Thailand Statistician
January 2016; 14(1): 25-33
http://statassoc.or.th
Contributed paper

A Note on Measure of Slope Rotatability for Second Order Response Surface Designs using a Pair of Balanced Incomplete Block Designs

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Received: 10 February 2014
Accepted: 30 January 2015

Abstract

A measure enables to assess the degree of slope-rotatability for a given response surface design. In this paper, a note on the construction of measure of slope rotatability for second order response surface designs using a pair of balanced incomplete block designs without any further set of points is suggested.

Keywords: Second order response surface designs, second order slope rotatable designs, measure of slope rotatability for second order response surface designs.

1. Introduction

Box and Hunter (1957) introduced rotatable designs for the exploration of response surfaces. The study of rotatable designs mainly emphasized on the estimation of absolute response. Estimation of differences in response at two different points in the factor space will often be of great importance. If differences at two points close together, estimation of local slope (rate of change) of the response is of interest. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in an animal etc. (Park 1987).

Hader and Park (1978) introduced slope rotatable central composite designs. Victorbabu and Narasimham (1991a) studied second order slope rotatable designs (SOSRD) and constructed SOSRD using balanced incomplete block designs (BIBD). Victorbabu and Narasimham (1991b) constructed SOSRD through a pair of incomplete block designs. Victorbabu and Surekha (2011) suggested a new method of construction of SOSRD using BIBD. Khuri (1988), Park et al. (1993) studied measure of rotatability for second order response surface designs. Specifically, Park and Kim (1992) introduced measure of slope rotatability for second order response surface experimental designs. Victorbabu and Surekha (2012a, 2012b, 2012c and 2013) constructed measure of SOSRD using BIBD, pairwise balanced designs, symmetrical unequal block arrangements with two unequal block sizes and partially balanced incomplete block type designs respectively. In this paper, a note on measure of slope rotatability for second order response surface designs using a pair of balanced incomplete block designs without any further set of points is suggested. Further, the methods of Park and Kim (1992), Victorbabu and Surekha (2012a), slope rotatability for second order response surface designs using central composite designs and balanced incomplete block designs are shown to be particular cases of this new method.

2. Conditions for Second Order Slope Rotatable Designs

Suppose we want to use the second order response surface design $D=((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u, \tag{1}$$

where X_{iu} denotes the level of the i^{th} factor $(i=1,2,\ldots,v)$ in the u^{th} run ($u=1,2,\ldots,N$) of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be SOSRD if the variance of the estimate of first order partial derivative of $Y_u(x_1,x_2,\ldots,x_v)$ with respect to each of independent variables (x_i) is only a function of the distance ($d^2=\sum x_i^2$) of the point $(x_1,x_2,\ldots,x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of slopes in the second order response surface is achieved, if the design points satisfy the following conditions (cf. Hader and Park 1978, and Victorbabu and Narasimham 1991a).

$$\sum x_{iu} = 0,\ \sum x_{iu}x_{ju} = 0,\ \sum x_{iu}x_{ju}^2 = 0,\ \sum x_{iu}x_{ju}x_{ku} = 0,\ \sum x_{iu}^3 = 0,$$
$$\sum x_{iu}x_{ju}^3 = 0,\ \sum x_{iu}x_{ju}x_{ku}^2 = 0,\ \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0;\ \text{for } i\neq j\neq k\neq l; \tag{2}$$

$$\sum x_{iu}^2 = \text{constant} = N\lambda_2; \tag{3}$$

$$\sum x_{iu}^4 = \text{constant} = cN\lambda_4;\ \text{for all } i \tag{4}$$

$$\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4;\ \text{for } i\neq j \tag{5}$$

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \tag{6}$$

$$\left[v(5-c)-(c-3)^2\right]\lambda_4 + \left[v(c-5)+4\right]\lambda_2^2 = 0, \tag{7}$$

where c, λ_2 and λ_4 are constants and summation is over the design points.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-\text{v}\lambda_2^2}\right],$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\lambda_2 \sigma^2}{N\left[\lambda_4(c+v-1) - v\lambda_2^2\right]},$$

$$Cov(\hat{b}_{ii}, \hat{b}_{ij}) = \frac{(\lambda_2^2 - \lambda_4)\sigma^2}{(c-1)N\lambda_4\left[\lambda_4(c+v-1) - v\lambda_2^2\right]} \text{ and other covariances vanish.} \quad (8)$$

3. SOSRD Using a Pair of BIBD (cf. Victorbabu and Narasimham 1991b)

The method of construction of SOSRD using a pair of BIBD is given in the following result (cf. Victorbabu and Narasimham 1991b).

Result: If $D_1 = (v, b_1, r_1, k_1, \lambda_1)$ and $D_2 = (v, b_2, r_2, k_2, \lambda_2)$ are two BIBDs, $2^{t(k_1)}$ and $2^{t(k_2)}$ denote Resolution V fractional replicates of 2^{k_1}, and 2^{k_2} factorials with levels ±1 then the design points, $\left[1-(v,b_1,r_1,k_1,\lambda_1)\right]2^{t(k_1)} \cup \left[a-(v,b_2,r_2,k_2,\lambda_2)\right]2^{t(k_2)} \cup (\mathrm{n}_0)$ give a v-dimensional SOSRD in $N = b_1 2^{t(k_1)} + b_2 2^{t(k_2)} + n_0$ design points, where a^2 is the positive real root of the biquadratic equation,

$$\begin{aligned}
&\left[2^{2t(k_2)} N\left\{\lambda_2^2(5v-9) + r_2\lambda_2(6-v) - r_2^2\right\} + 2^{3t(k_2)} r_2^2\left\{vr_2 - 5v\lambda_2 + 4\lambda_2\right\}\right] a^8 + \\
&\left[2^{t(k_1)+2t(k_2)+1} r_1 r_2 (vr_2 - 5v\lambda_2 + 4\lambda_2\right] a^6 + \\
&\begin{bmatrix} 2^{t(k_1)+2t(k_2)} N\left\{\lambda_1\lambda_2(10v-18) + (6-v)(r_1\lambda_2 + r_2\lambda_1) - 2r_1r_2\right\} + \\ 2^{t(k_1)+2t(k_2)} r_2^2 (vr_1 - 5v\lambda_1 + 4\lambda_1) + 2^{2t(k_1)+t(k_2)} r_1^2 (vr_2 - 5v\lambda_2 + 4\lambda_2) \end{bmatrix} a^4 + \\
&\left[2^{2t(2k_1)+t(k_2)+1} r_1 r_2 (vr_1 - 5v\lambda_1 + 4\lambda_1)\right] a^2 + \\
&\left[2^{2t(k_1)} N\left\{\lambda_1^2(5v-9) + r_1\lambda_1(6-v) - r_1^2\right\} + 2^{3t(k_1)} r_1^2\left\{vr_1 - 5v\lambda_1 + 4\lambda_1\right\}\right] = 0.
\end{aligned} \quad (9)$$

If at least one positive real root exists for a^2 in equation (9) then the design exists.

Note: Values of SOSRD using a pair of BIBD can be obtained by solving the above equation.

4. Conditions for Measure of Slope Rotatability for Second Order Response Surface Designs

Following Hader and Park (1978), Victorbabu and Narasimham (1991a), Park and Kim (1992), Equations (2-8) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs.

Further we have,

$$V(b_i) \text{ are equal for } i,$$

$$V(b_{ii}) \text{ are equal for } i,$$

$$V(b_{ij}) \text{ are equal for } i, j, \text{ where } i \neq j,$$

$$Cov(b_i, b_{ii}) = Cov(b_i, b_{ij}) = Cov(b_{ii}, b_{ij}) = Cov(b_{ij}, b_{il}) = 0 \text{ for all } i \neq j \neq l.$$

The measure of slope rotatability for second order response surface design can be obtained by using the following equation (Park and Kim 1992, p. 398),

$$Q_v(D)=\frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[(v(b_i)-\frac{1}{v}\sum_{i=1}^{v}v(b_i))+\frac{(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij}))-\frac{1}{v}\sum_{i=1}^{v}(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij}))}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}((4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij}))-\frac{1}{v}\sum_{i=1}^{v}(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})))^2+2\sum_{i=1}^{v}\left[\left[4v(b_{ii})-\frac{(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij}))}{v}\right]^2+\sum_{\substack{j=1\\ j\neq i}}^{v}\left[v(\mathrm{b}_{ij})-\frac{(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(\mathrm{b}_{ij}))}{v}\right]\right]$$

$$\left.+4(v+4)\left[4\operatorname{cov}^2(b_i,b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}\operatorname{cov}^2(b_i,b_{ij})\right]+4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\ j\neq i}}^{v}\operatorname{cov}^2(b_{ii},b_{ij})+\sum_{\substack{j<1\\ j,1\neq i}}^{v}\sum\operatorname{cov}^2(b_{ij},b_{i1})\right]\right\},$$

where $Q_v(D)$ is the proposed measure of slope-rotatability. Further it is greatly simplified to $Q_v(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-\mathrm{V}(\mathrm{b}_{ij})\right]^2$ (cf. Park and Kim 1992, p. 398).

5. Construction of Measure of Slope Rotatability for Second Order Response Surface Designs Using a Pair of BIBDs

The proposed measure of slope rotatability for second order response surface designs using a pair of BIBDs is suggested in this section. Let $D_1=(v,\mathrm{b}_1,r_1,k_1,\lambda_1)$ and $D_2=(v,\mathrm{b}_2,r_2,k_2,\lambda_2)$ are two BIBDs. $\left[1-(v,\mathrm{b}_1,r_1,k_1,\lambda_1)\right]$ denote the design points generated from the transpose of the incidence matrix of the design D_1. $\left[1-(v,\mathrm{b}_1,r_1,k_1,\lambda_1)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generated from D_1 by multiplication (see Das and Narasimham 1962). $\left[a-(v,\mathrm{b}_2,r_2,k_2,\lambda_2)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points generated from D_2 by multiplication. n_0 be the number of central points. Then with the above design points, we can obtain measure of slope rotatability for second order response surface designs as given in the below theorem.

Theorem. *The design points,*

$$\left[1-(v,\mathrm{b}_1,r_1,k_1,\lambda_1)\right]2^{t(k_1)}\cup\left[a-(v,\mathrm{b}_2,r_2,k_2,\lambda_2)\right]2^{t(k_2)}\cup(n_0)$$

give a v-dimensional measure of slope rotatability for second order response surface designs using a pair of BIBDs in $N=b_1 2^{t(k_1)}+b_2 2^{t(k_2)}+n_0$ *design points with level 'a' pre-fixed and*

$$c=\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4}{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}. \tag{10}$$

Proof: For the design points generated from a pair of BIBDs, conditions (2) to (5) are true. Conditions in (2) are true obviously. Conditions (3) to (5) are true as follows:

$$\sum x_{iu}^4 = r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2 = N\lambda_2 \tag{11}$$

$$\sum x_{iu}^4 = r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4 = cN\lambda_4 \tag{12}$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4 = N\lambda_4. \tag{13}$$

From equations (12) and (13), we get c.

Measure of slope rotatability for second order response surface designs using a pair of BIBDs can be obtained by solving the given equation (cf. Park and Kim 1992).

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e-V(b_{ij})\right]^2, \tag{14}$$

where

$$e=\frac{\begin{matrix}(v-1)\left[2^{t(k_1)}\lambda_1 n_0+2^{t(k_1)+t(k_2)}b_2\lambda_1-2r_1r_2 2^{t(k_1)+t(k_2)}a^2\right]+\\ \begin{bmatrix}(v-1)\left(2^{t(k_1)+t(k_2)}b_1\lambda_2+2^{2t(k_2)}b_2\lambda_2+2^{t(k_2)}n_0\lambda_2-2^{2t(k_2)}r_2^2\right)+\\ (r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)}b_1+2^{2t(k_2)}b_2+2^{t(k_2)}n_0\right)\end{bmatrix}a^4+\\ (r_1-\lambda_1)\left(2^{2t(k_1)}b_1+2^{t(k_1)+t(k_2)}b_2+2^{t(k_1)}n_0\right)+(vb_1\lambda_1-(v-1)r_1^2-b_1\lambda_1)2^{2t(k_1)}\end{matrix}}{\left[2^{t(k_1)}(r_1-\lambda_1)+2^{t(k_2)}(r_2-\lambda_2)a^4\right]\begin{bmatrix}(r_1-\lambda_1)\left(2^{2t(k_1)}b_1+2^{t(k_1)+t(k_2)}b_2+2^{t(k_1)}n_0\right)+\\ \begin{bmatrix}(r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)}b_1+2^{2t(k_2)}b_2+2^{t(k_2)}n_0\right)+\\ 2^{t(k_1)+t(k_2)}vb_1\lambda_2+2^{2t(k_2)}vb_2\lambda_2+2^{t(k_2)}vn_0\lambda_2-2^{2t(k_2)}vr_2^2\end{bmatrix}a^4+\\ (b_1\lambda_1-r_1^2)v2^{2t(k_1)}+2^{t(k_1)}v\lambda_1 n_0+(b_2\lambda_1-2r_1r_2a^2)v2^{t(k_1)+t(k_2)}\end{bmatrix}}.$$

The following Table 1 gives the values of measure of slope rotatability ($Q_v(D)$) for second order response surface designs using various parameters of pair of BIBDs for $n_0 = 1,2,3,4$ and 5 and level 'a'.

Corollary (i): *Taking* $D_1=(v=v, \mathrm{b}_1=1, r_1=1, k_1=1, \lambda_1=1)$ *and* $D_2=(v=v, \mathrm{b}_2=\mathrm{v}, r_2=1, k_2=1, \lambda_2=1)$*, we get Park and Kim (1992) measure of slope rotatability for second order response surface designs using central composite designs as a particular case.*

Corollary (ii): *Taking* $D_1=(v=v, \mathrm{b}_1=b, r_1=r, k_1=k, \lambda_1=\lambda)$ *and* $D_2=(v=v, \mathrm{b}_2=\mathrm{v}, r_2=1, k_2=1, \lambda_2=0)$*, we get Victorbabu and Surekha (2012a) measure of slope rotatability for second order response surface designs using BIBD's as a particular case.*

6. Conclusions

In this paper, measure of slope rotatability for second order response surface designs using a pair of BIBDs is suggested which enables us to assess the degree of slope rotatability for a given second order response surface design. Park and Kim (1992), Victorbabu and Surekha (2012a) measure of slope rotatability for second order response surface designs using central composite designs and

BIBD’s are shown to be obtainable using this method. It can be verified that measure of rotatability is zero if and only if a design D is a second order slope-rotatable design. Measure of rotatability becomes larger as D deviates from a second order slope rotatable design.

Acknowledgements

The authors are grateful to the referees and the Chief Editor for their constructive suggestions which have led to great improvement on the earlier version of the paper.

Table 1 Values of measure of slope rotatability ($Q_v(D)$) for second order response surface designs using pair of BIBDs

$D_1 = (v=7, b_1=7, r_1=3, k_1=3, \lambda_1=1), D_2 = (v=7, b_2=21, r_2=6, k_2=2, \lambda_2=1)$					
a	n_0=1 (N=141)	n_0=2 (N=142)	n_0=3 (N=143)	n_0=4 (N=144)	n_0=5 (N=145)
1.0	1.7343×10^{-4}	1.3286×10^{-4}	1.0532×10^{-4}	8.5734×10^{-5}	7.1281×10^{-5}
1.3	2.9393×10^{-3}	9.9885×10^{-4}	4.8657×10^{-4}	2.8212×10^{-4}	1.8134×10^{-4}
1.6	4.2406×10^{-5}	3.1389×10^{-5}	2.3688×10^{-5}	1.8158×10^{-5}	1.4095×10^{-5}
1.9	1.7108×10^{-6}	1.2409×10^{-6}	8.8818×10^{-7}	6.2422×10^{-7}	4.2800×10^{-7}
2.2	2.9026×10^{-8}	5.9439×10^{-8}	9.6002×10^{-8}	1.3640×10^{-7}	1.7888×10^{-7}
2.5	5.8116×10^{-7}	6.3465×10^{-7}	6.8447×10^{-7}	7.3057×10^{-7}	7.7298×10^{-7}
2.8	1.0872×10^{-6}	1.1231×10^{-6}	1.1549×10^{-6}	1.1828×10^{-6}	1.2071×10^{-6}
3.1	1.4166×10^{-6}	1.4359×10^{-6}	1.4516×10^{-6}	1.4642×10^{-6}	1.4738×10^{-6}
*	2.1461	2.1202	2.0952	2.0711	2.0479
$D_1 = (v=8, b_1=14, r_1=7, k_1=4, \lambda_1=3), D_2 = (v=8, b_2=28, r_2=7, k_2=2, \lambda_2=1)$					
a	n_0=1 (N=337)	n_0=2 (N=338)	n_0=3 (N=339)	n_0=4 (N=340)	n_0=5 (N=341)
1.0	3.3338×10^{-5}	3.2078×10^{-5}	3.0915×10^{-5}	2.9839×10^{-5}	2.8840×10^{-5}
1.3	5.2478×10^{-4}	3.1100×10^{-4}	2.0950×10^{-4}	1.5291×10^{-4}	1.1789×10^{-4}
1.6	1.2626×10^{-4}	9.3353×10^{-5}	7.2058×10^{-5}	5.7464×10^{-5}	4.7011×10^{-5}
1.9	2.9598×10^{-6}	2.7186×10^{-6}	2.5028×10^{-6}	2.3091×10^{-6}	2.1347×10^{-6}
2.2	8.3173×10^{-9}	6.1569×10^{-9}	4.3932×10^{-9}	2.9830×10^{-9}	1.8877×10^{-9}
2.5	2.3337×10^{-7}	2.3494×10^{-7}	2.3638×10^{-7}	2.3771×10^{-7}	2.3893×10^{-7}
2.8	5.1104×10^{-7}	5.0822×10^{-7}	5.0537×10^{-7}	5.0248×10^{-7}	4.9957×10^{-7}
3.1	6.5058×10^{-7}	6.4511×10^{-7}	6.3967×10^{-7}	6.3427×10^{-7}	6.2889×10^{-7}
*	2.2334	2.2284	2.2243	2.2204	2.2164
$D_1 = (v=9, b_1=12, r_1=4, k_1=3, \lambda_1=1), D_2 = (v=9, b_2=36, r_2=8, k_2=2, \lambda_2=1)$					
a	n_0=1 (N=241)	n_0=2 (N=242)	n_0=3 (N=243)	n_0=4 (N=244)	n_0=5 (N=245)
1.0	1.1553×10^{-5}	9.0417×10^{-6}	7.1744×10^{-6}	5.7583×10^{-6}	4.6663×10^{-6}
1.3	6.2963×10^{-4}	2.3329×10^{-4}	1.1318×10^{-4}	6.3108×10^{-5}	3.8261×10^{-5}
1.6	2.7465×10^{-7}	1.0518×10^{-7}	2.2945×10^{-8}	7.3916×10^{-13}	1.7213×10^{-8}
1.9	1.9234×10^{-6}	2.0011×10^{-6}	2.0719×10^{-6}	2.1363×10^{-6}	2.1946×10^{-6}
2.2	3.4210×10^{-6}	3.4259×10^{-6}	3.4277×10^{-6}	3.4269×10^{-6}	3.4235×10^{-6}
2.5	3.9654×10^{-6}	3.9388×10^{-6}	3.9111×10^{-6}	3.8825×10^{-6}	3.8530×10^{-6}
2.8	4.1290×10^{-6}	4.0888×10^{-6}	4.0483×10^{-6}	4.0075×10^{-6}	3.9667×10^{-6}
3.1	4.1475×10^{-6}	4.1009×10^{-6}	4.0544×10^{-6}	4.0081×10^{-6}	3.9620×10^{-6}
*	1.6446	1.6298	1.6149	1.6001	1.5854

Table 1 (Continued)

$D_1 = (v = 10, b_1 = 15, r_1 = 6, k_1 = 4, \lambda_1 = 2), D_2 = (v = 10, b_2 = 45, r_2 = 9, k_2 = 2, \lambda_2 = 1)$					
a	n_0=1 (N=421)	n_0=2 (N=422)	n_0=3 (N=423)	n_0=4 (N=424)	n_0=5 (N=425)
1.0	3.5689×10^{-6}	3.4562×10^{-6}	3.3500×10^{-6}	3.2498×10^{-6}	3.1551×10^{-6}
1.3	1.0827×10^{-4}	6.7723×10^{-5}	4.6262×10^{-5}	3.3557×10^{-5}	2.5423×10^{-5}
1.6	2.0763×10^{-5}	1.5256×10^{-5}	1.1529×10^{-5}	8.9072×10^{-6}	7.0054×10^{-6}
1.9	4.6775×10^{-8}	5.8574×10^{-8}	7.0795×10^{-8}	8.3275×10^{-8}	9.5879×10^{-8}
2.2	9.2965×10^{-7}	9.3223×10^{-7}	9.3445×10^{-7}	9.3631×10^{-7}	9.3785×10^{-7}
2.5	1.4042×10^{-6}	1.3965×10^{-6}	1.3888×10^{-6}	1.3811×10^{-6}	1.3733×10^{-6}
2.8	1.5509×10^{-6}	1.5395×10^{-6}	1.5281×10^{-6}	1.5168×10^{-6}	1.5056×10^{-6}
3.1	1.5557×10^{-6}	1.5431×10^{-6}	1.5306×10^{-6}	1.5182×10^{-6}	1.5059×10^{-6}
*	1.8580	1.8516	1.8451	1.8387	1.8323
$D_1 = (v = 11, b_1 = 11, r_1 = 5, k_1 = 5, \lambda_1 = 2), D_2 = (v = 11, b_2 = 55, r_2 = 10, k_2 = 2, \lambda_2 = 1)$					
a	n_0=1 (N=397)	n_0=2 (N=398)	n_0=3 (N=399)	n_0=4 (N=400)	n_0=5 (N=401)
1.0	2.7321×10^{-5}	2.6056×10^{-5}	2.4880×10^{-5}	2.3786×10^{-5}	2.2765×10^{-5}
1.3	4.7180×10^{-6}	4.1839×10^{-6}	3.7367×10^{-6}	3.3586×10^{-6}	3.0358×10^{-6}
1.6	9.2011×10^{-4}	2.2761×10^{-4}	9.6535×10^{-5}	5.1283×10^{-5}	3.0837×10^{-5}
1.9	8.1618×10^{-7}	5.3359×10^{-7}	3.3820×10^{-7}	2.0443×10^{-7}	1.1479×10^{-7}
2.2	9.5421×10^{-7}	9.8758×10^{-7}	1.0188×10^{-6}	1.0479×10^{-6}	1.0751×10^{-6}
2.5	2.0958×10^{-6}	2.0958×10^{-6}	2.0950×10^{-6}	2.1623×10^{-6}	2.0912×10^{-6}
2.8	2.5846×10^{-6}	2.5703×10^{-6}	2.5559×10^{-6}	2.5412×10^{-6}	2.5264×10^{-6}
3.1	2.7524×10^{-6}	2.7323×10^{-6}	2.7122×10^{-6}	2.6921×10^{-6}	2.6721×10^{-6}
*	1.9871	1.9762	1.9653	1.9544	1.9436
$D_1 = (v = 12, b_1 = 33, r_1 = 11, k_1 = 4, \lambda_1 = 3), D_2 = (v = 12, b_2 = 44, r_2 = 11, k_2 = 3, \lambda_2 = 1)$					
a	n_0=1 (N=881)	n_0=2 (N=882)	n_0=3 (N=883)	n_0=4 (N=884)	n_0=5 (N=885)
1.0	3.4064×10^{-6}	3.0778×10^{-6}	2.7977×10^{-6}	2.5568×10^{-6}	2.3482×10^{-6}
1.3	4.5030×10^{-6}	3.8925×10^{-6}	3.3986×10^{-6}	2.9933×10^{-6}	2.6566×10^{-6}
1.6	3.9219×10^{-8}	3.7893×10^{-8}	3.6620×10^{-8}	3.5399×10^{-8}	3.4226×10^{-8}
1.9	3.0526×10^{-10}	3.2426×10^{-10}	3.4350×10^{-10}	3.6297×10^{-10}	3.8263×10^{-10}
2.2	6.5887×10^{-9}	6.5957×10^{-9}	6.6023×10^{-9}	6.6086×10^{-9}	6.6145×10^{-9}
2.5	1.0663×10^{-8}	1.0641×10^{-8}	1.0618×10^{-8}	1.0595×10^{-8}	1.0572×10^{-8}
2.8	1.2268×10^{-8}	1.2231×10^{-8}	1.2193×10^{-8}	1.2156×10^{-8}	1.2119×10^{-8}
3.1	1.2693×10^{-8}	1.2650×10^{-8}	1.2606×10^{-8}	1.2562×10^{-8}	1.2519×10^{-8}
*	1.8546	1.8529	1.8513	1.8496	1.8480

Table 1 (Continued)

$D_1=(v=13,b_1=13,r_1=4,k_1=4,\lambda_1=1), D_2=(v=13,b_2=26,r_2=6,k_2=3,\lambda_2=1)$					
a	n_0=1 (N=417)	n_0=2 (N=418)	n_0=3 (N=419)	n_0=4 (N=420)	n_0=5 (N=421)
1.0	6.2691×10^{-6}	5.0975×10^{-6}	4.2238×10^{-6}	3.5552×10^{-6}	3.0322×10^{-6}
1.3	9.6676×10^{-6}	7.0418×10^{-6}	5.3040×10^{-6}	4.1008×10^{-6}	3.2374×10^{-6}
1.6	1.6782×10^{-10}	3.2115×10^{-14}	1.4044×10^{-10}	5.3778×10^{-10}	1.1046×10^{-8}
1.9	1.0787×10^{-7}	1.0874×10^{-7}	1.0955×10^{-7}	1.1031×10^{-7}	1.1101×10^{-7}
2.2	1.7266×10^{-7}	1.7206×10^{-7}	1.7144×10^{-7}	1.7081×10^{-7}	1.7016×10^{-7}
2.5	1.9243×10^{-7}	1.9125×10^{-7}	1.9007×10^{-7}	1.8889×10^{-7}	1.8771×10^{-7}
2.8	1.9504×10^{-7}	1.9364×10^{-7}	1.9225×10^{-7}	1.9086×10^{-7}	1.8949×10^{-7}
3.1	1.9180×10^{-7}	1.9032×10^{-7}	1.8886×10^{-7}	1.8741×10^{-7}	1.8596×10^{-7}
*	1.6058	1.6001	1.5944	1.5887	1.5831
$D_1=(v=15,b_1=15,r_1=7,k_1=7,\lambda_1=3), D_2=(v=15,b_2=35,r_2=7,k_2=3,\lambda_2=1)$					
a	n_0=1 (N=1241)	n_0=2 (N=1242)	n_0=3 (N=1243)	n_0=4 (N=1244)	n_0=5 (N=1245)
1.0	7.2536×10^{-6}	7.0898×10^{-6}	6.9331×10^{-6}	6.7832×10^{-6}	6.6396×10^{-6}
1.3	3.3312×10^{-6}	1.5723×10^{-6}	3.0174×10^{-6}	2.8856×10^{-6}	2.7672×10^{-6}
1.6	4.2997×10^{-5}	2.5359×10^{-5}	1.7041×10^{-5}	1.2424×10^{-5}	9.5777×10^{-6}
1.9	3.3017×10^{-7}	3.2190×10^{-7}	3.1403×10^{-7}	3.0654×10^{-7}	2.9939×10^{-7}
2.2	1.7412×10^{-8}	1.7183×10^{-8}	1.6957×10^{-8}	1.6737×10^{-8}	1.6520×10^{-8}
2.5	1.6089×10^{-9}	1.6173×10^{-9}	1.6257×10^{-9}	1.6339×10^{-9}	1.6420×10^{-9}
2.8	1.3897×10^{-8}	1.3868×10^{-8}	1.3839×10^{-8}	1.3810×10^{-8}	1.3781×10^{-8}
3.1	2.2863×10^{-8}	2.2800×10^{-8}	2.2736×10^{-8}	2.2673×10^{-8}	2.2610×10^{-8}
*	2.4059	2.4054	2.4049	2.4044	2.4039
$D_1=(v=16,b_1=16,r_1=6,k_1=6,\lambda_1=2), D_2=(v=16,b_2=80,r_2=15,k_2=3,\lambda_2=2)$					
a	n_0=1 (N=1153)	n_0=2 (N=1154)	n_0=3 (N=1155)	n_0=4 (N=1156)	n_0=5 (N=1157)
1.0	1.4581×10^{-7}	1.4469×10^{-7}	1.4358×10^{-7}	1.4248×10^{-7}	1.4141×10^{-7}
1.3	2.7705×10^{-6}	2.2363×10^{-6}	1.8408×10^{-6}	1.5401×10^{-6}	1.3062×10^{-6}
1.6	4.0886×10^{-7}	3.4974×10^{-7}	3.0066×10^{-7}	2.5961×10^{-7}	2.2502×10^{-7}
1.9	2.5718×10^{-8}	2.6254×10^{-8}	2.6778×10^{-8}	2.7291×10^{-8}	2.7792×10^{-8}
2.2	7.8540×10^{-8}	7.8534×10^{-8}	7.8525×10^{-8}	7.8512×10^{-8}	7.8496×10^{-8}
2.5	9.8550×10^{-8}	9.8341×10^{-8}	9.8131×10^{-8}	9.7921×10^{-8}	9.7710×10^{-8}
2.8	1.0388×10^{-7}	1.0360×10^{-7}	1.0332×10^{-7}	1.0304×10^{-7}	1.0276×10^{-7}
3.1	1.0358×10^{-7}	1.0328×10^{-7}	1.0298×10^{-7}	1.0267×10^{-7}	1.0237×10^{-7}
*	1.7607	1.7574	1.7540	1.7506	1.7473

***** indicates exact SOSRD using a pair of BIBDs.

References

Box GEP, Hunter JS. Multifactor experimental designs for exploring response surfaces. Ann Math Stat. 1957; 28: 195-241.

Das MN, Narasimham VL. Construction of rotatable designs through balanced incomplete block designs. Ann Math Stat. 1962; 33: 1421-1439.

Hader RJ, Park SH. Slope rotatable central composite designs. Technometrics. 1978; 20: 413-417.

Khuri AI. A measure of rotatability for response surface designs. Technometrics. 1988; 30: 95-104.

Park SH. A class of multi-factor designs for estimating the slope of response surface. Technometrics. 1987; 29: 449-453.

Park SH, Kim HJ. A measure of slope rotatability for second order response surface experimental designs. J Appl Stat. 1992; 19: 391-404.

Park SH, Lim JH, Yasumasa Baba. A measure of rotatability for second order response surface designs. Ann. Inst Statist Math. 1993; 45: 655-664.

Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs through balanced incomplete block designs. Commun Stat Theory M. 1991a; 20: 2467-2478.

Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs through a pair of incomplete block designs. J Indian Soc Agric Stat. 1991b; 43: 291-295.

Victorbabu BRe, Surekha ChVVS. A new method of construction of second order slope rotatable designs using balanced incomplete block designs. Pak J Stat. 2011; 27: 221-230.

Victorbabu BRe, Surekha, ChVVS. Construction of measure of second order slope rotatable designs using balanced incomplete block designs. J Stat. 2012a; 19: 1-10.

Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using pairwise balanced designs. Int J Statistics Anal. 2012b; 2: 97-106.

Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using symmetrical unequal block arrangements with two unequal block sizes. J Stat Manag Syst. 2012c; 15: 569-579.

Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using partially balanced incomplete block type designs. Thail Stat. 2013; 11: 159-171.