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A Matlab Program to Calculate Distribution with Maximum Renyi Entropy

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Abstract

Maximizing the Shannon entropy and Renyi entropy in a class of distributions subject to a set of constraints are the topics that play an important role in statistical inference. In this paper some distributions with maximum Renyi entropy under given constraints are presented. In this regard, we wrote a program in Matlab, that find out distributions with maximum Renyi entropy. In this program we have used the method of Lagrange. Using this program we have determined Lagrange multipliers and then obtained distributions with maximum Renyi entropy under a given constraint. In any case the results are summariezed in related table.

Keywords: Shannon entropy, Renyi entropy, maximum entropy principle, maximum Renyi entropy, lagrange method, moment condition.

1. Introduction

Optimization is an integral part in every field of study. This topic had its special position in information theory. One of the optimization methods is maximum entropy principle with Lagrange method. In many cases we seek the maximum entropy distribution and how to find that under some constraint is expressed by Kagan et al. (1979) and Kapur (1989). Maximum Renyi and Tsallis entropies subject to some conditions are extensions of this idea a larger class of Shannon entropy. Costa et al. (2004 and 2006), Bashkirov (2004 and 2006), Johnson and Vignat (2007), Haremouse (2006), Brody et al. (2008), Wilks and Wlodarrczyk (2008), Bercher (2008a and 2008b), Jose and Naik (2008) and Nagy and Romera (2009) have presented interpretations and characteristics of maximum Renyi entropy and Tsallis entropy in univariate and multivariate models. In this paper we give a Matlab program to find out distribution with maximum Renyi entropy. We have written it based on Djafari (1991). He has presented a Matlab program to calculate Lagrange multipliers and maximum entropy distributions. Here we were able to obtain Lagrange multipliers for an arbitrary number of conditions in connection with maximum Renyi entropy for different parameter α, where α is the Renyi parameter. In the limit $\alpha \to 1$, it is transformed to Shannon entropy.

In calculus of variations, Lagrange's equation is a second-order partial differential equation whose solutions are the functions for which a given functional is stationary. Because a differentiable functional is stationary at its local maxima and minima, the Lagrange equation is useful for solving optimization problems in which, given some functional, one seeks the function minimizing (or maximizing) it.

2. Maximum Renyi Entropy

Definition 2.1 Renyi entropy of a probability density function $P(x)$ is:

$$H_\alpha(P) = \frac{1}{1-\alpha}\log\int p^\alpha(x)dx, \quad \alpha > 0, \alpha \neq 1.$$

In this section we have consider maximizing $H_\alpha(P)$ under given conditions, the problem, in its general form is the following

$$\begin{cases} \max(H_\alpha(P(x))) = \max\left[\frac{1}{1-\alpha}\log\int p^\alpha(x)dx\right] \\ s.t \\ \int g_n(x)p(x)dx = u_n \ , \quad n = 0,1,\ldots,N, \end{cases} \tag{1}$$

where $g_0(x) = 1, u_0 = 1$ and in this case we have constraint

$$\int p(x)dx = 1, \tag{2}$$

$g_n(x); n = 0,1,\ldots,N,$ are known functions and $u_n; n = 0,1,\ldots,N$ are the given expectation data. We use Lagrange method for solving problem (1) we must make Lagrange equation.

$$\frac{1}{1-\alpha}\log\int p^\alpha(x)dx - \lambda_0\left(\int p(x)dx - 1\right) - \lambda_n\left(\int g_n(x)p(x)dx - u_n\right) = 0. \tag{3}$$

The derivative of Equation (3) yields

$$\begin{gathered} \frac{\partial}{\partial p}\left[\frac{1}{1-\alpha}\log\int p^\alpha(x)dx - \lambda_0\left(\int p(x)dx - 1\right) - \lambda_n\left(\int g_n(x)p(x)dx - u_n\right)\right] = 0 \\ \Rightarrow \frac{\alpha p^{\alpha-1}}{(1-\alpha)\int p^\alpha(x)dx} - \lambda_0 - \lambda_n g_n(x) = 0, \end{gathered} \tag{4}$$

where the Lagrange multipliers can be obtained from the constraints. We multiply both side of (4) by $P(x)$ and integrate the result. The relations (1) and (2) give the following results:

$$\frac{\alpha}{1-\alpha} - \lambda_0 - \lambda_n u_n = 0 \quad \Rightarrow \quad \lambda_0 = \frac{\alpha}{1-\alpha} - \lambda_n u_n. \tag{5}$$

With replacing (5) in (4) we have:

$$P(x) = \left[\int p^\alpha(x)dx\right]^{\frac{1}{\alpha-1}}\left[1 - \frac{1-\alpha}{\alpha}\lambda_n\left(u_n - g_n(x)\right)\right]^{\frac{1}{\alpha-1}}. \tag{6}$$

Assuming $Z_{\lambda_n} = \left[\int p^\alpha(x)dx\right]^{\frac{1}{\alpha-1}}$,

$$P(x)=\frac{1}{Z_{\lambda_n}}\left[1-\frac{1-\alpha}{\alpha}\lambda_n\left(u_n-g_n(x)\right)\right]^{\frac{1}{\alpha-1}}. \quad (7)$$

$P(x)$ in (7) is distribution with maximum Renyi entropy for a fixed α under constraint in the form (1). On the other hand, $P(x)$ must be density function and this gives

$$Z_{\lambda_n}=\int\left[1-\frac{1-\alpha}{\alpha}\lambda_n\left(u_n-g_n(x)\right)\right]^{\frac{1}{\alpha-1}}dx \quad (8)$$

In general, we can consider n constraints in the form (1), in this case relation (7) becomes the

$$p(x)=\left[\int p^{\alpha}(x)dx\right]^{\frac{1}{\alpha-1}}\left[1-\frac{1-\alpha}{\alpha}\sum_{i=1}^{n}\lambda_i\left(u_i-g_i(x)\right)\right]^{\frac{1}{\alpha-1}}. \quad (9)$$

As before assuming

$$B(\alpha,u_1,...,u_n)=\int 1-\left[\frac{1-\alpha}{\alpha}\sum_{i=1}^{n}\lambda_i(u_i-g_i(x))\right]^{\frac{1}{\alpha-1}}dx. \quad (10)$$

Then $p(x)$ with maximum Renyi entropy under n constraints is:

$$p(x)=\frac{1}{B(\alpha,u_1,...,u_n)}\left[1-\frac{1-\alpha}{\alpha}\sum_{i=1}^{n}\lambda_i(u_i-g_i(x))\right]^{\frac{1}{\alpha-1}}. \quad (11)$$

Similarly λ_0 in this case is obtained from:

$$\lambda_0=\frac{\alpha}{1-\alpha}-\sum_{i=1}^{n}\lambda_i u_i.$$

In particular we consider $g_i(x)=x^{k_i}$ and if $k_i=i,\ i=1,2,...,n$ we have moment constraints. This problem is solved by Brody et al. (2007). Before we present a Matlab program for estimate Lagrange multipliers, we remember their results. Let $p(x)$ be the unknown probability density function on the positive real line. We also assume that k-th moment of the distribution $p(x)$ is known and is given by u_k. So the problem is to find the $p(x)$ that maximizes the Renyi entropy with k-th moment u_k. Furthermore constraint (2) we have condition

$$\int x^k p(x)dx=u_k. \quad (12)$$

The resulting maximum Renyi entropy distribution takes the form

$$p(x)=\frac{1}{Z_{\lambda_k}}\left[1-\frac{1-\alpha}{\alpha}\lambda_k\left(u_k-x^k\right)\right]^{\frac{1}{\alpha-1}}, \quad (13)$$

where Z_β is the normalization factor such that (13) satisfies the condition (2). In view of the expression in (6) we observe that

$$Z_{\lambda_k}=\left[\int p^{\alpha}(x)dx\right]^{\frac{1}{\alpha-1}}=\int\left[1-\frac{1-\alpha}{\alpha}\lambda_k(u_k-x^k)\right]^{\frac{1}{\alpha-1}}dx\ . \quad (14)$$

Brody et al. (2007), in order to determine Z_{λ_k} in the right hand side of (13) have used the identity

$$\int \frac{x^{l-1}}{(a+bx^k)^{\mu}}dx = \frac{1}{ka^{\mu}}\left(\frac{a}{b}\right)^{\frac{l}{k}} \frac{\Gamma(\frac{1}{k})\Gamma(\mu-\frac{1}{k})}{\Gamma(\mu)}, \tag{15}$$

valid for $0<\frac{1}{k}<\mu,\ \mu>1.$

Compare the relation (14) with (15) and $\mu=\frac{1}{1-\alpha},\ a=1-\lambda_k u_k\left(\frac{1-\alpha}{\alpha}\right),\ l=1,\ b=\lambda_k\frac{1-\alpha}{\alpha}$ and with consider this point that, $\Gamma(x)$ denotes the standard gamma function we have:

$$Z_{\lambda_k} = \int \frac{1}{\left[1-\frac{1-\alpha}{\alpha}\lambda_k(u_k-x^k)\right]^{\frac{1}{\alpha-1}}}dx = \int \frac{1}{\left(a+bx^k\right)^{\frac{1}{\alpha-1}}}dx. \tag{16}$$

That concludes

$$Z_{\lambda_k} = \frac{1}{k(1-\lambda_k u_k(\frac{1-\alpha}{\alpha}))^{\frac{1}{\alpha-1}}}\left(\frac{\alpha}{\lambda_k(1-\alpha)}-u_k\right)^{\frac{1}{k}} \frac{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{1}{1-\alpha}-\frac{1}{k}\right)}{\Gamma\left(\frac{1}{1-\alpha}\right)}. \tag{17}$$

This relation valid for $\frac{1}{2}<\alpha<1,\ 0<\frac{1}{k}<\frac{1}{1-\alpha}.$ Now we multiply both sides of equation (13) by x^k and integrate of the result. Using Equation (12) we obtain:

$$\begin{aligned} u_k &= \int \frac{1}{Z_{\lambda_k}}\left[1-\left(\frac{1-\alpha}{\alpha}\right)\lambda_1(u_k-x^k)\right]^{\frac{1}{\alpha-1}} x^k dx \\ \Rightarrow u_k Z_{\lambda_k} &= \int \frac{x^k}{\left[1-\left(\frac{1-\alpha}{\alpha}\right)\lambda_k(u_k-x^k)\right]^{\frac{1}{1-\alpha}}}dx \\ &= \int \frac{x^k}{\left[1-\lambda_1 u_k\frac{1-\alpha}{\alpha}+\lambda_k\frac{1-\alpha}{\alpha}x^k\right]^{\frac{1}{1-\alpha}}}dx. \end{aligned} \tag{18}$$

Replace (17) in (18) and let $l=k+1,\ \mu=\frac{1}{1-\alpha},\ a=1-\lambda_k u_k\frac{1-\alpha}{\alpha}$ and $b=\lambda_k\frac{1-\alpha}{\alpha}$ and use of (15) after simple calculations we conclude

$$\begin{aligned} u_k\left(\frac{1}{1-\alpha}-\frac{1}{k}-1\right) &= \left(\frac{\alpha}{\lambda_k(1-\alpha)}-u_k\right)^{\frac{1}{k}} \\ \Rightarrow \lambda_k &= \frac{1}{ku_k}. \end{aligned} \tag{19}$$

We note that $\frac{1}{k+1}<\alpha<1$ the moment conditions are well defined for all $k\geq 1$ provided that, $\frac{1}{2}<\alpha<1.$ Density function in (13) is known as Renyi distribution or α distribution. When $\alpha\to 1$ the distribution p_i becomes the Gibbs canonical distribution which has maximum entropy. For maximum entropy Djafari (1991) presented a Matlab program. He used this program to find Lagrange multipliers and he obtained some distributions with maximum entropy. In this paper we could write a program in Matlab that gives distribution with maximum Renyi entropy under known constraints. This program determines Lagrange multipliers λ_i and with replace λ_i's in (13) we estimate $p(x)$ with maximum Renyi entropy. Some examples are presented below.

3. Some Examples of Maximum Renyi Entropy

In this section we consider some well-known distribution with maximum Renyi entropy under given constraints.

When we don't have any constraint the probability density function with maximum Renyi entropy is uniform distribution as well as Shannon entropy.

Let $g(x) = x$ that means $\int xp(x)dx = \theta$ then $p(x)$ is:

$$p(x) = \frac{1-\alpha}{(2\alpha-1)\theta}\left(\frac{2\alpha-1}{\alpha}\right)^{\frac{1}{1-\alpha}} \frac{\Gamma\left(\frac{1}{1-\alpha}\right)}{\Gamma\left(\frac{\alpha}{1-\alpha}\right)}\left[1-\frac{1-\alpha}{\beta}\beta(\theta-x)\right]^{\frac{1}{\alpha-1}}.$$

Thus function $p(x)$ under constraint $E(X) = \theta$ is generalized Pareto distribution with shape parameter $\frac{1}{\alpha}-1$ and scale parameter β..

In Table 1, we calculate Lagrange multipliers (λ_0, λ_1) under constraint $E(X) = u = 9$ for 4 sample sizes (400, 600, 1000, 1500). Since moment conditions are well defined for all $k \geq 1$ provided that $\frac{1}{2} < \alpha < 1$, we consider 4 different values for α (0.5, 0.75, 0.95, 0.999), α is the parameter of Renyi entropy. In addition this program being able to estimate $E(g(X))$ for that estimated probability $p(x)$ and in this example $E(X)$ for given sample. This value is shown with μ_1 in Table 1.

The last columns of Tables is dedicated to value Z_{λ_k}. With replacing λ_1 and Z_β in (13), we obtain density function $p(x)$ with maximum Renyi entropy under $E(X) = 9$.

Table 1 Estimate of λ_0, λ_1 under constraint $E(X) = 9$

Sample sizes	α	λ_0	λ_1	μ_0	μ_1	Z_{λ_k}
400	0.50	9.4825	-0.9425	1	9.7699	22.1222
	0.75	11.3691	-0.9299	1	9.0081	3.2975
	0.95	112.2076	-10.3564	1	9.9645	389730
	0.999	100.79	-0.9877	1	9.000	207498
600	0.50	16.7535	-1.8615	1	9.5344	953.9402
	0.75	25.3272	-2.7808	1	9.9063	86.5057
	0.95	94.9438	-8.4382	1	9.9381	9281.4
	0.999	218.2122	-13.2458	1	9.9412	126590
1000	0.50	5.58	-0.65	1	8.9076	4.1854
	0.75	22.9809	-2.2201	1	9.8317	26.6378
	0.95	74.3905	-6.1545	1	9.8887	287.4375
	0.999	1080.8	-9.0934	1	9.8954	1062.6

Table 1 (Continued)

Sample sizes	α	λ_0	λ_1	μ_0	μ_1	Z_{λ_k}
1500	0.50	13.6584	-1.5172	1	9.6576	2450.9
	0.75	54.4233	-5.7137	1	9.526	569910000
	0.95	60.4756	-4.6084	1	9.8233	41.7605
	0.999	1053.9	-6.1003	1	9.8339	73.1238

Johnson and Vignat (2005) are presented a Theorem for n dimensional probability density function with maximum Renyi entropy. Some of the results are presented as follow:

For $\frac{n}{n+1} < \alpha,\ \alpha \neq 1,$ Define the n-dimensional probability density $g_{\alpha,C}$ as

$$g_{\alpha,C}(x) = A_\alpha \left(1-(\alpha-1)\beta x^T C^{-1} x\right)_+^{\frac{1}{\alpha-1}} \tag{20}$$

with $\beta = \frac{1}{2\alpha - n(1-\alpha)}$ and normalization constants

$$A_\alpha = \begin{cases} \dfrac{\Gamma\left(\frac{1}{1-\alpha}\right)\left(\beta(1-\alpha)\right)^{\frac{n}{2}}}{\Gamma\left(\frac{1}{1-\alpha}-\frac{n}{2}\right)\pi^{\frac{n}{2}}|C|^{\frac{1}{2}}}, & \frac{n}{n+2} < \alpha < 1 \\ \dfrac{\Gamma\left(\frac{\alpha}{\alpha-1}\right)\left(\beta(\alpha-1)\right)^{\frac{n}{2}}}{\Gamma\left(\frac{\alpha}{\alpha-1}\right)\pi^{\frac{n}{2}}|C|^{\frac{1}{2}}}, & \alpha > 1. \end{cases} \tag{21}$$

We write $R_{\alpha,C}$ for a random variable with density $g_{\alpha,C}$ which has mean 0 and covariance C.

Theorem 3.1 *Given, any* $\alpha > \frac{n}{n+2}$, *and positive definite symmetric matrix* C, *among all probability densities* f *with mean 0 and* $\int_{\Omega_{\alpha,C}} p(x) x x^T dx = C$, *the Renyi entropy is uniquely maximized by* $g_{\alpha,C}$ *that is*

$$H_\alpha(p(x)) \le H_\alpha(g_{\alpha,C}),$$

with equality, if and only if $p(x) = g_{\alpha,C}$ *almost everywhere.*

For any, $\frac{n}{n+2} < \alpha < 1,$ writing $m = \frac{2}{1-\alpha} - n > 2$ we have

$$R_{\alpha,C} \sim \frac{Z_{(m-2)C}}{U}$$

where, $U \sim \chi_m$ and independent of Z. Thus $R_{\alpha,C}$ has t-student with m degrees of freedom.

Let $g(x) = x^2$ that means $E(X^2) = \theta$ then $p(x)$ is:

$$p(x)=2\left(\frac{1-\alpha}{(3\alpha-1)\theta}\right)^{\frac{1}{2}}\frac{\Gamma\left(\frac{1}{1-\alpha}\right)}{\sqrt{\pi}\Gamma\left(\frac{1}{1-\alpha}-\frac{1}{2}\right)}\left[1+\frac{1-\alpha}{(3\alpha-1)\theta}x^2\right]^{\frac{1}{\alpha-1}}. \tag{22}$$

This distribution is a special case *n* dimensional probability $g_{\alpha,C}$ when $n=l.$ In this case we have constraint $E(X^2)=u.$ The results are presented in Table 2. In this case our condition is $E(X^2)=25$ and this value is estimated for the obtained $p(x)$ and has presented in Table 2. For example when $n=400,$ $\alpha=0.5,$ the program has estimated $\mu_1=24.5395$ this value shows how correct our calculations has been.

Table 2 Estimate of λ_0,λ_1 under constraint $E(X^2)=25$

Sample sizes	α	λ_0	λ_1	μ_0	μ_1	Z_{λ_k}
400	0.50	3186	-127.4004	1	24.5394	0.00009
	0.75	161.3225	-6.3329	1	24.6768	0.0116
	0.95	127.1575	-4.3263	1	24.6981	0.0186
	0.99	202.32	-4.1328	1	24.7004	0.0197
600	0.50	458.4575	-18.2983	1	24.8148	0.0097
	0.75	107.3775	-4.1751	1	24.6797	0.0276
	0.95	91.2525	-2.8901	1	24.6649	0.0382
	0.99	168.0825	-2.7633	1	24.6634	0.0398
1000	0.50	223.3425	-8.8937	1	24.2927	0.0096
	0.75	66.22	-2.5288	1	24.3547	0.0381
	0.95	66.5675	-1.7427	1	24.3640	0.0561
	0.99	140.635	-1.6654	1	24.3650	0.0588
1500	0.50	133.8425	-5.3137	1	24.1042	0.0191
	0.75	46.2925	-1.7317	1	24.0929	0.0589
	0.95	48.725	-1.189	1	24.0929	0.0857
	0.99	127.3900	-1.1356	1	24.0930	0.0898

Let $g_1(x)=x$ and $g_2(x)=x^2$ in this case distribution with maximum Renyi entropy is:

$$p(x)=\frac{1}{Z_{\lambda_1,\lambda_2}}\left[1-\frac{1-\alpha}{\alpha}\lambda_1(\theta_1-x)-\frac{1-\alpha}{\alpha}\lambda_2(\theta_2-x^2)\right]^{\frac{1}{\alpha-1}}.$$

Now we consider $p(x)\sim\left(ax^2+bx+c\right)^{\frac{1}{\alpha-1}}$ and if we regard $y=x+\frac{b}{2a}$ then $p(x)\sim\frac{1}{y^2+c'}.$ Using

$$\int\frac{x^{l-1}}{(a+bx^k)^{\mu}}dx=\frac{1}{ka^2}\left(\frac{a}{b}\right)^{\frac{1}{k}}\frac{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\mu-\frac{1}{k}\right)}{\Gamma(\mu)}.$$

Now let $k=2, a=c', b=1, l=1, \mu=\frac{1}{1-\alpha}$ thus

$$Z_{(\lambda_1,\lambda_2)} = \frac{c'^{\frac{1}{2}-\frac{1}{1-\alpha}}}{2} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{1-\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{\alpha-1}\right)},$$

and $p(x)$ is Burr type (XII). Consider two constraints $E(X)=9,\ E(X^2)=0.8$ the results are summarized in the following table.

Table 3 Estimate of $\lambda_0,\lambda_1,\lambda_2$ under constraint $E(X)=9,\ E(X^2)=0.8$

Sample sizes	α	λ_0	λ_1	λ_2	μ_0	μ_1	μ_2	Z_{λ_1,λ_2}
400	0.50	8.9848	-4.3965	-5.0355	1	0.9677	0.9368	19.5960
	0.75	79.2658	-44.7857	-44.9483	1	0.92	0.8464	1667800
	0.95	45.6577	-8.9701	-23.2195	1	0.9874	0.9753	14.0568
	0.99	131.7677	-15.0077	-24.0759	1	0.9859	0.9722	11.8788
600	0.50	5.4367	-2.4485	-2.7913	1	0.9171	0.8649	0.6591
	0.75	12.0772	-4.3058	-6.5025	1	0.9594	0.9254	0.8010
	0.95	114.0232	-8.5529	-14.1570	1	0.9764	0.9540	1.6988
	0.99	120.9526	-9.2884	-16.9913	1	0.9772	0.9556	2.004
1000	0.50	4.7283	-1.9744	-2.4392	1	0.8736	0.7997	0.4812
	0.75	9.0520	-3.3878	-3.7537	1	0.9004	0.8307	0.3813
	0.95	31.1287	-5.3028	-9.1952	1	0.9583	0.9207	0.5553
	0.99	113.0195	-6.2958	-10.4416	1	0.9620	0.9270	0.6048
1500	0.50	-23.8026	13.9873	15.2675	1	0.8719	0.7601	143.6108
	0.75	8.3363	-2.8634	-3.4490	1	0.8817	0.8029	0.3601
	0.95	23.7768	0.8081	-6.8801	1	0.8965	0.8217	0.3316
	0.99	108.2797	-3.9030	-7.2088	1	0.9398	0.8876	0.3657

As mentioned previously sometimes we have generally constraint $E(g(X))=u$ for example $g(x)=\cos(x).$ For 4 sample sizes 400, 600, 1000, 1500 and different values for α $(\alpha = 0.75, 0.95, 1.25, 1.5)$, we determine Lagrange multipliers. The results are summarized in Table 4. When $\alpha > 1\,(\alpha = 1.25, 1.5)$ the values of μ_1 is equal 0.5.

Table 4 Estimate of λ_0,λ_1 under constraint $E(\cos(X))=0.5$

Sample sizes	α	λ_0	λ_1	μ_0	μ_1	Z_{λ_1}
400	0.75	1.9519	2.0963	1	-0.9299	29206000000
	0.95	34.2781	-30.5562	1	0.9967	28827000000000
	1.25	-4.3515	-1.2970	1	0.5003	4.5777
	1.50	-2.3188	-1.3624	1	0.5006	4.4029
600	0.75	5.8155	-5.6309	1	0.9934	17577
	0.95	32.9127	-27.8254	1	0.9950	6819000000000
	1.25	-4.3517	-1.2967	1	0.5005	4.5769
	1.50	-2.3186	-1.3627	1	0.5010	4.4021

Table 4 (Continued)

Sample sizes	α	λ_0	λ_1	μ_0	μ_1	Z_{λ_1}
1000	0.75	5.7036	-5.4073	1	0.9889	3423.7
	0.95	30.7988	-23.5977	1	0.9917	84535000
	1.25	-4.3513	-1.2974	1	0.5008	4.5764
	1.50	-2.3178	-1.3644	1	0.5016	4.4007
1500	0.75	5.5762	-5.1525	1	0.9832	1004.6
	0.95	28.9264	-19.8529	1	0.9786	1017900
	1.25	-4.3549	-1.2902	1	0.4988	4.5784
	1.50	-2.3167	-1.3665	1	0.5024	4.3989

4. Conclusions

In this paper while introducing the Renyi entropy, we were looking for distribution that maximizes the Renyi entropy under given constraints. Considering this constraints we have made nonlinear equation and this equations have been solved with Lagrange method. We wrote a program in Matlab that is given in appendix. Using this program, we have determined Lagrange multipliers. Then replacing this λ_i's in relation (11) we have obtained the distribution with maximum Renyi entropy.

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Appendix

```
function [lambda,p,beta,conditions] = reni2(mu,xprime,alpha)
%UNTITLED Summary of this function goes here
%   Detailed explanation goes here
%p has the values of PDF
Eps=1e-4;
Xmin=xprime(1);                %in order to avoid conflict we defined xprime instead of x
Xmax=xprime(length(xprime));
Dx=xprime(2)-xprime(1);
lambda=zeros(size(mu));
n=length(lambda);
phi2=fin_3();
r=1;
[row,column]=size(xprime);
finalx=xprime;
phin=zeros(n,column);
muprime=zeros(n,column);
lambdaprime=zeros(n,column);
phi1=zeros(n,1);
not=0;
y=0;
for myi=1:length(xprime)   %Calculation of muprime matrix which is n*r
muprime(:,myi)=mu';
end                        %End of muprime calculation
for ex=xmin:dx:xmax     %start of loop for calculating Phi-en
for i=1:n            %caculation of matrix phi1 in order to use in sigma
phi1(i,1)=phi2{i,1}(ex);
end                  %end of phi1 calculation
phin(:,r)=phi1(:);  %Calculation of Phi-en matrix which is r*n
r=r+1;
end                    %end of loop for calculating Phi-en
while 1         %first of while loop
```

```
for myi=1:length(xprime)  %Calculation of lambdaprime matrix which is n*r
lambdaprime(:,myi)=lambda';
end              %End of lambdaprime calculation
if (n==1)
lmphi=((1-alpha)/alpha)*(lambdaprime.*(muprime-phin));
%Calculation of sum of ((1-alpha)/alpha)*lambda.*(mu-phi) in which each column stands for a single x
else
lmphi=((1-alpha)/alpha)*sum(lambdaprime.*(muprime-phin));
end
beta=dx*sum((1-lmphi).^(1/(alpha-1)));           %calculation of beta
p=1/beta*((1-lmphi).^(1/(alpha-1)));          %Calculation of P(x)
a=(1-lmphi).^((1/(alpha-1))-1);
g=zeros(1,n);
for myi=1:n       %calculation of  g(1,i)
g(1,myi)=dx*sum(phin(myi,:).*p);
end %for end
if(isnan(g)|isinf(g)|(~isreal(g)))  %If any element in g was not a number or was infinite or was complex the
% calculation is broken.
not=1;
break
end
gmk=zeros(n,n);
for m=1:n
for k=1:n
first=(((-(1-lmphi)/beta)*sum((muprime(k,:)-phin(k,:)).*a)+(muprime(k,:)-phin(k,:))).*a)/(alpha*beta);
gmk=dx*sum(first.*phin(m,:));
end
end              %end of forming g(i,j)
v=(mu-g)';
delta=gmk\v;
lambda=lambda+delta';
if(abs(delta./(lambda'))<eps)
for myi=1:length(finalx)  %Calculation of lambdaprime matrix which is n*r
lambdaprime(:,myi)=lambda';
end              %End of lambdaprime calculation
if (n==1)
lmphi=((1-alpha)/alpha)*(lambdaprime.*(muprime-phin));
%Calculation of sum of ((1-alpha)/alpha)*lambda.*(mu-phi) in which each column stands for a single x
else
lmphi=((1-alpha)/alpha)*sum(lambdaprime.*(muprime-phin));
end              %Calculation of sum of ((1-alpha)/alpha)*lambda.*(mu-phi) in which each column stands for a single x
```

```
beta=dx*sum((1-lmphi).^(1/(alpha-1)));          %calculation of beta
p=1/beta*((1-lmphi).^(1/(alpha-1)));            %Calculation of P(x)
break,
end       %End of delta if
end%while end
if(not==0)
p_integral=dx*sum(p);
conditions=zeros(1,n);
for myj=1:n
conditions(1,myj)=dx*sum(phin(myj,:).*p);
end
conditions=[p_integral,conditions]; %Conditions shows how correct our calculations has been
if(size(p)==size(finalx))
plot(finalx,p)
disp('End of the program');
else
finalx(end)=[];
plot(finalx,p)
disp('End of the program');
end
else
disp('No answer exists')
end
end % function end
```