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Sub-state Life Expectancy Estimation Using the Methodology for Small Population

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Abstract

Life expectancy is one of the most preferred indicators in demographic and health analysis. In India, though the office of the Registrar General periodically publishes the life expectancies for the bigger states, for smaller states no figures are available. This paper attempts to estimate the life expectancy at birth along with its standard error for Kohima and Dimapur districts of Nagaland- a smaller tribal state of India. Silcocks and Chiang's revise methodology of life expectancy estimation is used to estimate the life expectancies. Both Silcocks and Chiang's revise method produced almost identical life expectancy estimates. However, Silcocks method estimate life expectancy at birth with a lower standard error. The Monte Carlo Simulation technique is used to generate the distribution of life expectancy at birth and its standard error. Estimated life expectancy at birth has an approximately normal distribution and the simulated result approximated very well to the Silcocks and Chiang's methodology of life expectancy estimation.

Keywords: Life expectancy, mortality, simulation.

1. Introduction

Life expectancy is a key characteristic of human longevity and it is considered as an important indicator to describe the level of mortality in a population. In small areas, especially in developing countries, due to non availability of reliable death registration data, estimation of life expectancy is a challenge for the demographers and public health planners. In India, estimation of life expectancy is usually provided by the Registrar General of India, using data of Sample Registration System (SRS) for 16 major states of India. However, SRS has not gone beyond the 16 major states for estimation of life expectancies and construction of life tables. Till recently, none of the surveys provide estimates of life expectancies at sub state level of India (Registrar General of India 2011b).

Public health authorities make extensive use of mortality data in monitoring the health of their local populations and drawing comparisons with other. The effectiveness of various public health policies designed for the improvement of population health is mostly monitored at sub state level. The Government of India and the state governments monitor the progress of implementation of most of the developmental activities at district level (Bhalotra 2007). Therefore, sub state level measures of mortality are helpful to assess the social and health progress, to identify the effectiveness of government programs, to locate high-risk groups, and even to understand the impact of health related behaviors.

Using regression analysis, Swanson (1989) had examined the life expectancy at birth in Ohio (USA). While Swanson's regression model was found to work well for small populations, it has one condition under which it can produce unreliable estimates (Swanson 1992). Swanson himself advised not to use his model in populations with very few deaths and in population where crude death rate fluctuates substantially from year to year (Swanson 1992).

However, small population also creates specific problems when estimating life expectancies in the usual way. In particular, the absence of deaths at certain age intervals brings the death rate to zero and may distort the construction of life table. Over the years, various attempts have been made by different demographers in this regards. The Chiang method of life table construction is long established and has been widely used by demographers all over the world. It is documented in a WHO publication (Chiang 1978). Using the Monte-Carlo simulation approach, Silcocks et al. (2001), Toson and Baker (2003), and Eayres and Williams (2004) have evaluated methodologies for estimation of life expectancy for small-areas of United Kingdom (UK). Silcocks et al. (2001) investigated the usefulness of life expectancy as a summary measure of mortality at small population level, and derives a formula for the variance of life expectancy results in UK. The Silcocks formula was designed with the intention of calculating the variance of the estimate of life expectancies and its standard error even when there are no deaths in some age intervals.

There is an obstacle in calculating the variance of the estimated life expectancies with the established Chiang (1978) method when there are no deaths in certain age intervals. In such a situation calculation of standard deviation fails. However, an alternative method of calculating the variance of the estimated probability of death, and the consequent possibility of calculating the standard error of the estimated life expectancies using the Chiang (1968) method was identified. Both Chiang's methods are identical in their calculation of life expectancy but differ in their calculation of variance. The original Chiang (1978) method is labeled as Chiang I and revised Chiang (1968) method is labeled as Chiang II.

In India, estimation of life expectancies at sub state level for its various states has been attempted by quite a few researchers. For instance, Sarma and Choudhury (2014) estimated district level life expectancy at birth in India, using data from Coale-Demeny West model life tables, United Nations South Asian model life tables and SRS life tables of India and its major states. Choudhury and Sarma (2014) estimated the life expectancy at birth for the district of the major states of India (2001-05) using infant mortality rate and proportion of population 65 years and above. However, in all these studies secondary data either from population census or from Sample Registration System has been used. Using primary data, no life table has so far been constructed in any part of Nagaland, India which could provide a reliable estimate of the mortality pattern prevailing in sub state areas of the state. So this paper attempts to estimate the life expectancies for Kohima and Dimapur districts of Nagaland (2010-2011), using Silcocks et al. (2001) and Chiang's (1968, 1978) methodology. Also to investigate the shape of the sampling distribution of the estimated life expectancy at birth using Monte Carlo simulation technique.

Nagaland the sixteenth state of the Indian union, with its population 1980602 comprising 1025707 males and 954895 females is one of the smaller states located in the far north-eastern part of India (Registrar General of India 2011a). The state is sharing an international border with the adjacent country of Myanmar on the extreme south-east. It lies geographically between 25 60 and 27 40 latitude North of equator and between the longitudinal lines 93 20 E and 95 15 E. The state is divided into eleven districts.

Kohima district is situated in the south corner of Nagaland at an altitude of 1444 meter above sea level and also the capital of the state. Dimapur district is situated in the plains with an average elevation of 260 meter above sea level. It is worth mentioning that, combining both Kohima and Dimapur district together constitutes an overall area, where more than 31.00% of the population of the state resides (Registrar General of India 2001). Moreover, people from all parts of the state migrate to these two districts. As such, it is assumed to be a proper representation of various segment of population residing in different part of the state.

2. Material and methods

The data used in this paper is a primary one collected through a household survey conducted in Kohima and Dimapur districts of Nagaland.

Both the district has urban as well as rural areas. Urban areas are covered under Municipal Corporation and rural areas are under Rural Development Authority. There are 19 wards under Kohima and 21 wards under Dimapur Municipal Corporation. From each wards, households were selected by systematic sampling techniques so that the entire wards can be covered and a representative sample can be obtained. The sample selection and implementation procedures were designed to ensure that the survey provides statistically valid estimates for population parameters. Accordingly, the data has been collected during May-August 2010 from 958 households covering a sample of size 4640 from 40 municipal wards.

There are four rural development blocks (RDB) each in the Kohima and Dimapur district of Nagaland. As a first stage sampling unit, we have selected two RDBs from each district by simple random sampling without replacement. Now from each of the selected RDBs villages are selected as second stage sampling unit. There are total of 154 inhabited villages under the selected RDBs of Kohima and Dimapur district from which 23 (14.94%) sampling villages are selected for investigation using probability proportional to size sampling. The total population of the selected 23 villages is 12344 (Registrar General of India 2001). The households under the selected villages are the third stage sampling units which are the ultimate sampling units. The households to be included in the sample for each village are selected by systematic sampling techniques. Data were collected during May-September 2010 from a sample of 1150 (550 from Kohima district and 600 from Dimapur district) households from 23 selected villages. We had interviewed 5414 respondents from 1150 selected households.

From each of the selected household, both in rural and urban, basic information such as age and gender of all the family members were collected. Additional information like, whether any family member died during last one year, his/her age at death, gender, medically certified cause of death etc. also was collected. Altogether 10054 respondents were interviewed from both the districts, out of which 5169 were males and 4885 were females, from 2108 households.

3. Estimation of Life Expectancy

The estimation of life expectancies requires the construction of a life table. In this paper, the life expectancies are estimated by the established methodology developed by Silcocks et al. (2001) and Chiang (1968, 1978). Silcocks assumes the cohort population of newborn $l_0 = 1$.

The proportion alive at age x is the product of the probabilities of surviving through each of the preceding age interval.

$$l_x = \prod_{i=0}^{x}(1-q_i); \quad x = 1, 5, 10, \ldots, 85_+$$

where q_i = Probability of dying in the i^{th} age interval. Assuming a constant mortality rate in each age interval implies that survival through the age interval is exponential (Silcocks et al. 2001), so the formula for l_x can be re-expressed as

$$l_x = \exp(-\sum_{i=0}^{x} w_i m_i); \quad x = 1, 5, 10, ..., 85+$$

where m_i = annual all-cause mortality rates in the age interval $i-1$ to i.

w_i = width of the i^{th} age interval.

If l_x values are plotted on the graph, the life expectancy at birth is given by the areas under the l_x curve from birth, divided by the number of person alive at age 0 (i.e. cohort size). For computational purposes, the area can be estimated by the Trapezium Rule (Silcocks et al. 2001).

$$e_0^0 = \frac{1}{2}w_1(l_0+l_1)+\frac{1}{2}w_2(l_1+l_2)+\frac{1}{2}w_3(l_2+l_3)+....+\frac{1}{2}w_n(l_{n-1}+l_n)$$

$$= \frac{1}{2}\sum_{i=1}^{n} w_i \times (l_{i-1}+l_i) = \sum_{i=1}^{n} A_i,$$

where A_i is the area of the i^{th} interval, $l_0 = 1$, $l_n = 0$ and the width of the n^{th} age interval (w_n) is taken to be twice the mean survival in this age group (assuming exponential survival) i.e. $w_n = 2/m_n$.

The calculation of Trapezium Rule is done with the help of a excel spreadsheet.

After obtaining the l_x values, the ${}_nL_x$, ${}_nT_x$ and e_x^0 can be estimated as follows.

The ${}_nL_x$ values which give the person year lived or life table population in the age group x to x $x+n$ can be estimated as: ${}_nL_x = n/2(l_x + l_{x+n})$ (Silcock et al. 2001).

The number of years lived beyond each age x, is obtained by summing ${}_nL_x$ column from the bottom upwards. i.e., ${}_nT_x = {}_nL_x + {}_nL_{x+n} + ... + {}_\infty L_{85+}$, $x = 0, 1, 5, 10, ..., 85+$.

Life expectancy at age x is estimated as: $e_x^0 = T_x / l_x$.

When comparing mortality at small area, for example, area within a district, sub division, block etc., it is important to consider the sampling variation to make a valid inference. Silcocks et al. (2001) derived an analytical formula for the variance of the estimated life expectancy at birth (applying the delta method).

$$Var(e_x^0) = \sum_{i=1}^{n}\left(\frac{\partial e_x^0}{\partial z_i}\right)^2 \times Var(z_i),$$

where
$$\frac{\partial e_x^0}{\partial z_i} = -\left(\frac{1}{2}w_i l_i + \sum_{j=i+1} A_j\right) \quad \text{for } i<n$$
$$= \frac{1}{2}l_{n-1} \quad \text{for } i=n$$
$$z_i = w_i m_i \quad \text{for } i<n \text{ with } Var(z_i) = w_i^2 r_i / N_i^2$$
$$= 2/m_n \quad \text{for } i=n \text{ with } Var(z_n) = 4/r_n m_n^2.$$

Chiang's method is based on the derivation of a relation for ${}_nL_x$ (number of years lived during the age interval x to $x + n$) in terms of ${}_na_x$, where ${}_na_x$ is the fraction of interval between x^{th} and $(x+n)^{th}$ birthdays lived on an average by those dying within that interval. ${}_nL_x$ is calculated as follows:

$${}_nL_x = n \times l_{x+n} + n \times {}_na_x \times {}_nd_x.$$

Assume that for all those dying in an age interval, the deaths are spread evenly throughout the period. The average number of years lived within each interval by those who died can be assumed to be half the width of the age band. Thus ${}_na_x$ = 0.5 is used for every age interval except for age under 1 year. The value of ${}_1a_0$ varies with infant mortality level. In general, as the mortality level improves for infant, the improvement is primarily in the latter part of the first year of life. Consequently, lower the infant death rate, the smaller is the value of ${}_1a_0$. Chiang (1984) suggested values for ${}_1a_0$ according to the level of infant mortality. There are 18 infant deaths per 1000 live births in Nagaland (Registrar General of India 2014), as such ${}_1a_0$ is assumes to be 0.09 (Chiang 1984).

The formula for computing the probability of dying within n years from the x^{th} birthday can be expressed as: ${}_nq_x = \dfrac{n \times {}_nm_x}{1 + n \times (1 - {}_na_x) \times {}_nm_x}$; ${}_np_x = 1 - {}_nq_x$,

where ${}_nm_x$ is the age-specific death rate in the age interval x to $x + n$.

Once the basic life table function ${}_nq_x$ has been estimated the remaining functions are calculated in a straightforward manner.

Cumulative number of years lived by the cohort population in the age interval and all subsequent age intervals are obtained as: ${}_nT_x = {}_nL_x + {}_nL_{x+n} + ... + {}_\infty L_{85+}$; $T_{85+} = l_{85+} / m_{85+}$.

The life expectancy at age x is estimated as: $e_x^0 = T_x / l_x$.

Sampling variation is especially important when the life tables are based on sample surveys. In that case, deaths are viewed as a random sample of the observed population of deaths that have occurred over time. Chiang (1968) assumed that ${}_nD_x$ (the count of actual death in an age-group) is a binomial random variable in ${}_nP_x$ (trials) with fixed probability of dying ${}_nq_x$. This leads to an estimate for the variance of the age specific probability of death as:

$$Var({}_nq_x) = \frac{n^2 \times {}_nm_x \times (1 - {}_na_x \times n \times {}_nm_x)}{{}_nP_x \times \{1 + (1 - {}_na_x) \times n \times {}_nm_x\}^3} \qquad \text{(Chiang 1968)}$$

The variance of life expectancy is

$$Var(e_x{}^0) = \frac{\sum l_x{}^2 \{(1 - {}_na_x) \times n + e^0{}_{x+n}\}^2 \times Var({}_nq_x)}{l_x{}^2} \qquad \text{(Chiang 1968)}$$

4. Simulation Study

In this study, The Monte Carlo simulation technique is carried out in a multi step procedure as follows.

Given the age group wise population size and number of deaths, let us assume that number of deaths in each age band is Poisson. Then one simulation consists of the following steps:

Step – I For each age group, a Poisson variate is generated with mean given by the original age specific number of deaths.

Step – II If the observed number of deaths is large (say 50 or more) then a normal approximation to a Poisson can be used, with rounding to give integer numbers of "deaths", otherwise we follow Step – I.

Step – III Combining these age specific death information along with the age wise populations a new set of age specific mortality rates is calculated.

Step – IV For each set of age specific mortality rate, corresponding life tables are constructed and estimated the area under the l_x curve (life expectancy at birth).

Simulation is repeated more than thousand times storing each result. A histogram of the stored results indicates the shape of the sampling distribution of the life expectancy at birth.

Computer programs using Microsoft excel were run to generate Monte Carlo simulations. For each combination of age-specific death rates and population size the simulations were repeated 10,000 times, for both males and females. Silcocks et al. (2001) and Toson and Baker (2003) had used number of simulations as 2000. A large number was chosen in order to reduce the statistical errors of the simulations outcome to an acceptable minimum.

As life expectancy at birth is distributed normally (Silcocks et al. 2001), the standard error of normal sample standard deviation S is given as $\sigma_S \approx \sigma \frac{1}{\sqrt{2(n-1)}}$

where σ is unknown standard deviation is estimated by S, and n is the sample size (Ahn and Fessler 2003). If number of simulation, n = 10,000, the standard error amounts to be about 0.71 percent of the standard deviation, which, being relatively small, may nonetheless considerably affect the outcome of the estimation.

The life expectancy results were calculated for each combination and the mean life expectancy at birth were compared with the corresponding life expectancies calculated using the Silcocks and Chiang's methodology. The same was done for standard error.

For small populations some age group may have zero deaths. This is a problem that arises when calculating life expectancy at local area level. For such age group, the Poisson distribution will be invalid. In such populations with no deaths in some age group, there will be a decrease in the estimated sampling variation. The standard error will be underestimated because the contribution to the variance for those intervals with no deaths is equal to zero. As a possible correction, Silcocks et al. (2001) explored the problem of zero deaths by imputing a small positive value (0.693). The value, 0.693 is the value for the Poisson rate for which the chance of observing 0 events is a half. In the present study no age band had more than 50 deaths and the Poisson model was simulated without a normal approximation.

5. Results and Discussion

The age and sex specific distribution of death in the sample population collected from Kohima and Dimapur districts of Nagaland are presented in Table 1, and also illustrated in Figures 1 and 2.

Tables 2, 3 and 4 show the estimation of life expectancies using Silcocks methodology. ${}_n m_x$ represents the cumulative age-specific death rate over the age interval x to $x + m$. T_x represents the cumulative number of years lived by the cohort population in the age interval and all subsequent age intervals. The various values of life expectancies along with its variance and standard error are presented for males, females and persons respectively. It is observed that life expectancy at birth for males, females and persons are 66.17, 67.84 and 67.07 years respectively.

Table 1 Age and sex specific distribution of death in the sample population

Age Group	Death			Population			ASDR		
	Male	Female	Person	Male	Female	Person	Male	Female	Person
0-1	1	1	2	43	29	72	0.0233	0.0345	0.0278
1-4	0	0	0	325	261	586	0.0000	0.0000	0.0000
5-9	1	0	1	465	414	879	0.0022	0.0000	0.0011
10-14	1	1	2	565	486	1051	0.0018	0.0021	0.0019
15-19	2	0	2	624	608	1232	0.0032	0.0000	0.0016
20-24	1	1	2	577	642	1219	0.0017	0.0016	0.0016
25-29	1	1	2	451	559	1010	0.0022	0.0018	0.0020
30-34	3	1	4	349	382	731	0.0086	0.0026	0.0055
35-39	2	1	3	349	368	717	0.0057	0.0027	0.0042
40-44	2	2	4	339	301	640	0.0059	0.0066	0.0063
45-49	3	2	5	301	285	586	0.0100	0.0070	0.0085
50-54	3	2	5	259	199	458	0.0116	0.0101	0.0109
55-59	2	3	5	189	122	311	0.0106	0.0246	0.0161
60-64	4	2	6	146	85	231	0.0274	0.0235	0.026
65-69	3	2	5	82	60	142	0.0366	0.0333	0.0352
70-74	2	2	4	49	34	83	0.0408	0.0588	0.0482
75-79	2	2	4	28	19	47	0.0714	0.1053	0.0851
80-84	2	1	3	15	17	32	0.1333	0.0588	0.0938
85+	1	2	3	9	18	27	0.1111	0.1111	0.1111
Total				5169	4885	10054			

Note: ASDR (Age Specific Death Rates)

Figure 1 Distribution of population by age

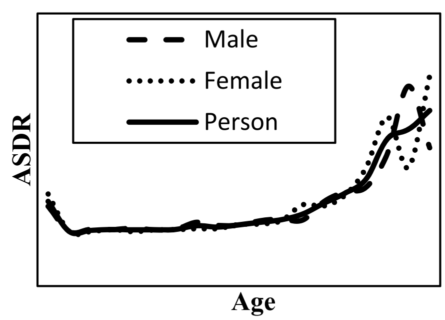

Figure 2 Distribution of death rates by age

Table 2 Life Expectancies for Kohima and Dimapur district of Nagaland, (Silcocks, Males)

Age Group	x	w_i	m_i	$w_i \times m_i$	l_x	L_x	T_x (or A_j)
0-1	0	1	0.0233	0.0233	1.0000	0.9885	66.1669
1-4	1	4	0.0000	0.0000	0.9770	3.9081	65.1784
5-9	5	5	0.0022	0.0108	0.9770	4.8589	61.2703
10-14	10	5	0.0018	0.0088	0.9666	4.8115	56.4114
15-19	15	5	0.0032	0.0160	0.9580	4.7522	51.5999
20-24	20	5	0.0017	0.0087	0.9428	4.6937	46.8477
25-29	25	5	0.0022	0.0111	0.9347	4.6476	42.1540
30-34	30	5	0.0086	0.0430	0.9244	4.5247	37.5063
35-39	35	5	0.0057	0.0287	0.8855	4.3649	32.9817
40-44	40	5	0.0059	0.0295	0.8605	4.2399	28.6168
45-49	45	5	0.0100	0.0498	0.8355	4.0758	24.3769
50-54	50	5	0.0116	0.0579	0.7949	3.8624	20.3011
55-59	55	5	0.0106	0.0529	0.7501	3.6540	16.4387
60-64	60	5	0.0274	0.1370	0.7115	3.3296	12.7847
65-69	65	5	0.0366	0.1829	0.6204	2.8427	9.4551
70-74	70	5	0.0408	0.2041	0.5167	2.3449	6.6124
75-79	75	5	0.0714	0.3571	0.4213	1.7902	4.2675
80-84	80	5	0.1333	0.6667	0.2948	1.1153	2.4773
85+	85	18	0.1111		0.1513	1.3621	1.3621
	111				0.0000		

Table 2 (Continued)

Age Group	e_x^0	dA/d(z_x)	(dA/d(z_x))^2	var(z_x)	var(z_x)×(dA/d(z_x))^2	Var(e_x^0)	SE(e_x^0)
0-1	66.17	-33.0777	1094.1346	0.000541	0.5917	2.82	1.68
1-4	66.71	-32.5892	1062.0559	0.000000	0.0000	2.23	1.49
5-9	62.71	-30.6221	937.7138	0.000116	0.1084	2.23	1.49
10-14	58.36	-28.1951	794.9614	0.000078	0.0623	2.12	1.46
15-19	53.86	-25.7809	664.6549	0.000128	0.0853	2.06	1.43
20-24	49.69	-23.4137	548.2010	0.000075	0.0412	1.97	1.40
25-29	45.10	-21.0641	443.6966	0.000123	0.0545	1.93	1.39
30-34	40.57	-18.7046	349.8604	0.000616	0.2154	1.88	1.37
35-39	37.25	-16.4596	270.9174	0.000411	0.1112	1.66	1.29
40-44	33.26	-14.2771	203.8359	0.000435	0.0887	1.55	1.24
45-49	29.18	-12.1377	147.3234	0.000828	0.1220	1.46	1.21
50-54	25.54	-10.0947	101.9020	0.001118	0.1139	1.34	1.16
55-59	21.91	-8.1710	66.7655	0.001400	0.0935	1.22	1.11
60-64	17.97	-6.2785	39.4196	0.004691	0.1849	1.13	1.06
65-69	15.24	-4.5979	21.1406	0.011154	0.2358	0.95	0.97
70-74	12.80	-3.1870	10.1568	0.020825	0.2115	0.71	0.84
75-79	10.13	-1.9756	3.9029	0.063776	0.2489	0.50	0.71
80-84	8.40	-1.0594	1.1223	0.222222	0.2494	0.25	0.50
85+	9.00	0.0757	0.0057	0.049383	0.0003	0.00	0.02

Table 3 Life Expectancies for Kohima and Dimapur district of Nagaland (Silcocks, Females)

Age Group	x	w_i	m_i	$w_i \times m_i$	l_x	L_x	T_x (or A_j)
0-1	0	1	0.0345	0.0345	1.0000	0.9831	67.8376
1-4	1	4	0.0000	0.0000	0.9661	3.8644	66.8546
5-9	5	5	0.0000	0.0000	0.9661	4.8305	62.9901
10-14	10	5	0.0021	0.0103	0.9661	4.8058	58.1596
15-19	15	5	0.0000	0.0000	0.9562	4.7811	53.3538
20-24	20	5	0.0016	0.0078	0.9562	4.7625	48.5727
25-29	25	5	0.0018	0.0089	0.9488	4.7229	43.8102
30-34	30	5	0.0026	0.0131	0.9403	4.6712	39.0873
35-39	35	5	0.0027	0.0136	0.9281	4.6093	34.4161
40-44	40	5	0.0066	0.0332	0.9156	4.5032	29.8069
45-49	45	5	0.0070	0.0351	0.8857	4.3520	25.3037
50-54	50	5	0.0101	0.0503	0.8551	4.1709	20.9516
55-59	55	5	0.0246	0.1230	0.8132	3.8309	16.7807
60-64	60	5	0.0235	0.1176	0.7191	3.3962	12.9498
65-69	65	5	0.0333	0.1667	0.6393	2.9513	9.5536
70-74	70	5	0.0588	0.2941	0.5412	2.3611	6.6023
75-79	75	5	0.1053	0.5263	0.4033	1.6038	4.2412
80-84	80	5	0.0588	0.2941	0.2382	1.0395	2.6373
85+	85	18	0.1111		0.1775	1.5979	1.5979
	99				0.0000		

Table 3 (Continued)

Age Group	e_x^0	dA/d(z_x)	(dA/d(z_x))^2	var(z_x)	var(z_x)×(dA/d(z_x))^2	Var(e_x^0)	SE(e_x^0)
0-1	67.84	-33.9103	1149.9108	0.001189	1.3673	4.00	1.99
1-4	69.20	-33.4273	1117.3832	0.000000	0.0000	2.63	1.62
5-9	65.20	-31.4951	991.9395	0.000000	0.0000	2.63	1.62
10-14	60.20	-29.0674	844.9166	0.000106	0.0894	2.63	1.62
15-19	55.80	-26.6769	711.6574	0.000000	0.0000	2.54	1.59
20-24	50.80	-24.2771	589.3772	0.000061	0.0357	2.54	1.59
25-29	46.17	-21.8945	479.3707	0.000080	0.0384	2.51	1.58
30-34	41.57	-19.5284	381.3575	0.000171	0.0653	2.47	1.57
35-39	37.08	-17.1924	295.5792	0.000185	0.0546	2.40	1.55
40-44	32.55	-14.8660	220.9988	0.000552	0.1220	2.35	1.53
45-49	28.57	-12.6137	159.1044	0.000616	0.0979	2.23	1.49
50-54	24.50	-10.4234	108.6477	0.001263	0.1372	2.13	1.46
55-59	20.63	-8.2727	68.4382	0.005039	0.3449	1.99	1.41
60-64	18.01	-6.3751	40.6420	0.006920	0.2813	1.65	1.28
65-69	14.94	-4.6541	21.6607	0.013889	0.3008	1.37	1.17
70-74	12.20	-3.1288	9.7893	0.043253	0.4234	1.06	1.03
75-79	10.52	-1.9143	3.6645	0.138504	0.5076	0.64	0.80
80-84	11.07	-1.2428	1.5445	0.086505	0.1336	0.13	0.37
85+	9.00	0.0888	0.0079	0.024691	0.0002	0.00	0.01

Table 4 Life Expectancies for Kohima and Dimapur district of Nagaland (Silcocks, Persons)

Age Group	x	w_i	m_i	$w_i \times m_i$	l_x	L_x	T_x (or A_j)
0-1	0	1	0.0278	0.0278	1.0000	0.9863	67.0676
1-4	1	4	0.0017	0.0068	0.9726	3.8772	66.0813
5-9	5	5	0.0011	0.0057	0.9660	4.8162	62.2041
10-14	10	5	0.0010	0.0048	0.9605	4.7911	57.3879
15-19	15	5	0.0016	0.0081	0.9560	4.7604	52.5967
20-24	20	5	0.0016	0.0082	0.9482	4.7217	47.8363
25-29	25	5	0.0020	0.0099	0.9405	4.6792	43.1146
30-34	30	5	0.0055	0.0274	0.9312	4.5932	38.4353
35-39	35	5	0.0042	0.0209	0.9061	4.4835	33.8421
40-44	40	5	0.0063	0.0313	0.8873	4.3683	29.3586
45-49	45	5	0.0085	0.0427	0.8600	4.2103	24.9903
50-54	50	5	0.0109	0.0546	0.8241	4.0111	20.7800
55-59	55	5	0.0161	0.0804	0.7803	3.7509	16.7689
60-64	60	5	0.0260	0.1299	0.7201	3.3810	13.0180
65-69	65	5	0.0352	0.1761	0.6324	2.9066	9.6370
70-74	70	5	0.0482	0.2410	0.5303	2.3675	6.7304
75-79	75	5	0.0851	0.4255	0.4167	1.7226	4.3629
80-84	80	5	0.0938	0.4688	0.2723	1.1067	2.6403
85+	85	18	0.1111		0.1704	1.5336	1.5336
	103				0.0000		

Table 4 (Continued)

Age Group	e_x^0	dA/d(z_x)	(dA/d(z_x))^2	var(z_x)	var(z_x)×(dA/d(z_x))^2	Var(e_x^0)	SE(e_x^0)
0-1	67.07	-33.5270	1124.0567	0.000386	0.4337	1.66	1.29
1-4	67.94	-33.0340	1091.2475	0.000047	0.0508	1.23	1.11
5-9	64.39	-31.0952	966.9121	0.000032	0.0313	1.17	1.08
10-14	59.75	-28.6882	823.0151	0.000023	0.0186	1.14	1.07
15-19	55.02	-26.2887	691.0960	0.000033	0.0228	1.12	1.06
20-24	50.45	-23.9085	571.6147	0.000034	0.0192	1.10	1.05
25-29	45.84	-21.5457	464.2169	0.000049	0.0228	1.08	1.04
30-34	41.27	-19.1863	368.1124	0.000187	0.0689	1.06	1.03
35-39	37.35	-16.8976	285.5293	0.000146	0.0417	0.99	1.00
40-44	33.09	-14.6452	214.4816	0.000244	0.0524	0.95	0.97
45-49	29.06	-12.4502	155.0086	0.000364	0.0564	0.90	0.95
50-54	25.22	-10.3353	106.8178	0.000596	0.0637	0.84	0.92
55-59	21.49	-8.3091	69.0416	0.001292	0.0892	0.78	0.88
60-64	18.08	-6.3994	40.9521	0.002811	0.1151	0.69	0.83
65-69	15.24	-4.6909	22.0045	0.006199	0.1364	0.57	0.76
70-74	12.69	-3.2233	10.3895	0.014516	0.1508	0.44	0.66
75-79	10.47	-2.0009	4.0037	0.045269	0.1812	0.29	0.53
80-84	9.70	-1.1928	1.4228	0.073242	0.1042	0.10	0.32
85+	9.00	0.0852	0.0073	0.016461	0.0001	0.00	0.01

Tables 5, 6 and 7 show the estimation of life expectancies using Chiang's methodology. The various values of life expectancies along with its variance and standard error are presented for males, females and persons respectively. It is observed that life expectancy at birth for males, females and persons are 66.69, 67.44 and 67.02 years respectively. Life expectancy at birth calculated by Silcocks and Chiang's methods are compared and found to be very similar.

A comparison of l_x values for males, females and persons are presented in Figure 3, 4 and 5 respectively. Life expectancy at birth is the area under the life table l_x plot from birth divided by the number of persons in the cohort (l_0). Likewise, life expectancy at age can be found. The plotted l_x values illustrate that Silcocks and Chiang's values are almost indistinguishable.

Figures 6, 7 and 8 show the sampling distribution of life expectancy at birth generated by Monte Carlo simulation technique with a superimposed normal curve (n = 10000) for males, females and persons respectively. The result indicates that life expectancy at birth has an approximately normal distribution. The mean life expectancy at birth is observed as 65.77 years for males, 66.67 years for females and 66.50 years for persons. The simulated result approximated well to Silcocks and Chiang's methodology of life expectancy estimation.

The standard error and lower and upper limit confidence interval for life expectancy at births are presented in Table 8. Ninety-five percent confidence limits were calculated (1.96×Standard Error) for life expectancy at birth based on the underlying death rates observed from the sample. On comparing the estimated life expectancy at birth with that of simulation based results it is observed that – for males, the Silcocks (Chiang) formula gave a life expectancy at birth of 66.17 (66.69) years with standard error 1.6789 (3.2667), while the simulation gave an average life expectancy at birth value of 65.77 years with standard error 0.0223. Similarly, on comparing the female life expectancy at birth, it is found that Silcocks (Chiang) formula gave a life expectancy at birth of 67.84 (67.44) years with standard error 1.999 (3.5366), while the simulation gave an average life expectancy at birth of 66.67 years with standard error 0.0267. The width of the confidence interval for life expectancy at birth using Silcocks formula is found to be 6.58 years for males and 7.84 years for females. However, the width of the confidence interval for life expectancy at birth using Chiang's formula is 12.8 years for males and 13.87 years for females.

Table 5 Life Expectancies for Kohima and Dimapur district of Nagaland, (Chiang II, Males)

Age Group	x	n	${}_na_x$	Pop (${}_nP_x$)	${}_nD_x$	${}_nm_x$	${}_nq_x$	${}_np_x$
0-1	0	1	0.09	43	1	0.023256	0.022774	0.977226
1-4	1	4	0.5	325	0	0.000000	0.000000	1.000000
5-9	5	5	0.5	465	1	0.002151	0.010695	0.989305
10-14	10	5	0.5	565	1	0.001770	0.008811	0.991189
15-19	15	5	0.5	624	2	0.003205	0.015898	0.984102
20-24	20	5	0.5	577	1	0.001733	0.008628	0.991372
25-29	25	5	0.5	451	1	0.002217	0.011025	0.988975
30-34	30	5	0.5	349	3	0.008596	0.042076	0.957924
35-39	35	5	0.5	349	2	0.005731	0.028249	0.971751
40-44	40	5	0.5	339	2	0.005900	0.029070	0.970930
45-49	45	5	0.5	301	3	0.009967	0.048622	0.951378
50-54	50	5	0.5	259	3	0.011583	0.056285	0.943715
55-59	55	5	0.5	189	2	0.010582	0.051546	0.948454
60-64	60	5	0.5	146	4	0.027397	0.128205	0.871795
65-69	65	5	0.5	82	3	0.036585	0.167598	0.832402
70-74	70	5	0.5	49	2	0.040816	0.185185	0.814815
75-79	75	5	0.5	28	2	0.071429	0.303030	0.696970
80-84	80	5	0.5	15	2	0.133333	0.500000	0.500000
85+	85	18	0.5	13	1	0.076923	1.000000	0.000000

Table 5 (Continued)

Age Group	l_x	${}_nL_x$	T_x	e_x^0	$Var(q_x)$	$Var(e_x^0)$	$SE(e_x^0)$
0-1	100000	97928	6668950	66.69	0.000507	10.6713	3.2667
1-4	97723	390890	6571022	67.24	0.000000	8.7094	2.9512
5-9	97723	486000	6180132	63.24	0.000113	8.7094	2.9512
10-14	96677	481258	5694131	58.90	0.000077	8.4628	2.9091
15-19	95826	475320	5212874	54.40	0.000124	8.3604	2.8914
20-24	94302	469477	4737554	50.24	0.000074	8.2755	2.8767
25-29	93489	464866	4268077	45.65	0.000120	8.2461	2.8716
30-34	92458	452563	3803211	41.13	0.000565	8.1969	2.8630
35-39	88568	436583	3350648	37.83	0.000388	7.9308	2.8162
40-44	86066	424074	2914064	33.86	0.000410	7.8558	2.8028
45-49	83564	407661	2489991	29.80	0.000750	7.8793	2.8070
50-54	79501	386317	2082330	26.19	0.000997	8.0233	2.8325
55-59	75026	365462	1696013	22.61	0.001260	8.3036	2.8816
60-64	71159	332986	1330551	18.70	0.003582	8.6013	2.9328
65-69	62036	284186	997565	16.08	0.007794	9.6898	3.1128
70-74	51639	234287	713379	13.81	0.013971	10.9906	3.3152
75-79	42076	178504	479092	11.39	0.032000	12.4962	3.5350
80-84	29326	109971	300588	10.25	0.062500	15.0156	3.8750
85+	14663	190617	190617	13.00	0.000000	0.0000	0.0000

Table 6 Life Expectancies for Kohima and Dimapur district of Nagaland, (Chiang II, Females)

Age Group	x	n	${}_na_x$	Pop (${}_nP_x$)	${}_nD_x$	${}_nm_x$	${}_nq_x$	${}_np_x$
0-1	0	1	0.09	29	1	0.034483	0.033434	0.966566
1-4	1	4	0.5	261	0	0.000000	0.000000	1.000000
5-9	5	5	0.5	414	0	0.000000	0.000000	1.000000
10-14	10	5	0.5	486	1	0.002058	0.010235	0.989765
15-19	15	5	0.5	608	0	0.000000	0.000000	1.000000
20-24	20	5	0.5	642	1	0.001558	0.007758	0.992242
25-29	25	5	0.5	559	1	0.001789	0.008905	0.991095
30-34	30	5	0.5	382	1	0.002618	0.013004	0.986996
35-39	35	5	0.5	368	1	0.002717	0.013495	0.986505
40-44	40	5	0.5	301	2	0.006645	0.032680	0.967320
45-49	45	5	0.5	285	2	0.007018	0.034483	0.965517
50-54	50	5	0.5	199	2	0.010050	0.049020	0.950980
55-59	55	5	0.5	122	3	0.024590	0.115830	0.884170
60-64	60	5	0.5	85	2	0.023529	0.111111	0.888889
65-69	65	5	0.5	60	2	0.033333	0.153846	0.846154
70-74	70	5	0.5	34	2	0.058824	0.256410	0.743590
75-79	75	5	0.5	19	2	0.105263	0.416667	0.583333
80-84	80	5	0.5	17	1	0.058824	0.256410	0.743590
85+	85	18	0.5	14	2	0.142857	1.000000	0.000000

Table 6 (Continued)

Age Group	l_x	${}_nL_x$	T_x	e_x^0	$Var(q_x)$	$Var(e_x^0)$	$SE(e_x^0)$
0-1	100000	96990	6744310	67.44	0.001081	12.5076	3.5366
1-4	96657	386622	6647320	68.77	0.000000	7.7723	2.7879
5-9	96657	483278	6260698	64.77	0.000000	7.7723	2.7879
10-14	96657	480804	5777420	59.77	0.000104	7.7723	2.7879
15-19	95667	478331	5296616	55.37	0.000000	7.5794	2.7531
20-24	95667	476476	4818285	50.37	0.000060	7.5794	2.7531
25-29	94925	472507	4341809	45.74	0.000079	7.5573	2.7490
30-34	94080	467335	3869302	41.13	0.000167	7.5414	2.7462
35-39	92856	461144	3401967	36.64	0.000180	7.4790	2.7348
40-44	91603	450527	2940823	32.10	0.000517	7.4639	2.7320
45-49	88610	435405	2490295	28.10	0.000574	7.4597	2.7313
50-54	85554	417282	2054890	24.02	0.001143	7.5691	2.7512
55-59	81360	383238	1637608	20.13	0.003954	7.7226	2.7790
60-64	71936	339696	1254371	17.44	0.005487	7.8679	2.8050
65-69	63943	295120	914675	14.30	0.010014	7.9968	2.8279
70-74	54106	235844	619555	11.45	0.024444	8.4470	2.9064
75-79	40233	159253	383711	9.54	0.050637	8.8711	2.9784
80-84	23469	102300	224458	9.56	0.048888	4.4122	2.1005
85+	17451	122158	122158	7.00	0.000000	0.0000	0.0000

Table 7 Life Expectancies for Kohima and Dimapur district of Nagaland, (Chiang II, Persons)

Age Group	x	n	${}_na_x$	Pop (${}_nP_x$)	${}_nD_x$	${}_nm_x$	${}_nq_x$	${}_np_x$
0-1	0	1	0.09	72	2	0.027778	0.027093	0.972907
1-4	1	4	0.5	586	1	0.001706	0.006803	0.993197
5-9	5	5	0.5	879	1	0.001138	0.005672	0.994328
10-14	10	5	0.5	1051	1	0.000951	0.004746	0.995254
15-19	15	5	0.5	1232	2	0.001623	0.008084	0.991916
20-24	20	5	0.5	1219	2	0.001641	0.008170	0.991830
25-29	25	5	0.5	1010	2	0.001980	0.009852	0.990148
30-34	30	5	0.5	731	4	0.005472	0.026991	0.973009
35-39	35	5	0.5	717	3	0.004184	0.020704	0.979296
40-44	40	5	0.5	640	4	0.006250	0.030769	0.969231
45-49	45	5	0.5	586	5	0.008532	0.041771	0.958229
50-54	50	5	0.5	458	5	0.010917	0.053135	0.946865
55-59	55	5	0.5	311	5	0.016077	0.077280	0.922720
60-64	60	5	0.5	231	6	0.025974	0.121951	0.878049
65-69	65	5	0.5	142	5	0.035211	0.161812	0.838188
70-74	70	5	0.5	83	4	0.048193	0.215054	0.784946
75-79	75	5	0.5	47	4	0.085106	0.350877	0.649123
80-84	80	5	0.5	32	3	0.093750	0.379747	0.620253
85+	85	18	0.5	27	3	0.111111	1.000000	0.000000

Table 7 (Continued)

Age Group	l_x	${}_nL_x$	T_x	e_x^0	$Var(q_x)$	$Var(e_x^0)$	$SE(e_x^0)$
0-1	100000	97535	6702492	67.02	0.000357	5.5189	2.3492
1-4	97291	387839	6604957	67.89	0.000046	4.0450	2.0112
5-9	96629	481774	6217118	64.34	0.000032	3.8955	1.9737
10-14	96081	479264	5735344	59.69	0.000022	3.8149	1.9532
15-19	95625	476191	5256080	54.97	0.000032	3.7766	1.9434
20-24	94852	472321	4779889	50.39	0.000033	3.7463	1.9355
25-29	94077	468067	4307568	45.79	0.000048	3.7298	1.9313
30-34	93150	459464	3839501	41.22	0.000177	3.7107	1.9263
35-39	90636	448488	3380037	37.29	0.000140	3.6231	1.9034
40-44	88759	436969	2931549	33.03	0.000229	3.5937	1.8957
45-49	86028	421157	2494581	29.00	0.000334	3.5832	1.8929
50-54	82435	401223	2073423	25.15	0.000535	3.6240	1.9037
55-59	78055	375193	1672200	21.42	0.001102	3.7008	1.9237
60-64	72022	338154	1297008	18.01	0.002176	3.8022	1.9499
65-69	63239	290614	958853	15.16	0.004389	4.0511	2.0127
70-74	53006	236534	668239	12.61	0.009076	4.3404	2.0834
75-79	41607	171538	431705	10.38	0.019979	4.6025	2.1453
80-84	27008	109400	260167	9.63	0.029815	3.9430	1.9857
85+	16752	150767	150767	9.00	0.000000	0.0000	0.0000

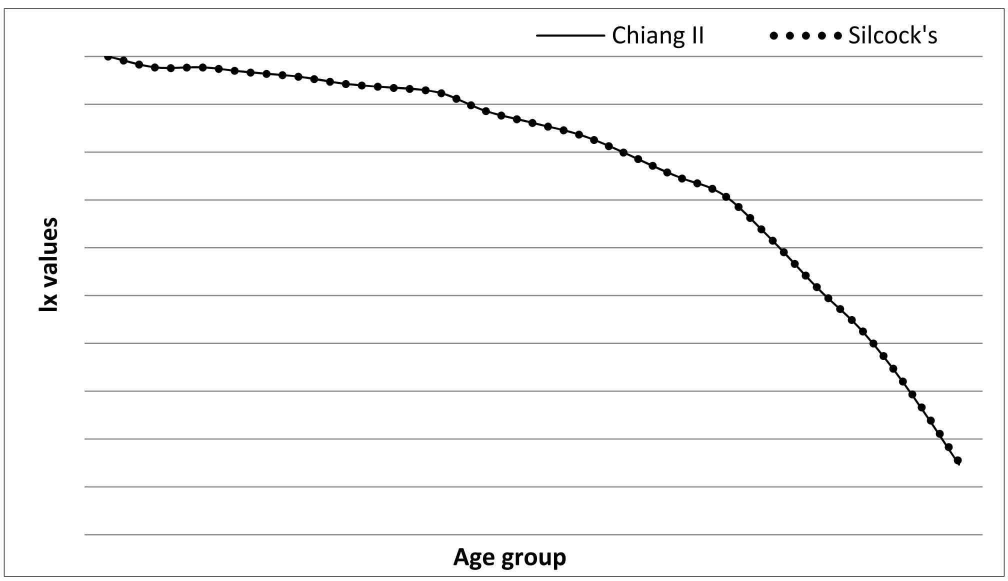

Figure 3 Comparison of l_x values - Chiang II and Silcocks methods (Males)

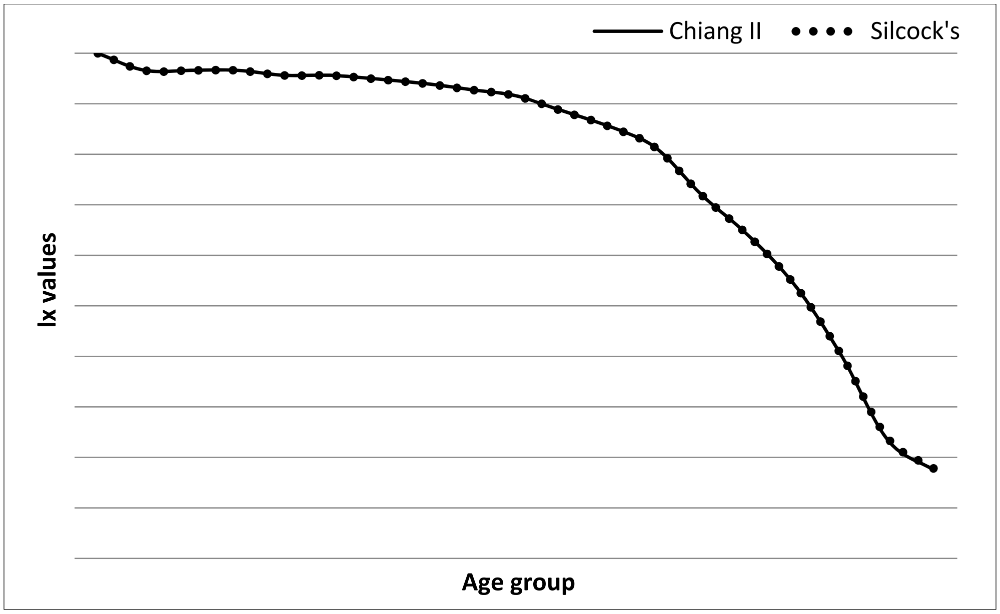

Figure 4 Comparison of l_x values - Chiang II and Silcocks methods (Females)

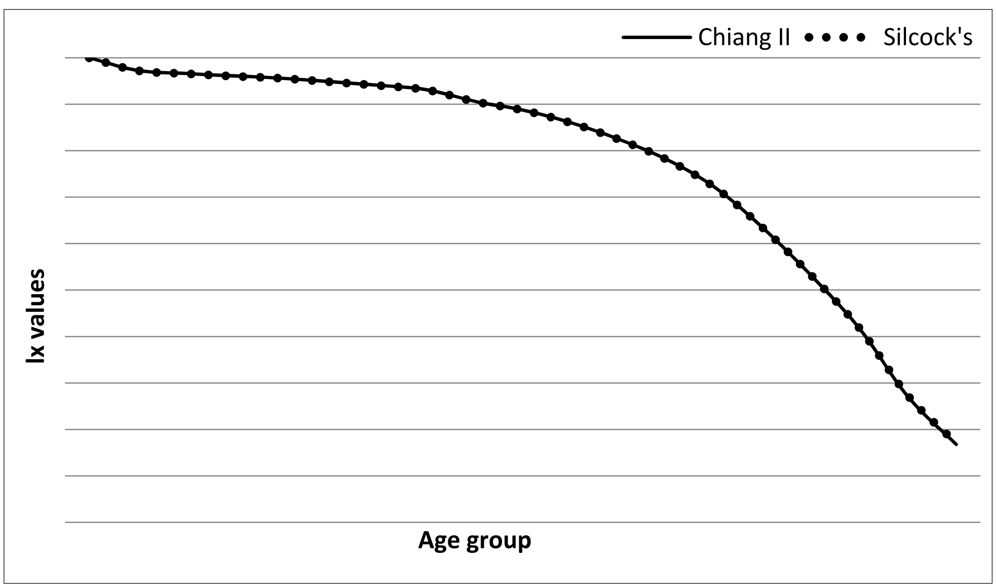

Figure 5 Comparison of l_x values - Chiang II and Silcocks methods (Persons)

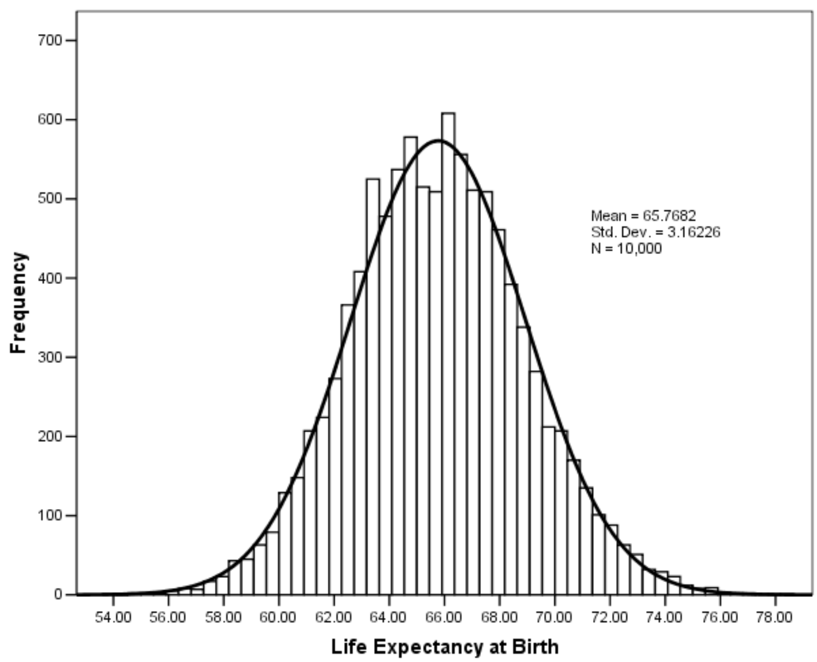

Figure 6 Simulated sampling distribution of life expectancy at birth with superimposed normal curve (Males)

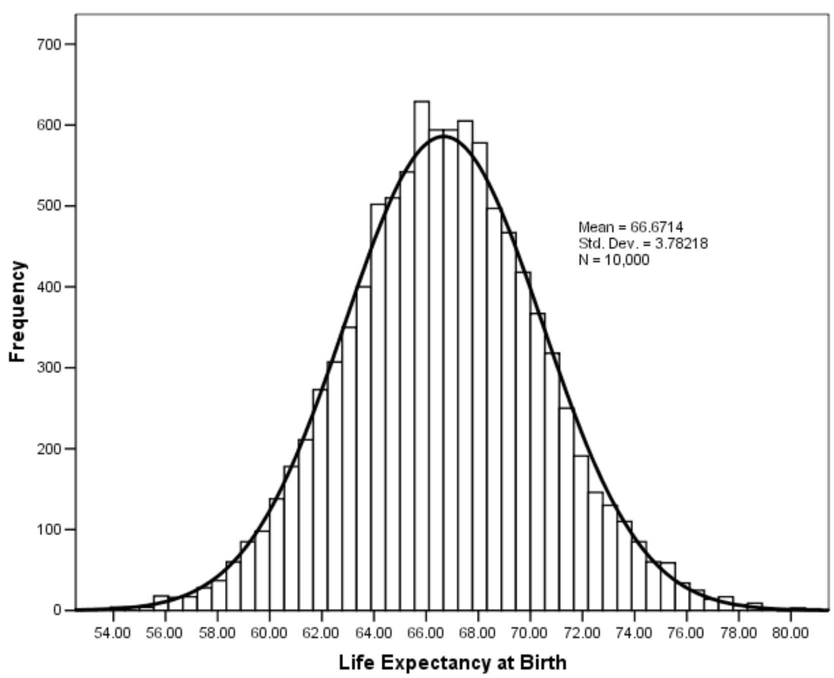

Figure 7 Simulated sampling distribution of life expectancy at birth with superimposed normal curve (Females)

Table 8 Comparison of life expectancy at birth and standard error

Sex	Life expectancy at birth (in years)			Standard Error			95% Interval for Life expectancy at birth			
							Silcocks		Chiang II	
	Silcocks	Chiang II	Simulation	Silcocks	Chiang II	Simulation	Lower	Upper	Lower	Upper
Males	66.17	66.69	65.77	1.6789	3.2667	0.0223	62.88	69.46	60.29	73.09
Females	67.84	67.44	66.67	1.9999	3.5366	0.0267	63.92	71.76	60.51	74.38
Persons	67.07	67.02	66. 50	1.2881	2.3492	0.0177	64.54	69.59	62.42	71.63

Theoretical considerations show that statistical efficiency of different methods is affected by its population size. Toson and Baker (2003) has illustrated the width of the 95% confidence intervals for different population sizes in their studies in United Kingdom. It was found that the width of the confidence interval is likely to be around 5 years if the sample size is approximately 10,000. For populations of less than 5,000 it was likely to be approaching eight years and for populations of 1,000 the likely interval width would be over 15 years. In our sample, we had interviewed 5169 males, resulting a 6.58 years of difference between the upper and lower confidence interval. For females, the width of the confidence interval is found to be 7.84 years when the sample size was 4885. The difference between the upper and lower confidence interval, observed in the present study coincides with the result obtained by Toson and Baker (2003). Silcocks and Chiang's methods produced almost identical life expectancy estimates. In the present study Silcocks methodology is in preference over Chiang's, as the former method estimated life expectancy at birth with a lower standard error and smaller confidence interval.

References

Ahn S, Fessler A. Standard errors of mean, variance, and standard deviation estimators. Technical Report. University of Michigan, [cited 2012 Jan 15], Available from: http://www.eecs.umich.edu/~fessler/papers/lists/files/tr/stderr.pdf [accessed Jan 15, 2012].

Bhalotra SR. Spending to save? State health expenditure and infant mortality in India. Discussion paper No. 2914. Germany: University of Bristol; 2007.

Chiang CL. Introduction to Stochastic Processes in Biostatistics. New York: Wiley; 1968.

Chiang CL. Life Table and Mortality Analysis. Switzerland: World Health Organization Geneva; 1978.

Chiang CL. The Life Table and its Applications. Florida: Robert E. Krieger Publication; 1984.

Choudhury L, Sarma R. Indirect estimation of life expectancy at birth for the district of the major states of India (2001-05). Indian J. Public Health. 2014; 5: 220-225.

Eayres D, Williams ES. Evaluation of methodologies for small area life expectancy estimation. J. Epidemiol. Commun. Health. 2004; 58: 243-249.

Registrar General of India. Social and Cultural Tables for North Eastern States. New Delhi: Ministry of Home Affairs; 2001.

Registrar General of India. Provisional population report. New Delhi: Ministry of Home Affairs; 2011a.

Registrar General of India. Annual Health Survey Bulletin 2010-11. New Delhi: Ministry of Home Affairs; 2011b.

Registrar General of India. Sample Registration System Bulletin. New Delhi: Ministry of Home Affairs; 2014.

Sarma R, Choudhury L. A New model for estimating district life expectancy at birth in India, with special reference to Assam State. Can. Stud. Popul. 2014; 41: 180-191.

Silcocks PBS, Jenner DA, Reza R. Life expectancy as a summary of mortality in a population: Statistical considerations and suitability for use by health authorities. J. Epidemiol. Commun. Health. 2001; 55: 38-43.

Swanson DA. A State based regression model for estimating sub state life expectancy. Demography. 1989; 26:161-170.

Swanson DA. The relationship between life expectancy and socioeconomic status in Arkansas: 1970 and 1990. J. Ark. Med. Soc. 1992; 89: 333-335.

Toson B, Baker A. Life expectancy at birth: Methodological options for small populations, National Statistics Methodological, Series No. 33(2003), [cited 2012 March 11] Available from: http://www.statistics.gov.uk/downloads/theme_other/GSSMethodology_No_33.pdf.