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An Approximate Formula for ARL in Moving Average Chart with ZINB Data

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Abstract

The objective of this paper is to propose an approximation of Average Run Length by deriving the explicit formula for Moving Average chart when observations are from Zero-Inflated Negative Binomial distribution so called ZINB MA chart. The new formula is simple and easy to implement and use by practitioners.

Keywords: moving average control chart, average run length, zero-inflated negative binomial model.

1. Introduction

Control charts are commonly used for monitoring the process in the manufacturing. The quality characteristics of a process can be usually measured on attributes, for example the number of nonconformities per unit. In this situation, it is necessary to use attribute control charts. The c chart is well-known for the attribute control charts whose control limits are based on the Poisson distribution. The Poisson distribution is generally used as the standard distribution for count data. It has well-known property that the mean of distribution is equal to the variance values. When the mean of a data set is often observed to be less than the variance of data, it is so called over-dispersion problem. If we ignored, it can lead to loss of efficiency or incorrect decision. The Negative Binomial distribution is a natural and more flexible extension of the Poisson distribution and allows for over-dispersion relative to the Poisson. Thus, the Negative Binomial distribution can be used instead of Poisson distribution when the data are overdispersed. Hoffman [1] proposed a simple approach to calculate exact and approximate control limits for count data based on the Negative Binomial distribution and illustrated by application to water bacteria count data taken from a water purification system.

Since, the technological advances in manufacturing have taking shape. There are large numbers of zero data for an attribute quality characteristic, for example, the number of nonconformities on the integrated circuit wafer (Yu et al. [2]). Then the Zero-Inflated models are used for modeling count data with zeros. For example, the Zero-Inflated Poisson (ZIP) model concerns a random event containing excess zero-count data in unit time. Lambert [3] introduced the ZIP model and applied model to the data collected from a quality control study. Mwalili et al. [4] explained the excess of zeros under the negative binomial model is called Zero-Inflated Negative Binomial (ZINB) model, which may be more appropriate than the Zero-Inflated Poisson model. If we use the Shewhart chart for monitoring observation based on ZINB model, the control chart often is underestimated of the values of mean and variance and may lead to higher false alarm rate in detecting out-of-control signal.

The practical in use of the Shewhart control charts are only very effective in the detecting of large shifts of process. When the small shifts of process with a trend occur, the Shewhart control chart cannot monitor the shift of the process, because the Shewhart control charts only use information regarding the process contained in the last observation and ignore the information given by the entire sequence of observations. The Cumulative Sum (CUSUM) control chart, the Exponentially Weighted Moving Average (EWMA) control chart and the Moving Average (MA) control chart are used for this problem because they use information about process contained in the entire sequence of points as opposed to the Shewhart control chart. There are many literatures available on control charts based on the Zero-Inflated distribution of count data. For example, Chen et al. [5] obtained attribute control charts using generalized Zero-Inflated Poisson distribution. Noorossana et al. [6] applied the EWMA chart for monitoring rare health events when some rare health events are Zero-Inflated Binomial (ZIB) distribution. He et al. [7] proposed a procedure using a CUSUM chart for monitoring the ZIP process. Katemee and Mayureesawan [8] developed the control limited of c-chart by using the Zero-Inflated Generalized Poisson (ZIGP) distribution.

The Average Run Length (ARL) is mostly used to measure the performance of control chart. The ARL_0 is representing to the performance of control chart when the process is in-control, it is an inverse of the probability for type I error when the null hypothesis is the process is in-control. When the process has a shift in process, the ARL_1 is representing to the performance of control chart when the process going to out-of-control. Many methods for evaluating the ARL for control charts have been studied in the literatures. A simple method as Monte Carlo (MC) simulation is often used to verity and accuracy. He et al. [7] used the MC for compute the ARL of Shewhart chart and CUSUM chart when the process is follow by ZIP distribution. Brook and Evans [9] propose the method to approximate the ARL by using the Markov Chain Approach (MCA) with finite state. Phengsalae et al. [10] proposed an approximation of ARL by MCA for Generally Weighted Moving Average control chart when the observations are from Poisson distribution. Sukparungsee and Novikov [11] introduced the Martingale techniques to compute ARL for EWMA charts for a variety of light-tailed distributions. Busaba et al. [12] derived the explicit formulas of ARL for CUSUM Procedure in the case of negative exponential data. Areepong [13] derived the explicit formulas of ARL for MA chart for monitoring the number of nonconforming items in sample.

In this paper, the analytical formulas are derived for ARL of MA chart for ZINB observations with arbitrary the values of w.

2. Zero-Inflated Negative Binomial (ZINB) model

The Zero-Inflated Negative Binomial mixed model contains components to model the probability of excess zero values and the negative binomial parameters. The appropriate probability distribution function to model the condition under study is given in (1). They are the relation of random successful observation with probability p and random number of failure until the successful occur or X which a negative binomial distribution with parameters π, λ and r. We define $X \sim \text{ZINB}\,(\pi, \lambda, r)$ with the probability mass function which can be written as

$$f(x)=\begin{cases}\pi+(1-\pi)\left(\dfrac{r}{r+\lambda}\right)^{r} & ,x=0\\[2ex] (1-\pi)\dfrac{\Gamma(x+r)}{\Gamma(r)\Gamma(x+1)}\left(\dfrac{r}{r+\lambda}\right)^{r}\left(\dfrac{\lambda}{r+\lambda}\right)^{x} & ,x=1,2,3,\ldots\end{cases} \tag{1}$$

where π is the probability of zero values, λ is the mean of the underlying Negative Binomial distribution, $\lambda > 0$, and r is the over-dispersion parameter where $r \in \mathbb{R}^{+}$ The Zero-Inflated Negative Binomial model reduce to Zero-Inflated Poisson model as $r \to \infty$. The mean and variance of the above distribution are as follows

$$E(X) = (1-\pi)\lambda \text{ and } Var(X) = (1-\pi)\lambda\left(1+\lambda\pi+\frac{\lambda}{r}\right).$$

3. The Moving Average Control Chart for ZINB observations

A Moving Average control chart is a type of memory control chart based on equal weighted moving average. Khoo [14] proposed the Moving Average control chart for the number of nonconformities in an inspection unit of product.

Let $x_1, x_2, \ldots x_i, \ldots$ are the number of nonconformities in an inspection unit of product and be a sequence of independently and identically distributed ZINB random variables. The moving average of width w at time i can be computed as

$$M_i = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} x_j & ; i < w \\ \frac{1}{w}\sum_{j=i-w+1}^{i} x_j & ; i \geq w. \end{cases} \tag{2}$$

The mean and the variance of the moving average, M_i for periods $i < w$ is

$$\begin{aligned} E(M_i) &= E\left(\frac{1}{i}\sum_{j=1}^{i} x_j\right) \\ &= \frac{1}{i}E\left(\sum_{j=1}^{i} x_j\right) \\ &= \frac{1}{i}\left(i\lambda(1-\pi)\right) \\ &= \lambda(1-\pi) \end{aligned} \tag{3}$$

and

$$\begin{aligned} Var(M_i) &= Var\left(\frac{1}{i}\sum_{j=1}^{i} x_j\right) \\ &= \frac{1}{i^2} Var\left(\sum_{j=1}^{i} x_j\right) \\ &= \frac{1}{i^2}\left(i\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)\right) \\ &= \frac{1}{i}\left(\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)\right). \end{aligned} \tag{4}$$

For periods $i \geq w$, the mean and the variance of the moving average are $\lambda(1-\pi)$ and $\frac{1}{w}\left(\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)\right)$, respectively.

The control limits are given

when $i < w$:

$$UCL_i \,/\, LCL_i = \lambda(1-\pi) \pm L\sqrt{\frac{1}{i}\left(\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)\right)} \tag{5}$$

and $i \geq w$:

$$UCL_w \,/\, LCL_w = \lambda(1-\pi) \pm L\sqrt{\frac{1}{w}\left(\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)\right)}, \tag{6}$$

where the value *L* is the width of control limit and determined based on a desired in-control ARL_0. The ZINB MA chart will signal to out-of-control when $M_i < LCL$ or $M_i > UCL$.

4. Explicit Formulas for Evaluating ARL for a ZINB MA Chart

The ARL values of a Moving Average control chart for Zero-Inflated Negative Binomial model can be derived as follows. Let $ARL = n$, then

$$\frac{1}{ARL} \cong \frac{1}{n}P\left(\text{o.o.c.signal at time } i < w\right) + \frac{1}{n}P\left(\text{o.o.c.signal at time } i \ge w\right)$$

$$\cong \frac{1}{n}\sum_{i=1}^{w-1}\left(P\left(M_i > UCL_i\right) + P\left(M_i < LCL_i\right)\right) + \frac{1}{n}\sum_{i=w}^{n}\left(P\left(M_i > UCL_w\right) + P\left(M_i < LCL_w\right)\right)$$

$$\cong \frac{1}{n}\sum_{i=1}^{w-1}\left(P\left(M_i > UCL_i\right) + P\left(M_i < LCL_i\right)\right) + \frac{n-w+1}{n}\left(P\left(M_i > UCL_w\right) + P\left(M_i < LCL_w\right)\right)$$

$$\cong \frac{1}{n}\sum_{i=1}^{w-1}\left(P\left(\frac{\sum_{j=1}^{i} x_j}{i} > \lambda_0\left(1-\pi\right) + L\sqrt{\frac{\lambda_0\left(1-\pi\right)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}\right) + P\left(\frac{\sum_{j=1}^{i} x_j}{i} < \lambda_0\left(1-\pi\right) - L\sqrt{\frac{\lambda_0\left(1-\pi\right)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}\right)\right)$$

$$+\frac{n-w+1}{n}\left(P\left(\frac{\sum_{j=i-w+1}^{i} x_j}{w} > \lambda_0\left(1-\pi\right) + L\sqrt{\frac{\lambda_0\left(1-\pi\right)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}\right) + P\left(\frac{\sum_{j=i-w+1}^{i} x_j}{w} < \lambda_0\left(1-\pi\right) - L\sqrt{\frac{\lambda_0\left(1-\pi\right)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}\right)\right)$$

$$\cong \frac{1}{n}\sum_{i=1}^{w-1}\left(P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{i}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{i}}}\right)\right)$$

$$+\frac{n-w+1}{n}\left(P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{w}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{w}}}\right)\right), \quad (7)$$

where L is the width of control limit. It is usually given L = 3 for the Shewhart control chart based on normal observation.

Let

$$A = \sum_{i=1}^{w-1}\left(P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{i}}}\right)\right)$$

$$+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{i}}}\right)\Bigg) \quad (8)$$

and

$$B =$$

$$P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{w}}}\right)$$

$$+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)\left(1+\lambda_0\pi+\lambda_0 r^{-1}\right)}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)\left(1+\lambda\pi+\lambda r^{-1}\right)}{w}}}\right)\Bigg). \quad (9)$$

Then, on substituting A and B into (7), we obtain

$$\frac{1}{n} \cong \frac{1}{n}(A)+\frac{n-w+1}{n}(B)$$

$$n \cong \frac{(1-A)}{B}+w-1.$$

Finally, the explicit formula of ARL is as following

$$ARL \cong \frac{(1-A)}{B}+w-1. \quad (10)$$

We substitute the values of A and B in (10). Then the explicit formula of the ARL of ZINB MA chart can be rewritten as

$$ARL \cong \left\{1-\sum_{i=1}^{w-1}\left(P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)(1+\lambda_0\pi+\lambda_0 r^{-1})}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)(1+\lambda\pi+\lambda r^{-1})}{i}}}\right)\right.\right.$$

$$\left.\left.+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)(1+\lambda_0\pi+\lambda_0 r^{-1})}{i}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)(1+\lambda\pi+\lambda r^{-1})}{i}}}\right)\right)\right\}$$

$$\times\left\{P\left(Z>\frac{\lambda_0(1-\pi)+L\sqrt{\frac{\lambda_0(1-\pi)(1+\lambda_0\pi+\lambda_0 r^{-1})}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)(1+\lambda\pi+\lambda r^{-1})}{w}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\lambda_0(1-\pi)-L\sqrt{\frac{\lambda_0(1-\pi)(1+\lambda_0\pi+\lambda_0 r^{-1})}{w}}-\lambda(1-\pi)}{\sqrt{\frac{\lambda(1-\pi)(1+\lambda\pi+\lambda r^{-1})}{w}}}\right)\right)^{-1}+w-1. \quad (11)$$

When the process is in-control then ARL = ARL_0, we substitute the value of the parameter λ with λ_0 and when the process is out-of-control then ARL = ARL_1, we substitute the value of the parameter λ with λ_1,

$$\lambda_1 = \lambda_0 + \delta\Big(\lambda_0\left(1-\pi\right)\Big(1+\lambda_0\pi+\lambda_0 r^{-1}\Big)\Big)^{1/2}.$$

5. Numerical Results

In this section, the numerical results for ARL_0 and ARL_1 for ZINB MA chart are calculated from (11). The parameter values for ZINB MA chart were chosen by given ARL0 = 370 and 500, the probability of the observation is zero, π = 0.2 the over-dispersion parameter, the constant r = 0.5, 20 and 100 and the shifts parameter, δ = 0.1, 0.2, 0.3, 0.4, 0.5, 1.0 (small shifts) 1.5, 2.0, 3.0 and 4.0 (large shifts). The calculations with explicit formula are shown in Table 1 and Table 2.

Table 1 show the ARL of ZINB MA chart the results show that the ZINB MA chart with $10 \le w \le 20$ has the best performance when shifts are small. For large shifts, MA ZINB chart with $2 \le w \le 5$ has the best performance.

Table 2 shows the ARL of MA ZINB chart the results show that the MA ZINB chart with $5 \le w \le 20$ has the best performance when shifts are small. For large shifts, MA ZINB chart with $2 \le w \le 4$ has the best performance.

5. Conclusion

Although many control charts rely on the Poisson distribution for monitoring variables of the defective data, when the process has developed the data has more zeros. The ZINB are used than Poisson distribution. The simple chart might be not detected the change in process. We propose the explicit formula of ARL for Moving Average chart when the probability of observations follows a Zero-Inflated Negative Binomial distribution. We have shown that the proposed formula is accurate, easy to calculate and simple to program. However, the traditional chart might not detect the change in the process. Furthermore, the ZINB MA chart has performed better as the value of w decreases. The over-dispersion parameter r has been increased, and then the ARL_1 also increased.

Table 1. The ARL results for the ZINB MA chart where π =0.2 and ARL_0 = 370.

r	δ	w = 1	w = 2	w = 3	w = 4	w = 5	w = 10	w = 15	w = 20
0.5	0.0	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
	0.1	121.262	118.990	116.813	114.728	112.729	103.897	96.707	**90.839**
	0.2	53.547	51.081	48.866	46.872	45.076	38.375	34.324	**31.959**
	0.3	29.031	27.097	25.465	24.084	22.914	19.269	17.882	**17.739**
	0.4	18.157	16.716	15.569	14.656	13.931	12.157	**12.126**	12.970
	0.5	12.571	11.489	10.676	10.070	9.625	**8.921**	9.569	10.850
	1.0	4.474	4.119	3.928	**3.853**	3.863	4.635	5.932	7.383
	1.5	2.827	2.659	**2.606**	2.633	2.719	3.597	4.736	5.873
	2.0	2.199	2.101	**2.094**	2.150	2.249	3.061	4.009	4.888
	3.0	1.691	**1.648**	1.667	1.729	1.820	2.468	3.142	3.707
	4.0	1.480	**1.456**	1.480	1.536	1.615	2.139	2.650	3.051
20	0.0	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
	0.1	234.697	228.782	223.156	217.800	212.698	190.479	172.697	**158.284**
	0.2	151.439	139.759	129.689	120.935	113.270	86.170	70.263	**60.359**
	0.3	100.658	87.446	77.203	69.072	62.498	42.999	34.304	**30.244**
	0.4	69.256	56.952	48.293	41.939	37.134	24.830	20.786	**19.817**
	0.5	49.356	38.778	31.936	27.240	23.889	16.443	**14.977**	15.522
	1.0	14.156	10.047	8.085	7.070	**6.554**	6.748	8.255	9.936
	1.5	6.685	4.817	4.135	**3.922**	3.934	5.017	6.266	7.223
	2.0	4.158	3.151	**2.896**	2.913	3.047	4.056	4.797	5.185
	3.0	2.432	2.034	**2.028**	2.138	2.281	2.833	3.023	3.067
	4.0	1.844	**1.645**	1.692	1.794	1.896	2.169	2.216	2.221
100	0.0	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
	0.1	241.602	235.439	229.579	224.004	218.694	195.592	177.124	**162.167**
	0.2	159.232	146.689	135.902	126.545	118.365	89.553	72.703	**62.219**
	0.3	107.396	92.907	81.735	72.905	65.789	44.795	35.457	**31.062**
	0.4	74.597	60.909	51.361	44.397	39.153	25.791	21.369	**20.231**
	0.5	53.462	41.586	33.992	28.817	25.141	16.990	**15.311**	15.776
	1.0	15.308	10.641	8.450	7.322	**6.744**	6.856	8.366	10.053
	1.5	7.104	4.997	4.240	**3.998**	3.998	5.080	6.322	7.254
	2.0	4.342	3.219	**2.936**	2.947	3.081	4.085	4.793	5.143
	3.0	2.477	2.043	**2.035**	2.147	2.289	2.806	2.963	2.994
	4.0	1.852	**1.640**	1.689	1.789	1.887	2.121	2.154	2.157

Table 2. The ARL results for the ZINB MA chart where π =0.2 and ARL_0 = 500.

r	δ	w = 1	w = 2	w = 3	w = 4	w = 5	w = 10	w = 15	w = 20
0.5	0.0	500.013	500.013	500.013	500.013	500.013	500.013	500.013	500.013
	0.1	153.772	150.730	147.815	145.022	142.344	130.492	120.794	**112.815**
	0.2	64.914	61.762	58.933	56.386	54.088	45.467	40.134	**36.865**
	0.3	34.047	31.657	29.640	27.930	26.475	21.855	19.911	**19.420**
	0.4	20.769	19.031	17.646	16.538	15.650	13.363	**13.074**	13.776
	0.5	14.105	12.825	11.858	11.130	10.587	**9.595**	10.117	11.342
	1.0	4.766	4.365	4.143	4.047	**4.043**	4.794	6.103	7.580
	1.5	2.947	2.760	**2.697**	2.719	2.802	3.690	4.853	6.019
	2.0	2.266	2.159	**2.148**	2.202	2.301	3.130	4.102	5.006
	3.0	1.724	**1.676**	1.695	1.757	1.851	2.515	3.208	3.790
	4.0	1.501	**1.474**	1.498	1.556	1.637	2.175	2.701	3.114
20	0.0	500.013	500.013	500.013	500.013	500.013	500.013	500.013	500.013
	0.1	308.798	300.587	292.787	285.370	278.311	247.633	223.121	**203.241**
	0.2	194.454	178.697	165.176	153.466	143.244	107.273	86.166	**72.895**
	0.3	126.389	109.031	95.669	85.116	76.612	51.441	40.057	**34.485**
	0.4	85.228	69.432	58.409	50.360	44.287	28.661	23.260	**21.604**
	0.5	59.659	46.350	37.813	31.976	27.811	18.403	**16.225**	16.447
	1.0	16.050	11.208	8.894	7.682	7.043	**7.019**	8.507	10.221
	1.5	7.302	5.171	4.381	4.113	**4.095**	5.160	6.450	7.461
	2.0	4.437	3.310	3.010	**3.008**	3.136	4.167	4.948	5.368
	3.0	2.529	2.091	**2.075**	2.184	2.330	2.907	3.112	3.161
	4.0	1.893	**1.676**	1.720	1.824	1.930	2.217	2.269	2.275
100	0.0	500.013	500.013	500.013	500.013	500.013	500.013	500.013	500.013
	0.1	318.400	309.832	301.696	293.963	286.606	254.666	229.181	**208.532**
	0.2	205.035	188.069	173.549	161.002	150.071	111.757	89.372	**75.318**
	0.3	135.339	116.238	101.622	90.130	80.903	53.749	41.519	**35.507**
	0.4	92.183	74.548	62.353	53.506	46.861	29.862	23.973	**22.097**
	0.5	64.915	49.913	40.407	33.956	29.376	19.069	**16.618**	16.735
	1.0	17.438	11.918	9.328	7.978	7.264	**7.136**	8.622	10.343
	1.5	7.791	5.381	4.501	4.198	**4.166**	5.227	6.512	7.498
	2.0	4.648	3.387	3.056	**3.046**	3.173	4.199	4.949	5.329
	3.0	2.580	2.102	**2.083**	2.194	2.339	2.882	3.052	3.088
	4.0	1.902	**1.671**	1.717	1.820	1.921	2.170	2.206	2.209

References

[1] Hoffman, D., Negative Binomial Control Limits for Count Data with Extra-Poisson Variation. *Pharmaceutical Statistics*. 2003, 2: 127-132.

[2] Yu, F.J., Yang, Y.Y., Wang, M.J. and Wu, Z., Using EWMA Control Schemes for Monitoring Wafer Quality in Negative Binomial Process. *Microelectronics Reliability*. 2011, 51: 400-405.

[3] Lambert, D., Zero-Inflated Poisson Regression with an Application to Defects in Manufacturing. *Technometrics*. 1992, 34: 1-14.

[4] Mwalili, S.M., Lesaffre, E. and Declerck, D., The Zero-Inflated Negative Binomial Regression Model with Correction for Misclassification: an Example in Caries Research. *Statistical Methods in Medical Research*. 2007, 1-17.

[5] Chen, N., Zhou, S., Chang, T-S. and Huang, H., Attribute Control Chart using Generalized Zero-Inflated Poisson Distribution. *Quality and Reliability Engineering International*. 2008, 24: 793-806.

[6] Noorossana, R., Fatahi, A.A., Dokouhaki, P., Babakhani, M., ZIB-EWMA Control Chart for Monitoring Rare Health Events. *Journal of Mechanics in Medicine and Biology*. 2011, 11: 881-895.

[7] He, S., Huang, W., Woodall, W.H., CUSUM Charts for Monitoring a Zero-Inflated Poisson Process. *Quality and Reliability Engineering International*. 2012, 28: 181-192.

[8] Katemee, N. and Mayureesawan, T., Control Charts for Monitoring the Zero-Inflated Generalized Poisson Processes. *Thai Journal of Mathematics*. 2013, 1: 237-249.

[9] Brook, D., Evans, D.A., An Approach to the Probability Distribution of CUSUM Run Length. *Biometrika*. 1972, 9: 539-548.

[10] Phengsalae, Y., Areepong, Y. and Sukparungsee, S., An Approximation of ARL of Poisson GWMA using Markov Chain Approach. *Thailand Statistician*. 2015, 13: 111-124.

[11] Sukparungsee, S. and Novikov, A.A., Analytical Approximations for Detection of a Change-Point in Case of Light-Tailed Distributions. *Journal of Quality Measurement and Analysis*. 2008, 4: 373-380.

[12] Busaba, J., Sukparungsee, S., Areepong, Y. and Mititelu, G., Analysis of Average Run Length for CUSUM Procedure with Negative Exponential Data. *Chiang Mai Journal of Science*. 2012, 39: 200-208.

[13] Areepong, Y. Explicit Formulas of Average Run Length for a Moving Average Control Chart for Monitoring the Number of Defective Products. *International Journal of Pure and Applied Mathematics*. 2012, 80: 331-343.

[14] Khoo, M.B.C., A Moving Average Control Chart for Monitoring the Fraction Non-Conforming. *Quality and Reliability Engineering International*. 2004. 20: 617-635.