Thailand Statistician
July 2016; 14(2): 147-164
http://statassoc.or.th
Contributed paper

Heuristics for Two-Dimensional Rectangular Guillotine Cutting Stock

Kimseng Tieng [a], Supphakorn Sumetthapiwat [b], Aussadavut Dumrongsiri [a] and Chawalit Jeenanunta* [a]
[a] Sirindhorn International Institute of Technology, Thammasat University, Pathumthani, Thailand.
[b] Department of Mathematics and Computer Science, Faculty of Science, Chulalongkorn University, Bangkok, Thailand.
*Corresponding author; e-mail: chawalit@siit.tu.ac.th

Received: 14 August 2015
Accepted: 28 December 2015

Abstract

Two-Dimensional Rectangular Guillotine Cutting Stock Problem (2DRGCSP) is one of the most significant problems in the manufacturing industries. A set of small rectangular size of paper, aluminum rolls, glasses, fiber glasses, or plastic are required to cut from a set of big size rectangular sheet of its raw materials. A guillotine cut is used, where the sheet is cut from one side to another side without changing the direction of the blade to produce the strips. Then each strip is cut again to produce the small rectangular size of panels that match with the required size, and the number of the panels must satisfy with the demand. This paper presents the heuristic techniques to solve the cutting problems. The heuristic techniques create the good cutting patterns such that the waste of the sheets is minimized and demands for each panel are satisfied. There are five proposed heuristics, developed to solve this problem. They are 2D simple heuristic cutting (2DSHC), 2D horizontal construction (2DHC), 2D vertical construction (2DVC), 2D horizontal improvement (2DHI), and 2D vertical improvement (2DVI). The testing instances are created from the real problems in the Printed Circuit Board (PCB) Company. The results are presented and compared with column generation (CG) method. The proposed heuristics provide good cutting patterns with a comparable waste to the column generation method. Moreover, the heuristics can produce solutions in a short computational time even in the large size instances, where the column generation method cannot find the solution. Therefore, these proposed heuristics techniques are applicable to solve the 2DRGCSP in the industry where the solutions from the exact algorithms cannot be found.

Keywords: Heuristics, 2D guillotine cutting.

1. Introduction

Two-Dimensional Rectangular Guillotine Cutting Stock Problem (2DRGCSP) is one of the significant problems, facing by manufacturing companies that produce products such as paper, aluminum rolls, glass, fiberglass plated, and printed circuit board (PCB). A set of small rectangle panels are required to cut from a set of large rectangular plates to match with the order quantities. The objective of cutting is to minimize the waste, trim loss, or cost, or maximize the plate area utilization. There are different methodologies that have been proposed by different researchers. Among of those methods, heuristic is another attractive method that have been proposed to solve this problem when the size of the instance is large and cannot be solved with an exact algorithm. The heuristic method does not guarantee the optimal solution for the created patterns, but it can solve a large instance in a reasonable computational time.

2D rectangular guillotine cutting can be classified into various cutting styles namely: 2D 2stage 1group, 2D 2stage 2group, and t-shape 3group, etc. Guillotine is a process of cutting, where the sheet is cut from one side to the other side without changing the direction. Group refers to the number of categories after the sheet cutting. Strip is a result of group cutting, where each strip can contain only one floor or one level of the panel. Many strips can be produced from each group. Stage refers to the number of times that a sheet is rotated to cut to produce the exact size of each panel type. In this paper, guillotine cutting is considered, and each group is cut, using 2D 2stage to produce many strips. Each strip is cut again to produce the panels. If there is some waste connected to the panel, one more step of cutting is required to get the exact size of the panels, but it is not counted as stage.

2D 2stage 1group cutting as shown in Figure 1(a), a parallel longitudinal guillotine cutting is applied to the sheet, which is regarded as group to produce the strip in the first stage. Then each strip is positioned to cut in parallel transversal guillotine cutting to produce the panels for the second stage. Figure 1(b) illustrated 2D 2stage 2group cutting. The sheet is cut into two sub-groups. Each sub-group is applied the same process of cutting as mentioned in 2D 2stage 1group cutting. Figure 1(c) shows t-shape 3group cutting. It is also one of the effective methods, where big sheet is cut vertically to produce two sub-groups first. Then one of the sub-groups is cut again to make other two sub-groups. Three sub-groups have been produced in total. Each sub-group is applied the same process of 2D 2stage 1group cutting method to get the exact size of the panels. This paper presents the heuristic algorithm to solve 2DRGCSP.

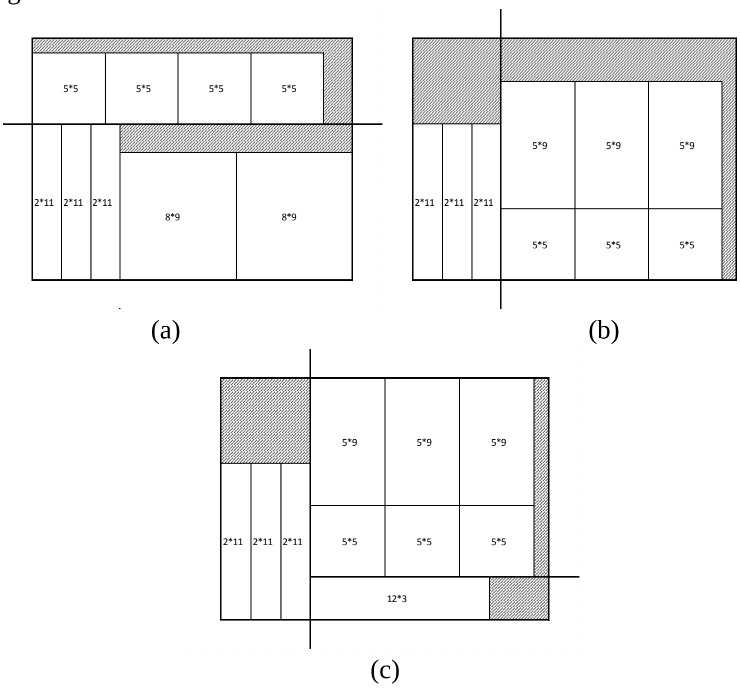


Figure 1 Different types of guillotine cutting methods (a) 2D 2stage 1 group cutting, (b) 2D 2stage 2group cutting, and (c) t-shape 3group cutting

The overall structure of this paper is organized as in the following. Section 2 is the literature review for cutting. Then problem description is presented in Section 3. Next in Section 4 is the proposed algorithm, which consists of six proposed techniques. Testing instances are explained in Section 5. Then Section 6 is the experimental result and discussion of those techniques. Finally, main conclusion of this work is drawn in Section 7.

2. Literature Review

Cutting stock is a widely recognized problem in operations research for both one-dimensional cutting and 2D cutting. It has been extensively treated in different literatures by different researchers to find the best patterns to cut. Gilmore and Gomory (1965) proposed a linear programming (LP) model by using knapsack problem to deal with 2D cutting stock problem. This technique generates a number of iterations to find the prominent cutting patterns such that it can maximize the material utilization. Lodi and Monaci (2003) proposed two mixed integer linear programming models for 2D 2stage knapsack problem, where the objective is to maximize number of panels used and the sum of the profits of the cut panels. Yanasse and Morabito (2008) have proposed a note on integer linear programming models to generate the pattern in 2group and 3group constrained and unconstrained 2D guillotine cutting stock. However, after implementing these models into GAMS modelling and CPLEX solver, these models are only effective and efficient to solve problems in small and moderate size.

Andrade and Birgin (2013) have proposed mix integer programming for solving the 2D 2stage guillotine cutting problems with usable leftover to get the optimal solution. Cui and Liu (2007) have solved T-shape homogenous block patterns for the 2D cutting problem by using dynamic programming recursion. The dynamic programming is to generate the optimal block, where the vertical cut divides the stock sheet into two segments, and each segment consists of panels that have the same length and direction or homogenous segments. A homogenous block consists of homogenous strips of the same panels' type. The computational results indicate that the algorithm is efficient in improving material utilization, and the computational time is reasonable. Cui (2012) later has proposed a 3stage patterns to solve the same problem by using the same method. He divides the panels into 3stage. Firstly, a vertical cutting divides the sheet into segments. Secondly, a horizontal cutting divides the segments into strips, and finally a vertical cutting divides the strips into panels.

Cui (2012) and Cerqueira and Yanasse (2009) have proposed 1-D Cutting stock by using a heuristic method to reduce the computational time and determine the group of patterns and its frequencies. Cui (2012) has proposed a fast heuristics for constrained homogenous T-shape (HTS) cutting problems. Cui (2004) has proposed generating optimal T-shape cutting pattern for rectangular blanks to cut rectangular pieces from the stock plate. For homogenous T-shape cutting, the plate is cut into homogenous strips at the first phase where these is only one panel type and one panel size in that strip, and the strips are cut into pieces at the second phase. Cui and Huang (2012) have implemented a heuristic for Constrained T-shape cutting patterns of rectangular pieces, where the objective function is to balance the cost and its complexity. Yanasse, Zinober (1991) proposed heuristic algorithm by using a pattern-building procedure, combined with an enumeration scheme to mix the boards. Suliman (2006) proposed a 3 stage sequential heuristic procedure for the 2D rectangular guillotine cutting stock problem. First, a width cutting pattern is determined. This width cutting pattern can produce the minimum width trim loss. Second, it determines the table length or sheet height and the associated layout of the pieces length or panels height to produce a good cutting pattern by minimize the trim loss of the length (height). Finally, a number of times, needed to cut for each generated pattern, is determined.

Alvarez-Valdes, Parajon (2002) proposed a computational study of LP-based heuristic algorithms for 2D guillotine cutting stock problems. They implemented three heuristic algorithms called a constructive algorithm, a GRASP algorithm, and a Tabu Search algorithm to solve the sub problem in the column generation technique. Yanasse and Limeira (2006) proposed a hybrid heuristic to reduce the number of different patterns in cutting stock problem such that a specific amount of patterns is generated. These patterns are repeatedly cut as much as possible but without overproducing any of the panels. They tradeoff between the waste and number of patterns. Cui, Yang (2013) have proposed sequential grouping heuristics to solve 2D cutting stock with pattern reduction. The objectives of this sequential heuristic are input minimization (main objective) and pattern reduction (auxiliary objective). The results indicate that a sequential grouping heuristic is effective and efficient in input minimization, pattern reduction, and computational time reduction. Cui and Zhao (2013) also have proposed heuristic for the rectangular 2D single stock size cutting stock problem with 2 stage patterns. They use column generation method to solve the residual problems repeatedly until the demands are satisfied. The computational results indicate that their algorithm can solve most instances to optimality. It is more efficient on average in reducing the number of plates used than a published algorithm and a commercial cutting stock software package.

3. Problem Description

The panels and sheets are grouped based on material types, thickness and copper. The mathematical notations in this problem are given is in the following. Given a set of panel $P=\{p_1,p_2,...,p_j\}$ along with its width set $w=\left\{w_1,w_2,...,w_j\right\},$ its height set $h=\left\{h_1,h_2,...,h_j\right\},$ and its demand set $d=\left\{d_1,d_2,...,d_j\right\}$ where $j=1,2,3,...,n.$ These panels are cut from a set of sheets $S=\{s_1,s_2,...,s_i\}$ that has its width set $W=\left\{W_1,W_2,...,W_i\right\},$ and its height set $H=\left\{H_1,H_2,...,H_i\right\},$ where $i=1,2,...,m.$ Sheet capacity is assumed to be unlimited. The demand's satisfaction for each panel is needed. The rotation of the sheets and the panels are not allowed. Lastly, only guillotine cutting is considered. Each size of panel must be less than or equal to each size of sheet. Over cut panels are regarded as waste. The objective of this paper is to look for the best patterns such that the total waste is minimized.

4. Proposed Algorithms

In this section, various techniques are presented. They are column generation (CG) technique, 2D simple heuristic cutting (2DSHC) technique, 2D horizontal cutting (2DHC) technique, 2D vertical cutting (2DVC) technique, 2D horizontal improvement (2DHI) technique, and 2D vertical improvement (2DVI) technique.

4.1. Column generation (CG)

Column generation technique is used to solve a problem with many decision variables and combinatorial problem. This technique is based on Danzig-Wolfe decomposition. Let $S'_i(i=1,...,\text{o})$ are a family of all feasible cutting pattern of sheet $i.$ The decision variable $x_p(p\in S'_i)\theta$ denote the number of times when the cutting pattern p is used in the solution. We define the waste from cutting pattern p by $A_p.$ In constraints (2), the coefficients C_p^j represent the number of panel $j(j=1,...,n)$ in the cutting pattern p and d_j represent the requirement of panel j. Z^+ denote an integer positive number.

Minimize $$\sum_{i=1}^{o}\sum_{s'_i\in S'_i} A_{s'_i} x_{s'_i} \tag{1}$$

Subject to

$$\sum_{i=1}^{o}\sum_{s'_i\in S'_i} C^j_{s'_i} x_{s'_i} = d_j ;\, j=1,...,n \tag{2}$$

$$x_{s'_i} \in Z^+ \quad ;\, i=1,...,o \;\; ,\, s'_i \in S'_i\,. \tag{3}$$

The above model is called master problem (MP). The objective function (1) is to minimize the total waste of the selected sheets. The constraints (2) ensure that the number of panel j must equal to the requirement. Finally, the constraints (3) are the integrality constraints.

In fact, we cannot generate all of feasible patterns for the large-size instances. A Restricted master problem (RMP) consists of a subset of patterns in Master problem. To obtain the optimal solution, we generated a sub-problem (for each sheet i), using the dual solution from the current solution in RMP. The sub-problem deals with two-dimensional two-staged knapsack problems with guillotine cuts (2DKP) that a mathematical model was proposed by Lodi and Monaci (2003). Therefore, solving this problem can give a pattern with the most negative reduced cost to prove the optimality. We show the model which involves integer variables x_{jk}, denoting the number of panels of type $j\,(j=1,...,n)$ in shelf $k(k=1,...,\alpha_j)$ and $q_k\,(k=1,...,\bar{n})$ denoting whether a shelf k is used (where $\bar{n}$ is the number of panels and $\alpha_j = \sum_{s'\leq j} d_s$).

Let π^*_j be a dual solution from the current optimal solution in RMP associated with panel j. The mathematical model for each sheet type is shown below:

Maximize $$\sum_{j=1}^{n} \pi^*{}_j (\sum_{k=1}^{\alpha_j} x_{jk} + \sum_{k=\alpha_{j-1}+1}^{\alpha_j} q_k) \tag{4}$$

subject to:

$$\sum_{k=1}^{\alpha_j} x_{jk} + \sum_{k=\alpha_{j-1}+1}^{\alpha_j} q_k \;\leq ub_j ;\, j=1,...,n \tag{5}$$

$$\sum_{j=\beta_k}^{n} \bar{w}_j x_{jk} \leq (W-\bar{w}_{\beta_k}) q_k ;\, k=1,...,\bar{n} \tag{6}$$

$$\sum_{k=1}^{\bar{n}} \bar{l}_{\beta_k} q_k \leq L \tag{7}$$

$$\sum_{s=k}^{\alpha_j} x_{js} \leq ub_j - (k-\alpha_{j-1});\, j=1,...,n;\, k\in[\alpha_{j-1}+1, \alpha_j] \tag{8}$$

$$0 \leq x_{jk} \leq d_j ;\, x_{jk} \in \text{integer} ;\, j=1,...,n;\, k\in[1,\alpha_j] \tag{9}$$

$$q_k \in \{0,1\};\, k=1,...,\bar{n}. \tag{10}$$

The objective function (4) is to maximize the sum of the cost of panel in pattern. Inequalities (5), (6), and (7) which impose the cardinality constraints, the width constraints, and the height constraint, respectively. An inequality (8) is to strengthen the bound on the x_{jk} variables (given by inequalities (9). If the feasible pattern with maximum profit, greater than zero is found, the column corresponding to this pattern is added to the current RMP. When no more feasible pattern with maximum profit greater than zero is found, that mean, the current optimal solution of RMP is the optimal solution of MP.

4.2. 2D simple heuristic cutting (2DSHC)

2DSHC is the simple heuristic technique that has been applied by the cutting company. In this heuristic, each sheet contains only one panel type, and the sheet with minimum waste will be selected. The algorithmic process of 2DSHC is implemented as follows:

Step 1: Given a set of sheets $S=\{s_1,s_2,...,s_i\}$ and a set of panels $P=\{p_1,p_2,...,p_j\}$.

Step 2: For each $p_j\in P,$ select $s_i\in S$ that has minimum waste to cut, where p_j is put in sheet s_i

Step 3: For each $p_j\in P,$ number of the sheet need to cut equal to the ceiling of the ratio between the demands and the number of panels used within the sheet

Step 4: Remove $p_j\in P$ that has been cut.

Step 5: If $P=\{0\}$, terminate cutting process; otherwise, go to Step 2.

For instance, given a set of panel $P=\{p_1,p_2,p_3,p_4\}$, coordinate with $w=\{2,8,5,4\}$, $h=\{11,9,5,4\}$, and $d=\{14,10,20,26\}$. These panels are cut from a sheet set $S=\{s_1,s_2\}$ coordinate with $H=\{30,17\}$ and $W=\{14,22\}$. For each panel type, selects the sheet that can give minimum waste. The number of sheet, needed to cut for each panel type in each pattern as illustrated in Figure 2 are 1, 4, 2, and 2, respectively. The total waste after fulfill all the demands are 1652 in^2. Note that the over cut of $p_2(8,9)$ are 2 panels, $p_3(5,5)$ are 4 panels, and $p_4(4,4)$ are 6 panels. They are considering as waste.

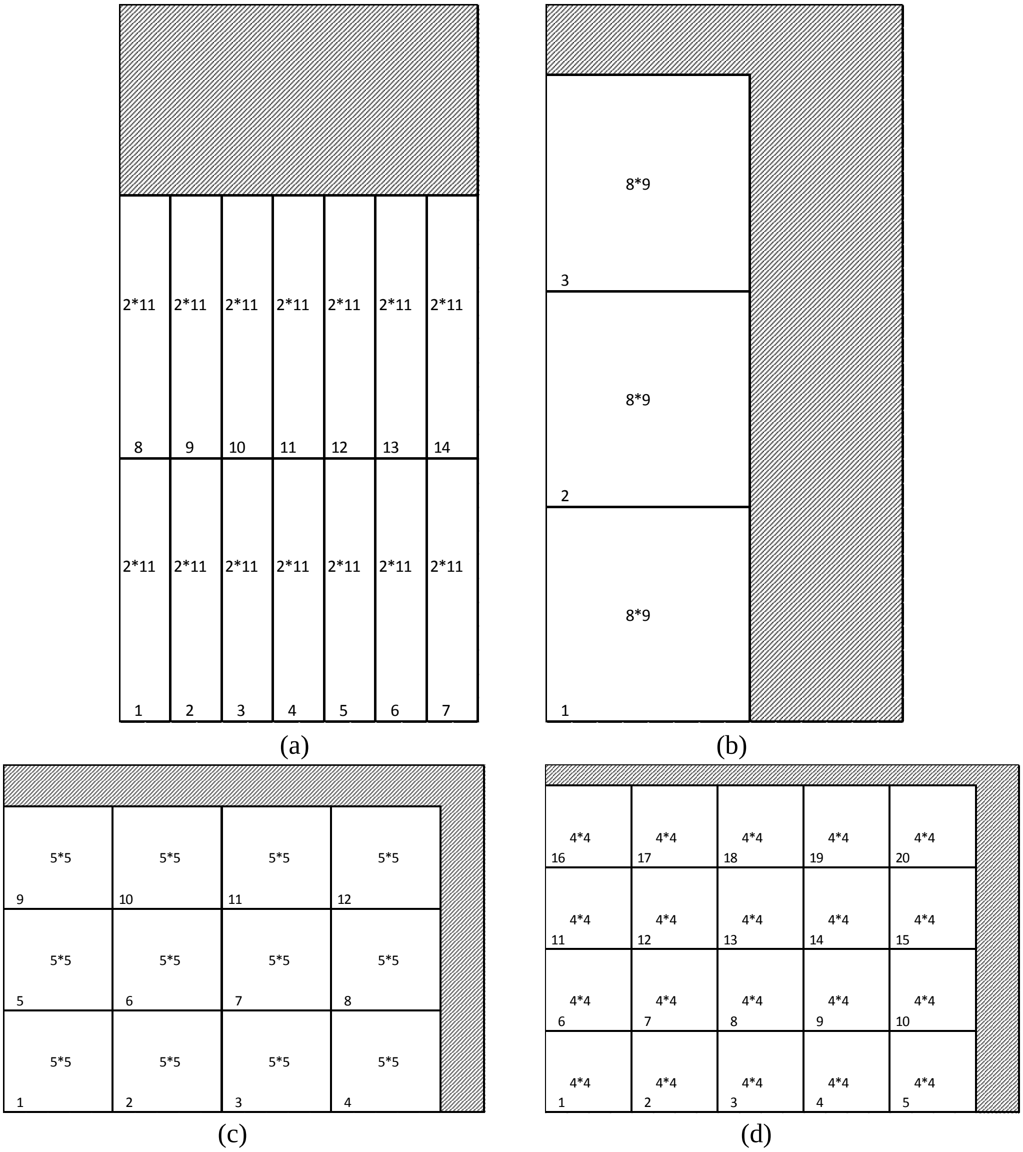
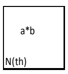
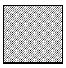


Figure 2 The example solution of 2DSHC



= panel size $(a \times b)$ put on N[th] time into the sheet


= the waste of the sheet

4.3. 2D horizontal construction (2DHC)

In this heuristic technique, the panels are arranged in the sheet to be cut horizontally. Multiple types of panels are allowed to put in each size of sheet. The process of this method is implemented as in the following.

Step 1: Given a set of sheets $S = \{s_1, s_2, ..., s_i\}$ and a set of panels $P = \{p_1, p_2, ..., p_j\}$. Then sort these two sets in descending order based on height.

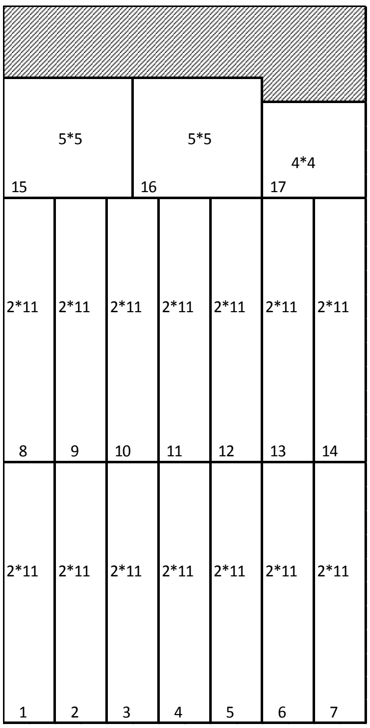
Step 2: P' is the set of panel where $p'_j \in P'$, such that the number of p'_j with the same width and height is equal to the demand d_j of that p_j with the same width and height. Thus, $|P'| = \sum_{j=1}^{n} d_j$.

Step 3: Each $p'_j \in P'$ is selected to put into the space next to the previous panel on the same strip $s_i \in S$ in horizontal line until no more panels types can be inserted in that strip, then move to next strip to cut horizontally. If there are not any strips available, move to the new sheet.

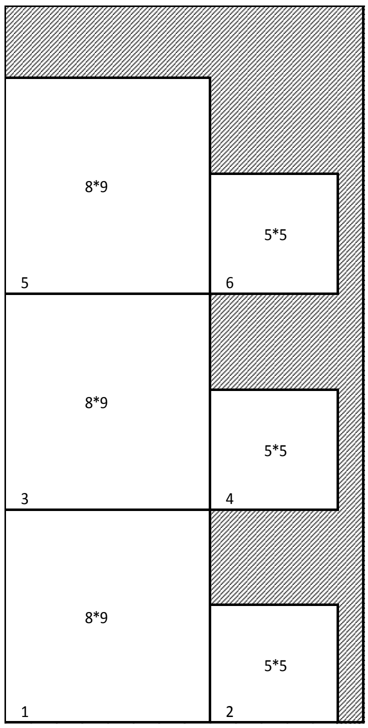
Step 4: Remove $p'_j \in P'$ that has been used.

Step 5: If $P' = \{0\}$, terminate the cutting process; otherwise, go to Step 3

From the same instance, mentioned in Section 4.2, the sheets and the panels are sorted in descending order based on height. Then each panel P' is picked to put into the sheet in the order. Panel $P_1(2\times 11)$ is put at the bottom left corner side of the first sheet $S_1(14\times 30)$ in the first level. Then, $P_2(2\times 11)$ is put next to P_1 horizontally. This process is repeated until no space to place any panels in the same level. Then we move to the next level of the same sheet. If the leftover area of the sheet cannot be put any panels, a new sheet is picked. These processes are repeated until all the demands are fulfilled. The amount of sheets, needed to cut for each pattern as illustrated in Figure 3, are 1, 3, 1, and 1, respectively. The total waste after fulfill all the demands are 576 in^2. Note that there is no over cut panels.



(a)



(b)

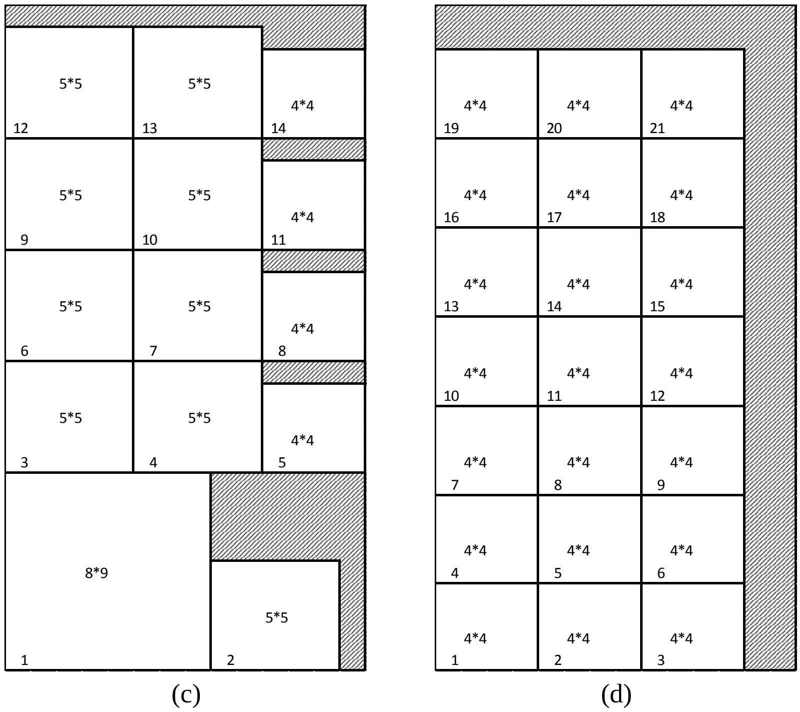


Figure 3 The example solution of 2DHC

4.4. 2D vertical construction (2DVC)

For this heuristic, the panels are arranged in a sheet to be cut vertically. Multiple types of panels are allowed to put in each size of sheet. The process of this method is implemented as follows:

Step 1: Given a set of sheets $S = \{s_1, s_2, ..., s_i\}$ and a set of panels $P = \{p_1, p_2, ..., p_j\}$. Then sort the panel set based on width and sheet set based height in descending order.

Step 2: P' is the set of panel where $p'_j \in P',$ such that the number of p'_j with the same width and height is equal to the demand d_j of that p_j with the same width and height. Thus, $|P'| = \sum_{j=1}^{n} d_j.$

Step 3: Each $p'_j \in P'$ is selected to put into the space next to the previous panel on the same strip $s_i \in S$ in horizontal line until no more panels types can be inserted in that strip, then move to next strip to cut horizontally. If there are not any strips available, move to the new sheet.

Step 4: Remove $p'_j \in P'$ that has been used.

Step 5: If $P' = \{0\},$ terminate the cutting process; otherwise, go to Step 3

From the same instance, mentioned Section 4.2, the sheets and the panels are sorted in descending order based on height. Then each panel P' is picked to put into the sheet in the order. Panel $P_1(8\times9)$ is put at the bottom left corner side of the first sheet $S_1(14\times30)$ in the first level. Then $P_2(8\times9)$ is put on P_1 vertically. This process is repeated until there is no space to place any panels in the same level. Then we move to the next level which located on right hand side of the first level. If we cannot put any panels into the leftover area of the same sheet, a new sheet is picked. These processes are repeated until all of the demands are fulfilled. The amount of sheet, needed to cut for each pattern as illustrated in Figure 4, are 3, 1, 1, and 1, respectively. The total waste after fulfill all the demands are 576 in^2. There is no over cut panels.

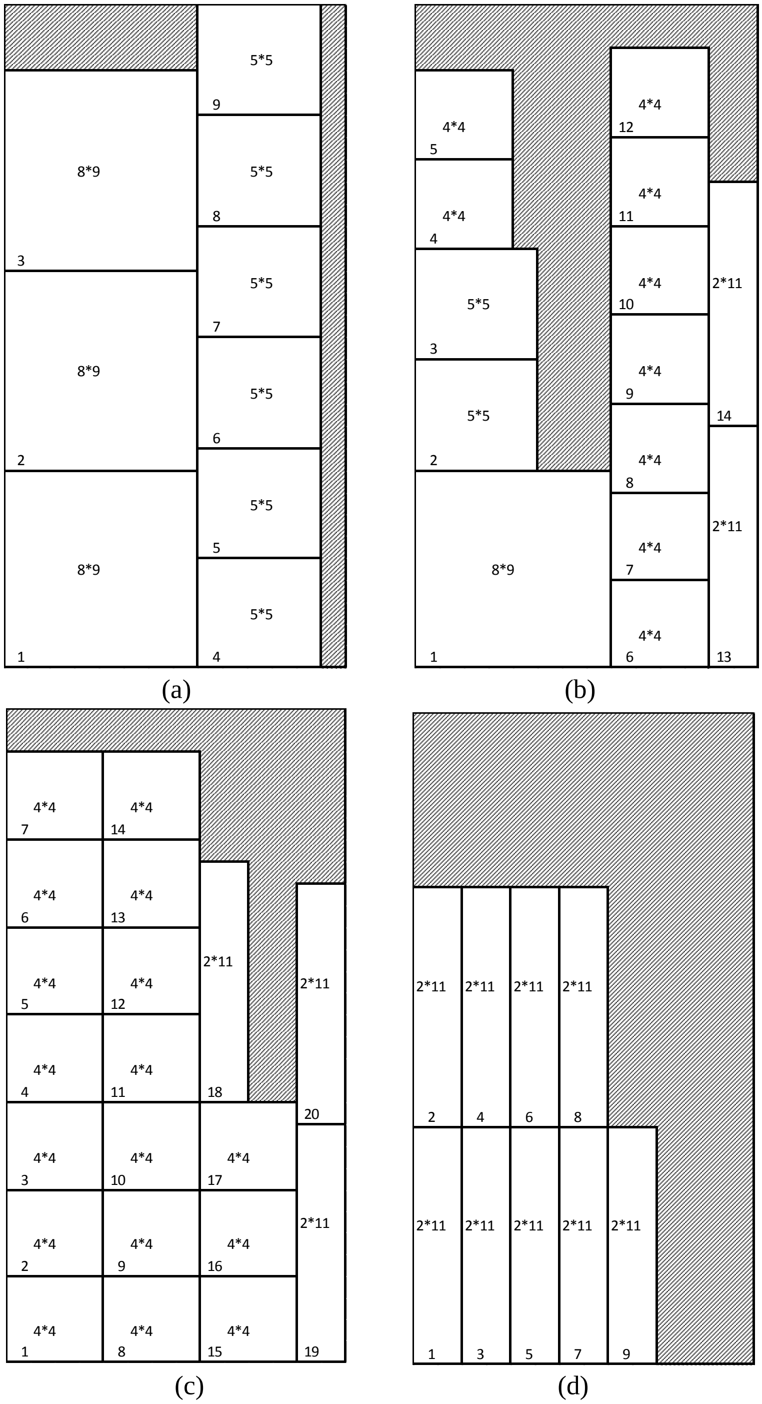


Figure 4 The example solution of 2DVC

4.5. 2D horizontal improvement (2DHI)

2DHI concept comes from a combination of 2DSHC and 2DHC. 2DSHC technique allows to put only one panel size into each sheet, and the sheet with the minimum waste is selected to cut. 2DHC technique allows to pick the panel to put horizontally into the sheet, and multiple sizes of panels are allowed to put into the same sheet. Therefore, in 2DHI technique, the panels are arranged to cut horizontally. Multiple sizes of panels are allowed to put in all each size of sheets. Each size of panels, located in the first panel set, is selected to put into each size of sheets first until no more panels in that size can fill up. Then the leftover area is tried to insert the other panel sizes. The pattern of the sheets with a minimum waste is selected to cut. The process of this technique is organized as in the following.

Step 1: Given a set of sheets $S = \{s_1, s_2, ..., s_i\}$ and a set of panels $P = \{p_1, p_2, ..., p_j\}$. Then sort the panel set in descending order based on height. P' is also the set of panel where $p'_j \in P'$, such that the number of p'_j with the same width and height is equal to the demand d_j with the same width and height. Thus, $|P'| = \sum_{j=1}^{n} d_j$.

Step 2: For each $p'_j \in P'$, put p'_j in each sheet s_i as much as possible in horizontal line. Once the horizontal strip is filled up, move to the next strip vertically until there is no more space to put p'_j.

Step 3: For each $s_i \in S$ used in Step 2, select the next $p'_j \in P'$ to put into the leftover area. Put p'_j in sheet s_i as much as possible in horizontal line. Once the horizontal strip is filled up, move to the next strip vertically until there is no space to put p'_j. Repeat this step for the next p'_j.

Step 4: Select $s_i \in S$, used in step 3 that can give minimum waste to cut.

Step 5: Number of the sheet need to cut is equal to the ceiling of the smallest ratio between the demands and the number of each panel type, used in the pattern

Step 6: Remove $p'_j \in P'$ that has been used.

Step 7: If $P' = \{0\}$, terminate the cutting process; otherwise, go to step 3

From the same instance, mentioned in section 4.2, the panels are sorted in descending order based on height. Then each panel P' is picked to put into each sheet. Panel $P_1(2\times11)$ is put at the bottom left corner side of the first sheet $S_1(14\times30)$ in the first level. Then $P_2(2\times11)$ is put close to P_1 horizontally. This process is repeated until there is no space to place any panels in the first level. Then we move to the next level which located on the first level. If the leftover area of the sheet $S_1(14\times30)$ cannot put any panels, sheet $S_2(22\times17)$ is used. The process for sheet $S_2(22\times17)$ is implemented the same as sheet S_1. The patterns in sheet $S_1(14\times30)$ and $S_2(22\times17)$ are given. In this sample, only the pattern in sheet $S_2(22\times17)$ is kept since this pattern can provide a minimum waste. New patterns are required to find until all the demands are fulfilled. The amount of sheets, needed to cut for each pattern as illustrated in Figure 5, are 2, 3, 2, and 1, respectively. The total waste after fulfill all the demands are 1048 in^2. Note that the over cut of $p_1(2,11)$ are 8 panels, $p_3(5,5)$ are 3 panels, and $p_4(4,4)$ are 16 panels. They are considering as waste.

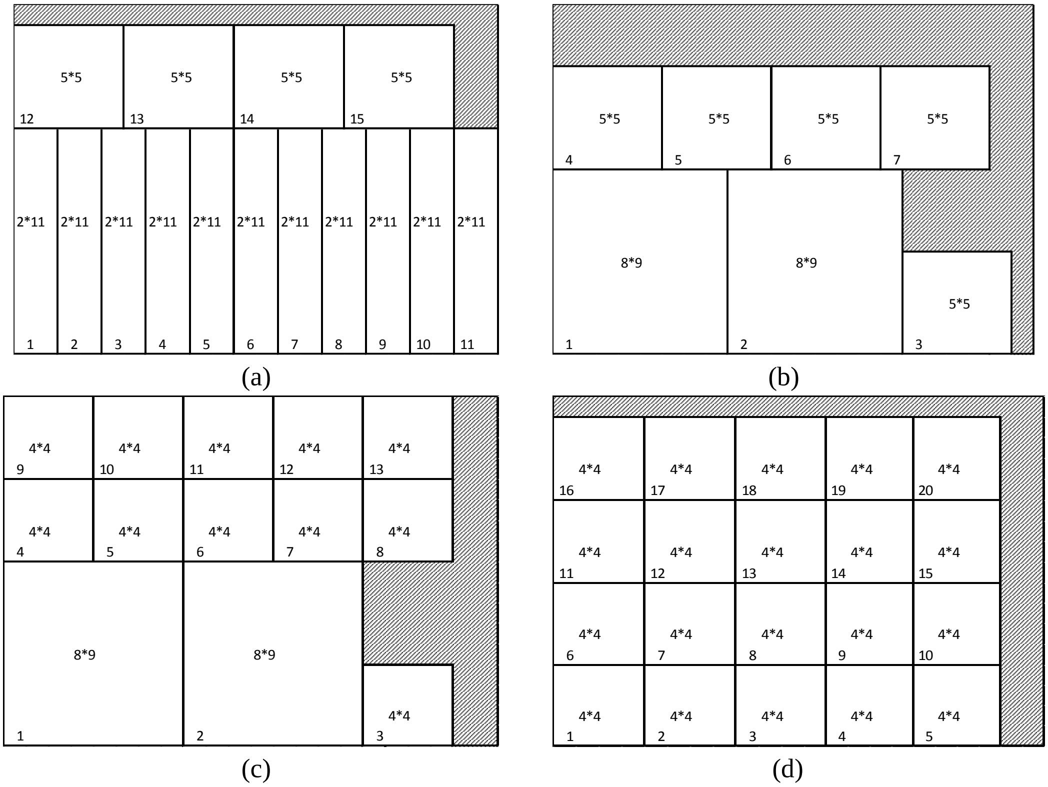


Figure 5 The example solution of 2DHI

4.6. 2D vertical improvement (2DVI)

2DVI comes from a combination of 2DSHC and 2DVC. Noted, first, 2DSHC technique is allowed to put only one panel size into each type of sheets, and the sheet with the minimum waste is selected to cut. Second, 2DVC technique is allowed to pick the panel to put vertically into the sheet in sheet set, and multiple sizes of panels are allowed to put into the sheet. Therefore, in 2DVI technique, the panels are arranged to cut vertically. Multiple sizes of panels are allowed to put in all each size of sheets. Each size of panels, located in the first panel set, is selected to put into each size of sheets first until no more panels in that size can fill up. Then the leftover area is tried to insert the other panel sizes. The pattern of the sheets with a minimum waste is selected to cut. The process of this technique is organized as in the following.

Step 1: Given a set of sheets $S = \{s_1, s_2, ..., s_i\}$ and a set of panels $P = \{p_1, p_2, ..., p_j\}$. Then sort the panel set in descending order based on width. P' is also the set of panel where $p'_j \in P'$, such that the number of p'_j with the same width and height is equal to the demand d_j with the same width and height. Thus, $|P'| = \sum_{j=1}^{n} d_j$.

Step 2: For each $p'_j \in P'$, put p'_j in each sheet s_i as much as possible in vertical line. Once the vertical strip is filled up, move to the next strip horizontally until there is no more space to put p'_j.

Step 3: For each $s_i \in S$ used in step 2, select the next $p'_j \in P'$ to put into the leftover area. Put p'_j in sheet s_i as much as possible in vertical line. Once the vertical strip is filled up, move to the next strip horizontally until there is no space to put p'_j. Repeat this step for the next p'_j.

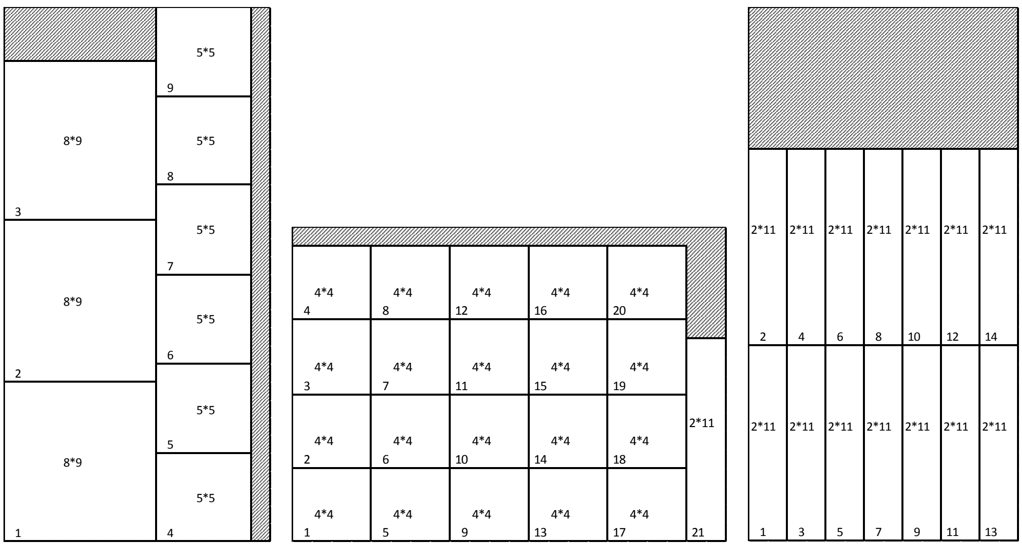
Step 4: Select $s_i \in S$ are used in step 3 that can give minimum waste to cut.

Step 5: Number of the sheet need to cut is equal to the ceiling of the smallest ratio between the demands and the number of each panel type, used in the pattern.

Step 6: Remove $p'_j \in P'$ that has been used.

Step 7: If $P' = \{0\}$, terminate the cutting process; otherwise, go to Step 3

From the same instance, mentioned Section 4.2, the panels are sorted in descending order based on width. Then each panel P' is picked to put into each sheet. Panel $P_1(8\times 9)$ is put at the bottom left corner side of the first sheet $S_1(14\times 30)$ in the first level. Then, $P_2(8\times 9)$ is put on P_1 vertically. This process is repeated until there is no space to place any panels in the first level. Then we move to the next level, which located on right hand side of the first level. If the leftover area of the sheet $S_1(14\times 30)$ cannot put any panels, sheet $S_2(22\times 17)$ is used. The process for sheet $S_2(22\times 17)$ is implemented the same as sheet S_1. The patterns in sheet $S_1(14\times 30)$ and $S_2(22\times 17)$ are given. In this sample, only the pattern in sheet $S_1(14\times 30)$ is kept since this pattern can provide a minimum waste. New patterns are required to find until all the demands are fulfilled. The amount of sheet, needed to cut for each pattern as illustrated in Figure **6** 6, are 4, 2, and 1, respectively. The total waste after fulfill all the demands are 904 in^2. Note that the over cut of $p_1(2,11)$ are 2 panels, $p_2(8,9)$ are 2 panels, $p_3(5,5)$ are 4 panels, and $p_4(4,4)$ are 14 panels. They are considering as waste.



(a) (b) (c)

Figure 6 The example solution of 2DVI

5. Testing Instances

Table 1 illustrated a list problem size of 20 instances, obtained from the Printed Circuit Board (PCB) Company. These instances are used to test with all the techniques, mentioned in section 4. These data are separated into 3 categories based on the total amount of sheets and panels. The problem size is regarded as a small size instance, medium size instance, and large size instance, when the total amount of sheet types and panel types are less than 6, between 7 and 20, and more than 20, respectively. The computational time and the total waste are compared to evaluate the efficient and effective of each technique.

Table 1 The input for small, medium, and large size instances

Size	No	#Sheet type	#Panel type	#Demand
Small Size	1	2	2	219
	2	2	2	301
	3	2	2	575
	4	2	3	537
	5	3	3	552
	6	4	2	300
	7	5	1	209
Medium Size	8	5	6	1,492
	9	5	8	2,240
	10	6	2	1,329
	11	6	3	219
	12	8	7	596
	13	9	3	2,192
	14	9	9	3,068
	15	11	4	4,299
	16	11	5	748
Large Size	17	15	12	4,429
	18	16	6	1,121
	19	17	27	10,611
	20	26	15	3,790

6. Experimental Result and Discussion

To evaluate the performance of these techniques, twenty different size of instances are run on Intel® Core™ i5-4200U CPU @ 1.60GHz 2.30GHz, installed memory (RAM) 4.00GB machine running under the Windows environment. CG technique is coded in IBM ILOG CPLEX Optimization Studio (64bit) 12.6, and the rest of the techniques are implemented on JAVA 7. All types of techniques are used to compare with the CG technique as illustrated in Table 2, where the gap is given by:

$$\text{Gap}(\%)=\frac{\text{waste of each technique-waste of CG}}{\text{waste of CG}}\times 100. \tag{11}$$

The results are presented in Table 2 which includes the total waste (sq. in), time (second), and the gap (%) of each technique. For 2DSHC technique, there are only 3 instances having the same waste as the CG technique, and the rest of the instances have higher waste. This 2DSHC can find the simple patterns in a short computational time, where each sheet contains only one panel size. Similarly, for 2DHC technique, there are only 2 instances having the same waste as the CG technique, and there are 18 instances with a higher waste. Moreover, for 2DVC technique, there are only 1 instances having the same waste as the CG technique, and the rest of the instances have higher waste. Both 2DHC and 2DVC cannot give acceptable solutions especially in medium and large size instances, since both 2DHC and 2DVC keep using the first sheet type without considering the waste. For example, in instance 15, the waste from both 2DHC and 2DVC are very high with the gap of 24,344.5% and 23,302.8%, respectively. Actually, there are other sheets type in the sheet set that can provide better patterns, but those sheets are not selected by these two techniques. The advantage is that the computational time given by 2DSHC, 2DHC and 2DVC techniques are very fast.

From the drawbacks of 2DSHC and 2DHC techniques, 2DHI is developed to improve the quality of the solutions. Its results indicate that there are 5 instances with the same waste as CG technique, 11 instances with higher waste, and 4 instances with lower waste. Similarly, from the disadvantages of 2DSHC and 2DVC techniques, 2DVI is developed to improve the quality of the solutions. Its results also indicate that there are 5 instances with the same waste, 11 instances with the higher waste, and 4 instances with lower waste. The advantages of 2DHI and 2DVI are; firstly, these techniques are very fast for all size of instances. Secondly, good patterns, closed to optimal, can be found in a short computational time. Thirdly, these techniques can put more than one type of panel into each type of sheet. Finally, they can explore for all possible sheets, and it always select the pattern that give the smallest waste to cut. Although in some instances, 2DHI and 2DVI cannot provide a better solution, the trade-off between the computational time and the solution are necessary.

Comparing between 2DHI and 2DVI, their results are not majorly different. Each technique sorts the panels in the panel set in descending order based on height and width to put horizontally or vertically in the sheet. For the large size instances, both 2DHI and 2DVI can even provide better solutions in very short computational time. Therefore, 2DHI and 2DVI are suitable to apply in real industry. It is observed that these two techniques can be complemented each other.

7. Conclusions

In this paper, only Two-Dimentional Rectangular Guillotine Cutting Stock Problem (2DRGCSP) is taken into consideration. There are five heuristics are proposed including Two-Dimensional Simple Heuristic Cutting (2DSHc), Two-Dimensional Horizontal Construction (2DHC), Two-Dimensional Vertical Construction (2DVC), Two-Dimensional Horizontal Improvement (2DHI), and Two-Dimensional Vertical Improvement (2DVI). From computational result, 2DSHC, 2DHC, and 2DVC techniques cannot give a good solution compare with the column generation technique. However, the solutions from 2DHI and 2DVI are close to the comlumn generation solution. Moreover, the 2DHI and 2DVI are faster than column generation especially in the large instance size. In addition, 2DHI and 2DVI can offer a better solution in all instances, comparing to the 2DSHC, 2DHC, and 2DVC . Therefore, 2DHI and 2DVI can provide good patterns in a short computational time efficiently and effectively even the size of the instances are large.

For further study, the addtional techniques should be considered such as the meta-heuristic methods namely: genetic algorithms (GAs), Harmony Search Algorithm (HSA), particle swarm optimization (PSO), ant colony (AC), or firefly algorithm (FA) to help exploring the solution further. These proposed heuristics may be applied to be used as initial solution for the column generation technique to explore further and find optimal solution.

Acknowledgments

This research is partially supported by Sirindhorn International Institute of Technology (SIIT), Thammasat University, Thailand.

Table 2 The output for small, medium, and large size instances

Size	No	Number of			CG		2DSHC			2DHC			2DVC			2DHI			2DVI		
		Sheet	Panel	Demand	Waste	Time	Waste	Time	Gap	Waste	Time	Gap	Waste	Time	Gap	Waste	Time	Gap	Waste	Time	Gap
Unit	0	(type)	(type)	(panel)	(Sq In)	(Sec)	(Sq In)	(Se	(%)	(Sq In)	(Se	(%)	(Sq In)	(Sec	(%)	(Sq In)	(Se	(%)	(Sq In)	(Sec	(%)
Small	1	2	2	219	58,228.90	1.2	58,228.90	1.3	0	59,362.90	1.4	1.9	59,362.90	1.4	1.9	58,228.90	1.4	0	58,228.90	1.3	0
	2	2	2	301	115,364.00	1.8	115,364.40	1.3	0	115,364.40	1.4	0	144,356.40	1.6	25.1	115,364.40	1.4	0	115,364.40	1.3	0
	3	2	2	575	98,370.80	3.2	185,445.80	1.4	88.5	252,918.80	1.8	157.1	252,918.80	1.6	157.1	98,370.80	1.3	0	98,370.80	1.3	0
	4	2	3	537	35,124.90	4.5	94,455.90	1.3	169	60,183.90	1.6	71.3	60,183.90	1.6	71.3	52,491.90	1.3	49.4	36,624.90	1.3	4.3
	5	3	3	552	29,108.20	9.2	29,108.20	1.4	0	128,180.20	1.5	340.4	128,180.20	1.6	340.4	29,108.20	1.4	0	29,108.20	1.4	0
	6	4	2	300	31,800.00	3.4	34,224.00	1.3	7.6	31,800.00	1.4	0	31,800.00	1.5	0	34,224.00	1.2	7.6	34,224.00	1.3	7.6
	7	5	1	209	70,101.60	1	70,376.60	1.3	0.4	75,626.60	1.5	7.9	75,626.60	1.4	7.9	70,376.60	1.2	0.4	70,376.60	1.3	0.4
Medium	8	5	6	1,492	274,683.00	66.8	558,711.60	1.4	103	1,534,059.60	2.1	458.5	2,058,123.60	1.9	649.3	274,904.40	1.5	0.1	558,711.60	1.5	103
	9	5	8	2,240	93,677.40	68	95,692.20	1.7	2.2	713,145.00	1.8	661.3	713,145.00	2.2	661.3	95,692.20	1.5	2.2	95,692.20	1.5	2.2
	10	6	2	1,329	194,561.00	28.4	351,808.50	1.5	80.8	361,528.50	1.6	85.8	361,528.50	1.8	85.8	192,808.50	1.3	-0.9	192,808.50	1.4	-0.9
	11	6	3	219	7,576.90	5.5	9,976.90	1.3	31.7	41,200.90	1.4	443.8	41,200.90	1.4	443.8	7,480.90	1.4	-1.3	9,976.90	1.3	31.7
	12	8	7	596	42,051.00	35	43,785.00	1.4	4.1	54,561.00	1.6	29.7	54,561.00	1.6	29.7	43,785.00	1.5	4.1	43,785.00	1.3	4.1
	13	9	3	2,192	115,340.00	88.8	310,399.60	1.5	169	322,483.60	2	179.6	322,483.60	2	179.6	256,366.60	1.5	122	98,779.60	1.5	-14.4
	14	9	9	3,068	963,234.00	205.5	1,270,337.80	1.6	31.9	1,473,185.80	2.1	52.9	1,473,185.80	2.3	52.9	1,262,081.8	1.6	31	1,264,961.80	1.6	31.3
	15	11	4	4,299	8,694.90	545.4	18,300.50	1.6	111	2,125,422.50	2.4	24,344.50	2,034,846.50	2.7	23,302.80	10,954.00	1.6	26	15,556.50	1.6	78.9
	16	11	5	748	28,908.00	67.9	51,962.50	1.5	79.8	113,155.00	1.6	291.4	113,155.00	1.7	291.4	28,908.00	1.4	0	28,908.00	1.4	0
Large	17	15	12	4,429	68,363.40	472.3	167,909.30	1.7	146	2,347,191.30	2.2	3,333.40	2,350,047.30	2.9	3,337.60	70,390.60	1.6	3	67,550.30	1.8	-1.2
	18	16	6	1,121	31,768.90	85.8	36,019.90	1.6	13.4	161,050.40	1.7	406.9	161,050.40	1.8	406.9	39,751.60	1.5	25.1	33,475.90	1.4	5.4
	19	17	27	10,611	451,778.80	1,467.00	567,036.00	2.1	25.5	943,605.60	3.3	108.9	971,685.60	3.8	115.1	371,064.50	2.2	-17.9	510,624.00	2.9	13
	20	26	15	3,790	149,754.00	869.4	189,450.20	1.9	26.5	365,469.20	2.2	144	1,218,523.20	2.6	713.7	141,894.20	1.7	-5.2	134,174.40	1.7	-10.4

References

Alvarez-Valdes R, Parajon A, Tamarit JM. A computational study of LP-based heuristic algorithms for two-dimensional guillotine cutting stock problems. OR Spectrum. 2002; 24(2): 179-192.

Andrade R, Birgin EG, Morabito R. Two-stage two-dimensional guillotine cutting problems with usable leftovers. Department of Computer Science, Institute of Mathematics and Statistics, University of Sao Paulo, Brazil. 2013.

Cerqueira GRL, Yanasse HH. A pattern reduction procedure in a one-dimensional cutting stock problem by grouping items according to their demands. J Comput Interdiscip Sci. 2009; 1(2): 159-164.

Cui Y. A new dynamic programming procedure for three-staged cutting patterns. J Global Optim. 2012; 55(2): 349-357.

Cui Y. A CAM system for one-dimensional stock cutting. Adv Eng Softw. 2012; 47(1): 7-16.

Cui Y. Fast heuristic for constrained homogenous T-shape cutting patterns. Appl Math Model. 2012; 36(8): 3696-3711.

Cui Y. Generating optimal T-shape cutting patterns for rectangular blanks. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture. 2004; 218(8): 857-866.

Cui Y, Huang B. Heuristic for constrained T-shape cutting patterns of rectangular pieces. Comput Oper Res. 2012; 39(12): 3031-3039.

Cui Y, Liu Z. T-shape homogenous block patterns for the two-dimensional cutting problem. J Global Optim. 2007; 41(2): 267-281.

Cui Y, Yang L, Zhao Z, Tang T, Yin M. Sequential grouping heuristic for the two-dimensional cutting stock problem with pattern reduction. Int J Prod Econ. 2013; 144(2): 432-439.

Cui Y, Zhao Z. Heuristic for the rectangular two-dimensional single stock size cutting stock problem with two-staged patterns. Eur J Oper Res. 2013; 231(2): 288-298.

Gilmore PC, Gomory RE. Multistage cutting stock problems of two and more dimensions. Oper Res. 1965; 13: 94-120.

Lodi A, Monaci M. Integer linear programming models for 2-staged two-dimensional Knapsack problems. Math Program. 2003; 94(2-3): 257-278.

Suliman SMA. A sequential heuristic procedure for the two-dimensional cutting-stock problem. Int J Prod Econ. 2006; 99(1-2): 177-185.

Yanasse HH, Limeira MS. A hybrid heuristic to reduce the number of different patterns in cutting stock problems. Comput Oper Res. 2006; 33(9): 2744-2756.

Yanasse HH, Morabito R. A note on linear models for two-group and three-group two-dimensional guillotine cutting problems. Int J Prod Res. 2008; 46(21): 6189-6206.

Yanasse HH, Zinober ASI, HARRIS RG. Two-dimensional cutting stock with multiple stock sizes. J Opl Res Soc. 1991; 42(8): 673-683.