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A New Mixture Pareto Distribution and Its Application

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Abstract

This paper introduces a new mixture Pareto distribution generated from logit of the weighted two-component mixture distribution which is mixed between a Pareto and a length biased Pareto distributions. Special sub-models include the Pareto, exponential, chi-square and logistic distributions. We contain derivation of the mixture Pareto types II, III, IV and provide various mathematical properties including its limit behavior, hazard rate and r^{th} moment. Discussion of the estimation procedure is based on maximum likelihood. Also, we display an application to Norwegian fire claim data, it shows that the mixture Pareto distribution give a better fit than other important lifetime models; a Weibull, the Pareto and the length biased Pareto distributions. In conclusion, the mixture Pareto distribution provides a rather general and flexible framework for the statistical lifetime data analysis.

Keywords: length biased, two-component mixture, mixture Pareto distribution, maximum likelihood estimation, lifetime data, hazard rate.

1. Introduction

The family of a Pareto distribution is well known in the literature for its capability in modeling the heavy-tailed data such as income data, exceedances of river flood data and fire claim data. The Pareto distribution is very versatile and a variety of uncertainties can be usefully modelled by it [1]. The Pareto distribution arises as tractable "life time" model in actuarial science, economics, finance, life testing, survival analysis and engineering. It is used in the frequency modeling of data with a right tail and no mode in the probability density. A random variable X is said to have the Pareto distribution, denoted by $X \sim Pareto(\alpha, \beta)$, its probability density function (pdf) is

$$g_P(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}, \qquad x \geq \beta; \alpha > 0, \beta > 0, \tag{1}$$

and the cumulative distribution function (cdf) is

$$G_P(x) = 1 - \left(\frac{x}{\beta}\right)^{-\alpha}. \tag{2}$$

For a thorough discussion on various properties and applications and different forms of the Pareto distribution [2-4], some mathematical properties such as the r^{th} moment and moment generating function (mgf) of the Pareto distribution are, respectively, given by

$$E_P(X^r) = \frac{\alpha\beta^r}{\alpha - r}, \qquad r = 1, 2, 3, \ldots; \alpha > r, \tag{3}$$

and,

$$M_X(t)_P = \alpha(-\beta t)^{\alpha}\,\Gamma(-\alpha, -\beta t), \qquad t < 0, \tag{4}$$

where, $\Gamma(a,b) = \int_b^{\infty} y^{a-1} e^{-y} dy$ denotes an incomplete gamma function.

Patil and Rao [5] presented a length biased Pareto (LP) distribution by concept of a weighted distribution. If X is Pareto random variable with pdf (1), then the pdf for the length biased distribution of random variable X is

$$g_{LP}(x) = \frac{(\alpha-1)}{\beta}\left(\frac{x}{\beta}\right)^{-\alpha}, \qquad x \geq \beta; \alpha > 1, \beta > 0. \tag{5}$$

By (5), it is not difficult to show that the cdf of the LP distribution is

$$G_{LP}(x) = 1 - \left(\frac{x}{\beta}\right)^{-(\alpha-1)}. \tag{6}$$

From (5), we can supply some mathematical properties for instance the r^{th} moment and mgf of the LP distribution are, respectively,

$$E_{LP}\left(X^r\right) = \frac{(\alpha-1)\beta^r}{\alpha-(r+1)}, \qquad r = 1,2,3,\ldots; \alpha > r+1, \tag{7}$$

and,

$$M_X(t)_{LP} = (\alpha-1)(-\beta t)^{(\alpha-1)}\Gamma\left(-(\alpha+1), -\beta t\right), \qquad t < 0. \tag{8}$$

Recently, attempts have been made to define new families of probability distributions that extend well-known families of distributions and at the same time provide great flexibility in modeling data in practice. One such class of distributions generated from the two-component mixture model of random variable which extends the original distribution with the length biased distribution provide powerful and popular tools for generating flexible distributions with attractive statistical and probabilistic properties, see McLachlan and Peel [6]. The two-component mixture model method is employed in many vocations. For example, Hall and Zhou [7] proposed nonparametric estimation for a mixture of two distributions in a multivariate mixture model. In addition, estimates of the mixing proportions, locations and variances for the components of a finite univariate mixture model were introduced by Cruz-Medina and Hettmansperger [8]. By assumed the assumptions of symmetric, different locations and parametric model are imposed on the components.

Furthermore, two real location parameters and the mixing proportion were presented by Bordes et al. [9]. Moreover, the problem of parameter estimation in finite mixtures is proposed by Hunter et al. [10], when comparing their method with the method of maximum likelihood using normal components, their method produces higher standard error estimates in the case where the components are truly normal. Their method dramatically outperforms the normal method when the components are heavy-tailed. Additionally, Leiva et al. [11] introduced a model that extends the inverse Gaussian distribution, their model is obtained when a parameter is incorporated into the logarithmic inverse Gaussian distribution producing great flexibility for fitting non-negative data. Moreover, several aspects of the mixture inverse Gaussian distributions are useful for modeling positive data that the empirical fit of the mixture inverse Gaussian distributions to the data is very good, introduced by Balakrishnan et al. [12]. Recently, Vandekerkhove [13] introduced the mixture of regression models which are generalization of the semi-parametric two-component mixture model.

In this paper, we propose a mixture Pareto (MP) distribution by method of the two-component mixture distribution. The main reasons for introducing it since its flexibility in accommodating mixture between original and length biased distributions. The MP distribution is an important model that can be used in a variety of problems in modeling lifetime data. The reminder of this paper is organized as follows: In Section 2, we approximate forms of the pdf and cdf for the MP distribution and provide special sub-models of it. The other types of the MP distribution are containing in this Section. Section 3, mathematical properties; limit behavior, hazard rate, the r^{th} moment and generating function are derived. Expressions for the mean, variance, skewness and kurtosis are discussed in Section 3. Moreover, in Section 4, we discuss estimation by the maximum likelihood method. An application using a real data set is presented in Section 5. Finally, in Section 6, we provide some conclusions, followed by the concluding remarks.

2. A new mixture Pareto distribution

In this section, we propose a new MP distribution produce widely flexible models with good statistical. Its some special sub-models and the other types are display in this section.

2.1 The probability density function and cumulative distribution function

Definition 1: Specifically, let $g_P(x)$ and $g_{LP}(x)$ are the pdf and length biased pdf of the random variable X, respectively. If ω is mixing parameter, $0 \le \omega \le 1$, then the weighted two-component mixture distribution produced by the mixture between $g_P(x)$ and $g_{LP}(x)$ is defined as

$$f(x) = (1-\omega)g_P(x) + \omega g_{LP}(x), \qquad x > 0.$$

Theorem 1: Let $X \sim MP(\alpha,\beta,\omega)$, then the pdf and cdf of random variable X, are, respectively,

$$f(x) = \frac{1}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}\left[(1-\omega)\alpha + \frac{\omega(\alpha-1)x}{\beta}\right], \qquad x \ge \beta; \alpha > 1, \beta > 0, 0 \le \omega \le 1, \tag{9}$$

and,

$$F(x) = 1 - \omega\left(\frac{x}{\beta}\right)^{-(\alpha-1)} - (1-\omega)\left(\frac{x}{\beta}\right)^{-\alpha}. \tag{10}$$

Proof: We then say that a random variable X follows the MP distribution with parameters α, β and ω, if its pdf is obtain by substitute (1) and (5) in Definition 1, can be obtained as

$$\begin{aligned} f(x) &= (1-\omega)\left[\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}\right] + \omega\left[\frac{\alpha-1}{\beta}\left(\frac{x}{\beta}\right)^{-\alpha}\right] \\ &= \left(\frac{x}{\beta}\right)^{-(\alpha+1)}\left[\frac{(1-\omega)\alpha}{\beta} + \frac{\omega(\alpha-1)x}{\beta^2}\right] \\ &= \frac{1}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}\left[(1-\omega)\alpha + \frac{\omega(\alpha-1)x}{\beta}\right]. \end{aligned}$$

Let $F(x)$ denote the cdf of a random variable X. The cdf for a generalized class of distribution, as defined by Definition 1, is generated by applying the cdf to the MP random variable to obtain

$$F(x) = (1-\omega)G_P(x) + \omega G_{LP}(x) \tag{11}$$

hence, obtain by substitute (2) and (6) in (11), can be written as

$$\begin{aligned} F(x) &= (1-\omega)\left[1-\left(\frac{x}{\beta}\right)^{-\alpha}\right] + \omega\left[1-\left(\frac{x}{\beta}\right)^{-(\alpha-1)}\right] \\ &= 1-\omega-(1-\omega)\left(\frac{x}{\beta}\right)^{-\alpha} + \omega - \omega\left(\frac{x}{\beta}\right)^{-(\alpha-1)} \\ &= 1-\omega\left(\frac{x}{\beta}\right)^{-(\alpha-1)} - (1-\omega)\left(\frac{x}{\beta}\right)^{-\alpha}. \end{aligned}$$

The MP distribution contains a large number of distributions. In Figures 1 and 2, we present the pdf and cdf of it.

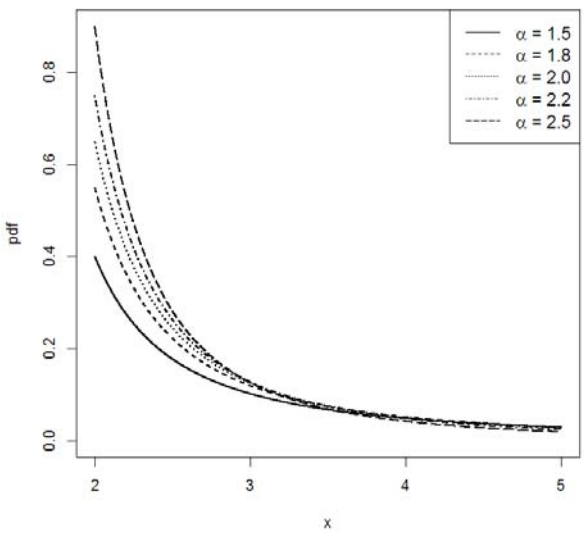

Figure 1. The pdf of the MP distribution, for different values of α where $\beta = 2$ and $\omega = 0.7$.

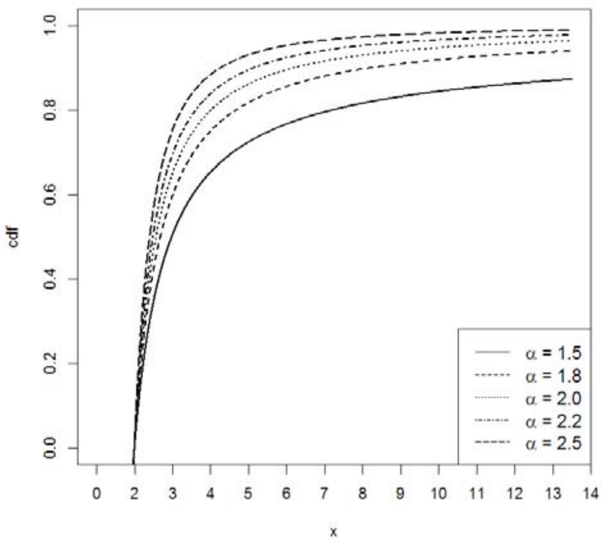

Figure 2. The cdf of the MP distribution, for different values of α where $\beta = 2$ and $\omega = 0.4$.

2.2 Special sub-models

We consider some special sub-models of the MP distribution, $X \sim MP(\alpha, \beta, \omega)$, in the following five corollaries.

Corollary 1: Where $\omega = 0$, the MP distribution reduces to the Pareto distribution with parameters α and β, is given by (1).

Corollary 2: Specify transformation technique by a new random variable $Y = 2\log\left(\frac{X}{\beta}\right)$ and let $\omega = 0$, the distribution of Y is the chi-square distribution with pdf $f(y) = \frac{1}{2}e^{\frac{-y}{2}}$.

Corollary 3: Let a new random variable $Y = -\log\left[\left(\frac{X}{\beta}\right)^{\alpha} - 1\right]$ and $\omega = 0$, the distribution of Y belongs to the logistic distribution with pdf $f(y) = \frac{e^{-y}}{\left(1+e^{-y}\right)^2}$, the mean and the standard deviation of Y are 0 and 1, respectively.

Corollary 4: Where $\omega = 1$, the MP distribution reduces to the LP distribution as (5).

Corollary 5: If $\omega = 1$ and let $Y = \log\left(\frac{X}{\beta}\right)$ by transformation technique, the distribution of Y is the exponential distribution with pdf $f(y) = (\alpha - 1)e^{-(\alpha-1)y}$.

2.3 Other types

Various types of the Pareto distribution other than the Pareto density in (1) were discussed by Nadarajah [14]. The density in (1) is called the Pareto type I. The cdf of Pareto types II , III and IV are, respectively, defined as

$$G_{II}(x) = 1 - \left(1 + \frac{x}{\beta}\right)^{-\alpha}, \qquad x > 0; \alpha, \beta > 0,$$

$$G_{III}(x) = 1 - \left[1 + \left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}}\right]^{-1}, \qquad x > \mu; \beta, \lambda > 0,$$

and,

$$G_{IV}(x) = 1 - \left[1 + \left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}}\right]^{-\alpha}, \qquad x > \mu; \alpha, \beta, \lambda > 0.$$

Note that, the Pareto types II also known as a Lomax distribution. The mixture distribution for the random variable X , as Definition 1, the pdf of the MP distribution in (9) is originated

$$f(x) = \left\{1 - \omega\left[1 - \frac{x}{E(X)}\right]\right\} g(x). \tag{12}$$

By applying $G_{II}(x)$, $G_{III}(x)$ and $G_{IV}(x)$ in (12), the corresponding types of MP density functions can be written, respectively, as

$$f_{II}(x)=\frac{\alpha}{\beta}\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\left\{1-\omega\left[1-\frac{x(\alpha-1)}{\beta}\right]\right\},$$

$$f_{III}(x)=\frac{1}{\lambda\beta}\left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}-1}\left[1+\left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}}\right]^{-2}\left\{1-\omega\left[1-\frac{x}{\beta\Gamma(1-\lambda)\Gamma(1+\lambda)}\right]\right\},$$

and,

$$f_{IV}(x)=\frac{\alpha}{\lambda\beta}\left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}-1}\left[1+\left(\frac{x-\mu}{\beta}\right)^{\frac{1}{\lambda}}\right]^{-\alpha-1}\left\{1-\omega\left[1-\frac{x\Gamma(\alpha)}{\beta\Gamma(\alpha-\alpha\lambda)\Gamma(1+\lambda)}\right]\right\}.$$

3. Mathematical properties of the MP distribution

The limit behavior of pdf and close form of the hazard rate for the MP distribution are studied in this section.

3.1 Limit behavior

The limit of pdf for the MP distribution as $X\to\infty$ is 0 and the limit as $X\to\beta$ is obtain by

$$\lim_{x\to\beta} f(x)=\begin{cases}\dfrac{\alpha}{\beta}, & \text{when } \omega=0,\\ \dfrac{\alpha-\omega}{\beta}, & \text{when } 0<\omega<1,\\ \dfrac{\alpha-1}{\beta}, & \text{when } \omega=1.\end{cases}$$

Proof: It is straightforward to show the above from the pdf of the MP distribution in (9) the result follows

$$\begin{aligned}\lim_{x\to\beta} f(x)&=\lim_{x\to\beta}\left\{\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}+\omega\left[\frac{(\alpha-1)}{\beta}\left(\frac{x}{\beta}\right)^{-\alpha}-\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}\right]\right\}\\ &=\frac{\alpha}{\beta}+\omega\left[\frac{(\alpha-1)}{\beta}-\frac{\alpha}{\beta}\right]\\ &=\frac{\alpha-\omega}{\beta}.\end{aligned}$$

3.2 Hazard rate

By definition, the hazard rate (or failure rate) of a random variable X with pdf $f(x)$ and cdf $F(x)$ is

$$h(x)=\frac{f(x)}{1-F(x)}.$$

Using (9) and (10), the hazard rate of the MP distribution may be expressed as

$$h(x)=\frac{\frac{1}{\beta}\left(\frac{x}{\beta}\right)^{-(\alpha+1)}\left[(1-\omega)\alpha+\frac{\omega(\alpha-1)x}{\beta}\right]}{1-\left[1-\omega\left(\frac{x}{\beta}\right)^{-(\alpha-1)}-(1-\omega)\left(\frac{x}{\beta}\right)^{-\alpha}\right]}$$
$$=\frac{\frac{1}{\beta}\left(\frac{x}{\beta}\right)^{-1}\left[(1-\omega)\alpha+\frac{\omega(\alpha-1)x}{\beta}\right]}{\omega\left(\frac{x}{\beta}\right)+(1-\omega)}$$

$$h(x)=\frac{\alpha\beta+\omega(\alpha x-x-\alpha\beta)}{\beta x+\omega(x^2-\beta x)}. \tag{13}$$

We display some hazard rate graphs of the MP distribution in Figure 3. It is noted that by setting $\omega=0$ in (13), we have the hazard rate of the Pareto distribution. In the like manner, by setting $\omega=1$, we have the hazard rate of the LP distribution. More generally, when modeling data with monotone hazard rate, right tail, high threshold and no mode in the probability density, the original distribution may be an initial choice because of its density shapes. However, in countering the phenomenon with non-monotone failure rate, it does not provide a reasonable parametric fit.

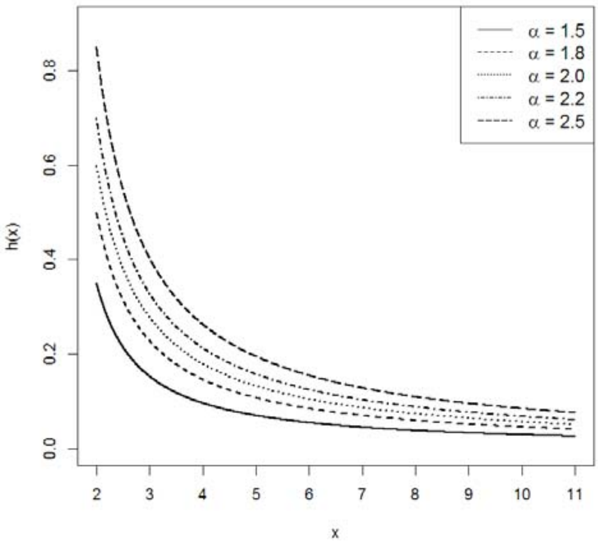

Figure 3. The hazard rates of random variable $X \sim MP\left(\alpha, \beta = 2, \omega = 0.8\right)$ for different values of α .

3.3 The r^{th} Moment

In this section we will consider the r^{th} moment of random variable $X \sim MP\left(\alpha, \beta, \omega\right)$.

Definition 2: Let $E_P\left(X^r\right)$ and $E_{LP}\left(X^r\right)$ are the r^{th} moments of original and length biased distributions of the random variable X , respectively. If $0 \le \omega \le 1$, then the r^{th} moments of two-component mixture of distribution produced by the mixture between $E_P\left(X^r\right)$ and $E_{LP}\left(X^r\right)$ is define by

$$E\left(X^r\right) = \left(1-\omega\right)E_P\left(X^r\right) + \omega E_{LP}\left(X^r\right), \qquad r = 1,2,3,\ldots; x > 0.$$

Theorem 2: Let $X \sim MP\left(\alpha, \beta, \omega\right)$, the r^{th} moment of random variable X as follows

$$E\left(X^r\right) = \frac{\beta^r\left[\alpha^2 - \left(r+1\right)\alpha + r\omega\right]}{\left(\alpha - r\right)\left[\alpha - \left(r+1\right)\right]}, \qquad r = 1,2,3,\ldots; \alpha > r+1, 0 \le \omega \le 1. \tag{14}$$

Proof: If $X \sim MP(\alpha,\beta,\omega)$, from Definition 2, by replace (3) and (7), then the r^{th} moment of random variable X is

$$\begin{aligned}E\left(X^r\right) &= (1-\omega)\frac{\alpha\beta^r}{\alpha-r}+\omega\left[\frac{(\alpha-1)\beta^r}{\alpha-(r+1)}\right] \\ &= \frac{\alpha\beta^r}{\alpha-r}-\frac{\omega\alpha\beta^r}{\alpha-r}+\frac{\omega(\alpha-1)\beta^r}{\alpha-(r+1)} \\ &= \frac{\beta^r\left[\alpha^2-(r+1)\alpha+r\omega\right]}{(\alpha-r)\left[\alpha-(r+1)\right]}.\end{aligned}$$

From (14) we can find the mean and variance of the MP distribution, are, respectively, follows as

$$E(X)=\frac{\beta\left(\alpha^2-2\alpha+\omega\right)}{(\alpha-1)(\alpha-2)}, \tag{15}$$

and,

$$Var(X)=\beta^2\left[\frac{\left(\alpha^2-3\alpha+2\omega\right)}{(\alpha-2)(\alpha-3)}-\frac{\left(\alpha^2-2\alpha+\omega\right)^2}{(\alpha-1)^2(\alpha-2)^2}\right]. \tag{16}$$

Moreover, the skewness and kurtosis of the MP distribution can be written, respectively, as

$$Skewness(X)=W^{-3}\left[\frac{\left(\alpha^2-4\alpha+3\omega\right)}{(\alpha-3)(\alpha-4)}+\frac{2\left(\alpha^2-2\alpha+\omega\right)^3}{(\alpha-1)^3(\alpha-2)^3}-\frac{3\left(\alpha^2-3\alpha+2\omega\right)\left(\alpha^2-2\alpha+\omega\right)}{(\alpha-1)(\alpha-2)^2(\alpha-3)}\right], \tag{17}$$

and,

$$Kurtosis(X)=W^{-4}\left[\frac{\left(\alpha^2-5\alpha+4\omega\right)}{(\alpha-4)(\alpha-5)}-\frac{3\left(\alpha^2-2\alpha+\omega\right)^4}{(\alpha-1)^4(\alpha-2)^4}-\frac{4\left(\alpha^2-4\alpha+3\omega\right)\left(\alpha^2-2\alpha+\omega\right)}{(\alpha-1)(\alpha-2)(\alpha-3)(\alpha-4)}\right.$$

$$+\frac{6\left(\alpha^2-3\alpha+2\omega\right)\left(\alpha^2-2\alpha+\omega\right)^2}{\left(\alpha-1\right)^2\left(\alpha-2\right)^3\left(\alpha-3\right)}\Bigg], \tag{18}$$

where, $W=\sqrt{\frac{\left(\alpha^2-3\alpha+2\omega\right)}{\left(\alpha-2\right)\left(\alpha-3\right)}-\frac{\left(\alpha^2-2\alpha+\omega\right)^2}{\left(\alpha-1\right)^2\left(\alpha-2\right)^2}}.$

From (15), the mean of the MP distribution is defined when $\alpha>2$ and from (16), the variance is defined when $\alpha>3$. The skewness of the MP distribution in (17) is defined when $\alpha>4$ and from (18) the kurtosis is defined when $\alpha>5$.

3.3 Moment generating function

The mgf corresponding to a random variable X for the Pareto distribution with parameters α and β is only defined for non-positive values of t. When mgf of the MP distribution is produced by mixing between $M_X\left(t\right)_P$ and $M_X\left(t\right)_{LP}$; thus, using (4) and (8), we can provide mgf of the MP distribution, is written by

$$\begin{aligned} M_X\left(t\right)&=\left(1-\omega\right)M_X\left(t\right)_P+\omega M_X\left(t\right)_{LP}\\ &=\alpha\left(-\beta t\right)^{\alpha}\Gamma\left(-\alpha,-\beta t\right)-\omega\left(-\beta t\right)^{\alpha}\left[\alpha\Gamma\left(-\alpha,-\beta t\right)+\frac{\left(\alpha-1\right)}{\beta t}\Gamma\left(-\left(\alpha-1\right),-\beta t\right)\right], \qquad t<0. \end{aligned}$$

4. Maximum likelihood estimates of the parameters

We discuss maximum likelihood estimation for the MP distribution in this section. Let $\Theta=\left(\alpha,\beta,\omega\right)^T$ be the vector of the model parameters and let $X_1,\ldots,X_n$ be a random sample from $X \sim MP\left(\Theta\right)$. The log-likelihood function for Θ reduces to

$$L\left(\Theta\right)=\prod_{i=1}^{n}\left\{\frac{1}{\beta}\left(\frac{x_i}{\beta}\right)^{-(\alpha+1)}\left[\left(1-\omega\right)\alpha+\frac{\omega\left(\alpha-1\right)x_i}{\beta}\right]\right\},$$

$$\log L\left(\Theta\right)=\sum_{i=1}^{n}\log\left[\left(1-\omega\right)\alpha+\frac{\omega\left(\alpha-1\right)x_i}{\beta}\right]^{-1}-n\log\beta-\left(\alpha+1\right)\sum_{i=1}^{n}\log\left(\frac{x_i}{\beta}\right). \tag{19}$$

The components corresponding to the model parameters are calculated by differentiating in (19) and setting the results equal to zero, we obtain by

$$\frac{\partial \log L(\Theta)}{\partial \alpha} = \sum_{i=1}^{n}\left[\alpha - \frac{\omega x_i}{\beta - \omega\beta + \omega x_i}\right]^{-1} - \sum_{i=1}^{n}\log\left(\frac{x_i}{\beta}\right), \tag{20}$$

and,

$$\frac{\partial \log L(\Theta)}{\partial \omega} = \sum_{i=1}^{n}\left[\omega + \frac{\alpha\beta}{\alpha x_i - x_i - \alpha\beta}\right]^{-1}. \tag{21}$$

Since $x \geq \beta$, the maximum likelihood estimate of β is the first-order statistic $x_{(1)}$. The maximum likelihood estimates of the parameters α and ω, which are solved iteratively (20) and (21). We use nlm function in statistical package of R program [15].

5. Application of the MP distribution

We provide an application of the MP distribution and compare the results of the fits for a Weibull, Pareto and LP distributions. We shall consider the data set in the field of insurance which has received extensive attention in the actuarial literature. This data set is one among the twenty sets of Norwegian fire claims (in millions of Norwegian krones) was presented in Fernández [16]. The results of parameter estimates for Norwegian fire claim data is shown in Table 1 while the density function is shown in Figure 4. In application, since the values of the K-S statistics are smaller for the MP distribution compared to those values of the Pareto, LP and 3-parameter Weibull distributions. Due to the fact that the MP distribution has flexibility in accommodating because parameter ω has plasticity interval 0-1. Note that, the 3-parameter Weibull distribution with pdf

$$g(x) = \frac{\alpha}{\theta}\left(\frac{x-\beta}{\theta}\right)^{\alpha-1} e^{-\left(\frac{x-\beta}{\theta}\right)^{\alpha}}.$$

Table 1. Parameter estimates and K-S statistics for Norwegian fire claim data.

Distribution	Pareto	LP	MP	Weibull
Parameter estimates	$\hat{\alpha}$ =1.2175 $\hat{\beta}$ =0.5	$\hat{\alpha}$ =2.2175 $\hat{\beta}$ =0.5	$\hat{\alpha}$ =2.1988 $\hat{\beta}$ =0.5 $\hat{\omega}$ =0.9661	$\hat{\alpha}$ =0.5792 $\hat{\beta}$ =0.5 $\hat{\theta}$ =0.8276
K-S statistics	0.0505	0.0505	0.0484	0.0983
p-value	0.862	0.862	0.894	0.128
AIC	268.487	268.487	270.479	284.586

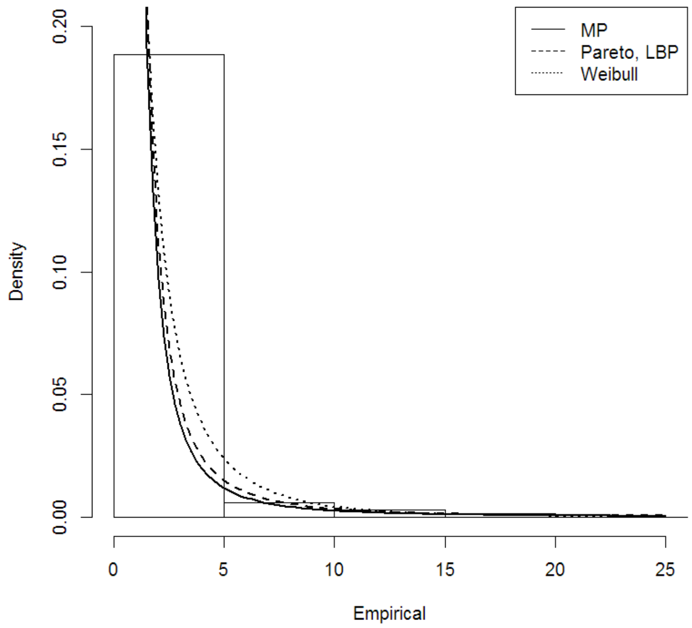

Figure 4. The density function of Norwegian fire claim data.

6. Conclusion

In this work, we introduced a new two-component mixture distribution, so-called a mixture Pareto distribution. It has some special sub-models, such as the Pareto, exponential, chi-square, logistic and the mixture Pareto types II , III , IV distributions.

We have derived various mathematical properties of the mixture Pareto distribution, including limit behavior, hazard rate and r^{th} moment. We demonstrate an application to Norwegian fire claim data by maximum likelihood estimation. An application to a real data set shows that the fit of the mixture Pareto distribution is best fit to the data with highest p-value. The mixture Pareto distribution provides a rather general and flexible framework for statistical analysis. We hope that the mixture Pareto distribution may attract wider application in lifetime data. The future research may consider in parameter estimation using Bayesian approach. In addition, a new mixture model between the beta and mixture Pareto distributions will be developed.

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