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# A Modified Poisson Exponentially Weighted Moving Average Chart Based on Improved Square Root Transformation

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**Abstract**

In this paper, the objective of the study is to propose a modified Poisson Exponentially Weighted Moving Average (EWMA) chart by Improving Square Root Transformation, namely ISRT $c$ EWMA in order to detection a change in Poisson observation. The performance of this control chart is compared with ISRT $c$ and EWMA charts in term of detection of a change in parameter by using the Average Run Length criterion. We compare the performance of the ISRT $c$ EWMA chart with ISRT $c$ chart and EWMA chart by using ARL criteria. The numerical results also indicated that ISRT $c$ EWMA chart is an effective alternative to the traditional EWMA control chart small shifts detection.

**Keywords**: Exponentially weighted moving average, improved square root transformation, average run length, nonconformities.

## 1. Introduction

Statistical Process Control (SPC) plays a vital role for controlling, measuring and detecting of a change in a process. In real applications, there are many applications for instance manufacturing and industry statistics, computer sciences and telecommunications, finance and economics, health care and epidemiology and environmental statistics. Since, the process improvement and modern technology are developed in order to decrease wasted products or produce a low defective level. The classical attribute control charts such as $p$ and $c$ charts for monitoring of a change in proportions of defects and number of nonconformities, respectively, are not suitable for this situation. Generally, the $c$ chart is developed by using the normal approximation with Poisson observation. The observations are frequently used to deal with the conditions $\lambda \geq 10$, where $\lambda$ denotes a mean of the underlying Poisson distribution. Schader and Schmid (1989) proposed that the performance of normal approximation to binomial distribution was very poor when $np \geq 5$ and $n(1-p) \geq 5$. If the traditional Shewhart $p$ and $c$ charts are used to detect a change in process on these conditions then they will provide a large false alarm rate. Later, Ryan and Schwertman (1997) presented an Arcsine $p$ chart where a lower tail probability is too small and an upper tail probability of $np$ chart is too large. Winterbottom (1993) and Chen (1998) applied the Cornish-Fisher expansion of quantiles to propose a modified $p$ chart. Recently, Tsai, T-R et al. (2002) proposed the Improved Square Root Transformation (ISRT) for attribute control charts, namely ISPRT $p,$ and ISRT $c$ control charts for binomial and Poisson observation, respectively. In general, the traditional attribute control chart as $c$ chart is often used when magnitudes of change is large (Montgomery (2009)). An Exponentially Weighted Moving Average (EWMA) control chart, however is an alternatives effective to traditional control chart to detect of small and moderate changes by Montgomery (2009). Sukparungsee (2014) presented the combined of ISRT and EWMA control charts together based on binomial observations, namely ISRT $p$ EWMA chart which the performance of this modified control chart is better than ISRT $p$ chart by measuring the Average Run Length (ARL).

They are many other criteria that have been used for optimality of SPC (see e.g., Shiryaey (1961, 1978), Lorden (1971)). The ARL is an important characteristic for comparison the performance of control charts. There are two criteria of ARL as in-control ARL $\left(ARL_0\right)$ and out-of-control ARL $\left(ARL_1\right)$. The $ARL_0$ is the expectation of the time before the control chart gives a false alarm that an in-control process has gone out-of-control which it should be large enough. The $ARL_1$ is the expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control which it should be minimal.

Consequently, the proposed control chart by modification EWMA chart with Improved Square Root Transformation (ISRT) method is applied to Poisson distribution, namely ISRT Poisson EWMA control chart in this paper. Furthermore, the performance of the proposed control chart is compared with the performance of ISRT $c$ and EWMA control charts in order to detect of a change in Poisson parameter.

## 2. Characteristics and Control Charts

Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed (i.i.d.) random variables with Poisson distribution $F(x,c),$ where $c$ is a parameter. It is usually assumed that under in-control condition the parameter is known $c = c_0.$ Then, at some unknown time $\nu,$ which is called the change-point time $(\nu < \infty),$ the parameter $c$ could be changed to an out-of-control value $c \neq c_0.$

As mentioned before, two criteria are commonly used to evaluate characteristics of control chart are the in-control Average Run Length $\left(ARL_0\right)$ and the out-of-control Average Run Length $\left(ARL_1\right)$. The $ARL_0$ is the average number of observations that will occur before an in-control process falsely gives an out-of-control signal. To reduce the number of false out-of-control signals a sufficiently large $ARL_0$ is required. The $ARL_1$ is a measure of the average number of observations that will occur before an out-of-control process correctly gives an out-of-control signal and to reduce the time that the process is out-of-control, a small $ARL_1$ is required.

The typical condition on choice of the stopping time $\tau$ is that

$$ARL_0 = E_\infty(\tau) = T, \qquad (1)$$

where $T$ is given (usually large) and $E_\infty(.)$ is the expectation under distribution $F(x,c_0)$ for the in-control state. The quantity $E_\infty(\tau)$ is usually called the in-control Average Run Length $\left(ARL_0\right)$.

Another typical characteristic of a control chart is obtained by minimizing the quantity

$$ARL_1 = E_\nu(\tau - \nu + 1 | \tau \geq \nu), \tag{2}$$

where $E_\nu(.)$ is the expectation under the assumption that change-point occurs at time $\nu < \infty$ and $c$ is the value of the parameter after change-point. In practice, the condition in Eq. (2) is usually calculated when $\nu = 1$. The quantity $E_1(\tau)$ is usually called the out-of-control Average Run Length $(ARL_1)$.

### 2.1. ISRT $c$ chart

Tsai et al. (2002) presented the Improved Square Root Transformation (ISRT) method for attribute control charts- ISPRT $p$, $np$ and $c$ charts for binomial and Poisson data, respectively to overcome the poor performance of the tradition control chart. Suppose that the process observations are taken from a Poisson distribution where $X$ is a Poisson random variable with parameter $c$, where $c$ is the average number of nonconformities product. The normal approximation could be poor, if $c$ is too small $(c < 10)$. In this paper, we engage Improved Square Root Transformation (ISRT) for Poisson observation, namely, ISRT $c$ chart. The upper and lower of $3\sigma$ control limits are shown by Tsai, et al. (2002) as following:

$$UCL_{ISRT} = \sqrt{c} + 3\left[\frac{3}{2} - \frac{1}{2}\left(\frac{1}{\sqrt{c}}\right)\right] \text{ and } LCL_{ISRT} = \sqrt{c} - 3\left[\frac{3}{2} - \frac{1}{2}\left(\frac{1}{\sqrt{c}}\right)\right]. \tag{3}$$

Therefore, the control limits can be rewritten as follows:

$$UCL_{ISRT} = \sqrt{c} + \left(\frac{1}{2} - \frac{1}{6\sqrt{c}}\right) \text{ and } LCL_{ISRT} = \sqrt{c} - \left(\frac{1}{2} - \frac{1}{6\sqrt{c}}\right). \tag{4}$$

These control limits are adequate for the low conformities product. If the parameter of Poisson distribution $c$ is unknown, then it is estimated by $\bar{c} = \frac{\sum_{i=1}^{n} D_i}{n}$, where $D_i$ is number of non-conforming product in the sample size $n$.

### 2.2. EWMA chart

Roberts (1959) proposed an Exponentially Weighted Moving Average (EWMA) control chart which is an effective alternative to the traditional Shewhart control chart small shifts detecting of a process mean. The EWMA statistics can be written as follows:

$$Z_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1}, \; i = 1, 2, \ldots$$

where $\lambda$ is weight of past information, $0 < \lambda < 1$. The control limits of Poisson EWMA control chart are

$$UCL_{EWMA} = \sqrt{c} + B\sqrt{c\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)}$$

and

$$LCL_{EWMA} = \sqrt{c} - B\sqrt{c\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)}$$

where $B$ is coefficient of EWMA control limit to correspond the value of $ARL_0$ and $i \to \infty$ the control limits of EWMA chart can be rewritten as the following:

$$UCL_{EWMA} = \sqrt{c} + B\sqrt{c\frac{\lambda}{2-\lambda}} \text{ and } LCL_{EWMA} = c - B\sqrt{c\frac{\lambda}{2-\lambda}}. \tag{5}$$

### 2.3. ISRT Poisson EWMA control chart

Consequently, we proposed the modified the control chart by engaging ISRT $c$ with Poisson EWMA control charts, is so-called ISRT Poisson EWMA control chart by integrating the ISRT and EWMA control charts together and found the control limits of this proposed control chart in this paper.

Therefore, the modified ISRT Poisson EWMA or ISRT $c$ EWMA chart can be presented the control limits as the following:

$$UCL_{ISRT\ c\ EWMA} = \sqrt{c} + H\left(\frac{1}{2} - \frac{1}{6\sqrt{c}}\right)\sqrt{c\frac{\lambda}{2-\lambda}} \tag{6}$$

and

$$LCL_{ISRT\ c\ EWMA} = \sqrt{c} - H\left(\frac{1}{2} - \frac{1}{6\sqrt{c}}\right)\sqrt{c\frac{\lambda}{2-\lambda}}. \tag{7}$$

where $H$ is coefficient of ISRT $c$ EWMA control limit to correspond the value of $ARL_0$.

## 3. Numerical Results

In this section, we compare the numerical results for $ARLs$ obtained from ISRT $c$ chart, EWMA and ISRT Poisson $(c)$ EWMA chart by Monte Carlo simulations as shown on Table 1 and 2. The comparison of the performance of ISRT $c$ chart, EWMA and ISRT $c$ EWMA chart are demonstrated when fixed $ARL_0$ = 370, $c_0$ = 5 and 9 and $\lambda$ = 0.05 and 0.1, respectively. On Table 1, given $ARL_0$ = 370 and $c_0$ = 5, the performance in detection of a change in parameter of ISRT $c$ EWMA control chart is superior to ISRT $c$ and EWMA charts when magnitudes of change are small to moderate $(\delta \leq 0.5)$. Otherwise, when $\delta > 0.5$, the ISRT $c$ chart is better than other control charts. We also study in the same manner on Table 1 by studying an in-control parameter where $c_0$ = 9 which found that the numerical results of the latter case are in good agreement of the former as shown on Table 2. In addition, the value of weights in EWMA and ISRT $c$ EWMA charts are selected to study the performance of control charts which $\lambda = 0.1$ is more appropriate than $\lambda = 0.05$ for both in-control parameter of $c_0$ = 5 and 9. This can be guaranteed that the proposed control chart as the modified ISRT $c$ EWMA chart be able to quickest detection of small and moderate shifts in the parameter for Poisson observation.

**Table 1** Comparison of $ARLs$ of ISRT $c$, EWMA and ISRT $c$ EWMA charts when $c_0 = 5$

| $\delta$ | $\lambda = 0.05$ | | | $\lambda = 0.10$ | | |
|---|---|---|---|---|---|---|
| | ISRT $c$ | EWMA | ISRT $c$ EWMA | ISRT $c$ | EWMA | ISRT $c$ EWMA |
| | UCL = 3.4652 | UCL = 2.3442 | UCL = 5.7635 | UCL = 3.6515 | UCL = 2.4522 | UCL = 6.2735 |
| 0.00 | 370.695 | 370.954 | 370.168 | 370.695 | 370.236 | 370.067 |
| 0.01 | 353.421 | 307.255 | 305.710 | 331.793 | 305.633 | 304.602 |
| 0.03 | 321.193 | 223.797 | 213.266 | 301.571 | 220.151 | 210.806 |
| 0.05 | 314.755 | 162.139 | 157.827 | 305.342 | 160.763 | 156.926 |
| 0.1 | 239.674 | 98.657 | 93.361 | 230.843 | 84.559 | 81.751 |
| 0.3 | 63.845 | 43.413 | 37.967 | 51.398 | 28.235 | 25.948 |
| 0.5 | 33.446 | 30.499 | 25.423 | 21.561 | 18.077 | 15.295 |
| 1.0 | 3.874 | 19.104 | 14.365 | 2.754 | 10.666 | 7.999 |
| 1.5 | 1.069 | 14.669 | 10.025 | 1.009 | 7.869 | 5.467 |
| 2.0 | 0.367 | 12.018 | 7.674 | 0.108 | 6.337 | 4.013 |

**Table 2** Comparison of $ARLs$ of ISRT $c$, EWMA and ISRT $c$ EWMA charts when $c_0 = 9$

| $\delta$ | $\lambda = 0.05$ | | | $\lambda = 0.10$ | | |
|---|---|---|---|---|---|---|
| | ISRT $c$ | EWMA | ISRT $c$ EWMA | ISRT $c$ | EWMA | ISRT $c$ EWMA |
| | UCL = 4.3120 | UCL = 3.1231 | UCL = 10.0107 | UCL = 4.4867 | UCL = 3.2322 | UCL = 10.6996 |
| 0.00 | 370.993 | 370.701 | 370.837 | 370.891 | 370.117 | 370.70 |
| 0.01 | 335.645 | 296.816 | 288.089 | 318. 835 | 292.626 | 287.181 |
| 0.03 | 281.238 | 191.851 | 188.616 | 261.962 | 189.120 | 182.941 |
| 0.05 | 239.851 | 142.773 | 133.572 | 217.457 | 131.724 | 124.418 |
| 0.1 | 154.328 | 87.283 | 79.349 | 108.861 | 68.638 | 65.729 |
| 0.3 | 38.551 | 42.655 | 35.563 | 28.538 | 25.299 | 21.536 |
| 0.5 | 29.683 | 31.554 | 24.563 | 20.897 | 17.437 | 13.84 |
| 1.0 | 1.335 | 20.591 | 14.288 | 1.146* | 10.798 | 7.711 |
| 1.5 | 0.240 | 16.053 | 10.154 | 0.216* | 8.248 | 5.228 |
| 2.0 | 0.049 | 13.351 | 7.839 | 0.039* | 6.763 | 3.958 |

## 4. Conclusions

In this paper, the modified Exponentially Weighted Moving Average (EWMA) control chart based on Improved Square Root Transformation (ISRT) with Poisson observations is proposed, namely ISRT $c$ EWMA control chart as an alternate control chart. The control limits of this chart are calculated and modified which is easy to implement and useful for monitoring a change in parameter when observations are Poisson distributed and could not be transformed to normal distribution due to restrictions. The numerical results show that the performance of ISRT $c$ EWMA chart is superior to the ISPRT $c$ chart and EWMA for small and moderate shifts, otherwise the ISRT $c$ chart performs better than the former. In addition, the performance of this chart is poor when magnitudes of change are large, however this control chart is flexible in order to vary the value of $\lambda$ in order to enhance the performance of control charts.

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