Thailand Statistician
July 2015; 13(2): 127-143
http://statassoc.or.th
Contributed paper

An Inventory Model for Deteriorating Items with Price Dependent Demand and Delay in Payments under Stochastic Inflation Rate

Manisha Pal* and Hare Krishna Maity
Department of Statistics, University of Calcutta, Kolkata-700019, West Bengal, India.
*Corresponding author; e-mail: manishapal2@gmail.com

Received: 3 November 2014
Accepted: 21 January 2015

Abstract

The paper studies an inventory model for deteriorating items under permissible delay in payment and stochastic inflation conditions. The deteriorating rate is assumed to be constant and the demand for the item is dependent on its selling price. A periodic review policy has been used over a finite planning period. The inflation rate changes from one periodic cycle to another following some probabilistic law. The optimal policy is determined so as to maximize the total profit over the planning horizon. Numerical examples are cited to illustrate the theoretical results, and a sensitivity analysis is carried out to examine the effect of the model parameters on the optimal policy.

Keywords: periodic review model, deteriorating items, permissible delay in payment, stochastic inflation rate.

1. Introduction

The classical inventory models have been developed under the assumption that inflation does not play a significant role on the inventory policy. However, from financial point of view, one may consider an inventory to be a capital investment, and, as such, it should compete with other assets for an organization's limited capital fund. It is, therefore, important to investigate how time-value of money influences various inventory policies. The first study in this direction has been reported by Buzacott [1], who considered EOQ model with inflation, subject to different types of pricing policies. Misra [2] developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan [3] surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some studies were also conducted with variable demand, see, for example, Uthayakumar and Geetha [4], Maity [5], Vrat and Padmanabhan [6], Dutta and Pal [7], Hariga [8], Hariga and Ben-Daya [9] and Chung [10]. In all these studies, it is assumed that the inflation rate remains constant over time and is known. However, in reality, the inflation rate is affected by many factors, which may be economical, political, social or cultural, like world inflation rate, unemployment rate, natural calamities, artificial scarcities, etc. As such, it is more justified to assume the rate of inflation to be random. A study in this direction has been carried out by Mirzazadeh [11], who considered an inventory model for deteriorating items allowing shortages when demand rate remains constant but inflation conditions are uncertain.

Another changing feature in today's business transactions is the permissible delay period allowed by the supplier to the inventory manager to pay his dues. If the manager pays up within the grace period allowed to him, he does not have to pay any interest. However, he is charged an interest if he settles his dues after the permissible period. In such a situation, it makes economical sense for the manager to delay his payment to the end of the grace period since during the period he can sell his stock and accumulate revenue on it. The first study along this line was carried out by Goyal [12]. Thereafter many authors investigated inventory models allowing permissible delay in payment. See, for example, Shinn et al. [13], Hwang and Shinn [14], Jamal et al. [15], Pal and Ghosh [16-17], Shah and Shah [18].

In this paper, we consider a periodic review inventory model for deteriorating items over a finite planning horizon allowing shortages when demand is price dependent. The inflation rate is assumed to be a random and the inventory manager is allowed a permissible delay in payment. The paper is organized as follows. Section 2, gives the notations used in the model and the assumptions made. The model is analyzed in section 3. In section 4, sensitivity of the optimal policy to change in the model parameters is examined. Finally, in section 5, a concluding discussion on the model is given.

2. Notations and assumptions

Notations:

$(0, H)$ = planning horizon;
A = ordering cost per order;
C = cost price per unit;
I_c = holding cost per unit per unit time;
s_1 = shortage cost per unit per unit time;
P = selling price per unit;
θ = deterioration rate;
I_e = interest earned per annum;
I_r = interest charged per annum;
M = permissible delay in payments;
T = complete inventory cycle length;
T_1 = time taken for stock on hand to become zero in a cycle;
$D(t)$ = demand rate at time t ;
r = inflation rate;
$f(r)$ = probability density function of r;
$M_r(t)$ = moment generating function of r;
D = discount rate representing the value of money;
k = discount rate net of inflation, i.e. $k = d\text{-}r$.

Assumptions:

1. The length of the planning period is $H = n\,T$, where n is an integer denoting the number of replenishments to be made in the period $(0, H)$ and T is the length of a reorder interval.
2. The demand rate is dependent on the selling price. It remains constant within an inventory cycle and for the s^{th} cycle it is given by $a - bpe^{r(s-1)T}$, $1 \le s \le n$, where $a, b \ge 0, a \succ\succ b$. This form of the demand rate arises from the fact that as the price of an item increases, its demand is likely to decrease, and, due to price inflation, the price p increases to $pe^{r(s-1)T}$ in the the $(s-1)^{th}$ reorder interval.

3. The inflation rate r is random but remains unchanged during an inventory cycle.
4. Shortages are allowed and backlogged during the first $(n-1)$ inventory cycles, but no shortage is allowed during the last cycle.
5. Replenishment is instantaneous on ordering.

3. THE MATHEMATICAL MODEL AND ITS ANALYSIS

We consider inventory of a single item which has a constant deterioration rate. A periodic review policy is used, and the inventory manager is allowed a fixed permissible delay in payment. The decision variables of the model are n, the number of replenishments, and Q, the order quantity at each reorder point, which are determined so as to maximize the total expected profit over $(0, H)$. During the s-th reorder interval, denoted by $[(s-1)T, sT]$, the inventory level becomes zero at the time point $((s-1)T+T_1)$ and thereafter shortage is allowed to accumulate till the end of the interval before they are backordered. However, in the last, that is the n-th reorder interval, no backlogging is allowed. The following figure illustrates the model:

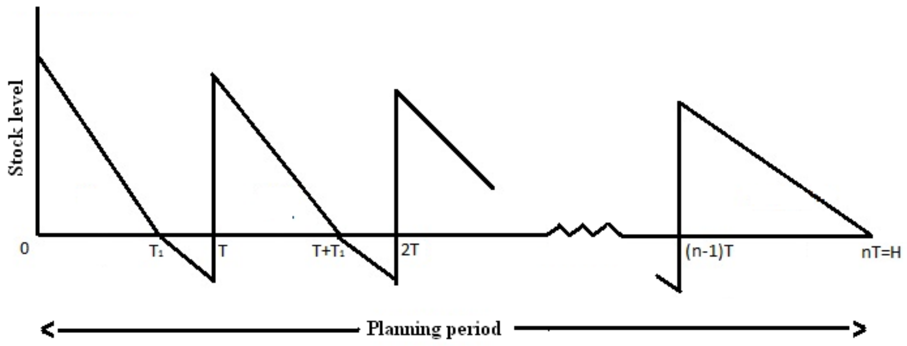

Figure 1. The inventory policy over a finite planning horizon.

Let $I_{s-1}(t)$ denote the stock on hand at time $(s\text{-}1)T+t$, $1 \le s \le n$, $0 \le t \le T$.

To obtain the differential equations defining transitions in the stock level, we note that in the s-th reorder interval, $1 \le s \le n-1$, depletion from stock occurs due to demand and deterioration of items in the interval $[0, T_1]$, and thereafter only due to demand as the stock size becomes zero at T_1. On the other hand, for the n-th interval, depletion of stock occurs due to both demand and deterioration in the interval $[(n-1)T, nT]$. Hence, the differential equations are as follows:

(i) for $1 \le s \le n-1$,

$$\frac{dI_{s-1}(t)}{dt} + \theta I_{s-1}(t) = -a + bpe^{r(s-1)T} \qquad 0 \le t \le T_1$$

$$\frac{dI_{s-1}(t)}{dt} = -a + bpe^{r(s-1)T} \qquad T_1 \le t \le T,$$

(ii) for $s = n$,

$$\frac{dI_{n-1}(t)}{dt} + \theta I_n(t) = -a + bpe^{r(n-1)T}, \qquad 0 \le t \le T.$$

The boundary condition is $I_{s-1}(T_1) = 0$ for $1 \le s \le n-1$, and $I_{n-1}(T) = 0$.

Defining $D_{s-1} = -a + bpe^{r(s-1)T}$, the above equations give

(i) for $1 \le s \le n-1$,

$$I_{s-1}(t) = \begin{cases} \frac{D_{s-1}}{\theta}\left(1 - e^{\theta(T_1 - t)}\right), & 0 \le t \le T_1 \\ D_{s-1}(T_1 - t), & T_1 \le t \le T \end{cases}$$

(ii) for *s* = *n*,

$$I_n(t) = \frac{D_{n-1}}{\theta}\left(1 - e^{\theta(T-t)}\right), \quad 0 \le t \le T.$$

Then, the maximum stock height for the *s*th cycle is $\frac{D_{s-1}}{\theta}\left(1 - e^{\theta T_1}\right)$, $1 \le s \le n-1$, and that for the last cycle is $\frac{D_{n-1}}{\theta}\left(1 - e^{\theta T}\right)$. Clearly these are functions of T_1 and T, respectively.

Profit Function:

We find the optimal values of T and T_1 that maximize the value of the expected total profit over $(0, H)$ at $t = H$.

In order to get the different components in the profit expression, we note that in the last cycle holding cost is incurred but the shortage cost is zero.

The different components of the present value of the expected profit are as follows:

expected ordering cost

$$C_0(T_1,T) = A\sum_{s=1}^{n} E(e^{-k(s-1)T}) = A\sum_{s=1}^{n} e^{-(s-1)dT} M_r((s-1)T) \text{ ;}$$

expected holding cost

$$C_h(T_1,T) = \sum_{s=1}^{n-1} E\left[\left(Ice^{-(s-1)kT}\int_0^{T_1} I_{s-1}(t)\,dt\right)\right] + E\left[\left(Ice^{-(n-1)kT}\int_0^{T} I_{n-1}(t)\,dt\right)\right]$$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r((s-1)T) - bpM_r(2(s-1)T)]$$

$$+ \frac{Ic}{\theta^2}\ (e^{\theta T} - \theta T - 1)[e^{-(n-1)dT}(aM((n-1)T) - bpM(2(n-1)T))]$$

expected deterioration cost $C_d(T_1,T) = \dfrac{\theta}{I} C_h(T_1,T)$

expected shortage cost $C_{sh}(T_1,T) = \sum_{s=1}^{n-1} E\left[s_1 e^{-k(s-1)T}\int_{T_1}^{T}\left(a - bpe^{rT}\right) tdt\right]$

$$= s_1\frac{(T^2 - T_1^2)}{2}\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r((s-1)T) - bpM_r(2(s-1)T))]$$

expected purchase cost

$$C_p(T_1,T) = \sum_{s=1}^{n-1} E\left[\left(ce^{-k(s-1)T}\left(I_{s-1}(0) + \int_{T_1}^{T}\left(a - bpe^{r(s-2)T}\right)dt\right)\right)\right]$$

$$+ E\left[\left(ce^{-k(n-1)T}\left(I_{n-1}(0) + \int_{T_1}^{T}\left(a - bpe^{r(n-2)T}\right)dt\right)\right)\right]$$

$$= \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r((s-1)T) - bpM_r(2(s-1)T)]$$

$$+ \frac{c}{\theta}\left(e^{\theta T} - 1\right)e^{-(n-1)dT}[aM_r((n-1)T) - bpM_r(2(n-1)T)]$$

$$+ c\left(T - T_1\right)\sum_{s=1}^{n} e^{-(s-1)dT}[aM_r((s-1)T) - bpM_r((2s-3)T)]$$

expected selling price

$$C_S^{(s)}(T_1,T)=\sum_{s=1}^{n-1}\ E\left[\left(pe^{-k(s-1)T}\left(\int_0^{T_1}\left(a-bpe^{r(s-1)T}\right)dt-I_{s-2}\left(T\right)\right)\right)\right]$$

$$+E\left[\left(pe^{-k(n-1)T}\left(\int_0^{T}\left(a-bpe^{r(s-1)T}\right)dt-I_{n-2}\left(T\right)\right)\right)\right]$$

$$=pT_1\sum_{s=1}^{n-1}\ e^{-(s-1)dT}[aM_r\left((s-1)T\right)-bpM_r\left(2(s-1)T\right)]$$

$$+pTe^{-(n-1)dT}[aM_r\left((n-1)T\right)-bpM_r\left(2(n-1)T\right)]$$

$$+p\left(T-T_1\right)\sum_{s=1}^{n}\ e^{-(s-1)dT}[aM_r\left((s-1)T\right)\text{-}bpM_r\left((2s-3)T\right)\big)].$$

In order to calculate the total interest earned or paid, we note that when $M\le T_1$, the inventory manager earns interest on his sales revenue in the interval $[(s-1)T,(s-1)T+M]$ and pays interest on the unsold stock in the interval $[(s-1)T+M,sT]$ for $1\le s\le n-1$. On the other hand, when $M\ge T_1$ he does not have to pay any interest. For $s = n$, he earns interest in $[(s-1)T,(s-1)T+M]$ and pays interest in the interval $[(n-1)T+M,nT]$.

Hence, we have the following:

Case 1: $M\le T_1$

Total interest earned

$$C_e^{(1)}(T_1,T)=\sum_{s=1}^{n-1}\ E\left[\left(pI_e\int_0^{T_1}\left(a-bpe^{r(s-1)T}\right)\left(T_1-t\right)dt\right)e^{-k(s-1)T}\right]+\ldots$$

$$+E\left[\left(pI_e\int_0^{M}\left(a-bpe^{(n-1)rT}\right)\left(M-t\right)dt\right)e^{-(n-1)kT}\right]$$

$$=pI_e\frac{T_1^2}{2}[\sum_{s=1}^{n-1}\ e^{-(s-1)dT}\{aM_r\left(\left(s-1\right)T\right)-bpM_r\left(2\left(s-1\right)T\right)\}]$$

$$+pI_e\frac{M^2}{2}[e^{-(n-1)dT}\{aM_r\left(\left(n-1\right)T\right)-bpM_r\left(2\left(n-1\right)T\right)\}].$$

Total interest payable

$$C_r^{(1)}(T_1,T)=\sum_{s=1}^{n-1} E\left[\left(cI_r\int_M^{T_1} I_1(t)\,dt\right)e^{-k(s-1)T}\right]$$
$$+E\left[\left(cI_r\int_M^{T} I_{n-1}(t)\,dt\right)e^{-k(n-1)T}\right]$$
$$=\frac{cI_r}{\theta}\left(e^{\theta(T_1-M)}-\theta(T_1-M)-1\right)\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r((s-1)T)$$
$$-bpM_r(2(s-1)T)]+\frac{cI_r}{\theta}\left(e^{\theta(T-M)}-\theta(T-M)-1\right)e^{-(n-1)dT}[aM_r((n-1)T)$$
$$-bpe^{-(n-1)dT}M_r(2(n-1)T)]$$

hence, for $M\le T_1$, the total profit in the interval $[0, H]$ is

$$C_1^M(T_1,T)=C_S(T_1,T)+C_e^{(1)}(T_1,T)-C_0(T_1,T)-C_h(T_1,T)-C_d(T_1,T)$$
$$-C_{sh}(T_1,T)-C_r^{(1)}(T_1,T)$$
$$=[pT_1+pI_e\frac{T_1^2}{2}-\left(e^{\theta T_1}-\theta T_1-1\right)\left(\frac{Ic}{\theta^2}+\frac{c}{\theta}\right)-\frac{cI_r}{\theta}\left(e^{\theta(T_1-M)}-\theta(T_1-M)-1\right)$$
$$-s_1\frac{T^2-T_1^2}{2}-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)]K_1(T)+(p-c)(T-T_1)K_2(T)-AK_3(T)$$
$$[pT+pI_e\frac{M^2}{2}-(\frac{Ic}{\theta^2}+\frac{c}{\theta})\,(e^{\theta T}-\theta T-1)-\frac{cI_r}{\theta}\left(e^{\theta(T-M)}-\theta(T-M)-1\right)$$
$$-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)][e^{-(n-1)dT}(aM((n-1)T)-bpM(2(n-1)T))],$$

where

$$K_1(T)=\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r((s-1)T)-bpM_r(2(s-1)T)]$$
$$K_2(T)=\sum_{s=1}^{n} e^{-(s-1)dT}[aM_r((s-1)T)-bpM_r((2s-3)T)]$$
$$K_3(T)=\sum_{s=1}^{n} e^{-(s-1)dT}M_r((s-1)T).$$

Case 2: $M \geq T_1$

Interest earned

$$C_e^{(2)}(T_1, T) =$$

$$\sum_{s=1}^{n-1} E\left[cI_e e^{-k(s-1)T}\left(\int_0^{T_1}\left(a - bpe^{r(s-1)T}\right)\left(T_1 - t\right)dt + \int_0^{T_1}\left(a - bpe^{r(s-1)T}\right)\left(M - T_1\right)dt\right)\right]$$

$$+E\left[cI_e e^{-k(n-1)T}\left(\int_0^{M}\left(a - bpe^{r(n-1)T}\right)\left(M - t\right)dt\right)\right]$$

$$= cI_e T_1(M - \frac{T_1}{2})\sum_{s=1}^{n-1} e^{-(s-1)dT}[aM_r\left(\left(s-1\right)T\right) - bpM_r\left(2\left(s-1\right)T\right)]$$

$$+ pI_e \frac{M^2}{2}[e^{-(n-1)dT}[aM_r\left(\left(n-1\right)T\right) - bpM_r\left(2\left(n-1\right)T\right)]$$

Interest payable $C_r^{(2)}(T_1, T) = E\left[\left(cI_r\int_M^T I_{n-1}\left(t\right)dt\right)e^{-k(n-1)T}\right]$

$$= \frac{cI_r}{\theta}\left(e^{\theta(T-M)} - \theta\left(T - M\right) - 1\right)e^{-(n-1)dT}[aM_r\left((n-1)T\right)$$

$$-bpe^{-(n-1)dT}M_r\left(2\left(n-1\right)T\right)]$$

Total profit in the interval $[0, H]$ is, therefore,

$$C_2^M\left(T_1, T\right) = C_S(T_1, T) + C_e^{(2)}(T_1, T) - C_0(T_1, T) - C_h(T_1, T) - C_d(T_1, T)$$

$$- C_{sh}(T_1, T) - C_r^{(2)}(T_1, T)$$

$$C_2^M\left(T_1, T\right) = [pT_1 + pI_e T_1\left(M - \frac{T_1}{2}\right) - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right)$$

$$- s\frac{T^2 - T_1^2}{2} - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]K_1(T) + \left(T - T_1\right)\left(p - c\right)K_2(T)$$

$$- AK_3(T) + [pT + pI_e\frac{M^2}{2} - (\frac{Ic}{\theta^2} + \frac{c}{\theta})\ (e^{\theta T} - \theta T - 1)$$

$$- \frac{cI_r}{\theta}\left(e^{\theta(T-M)} - \theta\left(T - M\right) - 1\right)$$

$$- \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)][e^{-(n-1)dT}(aM((n-1)T) - bpM(2(n-1)T))].$$

To obtain the optimal value of $\boldsymbol{T} = (T_1, T)$, we first obtain $\boldsymbol{T}_{(1)}$ and $\boldsymbol{T}_{(2)}$ that maximize $C_1^M(T_1,T)$ and $C_2^M(T_1,T)$ respectively. If $C_1^M(T_{(1)}) \le C_2^M(T_{(2)})$, $\boldsymbol{T} = \boldsymbol{T}_{(2)}$ is optimal, else $\boldsymbol{T} = \boldsymbol{T}_{(1)}$ is optimal.

For given T, the optimal value of T_1 minimizing $C_1^M(T_1,T)$ is a solution to

$$\frac{\partial C_1^M(\theta)}{\partial T_1} = 0,$$

which gives

$$\left(p + pI_eT_1 - \left(e^{\theta T_1} - 1\right)\left(\frac{Ic}{\theta} + c\right) - cI_r\left(e^{\theta(T_1 - M)} - 1\right) + s(T - T_1) - ce^{\theta T_1}\right)K_1(T)$$
$$+\left(p - c\right)K_2(T)e^{\theta T_1}\left(\frac{Ic}{\theta} + c + cI_re^{-\theta M} + cT_1\right) + (s - pI_e)T_1$$
$$= \frac{Ic}{\theta} + c + cI_r + p + sT + \left(p - c\right)\frac{K_2(T)}{K_1(T)} \tag{1}$$

i.e., $f(T_1 = g(T),$
where

$$f(T_1) = e^{\theta T_1}\left(\frac{Ic}{\theta} + c + cI_re^{-\theta M} + cT_1\right) + (s - pI_e)T_1$$

$f(T_1)$ is an increasing function of T_1 with $0 < f(0) < g(T)$. In order to get a solution to (1), we must have $f(T) \ge g(T)$, else $T_1 = T$ is the optimal value.

Theorem 1: For given T, a sufficient condition for $C_1^M(T_1,T)$ to be concave in T_1 is that

$$pI_e \le c\theta + s.$$

The proof follows from the fact that

$$\frac{\partial^2 C_1^M(T_1,T)}{\partial T_1^2} = \left(pI_e - e^{\theta T_1}\left(Ic + c\theta\right) - \theta cI_re^{\theta(T_1 - M)} - c\theta e^{\theta T_1} - s\right)K_1(T) \le 0,$$

since, $pI_e \le c\theta + s.$

In case 2, the optimal value of T_1 satisfies $\dfrac{\partial C_2^M(T_1,T)}{\partial T_1} = 0,$ which gives

$$2cI_rT_1 + e^{\theta T_1}\left(\frac{Ic}{\theta} + c\right) + cI_re^{\theta(T_1 - M)} + sT_1 - ce^{\theta T_1}$$
$$= p + cI_r(M + 1) + \frac{Ic}{\theta} + c + sT + \left(p - c\right)\frac{K_2(T)}{K_1(T)} \tag{2}$$

and,

$$\frac{\partial^2 C_2^M\left(T_1,T\right)}{\partial T_1^2}=-\left(2cI_r+e^{\theta T_1}\left(Ic+c\theta\right)+\theta cI_r e^{\theta(T_1-M)}+c\theta e^{\theta T_1}-s\right)K_1(T)<0,$$

which shows that $C_2^M\left(T_1,T\right)$ is concave in T_1.

To obtain the optimal value of T in each case, we first find optimum n that maximizes $C_i^M\left(T_1,T\right), i=1,2,$ where $T=H/n$, and hence find optimum T.

In the following examples and sensitivity analysis, we shall assume that $r\sim N\left(\mu,\sigma^2\right)$,

which gives $M_r(t)=e^{\mu t+\frac{1}{2}\sigma^2 t^2}$.

4. Examples and sensitivity analysis

In this section we give two examples based on our model, and carry out a sensitivity analysis of the model to changes in its parameters. The optimal values of the decision variables have been obtained using the software MATLAB.

Example 1: Consider the following parameter values: a = 2000; b = 0.1; c = Rs. 20, p = Rs. 25, θ = 0.3, A = Rs. 250, s_1 = Rs. 1.5, I = 0.05, I_e = 0.12; I_r = 0.15, d = 0.14, M = 0.1 year; H = 5 years, $r\sim N\left(\mu,\sigma^2\right)$, where $\mu=0.08, \sigma^2=0.04.$

For $M\le T_1$, optimal n = 33, optimal T = 0.15152 year, optimal T_1 = 0.15145 year, profit = Rs. 36,525.70.

For $M\ge T_1$, optimal n =21, optimal T = 0.15152 year, optimal T_1 = 0.09996 year, profit = Rs. 33,718.40.

Hence, optimal n = 33, optimal T = 0.15152 year, optimal T_1 = 0.15145 year, profit = Rs. 36,525.70.

Example 2: Suppose the values of s_1 and I in example 1 are changed to Re. 0.5 and 0.04, respectively.

For $M\le T_1$, optimal n = 12, optimal T = 0.41667 year, optimal T_1 = 0.100055 year, profit = Rs. 40,248.90.

For $M\ge T_1$, optimal n = 14, optimal T = 0.35714 year, optimal T_1 = 0.039835 year, profit = Rs. 41,008.10.

Hence, optimal n = 14, optimal T = 0. 35714 year, optimal T_1 = 0.039835 year, profit = Rs. 41,008.10.

To study the sensitivity of the model to changes in its parameters, we start with the following setup: a = 2000; b = 0.1; c = 200; p = 230; θ = 0.1; A = 250; s_1 = 4; I = 0.05; I_e = 0.12; I_r = 0.15; μ = 0.08; d = 0.14; σ^2 = 0.04; M = 0.1; H = 5.

The following tables show how the decision variables, namely n, T and T_1, change with change in the values of the model parameters, and also give the corresponding percentage change in the expected profit as compared to that for the above set of parameter values.

Table 1. Changes in the values of the decision variables with change in I, and the corresponding % change in the expected profit from that when I = 0.05 .

I	No of cycles (n)	T	T_1	Optimal profit	% change in profit
0.005	14	0.3571	0.1184	241,298.76	33.65
0.01	18	0.2778	0.1099	219,980.82	21.84
0.03	29	0.1724	0.1025	187,121.03	3.64
0.05	**32**	**0.1563**	**0.1018**	**180,548.89**	**0**
0.07	36	0.1389	0.1011	173,012.86	-4.17
0.09	40	0.1250	0.1007	166,190.48	-7.95
0.11	41	0.1220	0.1006	163,774.96	-9.29
0.13	41	0.1220	0.1006	162,736.72	-9.87

Table 2. Changes in the values of the decision variables with change in s_1, and the corresponding % change in the expected profit from that when s_1 = 4 .

s_1	No of cycles (n)	T	T_1	Optimal profit	% change in profit
0.5	19	0.2632	0.1086	216,698.54	20.02
2	25	0.2000	0.1040	196,345.51	8.75
4	**32**	**0.1562**	**0.1018**	**180,548.89**	**0**
6	35	0.1428	0.1013	175,079.92	-3.03
8	36	0.1389	0.1011	173,170.09	-4.09
10	40	0.1250	0.1007	167,437.16	-7.26
20	47	0.1064	0.1002	159,284.20	-11.78

Table 3. Changes in the values of the decision variables with change in I_e, and the corresponding % change in the expected profit from that when I_e = 0.12 .

I_e	No of cycles (n)	T	T_1	Optimal profit	% change in profit
0.01	34	0.1471	0.1014	171,661.67	-4.92
0.04	34	0.1471	0.1014	173,149.91	-4.10
0.08	33	0.1515	0.1016	176,864.08	-2.04
0.12	**32**	**0.1563**	**0.1018**	**180,548.89**	**0**
0.16	31	0.1613	0.1020	184,227.17	2.04
0.18	31	0.1613	0.1020	184,685.80	2.29
0.20	31	0.1613	0.1020	186,061.71	3.05

Table 4. Changes in the values of the decision variables with change in I_r, and the corresponding % change in the expected profit from that when I_r = 0.15 .

I_r	No of cycles (n)	T	T_1	Optimal profit	% change in profit
0.01	13	0.3846	0.1219	246,589.85	36.57
0.05	18	0.2778	0.1099	218,603.74	21.08
0.10	25	0.2000	0.1040	195,487.75	8.27
0.15	**32**	**0.1563**	**0.1018**	**180,548.89**	**0**
0.20	37	0.1351	0.1010	172,431.10	-4.50
0.30	47	0.1064	0.1002	159,679.68	-11.56
0.40	48	0.1042	0.1001	158,572.79	-12.17

Table 5. Changes in the values of the decision variables with change in p, and the corresponding % change in the expected profit from that when p = 230 .

p	No of cycles (n)	T	T_1	Optimal profit	% change in profit
210	21	0.2381	0.1066	81,069.29	-55.10
220	26	0.1923	0.1036	130,526.57	-27.71
230	**32**	**0.1563**	**0.1018**	**180,548.89**	**0**
240	40	0.1250	0.1007	229,836.87	27.30
250	42	0.1190	0.1005	288,654.66	59.88
260	45	0.1111	0.1003	346,201.91	91.75
270	46	0.1087	0.1002	406,266.31	125

Table 6. Changes in the values of the decision variables with change in μ, and the corresponding % change in the expected profit from that when μ = 0.08 .

μ	No of cycles (n)	T	T_1	Optimal profit	% change in profit
0.02	24	0.2083	0.1046	229,307.39	27.01
0.04	26	0.1923	0.1036	212,165.21	17.51
0.06	29	0.1724	0.1025	195,036.67	8.02
0.08	**32**	**0.1563**	**0.1018**	**180,548.89**	**0**
0.12	34	0.1471	0.1014	162,579.25	-9.95
0.16	36	0.1389	0.1011	147,349.50	-18.36
0.20	38	0.1316	0.1009	134,432.71	-25.54

Table 7. Changes in the values of the decision variables with change in the credit period M, and the corresponding % change in the expected profit from that when M = 0.1 .

M	No. of cycles (n)	T	T_1	Optimal profit	% change in profit
0.04	52	0.0962	0.0008	198,657.05	10.03
0.07	37	0.1351	0.0718	186,777.64	3.45
0.1	**32**	**0.1562**	**0.1018**	**180,548.89**	**0**
0.3	16	0.3125	0.0188	260,742.92	44.42
0.5	8	0.6250	0.0625	341,895.35	89.36
0.7	7	0.7143	0.1001	362,449.13	100.75
0.9	4	1.0000	0.9200	21,707.24	-87.98

Table 8. Changes in the values of the decision variables with change in θ, and the corresponding % change in the expected profit from that when θ = 0.1 .

θ	No. of cycles (n)	T	T_1	Optimal profit	% change in profit
0.01	19	0.2632	0.1086	226,884.74	25.66
0.04	22	0.2273	0.1058	214,857.70	19
0.07	26	0.1923	0.1036	202,598.97	12.21
0.1	**32**	**0.1562**	**0.1018**	**180,548.89**	**0**
0.2	41	0.1220	0.1006	166,889.66	-7.56
0.4	45	0.1111	0.1003	139,254.02	-22.87
0.6	46	0.1087	0.1002	114,205.22	-36.75

Tables 2 and 3 show that the model is fairly sensitive to changes in the shortage cost and the interest earned, while from the other tables, namely Tables 1, 4-8, we may conclude that the model is highly sensitive to changes in the corresponding parameters.

5. Conclusion

The existing literature on inventory policies with inflation generally assumes a constant inflation rate over time. This assumption is, however, violated in many real life situations since the time value of money may be subjected to change owing to change in environmental factors. In this paper we have therefore assumed a stochastic inflation rate to capture its change over time. We further consider a permissible delay in payment, which has not been investigated much for policies under inflation. Our model has usefulness when dealing with items like electronic components, fashion items and domestic goods, whose demands are affected by the selling prices, and customers for such goods are often allowed a grace period to repay their dues. The model may be extended to consider other types of demands, such as stock dependent demand, time dependent demand, etc., and also delay period dependent on the order quantity.

Acknowledgement

The authors thank the anonymous referees for their fruitful suggestions, which helped to improve the presentation of the paper.

References

[1] Buzacott, J.A., Economic Order Quantities with Inflation, *Operational Research Quarterly*, 1975; 26: 553–558.

[2] Misra, R.B., A Note on Optimal Inventory Management under Inflation, *Naval Research Logistics Quarterly*, 1979; 26(1): 161-165.

[3] Sarker, B.R. and Pan, H., Effects of Inflation and Time Value of Money on Order Quantity and Allowable Shortage, *International Journal of Production Economics*, 1994; 34: 65-72.

[4] Uthayakumar, R. and Geetha, K.V., Replenishment Policy for Single Item Inventory Model with Money Inflation, *Opsearch*, 2009; 46: 345-357.

[5] Maity, A.K., One Machine Multiple-Product Problem with Production-Inventory System under Fuzzy Inequality Constraint, *Applied Soft Computing,* 2010; 11: 1549-1555.

[6] Vrat, P. and Padmanabhan, G., An Inventory Model under Inflation for Stock-Dependent Consumption Rate Items, *Engineering Costs and Production Economics*, 1990; 19: 379-383.

[7] Dutta, T.K. and Pal, A.K., Effects of Inflation and Time-Value of Money on an Inventory Model with Linear Time-Dependent Demand Rate and Shortages, *European Journal of Operational Research*, 1991; 52: 326–333.

[8] Hariga, M.A., Effects of Inflation and Time-Value of Money on an Inventory Model with Time-Dependent Demand Rate and Shortages, *European Journal of Operational Research*, 1995; 81(3): 512-520.

[9] Hariga, M.A. and Ben-Daya, M., Optimal Time Varying Lot-Sizing Models under Inflationary Conditions, *European Journal of Operational Research,* 1996; 89(2): 313-325.

[10] Chung, K.J., An Algorithm for an Inventory Model with Inventory-Level-Dependent Demand Rate, *Computers and Operations Research*, 2003; 30: 1311-1317.

[11] Mirzazadeh, A., Effects of Uncertain Inflationary Conditions on an Inventory Model for Deteriorating Items with Shortages, *Journal of Applied Science,* 2010; 10(22): 2805-2813.

[12] Goyal, S.K., Economic Order Quantity under Conditions of Permissible Delay in Payments, *Journal of the Operational Research Society*, 1985; 36: 35–38.

[13] Shinn, S.W., Hwang, H.P. and Sung, S., Joint Price and Lot – Size Determination under Conditions of Permissible Delay in Payments and Quantity Discounts for Freight Cost, *European Journal of Operational Research,* 1996; 91: 528 – 542.

[14] Hwang, H. and Shinn, S.W., Retailer's Pricing and Lot Sizing Policy for Exponentially Deteriorating Products under the Condition of Permissible Delay in Payments, *Computers and Operations Research*, 1997; 24: 539-547.

[15] Jamal, A. M.M., Sarker, B.R. and Wang, S., An Ordering Policy for Deteriorating Items with Allowable Shortage and Permissible Delay in Payment, *Journal of the Operational Research Society*, 1997; 48: 826-833.

[16] Pal, M. and Ghosh, S.K., An Inventory Model with Shortage and Quantity Dependent Permissible Delay in Payment, *Australian Society of Operations Research Bulletin*, 2006; 25(3): 2-8.

[17] Pal, M. and Ghosh, S.K., An Inventory Model with Stock Dependent Demand and General Rate of Deterioration under Conditions of Permissible Delay in Payments, *Opsearch*, 2007; 44(3): 227-239.

[18] Shah, N.H. and Shah, Y.K., A Discrete-In-Time Probabilistic Inventory Model for Deteriorating Items under Conditions of Permissible Delay in Payments, *International Journal of System Science*, 1998; 29: 121-126.