mu}$ and $\hat{\rho}$, we obtain the predictor $\hat{\rho}(Y_T-\hat{\mu})$. For a non stationary process; $\rho=1$, the optimal predictor of Y_{T+1} is Y_T. These two predictors are unbiased forecasts, see e.g. Sanchez [5]. Following Sanchez [5], Phillips [4], and Diebold [2], we now construct the linear combination of two one-step-ahead forecasts which achieves a lower PMSE. The combined predictors can be expressed as

$$Y_{T+1}^{c}-\mu=\beta_0+\beta_1(Y_T-\mu)+(1-\beta_1)\rho(Y_T-\mu)+e_{T+1} \qquad \text{......................(3)}$$

where μ, ρ, β_0 and β_1 are the unknown parameters, $e_{T+1} \sim N(0,\sigma^2)$ and is independent of $e_1, e_2, ..., e_T$.

Sanchez [5] further pointed out that as ρ approaches one, (3) reduces to

$$Y_{T+1}^{c}=\beta Y_T+(1-\beta)(\mu(1-\rho)+\rho Y_T)+e_{T+1} \qquad \text{...............................(4)}$$

where $\beta = \dfrac{2(1-\rho^2)^2(1+\rho)}{T(1-\rho)^3 + 2(1-\rho^2)(1+\rho)}$.

The estimated combined predictor is obtained by replacing (μ, ρ, β) with $(\hat{\mu}, \hat{\rho}, \hat{\beta})$. Then

$$\hat{Y}^c_{T+1} = \hat{\beta} Y_T + (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho} Y_T) \qquad \text{...............................(5)}$$

where $\hat{\beta} = \dfrac{2(1-\hat{\rho}^2)^2(1+\hat{\rho})}{T(1-\hat{\rho})^3 + 2(1-\hat{\rho}^2)(1+\hat{\rho})}$.

Now we propose a one-step-ahead prediction interval for Y^c_{T+1} using model (4). From (4), it is straightforward to show that

$$e_{T+1} = Y^c_{T+1} - \beta Y_T - (1-\beta)(\mu(1-\rho) + \rho Y_T) \text{ and } \frac{e_{T+1}}{\sigma} \sim N(0,1).$$

Thus $\dfrac{e_{T+1}}{\sigma} \approx Z_{1-\frac{\alpha}{2}} = \dfrac{Y^c_{T+1} - \beta Y_T - (1-\beta)(\mu(1-\rho) + \rho Y_T)}{\sigma_0}$

where $\sigma_0{}^2 = \dfrac{1}{T} \sum_{t=2}^{T} (Y_t - \beta\, Y_{t-1} - (1-\beta)(\mu(1-\rho) + \rho Y_{t-1}))^2.$

Using $Z_{1-\frac{\alpha}{2}} = \dfrac{Y^c_{T+1} - \beta Y_T - (1-\beta)(\mu(1-\rho) + \rho Y_T)}{\sigma_0}$ as a pivotal quantity. The (1 - α)100% prediction interval for Y^c_{T+1} is therefore

$$\left[\beta Y_T + (1-\beta)(\mu(1-\rho) + \rho Y)_T - Z_{1-\frac{\alpha}{2}} \sigma_0, \beta Y_T + (1-\beta)(\mu(1-\rho) + \rho Y_T) + Z_{1-\frac{\alpha}{2}} \sigma_0 \right] \text{...(6)}$$

where $Z_{1-\frac{\alpha}{2}}$ is a $(1-\dfrac{\alpha}{2})th$ quantile of the standard normal distribution. Replacing the unknown parameters $(\mu, \rho, \beta, \sigma_0)$ in (6) by estimators ($\hat{\mu}, \hat{\rho}$, $\hat{\beta}$, $\hat{\sigma}_0$). The approximate (1 - α)100% prediction interval for Y^c_{T+1} is

$$PI_0 = \left[\hat{\beta}Y_T + (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_T) - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 , \hat{\beta}Y_T + (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_T) + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \right] \quad ...(7)$$

where $\hat{\sigma}_0{}^2 = \dfrac{1}{T-2}\sum_{t=2}^{T}(Y_t - \hat{\beta}\, Y_{t-1} - (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_{t-1}))^2.$

Similarly, from (1), the one-step-ahead prediction interval for Y_{T+1} is

$$\left[\rho(Y_T - \mu) - Z_{1-\frac{\alpha}{2}}\sigma_1 , \rho(Y_T - \mu) + Z_{1-\frac{\alpha}{2}}\sigma_1 \right] \quad ....(8)$$

where

$\sigma_1^2 = \dfrac{1}{T}\sum_{t=2}^{T}(y_t - \mu - \rho(Y_{t-1} - \mu))^2 .$

Replacing the unknown (μ, ρ, σ_1) in (8) by the estimators $(\hat{\mu}, \hat{\rho}, \hat{\sigma}_1)$. The approximate (1 - α)100% prediction interval for Y_{T+1} is therefore

$$PI_1 = \left[\hat{\rho}(Y_T - \hat{\mu}) - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1, \hat{\rho}(Y_T - \hat{\mu}) + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1 \right] \quad ....(9)$$

where $\hat{\sigma}_1^2 = \dfrac{1}{T-2}\sum_{t=2}^{T}(y_t - \hat{\mu} - \hat{\rho}(Y_{t-1} - \hat{\mu}))^2$. We call a prediction interval PI_1 as a standard prediction interval for Y_{T+1}.

3. The Coverage Probabilities of Prediction Intervals

The unconditional coverage probability for Y_{T+1}^c of PI_0 in (7) is

$$P\left[\hat{\beta}Y_T + (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_T) - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \le Y_{T+1}^c \le \hat{\beta}Y_T + (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_T) + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \right]$$

$$= P\left[\hat{\beta}Y_T + k_1 - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \le \beta Y_T + (1-\beta)(\mu(1-\rho) + \rho Y_T) + e_{T+1} \le \hat{\beta}Y_T + k_1 + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \right]$$

$$(when \quad k_1 = (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_T))$$

$$= P\left[\hat{\beta}Y_T + k_1 - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \le \mu + (\beta + (1-\beta)\rho)(Y_T - \mu) + e_{T+1} \le \hat{\beta}Y_T + k_1 + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \right]$$

$$= P\left[(\hat{\beta} + (1-\hat{\beta})\hat{\rho})(Y_T - \mu) + k_2 - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \le \mu + (\beta + (1-\beta)\rho)(Y_T - \mu) + e_{T+1} \le (\hat{\beta} + (1-\hat{\beta})\hat{\rho})(Y_T - \mu) + k_2 + Z_{1-\frac{\alpha}{2}}\hat{\sigma} \right]$$

$$(when \quad k_2 = \hat{\mu} - (\hat{\beta} + (1-\hat{\beta})\hat{\rho})(\hat{\mu} - \mu))$$

$$= P\left[k_3(Y_T - \mu) + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})(\hat{\mu} - \mu) - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_0 \le e_{T+1} \le k_3(Y_T - \mu) + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})(\hat{\mu} - \mu) + Z_{1-\frac{\alpha}{2}}\hat{\sigma} \right]$$

$$(when \quad k_3 = \hat{\beta} + (1-\hat{\beta})\hat{\rho} - \beta - (1-\beta)\rho)$$

$$= P\left[k_3\frac{(Y_T - \mu)}{\sigma} + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\frac{(\hat{\mu} - \mu)}{\sigma} - Z_{1-\frac{\alpha}{2}}\frac{\hat{\sigma}_0}{\sigma} \le \frac{e_{T+1}}{\sigma} \le k_3\frac{(Y_T - \mu)}{\sigma} + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\frac{(\hat{\mu} - \mu)}{\sigma} + Z_{1-\frac{\alpha}{2}}\frac{\hat{\sigma}_0}{\sigma} \right]$$

$$= P\left[k_3X_T + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\bar{X} - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X \le \frac{e_{T+1}}{\sigma} \le k_3X_T + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\bar{X} + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X \right]$$

$$= \Phi(k_3X_T + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\bar{X} + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X) - \Phi(k_3X_T + (1 - \hat{\beta} + (1-\hat{\beta})\hat{\rho})\bar{X} - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X) \cdot \quad ...(10)$$

where

$X_T = \dfrac{Y_T - \mu}{\sigma}$, $\dfrac{(\hat{\mu} - \mu)}{\sigma} = \dfrac{\sum Y_T - T\mu}{\sigma T} = \dfrac{\sum((Y_T - \mu)/\sigma)}{T} = \dfrac{\sum X_T}{T} = \bar{X}$ and

$\Phi(.)$ is a cumulative standard Normal distribution.

Observe from (1) that

$$X_t = \rho X_{t-1} + \eta_t$$

where $\eta_t = \dfrac{e_t}{\sigma}$ so that $\{\eta_t\}$ is a sequence of independent and identically distributed

N(0,1) random variables. Therefore ($X_{1,} X_2, \ldots, X_T$) has a probability distribution that does not depend on (μ, σ^2) , i.e. it is a function of ρ only. Also,

$$\hat{\sigma}_X^2 = \frac{\hat{\sigma}_0^2}{\sigma^2} = \frac{1}{T-2}\sum_{t=2}^{T}\left[(Y_t - \hat{\beta}Y_{t-1} - (1-\hat{\beta})(\hat{\mu}(1-\hat{\rho}) + \hat{\rho}Y_{t-1}))\sigma^{-1} \right]^2$$

$$= \frac{1}{T-2}\sum_{t=2}^{T}\left[((Y_t-\mu)-(\hat{\mu}-\mu)-\hat{\beta}((Y_{t-1}-\mu)-(\hat{\mu}-\mu))-(1-\hat{\beta})\hat{\rho}((Y_{t-1}-\mu)-(\hat{\mu}-\mu)))\sigma^{-1}\right]^2$$

$$= \frac{1}{T-2}\sum_{t=2}^{T}\left[(X_t-\bar{X}-\hat{\beta}(X_{t-1}-\bar{X})-(1-\hat{\beta})\hat{\rho}(X_{t-1}-\bar{X}))\right]^2$$

and

$$\hat{\rho}=\frac{\sum_{t=2}^{T}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=2}^{T}(Y_{t-1}-\bar{Y})^2}=\frac{\sum_{t=2}^{T}(\sigma^{-1}((Y_t-\mu)-(\bar{Y}-\mu)))(\sigma^{-1}((Y_{t-1}-\mu)-(\bar{Y}-\mu)))}{\sum_{t=2}^{T}(\sigma^{-1}((Y_{t-1}-\mu)-(\bar{Y}-\mu)))^2}=\frac{\sum_{t=2}^{T}(X_t-\bar{X})(X_{t-1}-\bar{X})}{\sum_{t=2}^{T}(X_{t-1}-\bar{X})^2}.$$

are functions of $(X_{1,}X_2,....,X_T)$.

From the previous section, $\hat{\beta}$ is a function of $\hat{\rho}$. Therefore the coverage probability of PI_0 is functionally independent of (μ,σ^2) , i.e. it is a function of ρ only.

Similarly, from (9), the coverage probability of PI_1 is therefore

$$P\left[\hat{\mu}+\hat{\rho}(Y_T-\hat{\mu})-Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\le Y_{T+1}\le\hat{\mu}+\hat{\rho}(Y_T-\hat{\mu})+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\right]$$

$$=P\left[\hat{\mu}+\hat{\rho}(Y_T-\hat{\mu})-Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\le \mu+\rho(Y_T-\mu)+e_{T+1}\le\hat{\mu}+\hat{\rho}(Y_T-\hat{\mu})+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\right]$$

$$=P\left[(1-\hat{\rho})(\hat{\mu}-\mu)+(\hat{\rho}-\rho)(Y_T-\mu)-Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\le e_{T+1}\le(1-\hat{\rho})(\hat{\mu}-\mu)+(\hat{\rho}-\rho)(Y_T-\mu)+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\right]$$

$$=P\left[(1-\hat{\rho})\frac{(\hat{\mu}-\mu)}{\sigma}+(\hat{\rho}-\rho)\frac{(Y_T-\mu)}{\sigma}-Z_{1-\frac{\alpha}{2}}\frac{\hat{\sigma}_1}{\sigma}\le(1-\hat{\rho})\frac{(\hat{\mu}-\mu)}{\sigma}+(\hat{\rho}-\rho)\frac{(Y_T-\mu)}{\sigma}+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_1\right]$$

$$=P\left[(1-\hat{\rho})\bar{X}+(\hat{\rho}-\rho)X_T-Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X}\le\frac{e_{T+1}}{\sigma}\le(1-\hat{\rho})\bar{X}+(\hat{\rho}-\rho)X_T+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X}\right]$$

$$=\Phi((1-\hat{\rho})\bar{X}+(\hat{\rho}-\rho)X_T+Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X})-\Phi((1-\hat{\rho})\bar{X}+(\hat{\rho}-\rho)X_T-Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X}). \quad (11)$$

It is also straightforward to see that

$$\hat{\sigma}_{1X}^2 = \frac{\hat{\sigma}_1^2}{\sigma^2} = \frac{1}{T-2}\sum_{t=2}^{T}\frac{(Y_t-\hat{\mu}-\hat{\rho}(Y_{t-1}-\hat{\mu}))^2}{\sigma^2} = \frac{1}{T-2}\sum_{t=2}^{T}\left[\frac{(Y_t-\mu)-(\hat{\mu}-\mu)-\hat{\rho}((Y_{t-1}-\mu)-(\hat{\mu}-\mu))}{\sigma}\right]^2$$

Thus,

$$\hat{\sigma}_{1X}^2 = \frac{1}{T-2}\sum_{t=2}^{T}(X_t-\overline{X}-\hat{\rho}(X_{t-1}-\overline{X}))^2.$$

Therefore, the coverage probability of PI_1 is functionally independent of (μ,σ^2), i.e. it is a function of ρ only.

4. Monte Carlo Simulation

In this section, the coverage probabilities of PI_0 and PI_1 are compared using Monte Carlo simulation. Naturally, we prefer a prediction interval which has a large coverage probability. In this paper, we also prefer a prediction interval which has a desired minimum coverage probability $1-\alpha$.

Suppose the indicator $I_{PI_j}(X_{n+1})$ defined by $I_{PI_j}(X_{n+1})=1$ if $X_{n+1}\in PI_j$ ($j=0,1$) and 0 otherwise. We suppose that each Monte Carlo simulation consists of M independent runs.

Let the observed values of $X_t, \overline{X}, k_3, \hat{\beta}, \hat{\rho}, \hat{\sigma}_X, \hat{\sigma}_{1X}$ be denoted by $x_t^{(k)}, \overline{x}^{(k)}, k_3^{(k)}, \hat{\beta}^{(k)}, \hat{\rho}^{(k)}, \hat{\sigma}_X^{(k)}, \hat{\sigma}_{1X}^{(k)}$ for the k th run. Thus, from (10) and (11), we have

$$\Pr(X_{n+1}\in PI_0) = E(I_{PI_0}(X_{n+1})) \approx \sum_{k=1}^{M}\left[A-B\right]$$

when

$$A = \Phi(k_3^{(k)}x_T + (1-\hat{\beta}^{(k)} + (1-\hat{\beta}^{(k)})\hat{\rho}^{(k)})\overline{x}^{(k)} + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X^{(k)})$$

$$B = \Phi(k_3^{(k)}x_T + (1-\hat{\beta}^{(k)} + (1-\hat{\beta}^{(k)})\hat{\rho}^{(k)})\overline{x}^{(k)} - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_X^{(k)})$$

and $\Pr(X_{n+1} \in PI_1) = E(I_{PI_1}(X_{n+1}))$

$$\approx \sum_{k=1}^{M}\left[\Phi((1-\hat{\rho}^{(k)})\overline{x}^{(k)} + (\hat{\rho}^{(k)} - \rho)x_T + Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X}^{(k)}) - \Phi((1-\hat{\rho}^{(k)})\overline{x}^{(k)} + (\hat{\rho}^{(k)} - \rho)x_T - Z_{1-\frac{\alpha}{2}}\hat{\sigma}_{1X}^{(k)})\right].$$

We chose ρ = 0.1, 0.3, 0.6, 0.8, 0.9, 0.95, 0.97 and 0.99, T = 30, 50, 100 and 200, $\alpha = 0.01, 0.05,$ and 0.10, $\mu = 0, 1, 5$ and $\sigma^2 = 1, 5,$ 10. Here, we emphasize ρ approaches one since our predictor is a linear combination of predictors from a stationary process and a non stationary process. All simulations were performed using programs written in S-PLUS with *M* =1000.

The estimated coverage probabilities for a prediction interval PI_0 and a prediction interval PI_1 are reported in Tables 1-4. As can be seen from these tables, the new prediction interval, PI_0, has a minimum (estimated) coverage probability $1-\alpha,$ for all sample sizes and values of ρ considered here. The standard prediction interval PI_1, however, does not have a minimum coverage probability $1-\alpha,$ for any sample sizes and values of ρ considered here. Therefore the new prediction interval PI_0 is preferable to the prediction interval PI_1.

Table1. The estimate coverage probabilities of a prediction interval PI_1 compared to a prediction interval PI_0 for *M* = 1000, $\alpha = 0.01, \mu = 0, \sigma = 1.$

	T = 30		**T = 50**		**T =100**		**T =200**	
ρ	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1
0.990	0.9941	0.9884	0.9944	0.9890	0.9949	0.9896	0.9948	0.9896
0.970	0.9925	0.9859	0.9937	0.9870	0.9944	0.9888	0.9947	0.9893
0.950	0.9934	0.9857	0.9932	0.9867	0.9945	0.9887	0.9948	0.9895
0.900	0.9927	0.9847	0.9934	0.9863	0.9946	0.9886	0.9947	0.9894
0.800	0.9919	0.9822	0.9935	0.9865	0.9943	0.9882	0.9947	0.9892
0.600	0.9923	0.9831	0.9935	0.9859	0.9942	0.9881	0.9946	0.9891
0.300	0.9922	0.9828	0.9934	0.9855	0.9941	0.9878	0.9947	0.9892
0.100	0.9924	0.9836	0.9933	0.9859	0.9943	0.9884	0.9948	0.9894

Table 2. The estimate coverage probabilities of a prediction interval PI_1 compared to a prediction interval PI_0 for *M* = 1000, $\alpha = 0.05, \mu = 0, \sigma = 1.$

	T = 30		T = 50		T =100		T =200	
ρ	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1
0.990	0.9744	0.9504	0.9748	0.9521	0.9758	0.9486	0.9746	0.9489
0.970	0.9709	0.9450	0.9726	0.9451	0.9735	0.9477	0.9748	0.9490
0.950	0.9667	0.9355	0.9718	0.9433	0.9736	0.9471	0.9742	0.9485
0.900	0.9764	0.9479	0.9717	0.9431	0.9730	0.9455	0.9743	0.9484
0.800	0.9739	0.9411	0.9718	0.9423	0.9726	0.9445	0.9744	0.9485
0.600	0.9671	0.9321	0.9727	0.9435	0.9733	0.9454	0.9742	0.9486
0.300	0.9679	0.9349	0.9726	0.9425	0.9734	0.9458	0.9742	0.9483
0.100	0.9669	0.9316	0.9725	0.9424	0.9735	0.9461	0.9744	0.9484

Table 3. The estimate coverage probabilities of a prediction interval PI_1 compared to a prediction interval PI_0 for *M* = 1000, $\alpha = 0.1, \mu = 0, \sigma = 1.$

	T = 30		T = 50		T =100		T =200	
ρ	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1
0.990	0.9529	0.9067	0.9515	0.9033	0.9496	0.8973	0.9494	0.8983
0.970	0.9476	0.8935	0.9467	0.8937	0.9482	0.8960	0.9492	0.8985
0.950	0.9453	0.8901	0.9478	0.8935	0.9491	0.8952	0.9500	0.8988
0.900	0.9476	0.8893	0.9473	0.8924	0.9484	0.8950	0.9495	0.8982
0.800	0.9455	0.8865	0.9455	0.8903	0.9488	0.8958	0.9490	0.8976
0.600	0.9463	0.8834	0.9471	0.8911	0.9486	0.8953	0.9493	0.8980
0.300	0.9426	0.8816	0.9458	0.8899	0.9480	0.8945	0.9491	0.8981
0.100	0.9447	0.8830	0.9473	0.8918	0.9489	0.8964	0.9492	0.8948

Table 4. The estimate coverage probabilities of a prediction interval PI_1 compared to a prediction interval PI_0 for *M* = 1000, $\alpha = 0.05, \rho = 0.50.$

	T = 30		T = 50		T =100		T =200	
(μ, σ)	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1	PI_0	PI_1
(1, 1)	0.9712	0.9375	0.9723	0.9430	0.9732	0.9450	0.9744	0.9483
(1, 5)	0.9713	0.9381	0.9720	0.9415	0.9733	0.9462	0.9750	0.9494
(1, 10)	0.9711	0.9380	0.9725	0.9429	0.9733	0.9451	0.9743	0.9478
(5, 1)	0.9711	0.9373	0.9733	0.9433	0.9736	0.9452	0.9742	0.9481
(5, 5)	0.9712	0.9377	0.9715	0.9413	0.9737	0.9466	0.9743	0.9482
(5, 10)	0.9713	0.9387	0.9734	0.9431	0.9736	0.9456	0.9743	0.9483

5. Conclusion

We have proposed a new prediction interval for an AR(1) process based on the combined predictors of a stationary process and a non stationary process. The coverage probability of this new prediction interval is shown to be functionally independent of (μ, σ^2). This result allows us to set $\mu = 0$ and σ^2 = 1 in Monte Carlo simulation. This leads to a great reduction in computational effort. The new prediction interval is preferable to the standard prediction interval since it has a desired minimum coverage probability $1-\alpha,$ for all values of ρ and all sample sizes considered here.

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