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Estimating the Mean of the Logistic Distribution with Known Coefficient of Variation by U-Statistics

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Abstract

In this work we propose a technique of estimating the mean μ of a logistic distribution with known coefficient of variation by U-statistics constructed by taking best linear functions of order statistics as kernels. The efficiency comparison of the proposed estimator with respect to moment estimator is also considered.

Keywords: Order statistics, location-scale family of distributions, best linear unbiased estimator, U-statistics, coefficient of variation, logistic distribution.

1. Introduction

In some of the problems of biological and physical sciences, situations where the scale parameter is proportional to the location parameter are seen reported in the literature, then knowing the proportionality constant is equivalent to knowing the population coefficient of variation. The interest for inferentialists comes from the fact that, the inferential procedures instead of getting simplified becomes more intricate. Also the complete sufficient statistic no longer holds in this case. Therefore uniformly minimum variance unbiased estimator (UMVUE) does not exist in this case. Several authors, have studied this kind of inference problems. If the parent distribution is a normal with mean μ and standard deviation $c\mu$ and if it is denoted by $N(\mu, c\mu)$, then the problem of estimating μ has been extensively discussed in the available literature, for example, see, Gleser and Healy (1976), Arnholt and Hebert (1995), Kunte (2000) and Guo and Pal (2003). The above mentioned normal model with known coefficient of variation is useful in environmental studies, where μ explains the average concentration level of particular pollutant and the standard deviation $c\mu$ is directly related to mean. In this case we know the value of coefficient of variation from past studies. The best linear unbiased estimator (BLUE) of μ for $N(\mu, c\mu)$ distribution for different values of c using order statistics are discussed in Thomas and Sajeevkumar (2003).

Estimating the location parameter of an exponential distribution with known coefficient of variation are discussed in Ghosh and Razmpour (1982) and Samanta (1984). Hirano and Iwase (1989) and Farsipour (1997) have studied the problem of estimation of mean of inverse Gaussian distribution with known coefficient of variation. Estimating the location parameter μ of the exponential distribution with known coefficient of variation by ranked set sampling are discussed by Irshad and Sajeevkumar (2011). Also Sajeevkumar and Irshad (2011) discussed estimation of the location parameter μ of the exponential distribution using censored samples by order statistics. It is well known that logistic distribution is having more or less similar properties of a normal distribution and hence it is known as an alternative model to normal distribution. Now consider the logistic distribution with mean μ and standard deviation $c\mu$, where c is the known coefficient of variation, whose probability density function (pdf) is given by

$$f(x;\mu,c\mu)=\frac{\pi}{\sqrt{3}}\frac{e^{-\frac{\pi}{\sqrt{3}}\left(\frac{x-\mu}{c\mu}\right)}}{\left[1+e^{-\frac{\pi}{\sqrt{3}}\left(\frac{x-\mu}{c\mu}\right)}\right]^2}\ \mu,c>0,x\in R. \tag{1}$$

We will write $LD(\mu,c\mu)$ to denote the logistic distribution defined by (1). The best linear unbiased estimator (BLUE) of μ using order statistics arising from $LD(\mu,c\mu)$ are obtained by Sajeevkumar and Thomas (2005).

Eventhough best linear unbiased estimation of location and scale parameters using order statistics (see Lloyd 1952) is a widely accepted method of estimation, one serious problem in the application of this method is that, in order to obtain these estimators one requires the values of means, variances and covariances of the entire order statistics of a random sample of size n arising from the corresponding standard distribution. However, if one obtains the BLUEs of order statistics based on small or moderate sample of size m and use this as kernel of degree m to construct appropriate U-statistics to estimate μ and σ, then these U-statistics are highly useful as they estimate the parameters explicitly. Moreover these estimators are highly preferred as they utilize the optimality conditions of BLUE as well as U-statistics. Thomas and Sreekumar (2004) developed the concept of U-statistics by taking BLUE based on the order statistics of a random sample of size two as kernel of degree two to estimate the scale parameter of generalized exponential distribution. Again Thomas and Sreekumar (2008) generalized the results of Thomas and Sreekumar (2004) to generate estimators based on U-statistics for the location and scale parameters of any distribution, by taking best linear functions of order statistics of a sample of size $m<n$ as kernels. Recently Sajeevkumar and Irshad (2012) estimating the mean of the double exponential distribution with known coefficient of variation by U-statistics.

In this article our aim is to estimate the mean μ of the $LD(\mu,c\mu)$ distribution with known coefficient of variation by U-statistics based on kernels of degree m defined by the respective BLUE of μ based on order statistics of a random sample of size m.

2. Some Basic Results

2.1. BLUE of the location parameter μ when the scale parameter is proportional to the location parameter

In this section we consider the family G of all absolutely continuous distributions which depends on a location parameter μ and a scale parameter $\sigma=c\mu$, where c is the known coefficient of variation. Then any distribution belongs to G has a pdf of the form

$$f(x;\mu,c\mu)=\frac{1}{c\mu}f_0\left(\frac{x-\mu}{c\mu}\right),\mu>0,x\in R. \tag{2}$$

Let $\boldsymbol{X}=(X_{1:m},X_{2:m},\ldots,X_{m:m})'$ be the vector of order statics of a random sample of size m drawn from (2). Define $Y_{r:m}=\dfrac{X_{r:m}-\mu}{c\mu}$, $r=1,2,\ldots,m.$ Then $Y_{r;m}$, $r=1,2,\ldots,m$ are distributed as the order statistics of a random sample of size m drawn from the standard form of (2) with pdf $f_0(y)$. Let $\boldsymbol{\alpha}=(\alpha_{1:m},\alpha_{2:m},\ldots,\alpha_{m:m})'$ and $\boldsymbol{V}=((v_{r,s:m}))$ be the vector of means and dispersion matrix of the vector of order statistics of a random sample of size m drawn from $f_0(y)$. Then the BLUE $\tilde{\mu}$ of μ based on order statistics is given by (see Sajeevkumar and Thomas 2005),

$$\tilde{\mu}=\frac{(c\boldsymbol{\alpha}+1)'\boldsymbol{V}^{-1}}{(c\boldsymbol{\alpha}+1)'\boldsymbol{V}^{-1}(c\boldsymbol{\alpha}+1)}\boldsymbol{X}, \tag{3}$$

and

$$Var(\tilde{\mu})=\frac{c^2\mu^2}{(\mathbf{c\alpha}+1)'\mathbf{V}^{-1}(\mathbf{c\alpha}+1)}. \tag{4}$$

2.2. U-statistics

Let $X_1,X_2,...,X_n$ be independent observations drawn from a population with cumulative distribution function $F(x;\theta)$. The U-statistic for the estimable parameter θ with the symmetric kernel $h(\cdot)$ of degree m is given by

$$U(X_1,X_2,\ldots,X_n)=\frac{1}{\binom{n}{m}}\sum_{\beta\in B}h\left(X_{\beta_1},X_{\beta_2},\ldots,X_{\beta_m}\right), \tag{5}$$

where $B=\{\beta\mid\beta=(\beta_1,\beta_2,\ldots,\beta_m),\beta_1<\beta_2<\ldots<\beta_m\}$ is one of the $\binom{n}{m}$ combinations of m integers chosen without replacement from the set $(1,2,\ldots,n)$. Suppose that,

$$E[h(X_1,X_2,\ldots X_m)]=\theta \text{ and } E[h^2(X_1,X_2,...X_m)]<\infty.$$

Let $h(X_1,X_2,\ldots,X_\omega,X_{\omega+1},\ldots,X_m)$ and $h(X_1,X_{2,}\ldots X_\omega,X_{m+1},\ldots,X_{2m-\omega})$ be two random variables having exactly ω sample observations in common, $\omega=1,2,\ldots,m.$ Let $\xi_\omega^{(m)}$ be the covariance between these two random variables. Then Hoeffding (1948) has obtained the variance of the U-statistics given in (5) as

$$Var[U(X_1,X_2,\ldots,X_n)]=\frac{1}{\binom{n}{m}}\sum_{\omega=1}^{m}\binom{m}{\omega}\binom{n-m}{m-\omega}\xi_\omega^{(m)}. \tag{6}$$

Clearly the U-statistic defined in (5) is an unbiased estimator of $\theta.$ For a detailed survey on the optimal properties of U-statistics, (see Serfling 1980).

3. Estimating the Location Parameter μ of a Distribution When the Scale Parameter is cμ by U-statistics

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from

$$f(x;\mu,c\mu) = \frac{1}{c\mu} f_0\left(\frac{x-\mu}{c\mu}\right), \mu > 0, x \in R.$$

Let the BLUE of μ as given in (3) can be written as

$$h_1(X_1, X_2, \ldots, X_m) = a_1 X_{1:m} + a_2 X_{2:m} + \ldots + a_m X_{m:m} \tag{7}$$

where $a_1, a_2, \ldots, a_m$ are constants. Now we can easily write

$$U_{1:n}^{(m)} = \frac{1}{\binom{n}{m}} \sum_{r=1}^{n} \left[\sum_{i=0}^{m-1} \binom{n-r}{m-1-i} \binom{r-1}{i} a_{i+1} \right] X_{r:n} \tag{8}$$

as the U-statistics for estimating μ based on kernel given in (7), where we define $\binom{r-1}{i} = 0$ for $i \geq r$ and $\binom{n-r}{m-1-i} = 0$ for $n-r < m-1-i.$ If we write

$$\xi_{\omega}^{(m)} = \operatorname{cov}\left[h_1(X_1, X_2, \ldots, X_{\omega}, X_{\omega+1}, \ldots, X_m), h_1(X_1, X_2, \ldots, X_{\omega}, X_{m+1}, \ldots, X_{2m-\omega})\right],$$

for $\omega = 1, 2, \ldots, m,$ then the variances of $U_{1:n}^{(m)}$ is given by

$$Var\left[U_{1:n}^{(m)}\right] = \frac{1}{\binom{n}{m}} \sum_{\omega=1}^{m} \binom{m}{\omega} \binom{n-m}{m-\omega} \xi_{\omega}^{(m)}, \tag{9}$$

where

$$\xi_m^{(m)} = Var\left[h_1(X_1, X_2, \ldots, X_m)\right]. \tag{10}$$

Clearly $\xi_m^{(m)}$ is equal to $Var(\tilde{\mu})$ given in (4).

We now explain the procedure of obtaining the values of $\xi_{\omega}^{(m)}, \omega = 1, 2, \ldots, m-1.$ If we put $n = m+k$ in (9), we have the following:

$$Var\left[U_{1:m+k}^{(m)}\right] = \frac{1}{\binom{m+k}{m}} \left[\binom{m}{m-k} \binom{k}{k} \xi_{m-k}^{(m)} + \binom{m}{m-k+1} \binom{k}{k-1} \xi_{m-k+1}^{(m)} + \ldots + \binom{m}{m} \binom{k}{0} \xi_m^{(m)} \right] \tag{11}$$

for $k = 1, 2, \ldots, m-1.$ The expression for $U_{1:m+k}^{(m)}$ for $k = 1, 2, \ldots, m-1.$ is given by (putting $n = m+k$ in (8)),

$$U_{1:m+k}^{(m)}=\frac{1}{\binom{m+k}{m}}\left[\sum_{i=0}^{m-1}\binom{m+k-1}{m-1-i}\binom{0}{i}a_{i+1}\right]X_{1:m+k}$$
$$+\frac{1}{\binom{m+k}{m}}\left[\sum_{i=0}^{m-1}\binom{m+k-2}{m-1-i}\binom{1}{i}a_{i+1}\right]X_{2:m+k}+\ldots \tag{12}$$
$$+\frac{1}{\binom{m+k}{m}}\left[\sum_{i=0}^{m-1}\binom{0}{m-1-i}\binom{m+k-1}{i}a_{i+1}\right]X_{m+k:m+k}.$$

The above expression can also written as

$$U_{1:m+k}^{(m)}=b'_{m+k}X_{m+k}, \tag{13}$$

where vector b_{m+k} is given by

$$b'_{m+k}=\left[\frac{\sum_{i=0}^{m-1}\binom{m+k-1}{m-1-i}\binom{0}{i}a_{i+1}}{\binom{m+k}{m}},\frac{\sum_{i=0}^{m-1}\binom{m+k-2}{m-1-i}\binom{1}{i}a_{i+1}}{\binom{m+k}{m}},\ldots,\frac{\sum_{i=0}^{m-1}\binom{0}{m-1-i}\binom{m+k-1}{i}a_{i+1}}{\binom{m+k}{m}}\right] \tag{14}$$

and

$$X_{m+k}=\left(X_{1:m+k},X_{2:m+k},\ldots,X_{m+k:m+k}\right)'. \tag{15}$$

Hence,

$$Var\left[U_{1:m+k}^{(m)}\right]=\left(b'_{m+k}V_{m+k}b_{m+k}\right)c^2\mu^2. \tag{16}$$

where V_{m+k} is the variance covariance matrix of the vector of order statistics of random sample of size $m+k$ drawn from the distribution with pdf $f(x;0,1)$.

Equations (11) and (16) are identically equal and consequently we can write

$$\binom{m}{m-k}\binom{k}{k}\xi_{m-k}^{(m)}+\binom{m}{m-k+1}\binom{k}{k-1}\xi_{m-k+1}^{(m)}+\ldots$$
$$+\binom{m}{m-1}\binom{k}{1}\xi_{m-1}^{(m)}=\binom{m+k}{m}\left(b'_{m+k}V_{m+k}b_{m+k}\right)c^2\mu^2-\xi_m^{(m)}, \tag{17}$$

$k=1,2,\ldots,m-1.$ The above system of equations can be written by the following matrix equation,

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \binom{m}{m-1}\binom{1}{1} \\ 0 & 0 & \cdots & \binom{m}{m-2}\binom{2}{2} & \binom{m}{m-1}\binom{2}{1} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \binom{m}{1}\binom{m-1}{m-1} & \binom{m}{2}\binom{m-1}{m-2} & \cdots & \binom{m}{m-2}\binom{m-1}{2} & \binom{m}{m-1}\binom{m-1}{1} \end{bmatrix} \times \begin{bmatrix} \xi_1^{(m)} \\ \xi_2^{(m)} \\ \vdots \\ \xi_{m-1}^{(m)} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{m-1} \end{bmatrix} \quad (18)$$

where $w_k = \binom{m+k}{m}\left(b'_{m+k}V_{m+k}b_{m+k}\right)c^2\mu^2 - \xi_m^{(m)}, k = 1,2,\ldots,m-1.$ If we write H to denote the coefficient matrix of the left side of (18) and W to denote the vector in the right side of (18), then we have, $\begin{bmatrix} \xi_1^{(m)} \\ \xi_2^{(m)} \\ \vdots \\ \xi_{m-1}^{(m)} \end{bmatrix} = H^{-1}W.$

Once we obtain the values of $\xi_\omega^{(m)}, \omega = 1,2,\ldots,m-1$ then the exact variances of the U-statistics for estimating the location parameter μ when scale parameter is proportional to the location parameter based on any sample of size n can be obtained by using (9) without any further direct evaluation of moments of order statistics.

The main advantage of this method is that if one uses the BLUE based on sample of size m as the kernel, then the evaluation of variances and covariances of order statistics of sample sizes up to $2m-1$ arising from (2) alone are necessary to obtain the explicit expression for the variance of the U-statistics $U_{1:n}^{(m)}$ whatever is the sample size.

4. Estimation of the Mean of a Logistic Distribution LD(μ,cμ) with Known Coefficient of Variation by U-statistics

We have evaluated the means, variances and covariances of the order statistics arising from standard form of the distribution defined in (1). Using these values and also using the results of Sajeevkumar and Thomas (2005) we have evaluated the coefficients of BLUE of μ and variances of the BLUE of μ for known values of c and are given by Table 1. Now using the coefficients of BLUE of μ, we have also evaluated the values of $\xi_\omega^{(m)}$ involved in (9) for $\omega = 1,2,\ldots,m-1; m = 2,3,4$ for the $LD(\mu, c\mu)$ distribution and are given in Table 2. Using the values of $\xi_\omega^{(m)}$ given in Table 2, we have obtained the variance of the U-statistic estimator of μ namely $U_{1:n}^{(m)}$ given in (8) of the $LD(\mu, c\mu)$ distribution for $m = 2,3,4$; $n = 5(5)20(10)40(20)100$ and are given in Table 3.

5. Comparison of the U-statistic Estimator with Moment Estimator

Let μ_1' be the first population raw moment of the logistic distribution $LD(\mu, c\mu)$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the first raw moment of the sample. Equate

$$\mu_1' = \bar{X}, \tag{19}$$

we get the unbiased moment estimator of μ, namely $\hat{\mu} = \bar{X}$, and $Var(\hat{\mu}) = Var(\bar{X})$, that is

$$Var(\hat{\mu}) = \frac{c^2\mu^2}{n}. \tag{20}$$

Using (20) we have evaluated the variance $Var(\hat{\mu})$ for $c = 0.20(0.05)0.30$ and for $n = 5(5)20(10)40(20)100$. The relative efficiency (RE) of $U_{1:n}^{(m)}$ relative to $\hat{\mu}$ is defined as $e\left(U_{1:n}^{(m)}|\hat{\mu}\right) = \frac{Var(\hat{\mu})}{Var\left(U_{1:n}^{(m)}\right)}$ and are also calculated for $m = 2,3,4$ and for $n = 5(5)20(10)40(20)100$. These values are given in table 3. From this table it is clear that all efficiencies are greater than one, that means $Var(\hat{\mu}) > Var\left(U_{1:n}^{(m)}\right)$. From this, it can be concluded that, estimator of μ based on U-statistics is much better than that of the moment estimator.

6. Asymptotic Relative Efficiency Study of U-statistic Estimator with Known Coefficient of Variation

Asymptotic variance of the U-statistic estimator $U_{1:n}^{(m)}$ is given by $\frac{m^2\xi_1^{(m)}}{n}$. Therefore the asymptotic relative efficiency (ARE) $E\left(U_{1:n}^{(m)}|\hat{\mu}\right)$ of $U_{1:n}^{(m)}$ relative to $\hat{\mu}$ is given by,

$$E\left(U_{1:n}^{(m)}|\hat{\mu}\right) = \lim\left[\frac{Var(\hat{\mu})}{Var\left(U_{1:n}^{(m)}\right)}\right] = \frac{c^2}{m^2\xi_1^{(m)}\mu^{-2}}. \tag{21}$$

Consequently the numerical values of $E\left(U_{1:n}^{(m)} \mid \hat{\mu}\right)$ are calculated for $m = 2(1)4, c = 0.20(0.05)0.3$ and are also given in the last row of Table 3. From this, it can be noted that efficiencies are greater than one. So it can be concluded that, U-statistic estimator is much better than that of the moment estimator even for large sample size.

Table 1 Coefficients of BLUE $\tilde{\mu}$ and $\mu^{-2}Var(\tilde{\mu})$

n	c	Coefficients				
		a_1	a_2	a_3	a_4	$\mu^{-2}Var(\tilde{\mu})$
2	0.20	0.34857	0.62135			0.01940
	0.25	0.30924	0.64454			0.02981
	0.30	0.27022	0.66456			0.04206
3	0.20	0.12678	0.48119	0.35439		0.01255
	0.25	0.09629	0.47121	0.37490		0.01920
	0.30	0.06674	0.45956	0.39280		0.02697
4	0.20	0.15156	0.29679	0.37844	0.23219	0.00924
	0.25	0.02833	0.28012	0.37988	0.24902	0.01412
	0.30	0.00609	0.26290	0.37940	0.26382	0.01979

Table 2 Values of $\xi_{\omega}^{(m)}$, for $\omega = 1, 2, ..., m$ and $m = 2(1)4$

m	ω	$c = 0.20$	$c = 0.25$	$c = 0.30$
		$\mu^{-2}\xi_{\omega}^{(m)}$	$\mu^{-2}\xi_{\omega}^{(m)}$	$\mu^{-2}\xi_{\omega}^{(m)}$
2	1	0.00957	0.01459	0.02042
	2	0.01940	0.02981	0.04206
3	1	0.00408	0.00618	0.00864
	2	0.00821	0.01254	0.01755
	3	0.01255	0.01920	0.02697
4	1	0.00226	0.00341	0.00475
	2	0.00451	0.00688	0.00961
	3	0.00685	0.01044	0.01461
	4	0.00924	0.01412	0.01979

Table 3 $Var(\hat{\mu})$, $Var\left(U_{1:n}^{(m)}\right)$, Relative Efficiency (RE) and ARE

m	n	$c = 0.20$		
		$\mu^{-2}Var(\hat{\mu})$	$\mu^{-2}Var\left(U_{1:n}^{(m)}\right)$	RE
	5	0.00800	0.00768	1.0416
	10	0.00400	0.00383	1.0443
	15	0.00267	0.00255	1.0470
	20	0.00200	0.00192	1.0416
2	30	0.00133	0.00128	1.0390
	40	0.00100	0.00096	1.0416
	60	0.00067	0.00064	1.0468
	80	0.00050	0.00048	1.0416
	100	0.00040	0.00038	1.0526
	∞	…	….	1.0449
	5	0.00800	0.00741	1.0796
	10	0.00400	0.00368	1.0869
	15	0.00267	0.00245	1.0898
	20	0.00200	0.00184	1.0869
3	30	0.00133	0.00123	1.0813
	40	0.00100	0.00092	1.0869
	60	0.00067	0.00061	1.0983
	80	0.00050	0.00046	1.0869
	100	0.00040	0.00037	1.0810
	∞	…	…	1.0893
	5	0.00800	0.00733	1.0914
	10	0.00400	0.00362	1.1049
	15	0.00267	0.00241	1.1078
	20	0.00200	0.00181	1.1049
4	30	0.00133	0.00120	1.1083
	40	0.00100	0.00090	1.1111
	60	0.00067	0.00060	1.1166
	80	0.00050	0.00045	1.1111
	100	0.00040	0.00036	1.1111
	∞	….	…	1.1061

Table 3 (Continued)

m	n	$c = 0.25$		
		$\mu^{-2}Var\left(\hat{\mu}\right)$	$\mu^{-2}Var\left(U_{1:n}^{(m)}\right)$	RE
2	5	0.01250	0.01174	1.0647
	10	0.00625	0.00585	1.0684
	15	0.00417	0.00390	1.0692
	20	0.00313	0.00292	1.0719
	30	0.00208	0.00195	1.0667
	40	0.00156	0.00146	1.0685
	60	0.00104	0.00097	1.0721
	80	0.00078	0.00073	1.0685
	100	0.00063	0.00058	1.0862
	∞	…	…	1.0709
3	5	0.01250	0.01130	1.1062
	10	0.00625	0.00560	1.1161
	15	0.00417	0.00372	1.1210
	20	0.00313	0.00279	1.1219
	30	0.00208	0.00186	1.1183
	40	0.00156	0.00139	1.1223
	60	0.00104	0.00093	1.1183
	80	0.00078	0.00070	1.1143
	100	0.00063	0.00056	1.1250
	∞	…	…	1.1237
4	5	0.01250	0.01118	1.1181
	10	0.00625	0.00551	1.1343
	15	0.00417	0.00366	1.1393
	20	0.00313	0.00274	1.1423
	30	0.00208	0.00182	1.1429
	40	0.00156	0.00137	1.1387
	60	0.00104	0.00091	1.1429
	80	0.00078	0.00068	1.1471
	100	0.00063	0.00055	1.1454
	∞	…	…	1.1455

Table 3 (Continued)

m	n	$c = 0.30$		
		$\mu^{-2}Var\left(\hat{\mu}\right)$	$\mu^{-2}Var\left(U_{1:n}^{(m)}\right)$	RE
2	5	0.01800	0.01646	1.0936
	10	0.00900	0.00820	1.0976
	15	0.00600	0.00546	1.0989
	20	0.00450	0.00409	1.1002
	30	0.00300	0.00273	1.0989
	40	0.00225	0.00204	1.1029
	60	0.00150	0.00136	1.1029
	80	0.00113	0.00102	1.1078
	100	0.00090	0.00082	1.0976
	∞	…	…	1.1019
3	5	0.01800	0.01582	1.1378
	10	0.00900	0.00783	1.1494
	15	0.00600	0.00521	1.1516
	20	0.00450	0.00390	1.1539
	30	0.00300	0.00260	1.1539
	40	0.00225	0.00195	1.1539
	60	0.00150	0.00130	1.1539
	80	0.00113	0.00097	1.1641
	100	0.00090	0.00078	1.1539
	∞	…	…	1.1574
4	5	0.00800	0.00733	1.0914
	10	0.00400	0.00362	1.1049
	15	0.00600	0.00511	1.1742
	20	0.00450	0.00382	1.1780
	30	0.00300	0.00254	1.1811
	40	0.00225	0.00191	1.1780
	60	0.00150	0.00127	1.1811
	80	0.00113	0.00095	1.1895
	100	0.00090	0.00076	1.1842
	∞	…	…	1.1842

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References

Arnholt AT, Hebert JL. Estimating the mean with known coefficient of variation. The Amer Statis. 1995; 49: 367-369.

Farsipour NS. Estimating the mean of inverse Gaussian distribution with known coefficient of variation under entropy loss. J Sci I.R. Iran. 1997; 8: 61-65.

Ghosh M, Razmpour A. Estimating the location parameter of an exponential distribution with known coefficient of variation. Cal Statist Assoc Bulle. 1982; 31: 137-150.

Gleser LJ, Healy JD. Estimating the mean of a normal distribution with known coefficient of variation. J Amer Statist Assoc. 1976; 71: 977-981.

Guo H, Pal N. On a normal mean with known coefficient of variation. Cal Statist Assoc Bull. 2003; 54: 17-29.

Hoeffding W. A class of statistics with asymptotically normal distributions. Ann Math Statist. 1948; 19: 293-325.

Hirano K, Iwase K. Conditional information for an inverse Gaussian distribution with known coefficient of variation. Ann. Inst. Statist. Math. 1989; 41: 279-287.

Irshad MR, Sajeevkumar NK. Estimating a parameter of the exponential distribution with known coefficient of variation by ranked set sampling. J. Kerala Statist. Assoc. 2011; 22: 41-52.

Kunte S. A note on consistent maximum likelihood estimation for $N(\theta,\theta^2)$ family. Cal Statist Assoc Bull. 2000; 50: 325-328.

Lloyd EH. Least squares estimation of location and scale parameter using order statistics. Biometrika. 1952; 39: 88-95.

Sajeevkumar NK, Thomas PY. Estimating the mean of logistic distribution with known coefficient of variation by order statistics. Recent Adv. In. Statist. The. and. Appli; ISPS Proceedings. 2005; 1:170-176.

Sajeevkumar NK, Irshad MR. Estimating the parameter μ of the exponential distribution with known coefficient of variation using censored sample by order statistics. IAPQR, Trans. 2011; 36: 155-169.

Sajeevkumar NK, Irshad MR. Estimation of the mean of double exponential distribution with known coefficient of variation by U-statistics. J Kerala Statist Assoc. 2012; 23: 61-71.

Samanta M. Estimation of the location parameter of an exponential distribution with known coefficient of variation. Comm Statist-The and Meth. 1984; 13: 1357-1364.

Serfling RJ. Approximation theorems of mathematical statistics. Wiley: New York; 1980.

Thomas PY, Sajeevkumar NK. Estimating the mean of normal distribution with known coefficient of variation by order statistics. J Kerala Statist Assoc. 2003; 14: 26-32.

Thomas PY, Sreekumar NV. Estimation of the scale parameter of generalized exponential distribution using order statistics. Cal Statist Assoc Bull. 2004; 55: 199-208.

Thomas PY, Sreekumar NV. Estimation of the location and scale parameters of a distribution by U-statistics based on best linear functions of order statistics. J Satist Plan Inf. 2008; 138: 2190-2200.