บทความวิจัย

ผลของโปรแกรมการให้ความรู้โดยใช้สื่อที่มีต่อตัวแบบเชิงคณิตศาสตร์การระบาดของโรคเลปโตสไปโรซิส

The Effect of Awareness Program by Media on the Transmission Mathematical Model of Leptospirosis

กันญารัตน์ หนูชุม[1]
Kanyarat Noochum[1]

บทคัดย่อ

บทความนี้มีวัตถุประสงค์เพื่อพัฒนาและวิเคราะห์ตัวแบบเชิงคณิตศาสตร์แบบไม่เชิงเส้นสำหรับการระบาดของโรคเลปโตสไปโรซิสที่มีผลมาจากโปรแกรมการให้ความรู้โดยใช้สื่อ ซึ่งจะใช้โปรแกรมการให้ความรู้โดยใช้สื่อทันทีที่เกิดโรคระบาด ในการศึกษาได้แบ่งประชากรคนที่เสี่ยงต่อการติดเชื้อออกเป็น 2 กลุ่ม คือ คนที่เสี่ยงต่อการติดเชื้อที่ไม่ได้รับสื่อ (S_H) และคนที่เสี่ยงต่อการติดเชื้อที่ได้รับสื่อ (S_M) โดยสมมติว่าทุกคนเป็นคนกลุ่มเสี่ยง แต่ถ้าได้รับความรู้จากสื่อจะกลายเป็นคนกลุ่มเสี่ยงที่ได้รับสื่อซึ่งจะมีความตระหนักและหลีกเลี่ยงการสัมผัสกับคนที่ติดเชื้อ จากการวิเคราะห์ตัวแบบเชิงคณิตศาสตร์โดยใช้ทฤษฎีเสถียรภาพของสมการเชิงอนุพันธ์ ผลการวิจัยพบว่า ถ้าค่าระดับการติดเชื้อ $R_0 < 1$ แล้วจะไม่เกิดการระบาดของโรค และถ้าค่าระดับการติดเชื้อ $R_0 > 1$ แล้วจะเกิดการระบาดของโรค นั่นคือเมื่อเพิ่มโปรแกรมการให้ความรู้โดยใช้สื่อให้มากขึ้นจะส่งผลให้จำนวนผู้ติดเชื้อในประชากรคนกลุ่มเป้าหมายมีค่าลดลง

คำสำคัญ: ตัวแบบเชิงคณิตศาสตร์ โรคเลปโตสไปโรซิส โปรแกรมการให้ความรู้

Abstract

This paper, aimed to develope and analyze a non-linear mathematical model for the effect of awareness programs driven by the media on the spread of Leptospirosis. The awareness program are treated as an intervention for the emergent disease. We divided the susceptible populations into two sub-classes: susceptible class and aware susceptible class. We assumed that susceptible individuals who come across with media campaign move to aware susceptible class and in general avoid contact with infection. This model is analyzed by using stability theory of differential equations. The results show that if the basic reproduction number $R_0 < 1$, then the disease dies out, however, if $R_0 > 1$, then the disease becomes endemic. Our analysis shows that the awareness programs driven by the media have positive impact in reducing the infection prevalence of the infective population.

Keywords: Mathematical Model, Leptospirosis, Awareness Program

---

[1] อ., สาขาวิชาคณิตศาสตร์ คณะวิทยาศาสตร์และเทคโนโลยี มหาวิทยาลัยราชภัฏสุราษฎร์ธานี 84000

[1] Lecturer, Department of Mathematics, Faculty of Science and Technology, Surat thani Rajabhat Univesity, Surat Thani, 84000

[*] Corresponding author: E-mail address: nok.math@hotmail.com

Introduction

Leptospirosis is a zoonotic bacteriological disease, caused by members of the genus Leptospira. The human is infected by drinking water in which a rat was found dead, and cattle that drink this water become infectious. The human whose urine is used by other animals and cattle is also infected because the leptospirosis disease germs come out in urine. Those who wade through dirty water are mostly infected from this disease [1]. Infection induces antibody production in the animal renal carriers, shedding leptospirosis in their urine several weeks or even months after infection. The symptoms of leptospirosis are generally flu-like, but the disease can result in liver damage and renal failure [2]. In Thailand, leptospirosis occurs mainly in the rainy season, with an increase in cases beginning in August, reaching a peak in October, and beginning to fall in November [3].

Mathematical model play an important role to present the transmission dynamics of different diseases. Many models have been proposed to represent the dynamics of both human and vector population. Pongsuumpun *et al*. [4] developed mathematical models to study the behavior of leptospirosis disease. They represent the rate of change for both human and rats population. The human population is further divided into two main groups: juveniles and adults. The dynamical interaction including local and global stability of leptospirosis infected human and vector population can be found in Zaman *et al*. [5]

The media may be the most important source of health information for the general public; it can play a special role in providing a voice for people to express their experiences of illness [6]. In 1998 American 'National Health Council' conducted a study, where it is was observed that 75% of the people receive health news via the media (40% from the TV, 35% - by magazines or journals, 16% from newspapers, and 2% through the internet) [7]. Recently some scholars used mathematical models to discuss the impact of awareness programs on the diseases spreading and controlling in a given region. Statistical analysis on AIDS awareness programs shows that public awareness can play an appreciable role in preventing the AIDS epidemic [8]. Misra *et al*. [9] investigated the effects of awareness programs on the spread of infectious diseases and concluded that the spread of an infectious disease can be controlled by using awareness programs but the disease remains endemic due to immigration. Kaur *et al*. [10] studied modeling and analysis of an SIRS epidemic model with effect of awareness programs by media found that the awareness programs driven by the media have positive impact in reducing the infection prevalence of the infective population in the region under consideration.

In this present study, we have proposed and analyzed epidemic model to study the effect of awareness programs conducted through media campaigning on the spread of Leptospirosis. This some of the human beings, if someone is not listen to any of the media awareness programs and will interact with infectives in anyway. Hence, sending them back to unaware susceptible class is not reasonable as once someone is aware. In our model we have kept such kind of individual in aware class only but we assume that they interact with infectives. This leads to the assumption that some fraction of total aware susceptible population is interacting with infectives.

The paper is organized as follows. In Section 2, we present the mathematical model. Section 3, we show the basic properties of the model. Section 4, our model is analyzed with standard dynamical method. Section 5, we present numerical simulation of the model. Section 6, we conclude with a short discussion.

Mathematical Model

We consider human population of size N_H divided into four subclasses; susceptible class S_H , infective class I_H , recovered class R_H and aware susceptible class S_M . Similarly, the rat population of size N_R is divided into two subclasses; susceptible class S_R and infective class I_R . Let M be the cumulative density of the awareness programs driven by media in the region under consideration. It is assumed that susceptible individuals who come across with media campaign move to aware susceptible class, in general, they avoid contact with infectives.

The disease is described by the following system of non-linear differential equations:

$$\frac{dS_H}{dt} = b_H + \lambda_H R_H - \mu_H S_H - \beta_1 S_H I_H - \beta_2 S_H I_R - \alpha S_H M \tag{1}$$

$$\frac{dI_H}{dt} = \beta_1 S_H I_H + \beta_2 S_H I_R + \beta_4 S_M I_H - \mu_H I_H - \delta_H I_H - \gamma_H I_H \tag{2}$$

$$\frac{dR_H}{dt} = \gamma_H I_H - \mu_H R_H - \lambda_H R_H \tag{3}$$

$$\frac{dS_M}{dt} = \alpha S_H M - \beta_4 S_M I_H - \mu_H S_M \tag{4}$$

$$\frac{dS_R}{dt} = b_R - \mu_R S_R - \beta_3 S_R I_H \tag{5}$$

$$\frac{dI_R}{dt} = \beta_3 S_R I_H - \mu_R I_R - \delta_R I_R \tag{6}$$

$$\frac{dM}{dt} = \mu + \mu_1 I_H - \mu_0 M \tag{7}$$

$$S_H(0) > 0,\ I_H(0) \ge 0,\ R_H(0) \ge 0,\ S_M(0) \ge 0,\ S_R(0) > 0,\ I_R(0) \ge 0,\ M(0) \ge 0 .$$

with initials conditions .

The flow diagram describing population movements between the compartments is shown in Fig. 1.

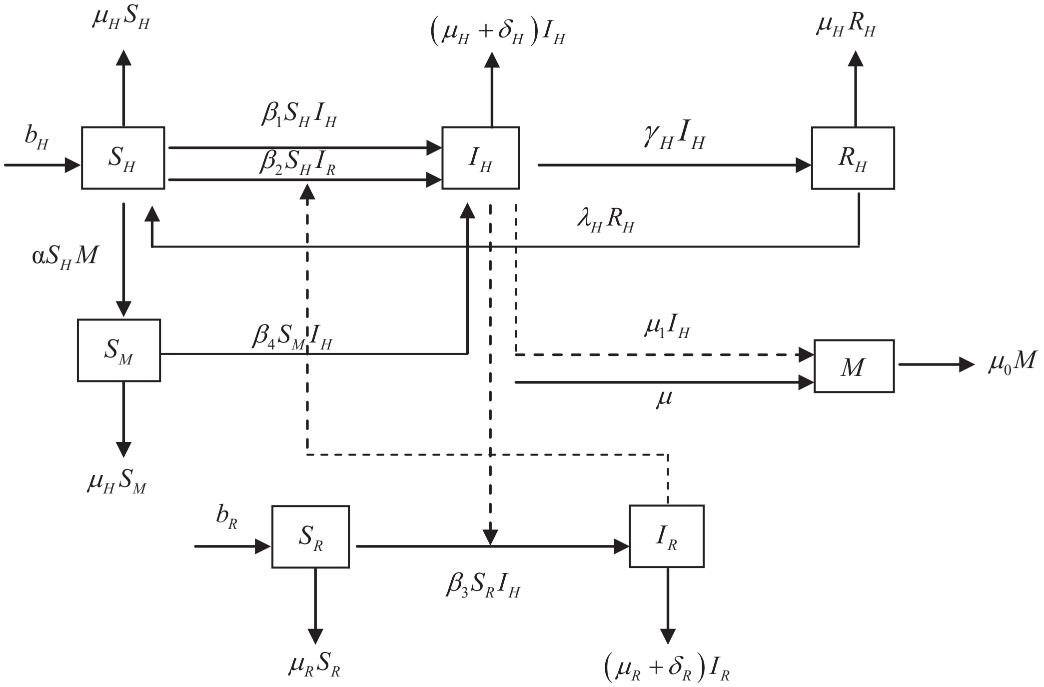


Fig. 1 Flow diagram for the spread of Leptospirosis

Here, b_H is the recruitment rate of human population ; b_R is the recruitment rate of rat population ; β_1 , β_2 , β_3 and β_4 are respectively represent the transmission coefficient between human, susceptible human and infective human, susceptible human and infective rat, susceptible rat and infective human, and aware susceptible human and infective human ; á is the dissemination rate of awareness among susceptibles of human population due to media awareness programs ; μ_H is the natural death rate of human population ; μ_R is the natural death rate of rat population; λ_H is the rate at which individual from recovered class move to susceptible class again after loosing immunity; γ_H is the natural recovery rate constant; δ_H is the disease related death rate of human population; δ_R is the disease related death rate of human population; μ is the rate constant corresponding to regular media coverage, μ_1 is the rate constant influenced by number of infectives and μ_0 is the natural decay rate constant of media coverage/awareness programs.

The total dynamics of human population (N_H) and the total of rat population (N_R) are given by

$N_H = S_H + S_M + I_H + R_H$ and $N_R = S_R + I_R$, respectively.

$$\begin{aligned}
\frac{dN_H}{dt} &= b_H - \mu_H N_H - \delta_H I_H \\
\frac{dI_H}{dt} &= (\beta_1 I_H + \beta_2 I_R)(N_H - S_M - I_H - R_H) + \beta_4 S_M I_H - \mu_H I_H - \delta_H I_H - \gamma_H I_H \\
\frac{dR_H}{dt} &= \gamma_H I_H - \mu_H R_H - \lambda_H R_H \\
\frac{dS_M}{dt} &= \alpha M(N_H - S_M - I_H - R_H) - \beta_4 S_M I_H - \mu_H S_M \\
\frac{dN_R}{dt} &= b_R - \mu_R N_R - \delta_R I_R
\end{aligned} \tag{8}$$

$$\frac{dI_R}{dt} = \beta_3 I_H \left(N_R - I_R\right) - \mu_R I_R - \delta_R I_R$$
$$\frac{dM}{dt} = \mu + \mu_1 I_H - \mu_0 M$$

Basic Properties

Since the model monitors human population, rat popalation and media coverage, it is assumed that all the state variables are non-negative at time $t = 0$. All parameters of the model are assumed to be non-negative. It then follows from the differential equations that the variables are non-negative for all $t \geq 0$. In the absence of leptospirosis infection, $N_H \leq \frac{b_H}{\mu_H}$, $N_R \leq \frac{b_R}{\mu_R}$ and $M \leq \frac{\mu}{\mu_0}$ $as\ t \to \infty$. The solution of system (8) with initiate in Ω remain in the region Ω. This result can be summarized in the following theorem:

Theorem 1 Let $(S_H, I_H, R_H, S_M, S_R, I_R, M)$ be the solution of the system (8) with initial conditions

$S_H(0) > 0,\ I_H(0) \geq 0,\ R_H(0) \geq 0,\ S_M(0) \geq 0,\ S_R(0) > 0,\ I_R(0) \geq 0,\ M(0) \geq 0$ and closed set

.

$$\Omega = \left\{ \left(S_H, I_H, R_H, S_M, S_R, I_R, M\right) \in R_+^7 ; N_H \leq \frac{b_H}{\mu_H}, N_R \leq \frac{b_R}{\mu_R}, M \leq \frac{\mu}{\mu_0} \right\}.$$

Then Ω is positively – invariant and attracting under the flow described by the system (8).

Proof Consider the Lyapunov function [11]

$V(t) = (N_H(t), N_R(t), M) = (S_H + I_H + R_H + S_M, S_R + I_R, M)$

The time derivative of this equation is given by:

$$\frac{dV}{dt} = (b_H - \mu_H N_H - \delta_H I_H, b_R - \mu_R N_R - \delta_R I_R, \mu + \mu_1 I_H - \mu_0 M)$$

Now it is easy to prove that

$$\frac{dN_H}{dt} \leq b_H - \mu_H N_H \qquad for \qquad N_H \geq \frac{b_H}{\mu_H}$$
$$\frac{dN_R}{dt} \leq b_R - \mu_R N_R \qquad for \qquad N_R \geq \frac{b_R}{\mu_R}$$
$$\frac{dM}{dt} \leq \mu - \mu_0 M \qquad for \qquad M \geq \frac{\mu}{\mu_0}$$

Thus is follows that $\frac{dV}{dt} \leq 0$ which implies that Ω is positively – invariant set and a standard comparison theorem [12] is used to show that

Thus as $t \to \infty$, $0 \le (N_H, N_R, M) \le \left(\dfrac{b_H}{\mu_H}, \dfrac{b_R}{\mu_R}, \dfrac{\mu}{\mu_0}\right)$ and so Ω is an attracting set.

Mathematical Analysis

Firstly, we consider the equilibrium points for the model. Since the local stability of disease free equilibrium point and disease endemic equilibrium point is governed by the reproduction number of the model.

4.1 Equilibrium Points

In order to obtain equilibrium points, we set all equations of the system (8) equal to zero. Our model has two non – negative equilibrium.

1) Disease Free Equilibrium: DFE

Theorem 4.1 – 1 There exists disease free equilibrium point $E_0(N_H, I_H, R_H, S_M, N_R, I_R, M)$ of the system (8).

Proof In the absence of disease, that is $I_H = 0$ and $I_R = 0$, we obtained $N_H = \dfrac{b_H}{\mu_H}$, $R_H = 0$, $S_M = \dfrac{\alpha \mu b_H a_1}{\alpha \mu a_2 + \mu_H a_3}$, $N_R = \dfrac{b_R}{\mu_R}$ and $M = \dfrac{\mu}{\mu_0}$ where $a_1 = \mu_H + \lambda_H$, $a_2 = \mu_H\left(\mu_H + \lambda_H\right)$ and $a_3 = \mu_0 \mu_H \left(\mu_H + \lambda_H\right)$.

Then $E_0(N_H, I_H, R_H, S_M, N_R, I_R, M) = \left(\dfrac{b_H}{\mu_H}, 0, 0, \dfrac{\alpha \mu b_H a_1}{\alpha \mu a_2 + \mu_H a_3}, \dfrac{b_R}{\mu_R}, 0, \dfrac{\mu}{\mu_0}\right)$.

2) Disease Endemic Equilibrium: DEE

Theorem 4.1 – 2 There exists disease endemic equilibrium point $E_1(N_H^*, I_H^*, R_H^*, S_M^*, N_R^*, I_R^*, M^*)$ of the system (8) if $A_1\left(I_H^*\right)^3 + A_2\left(I_H^*\right)^2 + A_3 I_H^* + A_4 = 0$ and $A_2 A_3 - A_4 > 0$ where I_H^* has positive roots.

Proof This is disease endemic equilibrium point as $E_1(N_H^*, I_H^*, R_H^*, S_M^*, N_R^*, I_R^*, M^*)$

where $N_H^* = \dfrac{b_H - \delta_H I_H^*}{\mu_H}$, $R_H^* = \dfrac{\gamma_H I_H^*}{\mu_H + \lambda_H}$, $S_M^* = \dfrac{a_4\left(I_H^*\right)^2 + a_5 I_H^* + a_6}{a_7 I_H^* + a_8}$,

$$N_R^* = \frac{b_R\left(\beta_3 I_H^* + \mu_R + \delta_R\right)}{\beta_3 \delta_R I_H^* + \beta_3 \mu_R I_H^* + \mu_R^2 + \mu_R \delta_R}, \quad \text{and } M^* = \frac{\mu + \mu_1 I_H^*}{\mu_0}.$$

Let $a_1 = \mu_H + \lambda_H, a_2 = \mu_H\left(\mu_H + \lambda_H\right), a_3 = \mu_0 \mu_H\left(\mu_H + \lambda_H\right), a_4 = \left(-\alpha \mu_1 \delta_H a_1 - \alpha \mu_1 a_2 - \alpha \mu_1 \mu_H \gamma_H\right)$

$a_5 = \left(\alpha \mu_1 b_H a_1 - \alpha \mu \delta_H a_1 - \alpha \mu a_2 - \alpha \mu \mu_H \gamma_H\right), a_6 = \alpha \mu b_H a_1, a_7 = \alpha \mu_1 a_2 + \beta_4 a_3, a_8 = \alpha \mu a_2 + \mu_H a_3$,

$$A_1 = \left(\mu_H \beta_4 - \mu_H \beta_1\right) a_1 a_4 - \left(\delta_H \beta_1 + \mu_H \beta_1 + \mu_H^2 - \delta_H \mu_H\right) a_1 a_7 - \mu_H \gamma_H \beta_1 a_7,$$

$$\begin{aligned} A_2 = {} & \left(\mu_H \beta_4 - \mu_H \beta_1\right) a_1 a_5 + \left(b_H \beta_1 - \gamma_H \mu_H\right) a_1 a_7 - \left(\delta_H \beta_1 + \delta_H \mu_H + \mu_H \beta_1 a_1 a_8 + \mu_H \mu_H a_1 a_8\right) a_1 a_8 - \\ & \mu_H \gamma_H \beta_1 a_8 - \left(\delta_H \beta_2 a_1 a_7 + \mu_H \beta_2 a_1 a_4 + \mu_H \beta_2 a_1 a_7 + \mu_H \gamma_H \beta_2 a_7\right) I_R, \end{aligned}$$

$$A_3 = \left(\mu_H \beta_4 - \mu_H \beta_1\right) a_1 a_6 + \left(b_H \beta_1 - \gamma_H \mu_H\right) a_1 a_8 + \begin{pmatrix} b_H \beta_2 a_1 a_7 - \mu_H \beta_2 a_1 a_5 - \delta_H \beta_2 a_1 a_8 - \\ \mu_H \beta_2 a_1 a_8 - \mu_H \gamma_H \beta_2 a_8 \end{pmatrix} I_R,$$

$$A_3=\left(\mu_H\beta_4-\mu_H\beta_1\right)a_1a_6+\left(b_H\beta_1-\gamma_H\mu_H\right)a_1a_8+\begin{pmatrix}b_H\beta_2a_1a_7-\mu_H\beta_2a_1a_5-\delta_H\beta_2a_1a_8-\\ \mu_H\beta_2a_1a_8-\mu_H\gamma_H\beta_2a_8\end{pmatrix}I_R,$$

and $A_4=\left(b_H\beta_2a_1a_8-\mu_H\beta_2a_1a_6\right)I_R$.

Then we have $A_1\left(I_H^*\right)^3+A_2\left(I_H^*\right)^2+A_3I_H^*+A_4=0$ (9)

It is clear that if $A_2A_3-A_4>0$ then one of the roots of cubic equation (9) will be positive.

4.2 Basic Reproduction Number

In the following, the basic reproduction number of system (8) will be obtained by the next generation matrix method [13].

We rewritten the system in matrix form $\frac{dx}{dt}=F(x)-V(x)$, Here $F(x)$ represent the rate appearance of new infections and $V(x)$ represent the transfer of individuals. The matrices $F(x)$ and $V(x)$ for the system are computed as follows.

$$F(x)=\begin{bmatrix}0\\ \left(\beta_1I_H+\beta_2I_R\right)\left(N_H-S_M-I_H-R_H\right)+\beta_4S_MI_H\\ 0\\ 0\\ 0\\ \beta_3I_H\left(N_R-I_R\right)\\ 0\end{bmatrix},$$

$$V(x)=\begin{bmatrix}\mu_HN_H+\delta_HI_H-b_H\\ (\mu_H+\delta_H+\gamma_H)I_H\\ (\mu_H+\lambda_H)R_H-\gamma_HI_H\\ \beta_4S_MI_H+\mu_HS_M-\alpha M\left(N_H-S_M-I_H-R_H\right)\\ \mu_RN_R+\delta_RI_R-b_R\\ (\mu_R+\delta_R)I_R\\ \mu_0M-\mu-\mu_1I_H\end{bmatrix}\text{W}$$

e get Jacobain matrix of $F(x)$ and $V(x)$ evaluated at E_0,

$$J\left(F\left(E_0\right)\right)=\begin{bmatrix}0&0&0&0&0&0&0\\ 0&\begin{matrix}\left(\frac{b_H}{\mu_H}-\frac{\alpha\mu b_Ha_1}{\alpha\mu a_2+\mu_Ha_3}\right)\beta_1\\ +\frac{\beta_4\alpha\mu b_Ha_1}{\alpha\mu a_2+\mu_Ha_3}\end{matrix}&0&0&0&\left(\frac{b_H}{\mu_H}-\frac{\alpha\mu b_Ha_1}{\alpha\mu a_2+\mu_Ha_3}\right)\beta_2&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&\frac{\beta_3b_R}{\mu_R}&0&0&0&0&0\\ 0&0&0&0&0&0&0\end{bmatrix}$$

$$J\left(V\left(E_0\right)\right)=\begin{bmatrix} \mu_H & \delta_H & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_H+\delta_H+\gamma_H & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma_H & \mu_H+\lambda_H & 0 & 0 & 0 & 0 \\ \dfrac{-\alpha\mu}{\mu_0} & \left(\dfrac{\beta_4\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}+\dfrac{\mu}{\mu_0}\right)\lambda_H & \dfrac{\lambda\mu}{\mu_0} & \mu_H+\dfrac{\alpha\mu}{\mu_0} & 0 & 0 & \left(\dfrac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}-\dfrac{b_H}{\mu_H}\right)\lambda_H \\ 0 & 0 & 0 & 0 & \mu_R & \delta_R & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_R+\delta_R & 0 \\ 0 & -\mu_1 & 0 & 0 & 0 & 0 & \mu_0 \end{bmatrix}$$

The basic reproduction number R_0 is given by the spectral radius of the next generation matrix FV^{-1}. Hence $R_0=\rho\left(FV^{-1}\right)$, where

$$FV^{-1}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\left(\dfrac{b_H}{\mu_H}-\dfrac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}\right)\beta_2}{\mu_R+\delta_R} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Thus, } R_0=\sqrt{\frac{\left(\dfrac{b_H}{\mu_H}-\dfrac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}\right)\beta_2}{\mu_R+\delta_R}} \qquad (10)$$

4.3 Stability Analysis

The local asymptotic stability of the disease free equilibrium point E_0 and disease endemic equilibrium point E_1 is established using variational matrix method and stated in the following theorem.

1) Stability of Disease Free State

Theorem 4.3 – 1 The system (8) with all positive parameters has a unique disease free equilibrium point E_0 and if $R_0<1$ then E_0 is locally asymptotically stable

Proof Hence $I_H=0$ and $I_R=0$ the disease free equilibrium is

$$E_0(N_H,I_H,R_H,S_M,N_R,I_R,M)=\left(\frac{b_H}{\mu_H},0,0,\frac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3},\frac{b_R}{\mu_R},0,\frac{\mu}{\mu_0}\right)$$

The Jacobian matrix of the model (8) at E_0 is given by

$$J(E_0)=J_0=\begin{bmatrix} -\mu_H & -\delta_H & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & \left(\frac{b_H}{\mu_H}-\frac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}\right)\beta_2 & 0 \\ 0 & \gamma_H & -(\mu_H+\lambda_H) & 0 & 0 & 0 & 0 \\ \frac{\alpha\mu}{\mu_0} & -\left(\frac{\alpha\mu}{\mu_0}+\frac{\alpha\mu b_H a_1\beta_4}{\alpha\mu a_2+\mu_H a_3}\right) & -\frac{\alpha\mu}{\mu_0} & -\left(\frac{\alpha\mu}{\mu_0}+\mu_H\right) & 0 & 0 & \alpha\left(\frac{b_H}{\mu_H}-\frac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}\right) \\ 0 & 0 & 0 & 0 & -\mu_R & -\delta_R & 0 \\ 0 & \frac{\beta_3 b_R}{\mu_R} & 0 & 0 & 0 & -(\mu_R+\delta_R) & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & -\mu_0 \end{bmatrix}$$

where $a=\left(\frac{b_H}{\mu_H}-\frac{\alpha\mu b_H a_1}{\alpha\mu a_2+\mu_H a_3}\right)\beta_1+\left(\frac{\alpha\mu b_H a_1\beta_4}{\alpha\mu a_2+\mu_H a_3}-\mu_H-\delta_H-\gamma_H\right)$.

Using elementary row operation the characteristics equation about E_0 is given by

$$(\lambda+\mu_R)\left(\lambda+\left(\frac{\alpha\mu}{\mu_0}+\mu_H\right)\right)(\lambda+(\mu_H+\lambda_H))(\lambda+\mu_0)(\lambda+\mu_H)(\lambda^2+B_1\lambda+B_2)=0 \qquad (11)$$

where $B_1=-\left(\frac{b_H}{\mu_H}-\frac{\lambda\mu b_H a_1}{\lambda\mu a_2+\mu_H a_3}\right)\beta_1-\left(\frac{\lambda\mu b_H a_1\beta_4}{\lambda\mu a_2+\mu_H a_3}-\mu_H-\delta_H-\gamma_H\right)$ and

$B_2=-(\mu_R+\delta_R)\left(\frac{b_H}{\mu_H}-\frac{\lambda\mu b_H a_1}{\lambda\mu a_2+\mu_H a_3}\right)\beta_1-(\mu_R+\delta_R)\left(\frac{\lambda\mu b_H a_1\beta_4}{\lambda\mu a_2+\mu_H a_3}-\mu_H-\delta_H-\gamma_H\right)-\left(\frac{\beta_3 b_R}{\mu_R}\right)\left(\left(\frac{b_H}{\mu_H}-\frac{\lambda\mu b_H a_1}{\lambda\mu a_2+\mu_H a_3}\right)\beta_2\right)$. T

he eigenvalues of this matrix are $\lambda_1=-\mu_R$, $\lambda_2=-\left(\frac{\lambda\mu}{\mu_0}+\mu_H\right)$, $\lambda_3=-(\mu_H+\lambda_H)$,

$\lambda_4 = -\mu_0$, $\lambda_5 = -\mu_H$, λ_6 and λ_7 are solution of quadratic equation $\lambda^2 + B_1\lambda + B_2 = 0$. By the Routh-Herwitz criteria [14], if $B_1 > 0$ and $B_2 > 0$, then roots of the quadratic equation have negative real part. Hence, the disease free equilibrium point is locally asymptotically stable.

2) Stability of Disease Endemic State

Theorem 4.3 – 2 Suppose $R_0 > 1$ then there is a unique disease endemic equilibrium point E_1 of (8) which is locally asymptotically stable.

Proof The disease endemic equilibrium is $N_H^* = \dfrac{b_H - \delta_H I_H^*}{\mu_H}$, $R_H^* = \dfrac{\gamma_H I_H^*}{\mu_H + \lambda_H}$,

$$S_M^* = \frac{a_4\left(I_H^*\right)^2 + a_5 I_H^* + a_6}{a_7 I_H^* + a_8}, N_R^* = \frac{b_R\left(\beta_3 I_H^* + \mu_R + \delta_R\right)}{\beta_3\delta_R I_H^* + \beta_3\mu_R I_H^* + \mu_R^2 + \mu_R\delta_R}, M^* = \frac{\mu + \mu_1 I_H^*}{\mu_0}$$

and I_H^* is the solution of $A_1\left(I_H^*\right)^3 + A_2\left(I_H^*\right)^2 + A_3 I_H^* + A_4 = 0$

The Jacobian matrix of the system (8) at E_1 is

$$J(E_1) = J_1$$

$$= \begin{bmatrix} -\mu_H & -\delta_H & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_H^* + \beta_2 I_R^* & \begin{pmatrix} N_H^* - S_M^* - \\ I_H^* - R_H^* \end{pmatrix}\beta_1 + \begin{pmatrix} \beta_4 S_M^* - \beta_2 I_R^* \\ -\mu_H - \delta_H - \gamma_H \end{pmatrix} & -\left(\beta_1 I_H^* + \beta_2 I_R^*\right) & -\begin{pmatrix} \beta_1 I_H^* + \beta_2 I_R^* \\ -\beta_4 I_H^* \end{pmatrix} & 0 & \begin{pmatrix} N_H^* - S_M^* - \\ I_H^* - R_H^* \end{pmatrix}\beta_2 & 0 \\ 0 & \gamma_H & -\left(\mu_H + \lambda_H\right) & 0 & 0 & 0 & 0 \\ \alpha M^* & -\left(\alpha M^* + \beta_4 S_M^*\right) & -\alpha M^* & -\begin{pmatrix} \alpha M^* + \beta_4 I_H^* \\ +\mu_H \end{pmatrix} & 0 & 0 & \alpha\begin{pmatrix} N_H^* - S_M^* - \\ I_H^* - R_H^* \end{pmatrix} \\ 0 & 0 & 0 & 0 & -\mu_R & -\delta_R & 0 \\ 0 & \beta_3\left(N_R^* - I_R^*\right) & 0 & 0 & \beta_3 I_H^* & -\left(\beta_3 I_H^* + \mu_R + \delta_R\right) & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & -\mu_0 \end{bmatrix}$$

Using elementary row operation the characteristics equation about E_1 is given by

$$\lambda^7 + H_1\lambda^6 + H_2\lambda^5 + H_3\lambda^4 + H_4\lambda^3 + H_5\lambda^2 + H_6\lambda + H_7 = 0 \qquad (12)$$

where $H_1 = -\left(G_1 + G_4 + D_{21}\right)$, $H_2 = -\left(D_2D_3 + G_5 + G_{11} - D_{21}G_1 - D_{21}G_4 - G_1G_4 - G_2\right)$,

$H_3 = -\left(D_2G_{14} + G_2G_4 + G_6 + G_3 + D_{21}G_1G_4 + D_{21}G_2 - D_3G_{12} - G_1G_5 - G_7G_{11} - G_9G_{11} - D_2D_{21}D_3 - D_{21}G_5 - D_{21}G_{11} - G_{16}\right)$,

$$H_4 = -\begin{pmatrix} D_3D_{21}G_{12} + D_{21}G_1G_5 + D_{21}G_7G_{11} + D_{21}G_9G_{11} + D_2G_{15} + D_3G_{13} + G_2G_5 + G_7G_9G_{11} + G_{10}G_{11} \\ +G_8G_{11} + G_{16}G_{17} + D_1G_{16} - D_2D_{21}G_{14} - D_{21}G_2G_4 - D_{21}G_6 - D_{21}G_3 - G_1G_6 - G_3G_4 - G_{12}G_{14} \end{pmatrix},$$

$$H_5 = -\begin{pmatrix} D_{21}G_1G_6 + D_{21}G_3G_4 + D_{21}G_{12}G_{14} + G_2G_6 + G_{13}G_{14} + G_{16}G_{18} - D_2D_{21}G_{15} - D_3D_{21}G_{13} - D_{21}G_2G_5 \\ -D_{21}G_7G_9G_{11} - D_{21}G_{10}G_{11} - D_{21}G_8G_{11} - G_7G_{10}G_{11} - G_8G_9G_{11} - G_3G_5 - G_{12}G_{15} - D_1G_{16}G_{17} \end{pmatrix},$$

$H_6 = -\left(D_{21}G_7G_{10}G_{11} + D_{21}G_8G_9G_{11} + D_{21}G_3G_5 + D_{21}G_{12}G_{15} + G_8G_{10}G_{11} + G_{13}G_{15} + G_{16}G_{19} - D_{21}G_2G_6 - D_{21}G_{13}G_{14} - G_3G_6 - D_1G_{16}G_{18}\right)$,

$H_7 = -D_{21}G_3G_6 + D_{21}G_8G_{10}G_{11} + D_{21}G_{13}G_{15} + D_1G_{16}G_{19}$,

$G_1 = D_1 + D_{15} + D_{19}$, $G_2 = D_{15}D_{19} + D_{16}D_{18} + D_1D_{15} + D_1D_{19}$, $G_3 = D_1D_{15}D_{19} - D_1D_{16}D_{18}$, $G_4 = D_4 + D_9 + D_{13}$,

$G_5 = D_6D_{11} + D_5D_8 - D_9D_{13} - D_4D_9 - D_4D_{13}$, $G_6 = D_4D_9D_{13} + D_6D_8D_{12} - D_6D_9D_{11} - D_5D_8D_{13}$, $G_7 = D_1 + D_{15}$, $G_8 = D_1D_{15}$,

$G_9 = D_9 + D_{13}$, $G_{10} = D_9D_{13}$, $G_{11} = D_7D_{17}$, $G_{12} = D_2D_{15} + D_2D_{19}$, $G_{13} = D_2D_{15}D_{19} + D_2D_{16}D_{18}$, $G_{14} = D_6D_{10} - D_3D_9 - D_3D_{13}$,

$G_{15} = D_3D_9D_{13} - D_6D_9D_{10}$, $G_{16} = D_6D_{14}D_{20}$, $G_{17} = D_{15} + D_{19} + D_9$, $G_{18} = D_{16}D_{18} - D_9D_{15} - D_9D_{19} - D_{15}D_{19}$,

and $G_{19} = D_9D_{15}D_{19} - D_9D_{16}D_{18}$.

By Routh-Hurwitz criteria [14] since $H_i > 0$ for all $i = 1, 2, 3, 4, 5, 6, 7$, each eigenvalues of (12) has the negative parts.

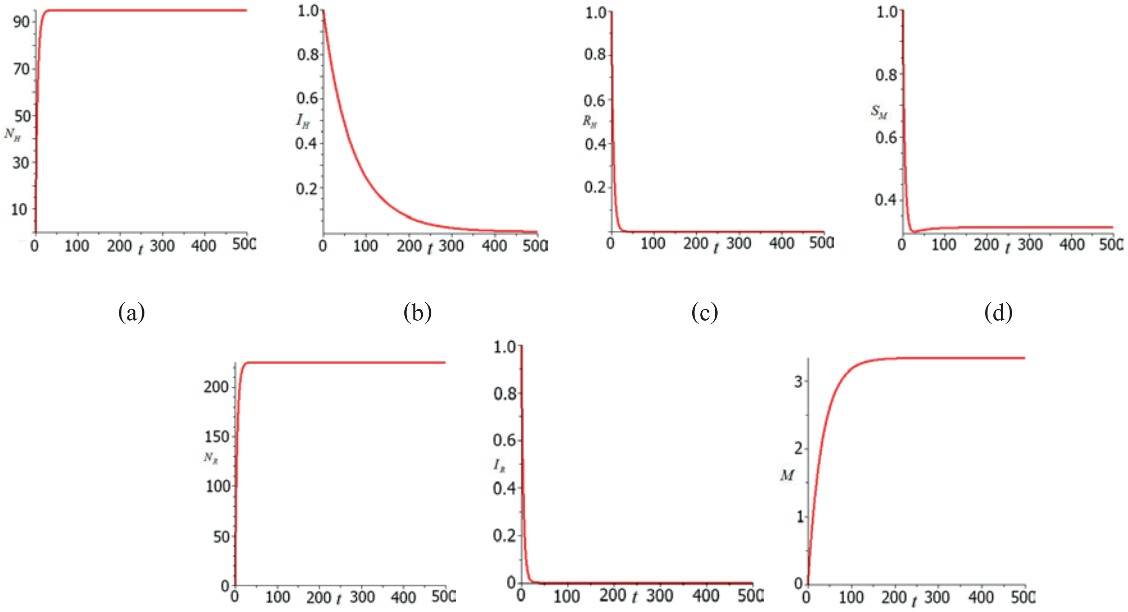
Numerical simulation

In this section, we present the numerical simulation of the dynamical behavior of model. The set of parameter values are given in Table 1.

Table 1 Parameter value used in the numerical simulation

Parameters	b_H	b_R	μ_H	μ_R	δ_H	δ_R	λ_H	γ_H	β_1	β_2	β_3
values (day^{-1})	19	45	0.2	0.2	0.002	0.025	0.005	0.003	0.000075	0.00002	0.000005
Reference	[5]								Assumed		

The stability of disease free equilibrium point E_0 is show in Fig. 2, we choose $\beta_4 = 0.00009$, $\mu = 0.1$, $\mu_0 = 0.03$, $\mu_1 = 0.001$ and á = 0.0002 and keep the other values of parameters to be those given in Table 1. The disease free equilibrium point E_0 for above set of parameters is as follow : $N_H = 95, I_H = 0, R_H = 0,$ $S_M = 0.315615, N_R = 225, I_R = 0, M = 3.33333$. The eigenvalues and the basic reproduction number are obtained as : $\lambda_1 = -\ 0.205, \lambda_2 = -\ 0.200667, \lambda_3 = -\ 0.2,\ \lambda_4 = -\ 0.2, \lambda_5 = -\ 0.03, \lambda_6 = -\ 0.0989485,$ $\lambda_7 = -\ 0.0989485,\ R_0 = 0.197897$. Since all the eigenvalues corresponding to E_0 be negative and basic reproduction number is less than one, the disease free equilibrium point E_0 will be local asymptotically stable.



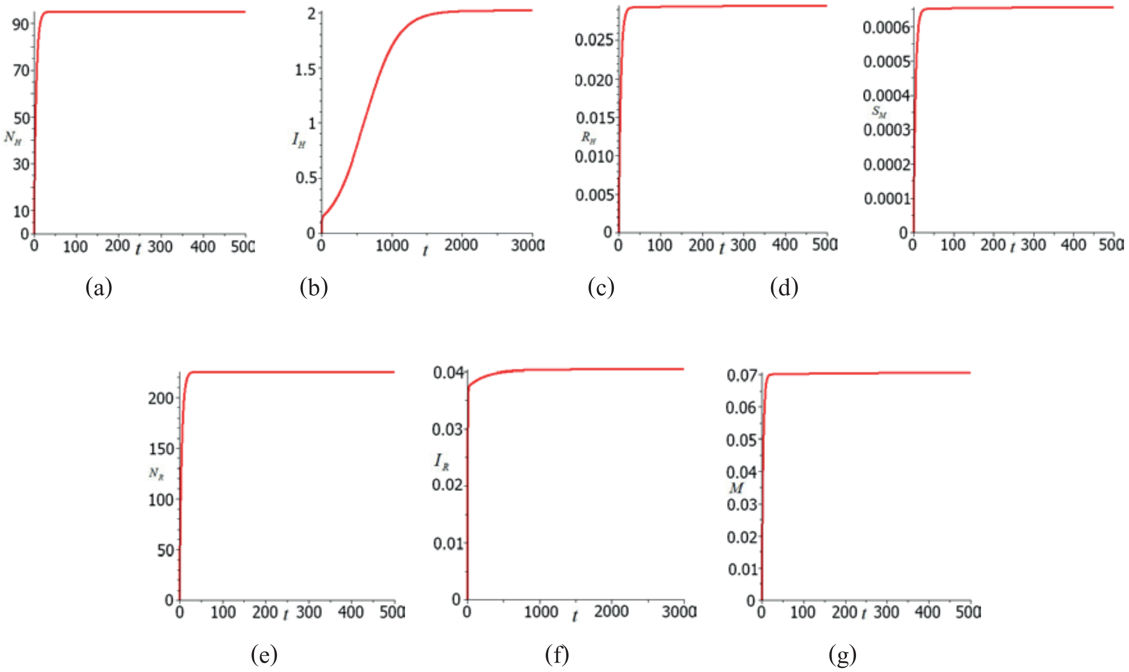
(a) (b) (c) (d)

(e) (f) (g)

Fig. 2 Time series of the (a) total of human population, (b) infective human population, (c) recovered human population, (d) aware susceptible human population, (e) total of rat population, (f) infective rat population and (g) the awareness programs driven by media. The fractions of populations approach to the free disease equilibrium state $E_0(95,0,0,0.315615,225,0,3.33333)$.

The stability of disease endemic equilibrium point E_1 is show in Fig. 3, we choose β_4 =0.009, μ =0.001, μ_0 =0.3, μ_1 =0.01 and á =0.00002 and keep the other values of parameters to be those given in Table 1. The interior equilibrium point E_1 for above set of parameters is as follow : $N_H^* = 94.979$, $I_H^* = 2.09713, R_H^* = 0.0306897, S_M^* = 0.000679979, N_R^* = 224.995,\ I_R^* = 0.0419338,\ M^* = 0.0732376$

The eigenvalues and the basic reproduction number are obtained as : $\lambda_1 = -\ 0.299998, \lambda_2 = -\ 0.274514,$ $\lambda_3 = -\ 0.204985, \lambda_4 = -\ 0.200013, \lambda_5 = -\ 0.199945, \lambda_6 = -\ 0.199945, \lambda_7 = -\ 0.149068,\ R_0$=1.94936. Since all the eigenvalues corresponding to E_1 be negative and basic reproduction number is more than one, the disease endemic equilibrium point E_1 will be local asymptotically stable.



(a) (b) (c) (d)

(e) (f) (g)

Fig. 3 Time series of the (a) total of human population (b) infective human population (c) recovered human population (d) aware susceptible human population (e) total of rat population (f) infective rat population and (g) the awareness programs driven by media. The fractions of populations approach to the disease endemic equilibrium state $E_1(94.979, 2.09713, 0.0306897, 0.000679979, 224.995, 0.0419338, 0.0732376)$.

Conclusion

In this paper, a nonlinear mathematical model is proposed and analyzed to determine the effects of awareness programs driven by the media. The model is divided the human population into four subclasses and rat population into two subclasses. It has been considered that the growth rate of awareness programs impacting the population is assumed to be proportional to the numbers of infective class. It has been further assumed that due to the effect of media, susceptible class individuals form a separate class of aware susceptible class individuals. By analysis the model, we have found parameter R_0. It is noted that when $R_0 < 1$ then the disease dies out and when $R_0 > 1$, the disease becomes endemic. The model has two non-negative equilibrium namely the disease free equilibrium E_0 and the disease endemic equilibrium E_1. It is found if $R_0 < 1$, the disease free equilibrium E_0 is locally asymptotically stable and if $R_0 > 1$, then E_0 is unstable and the infection is maintained in the population. The endemic equilibrium E_1 is always locally asymptotically stable when $R_0 > 1$. The numerical simulation We found the value of R_0 were 0.197897, 1.94936 when $\mu = 0.1, \mu_0 = 0.03, \mu_1 = 0.001$ and $\mu = 0.001,\ \mu_0 = 0.3,\ \mu_1 = 0.01$ respectively. The analysis showed that awareness programs through the media campaigning are helpful in decreasing the infective class in a human population. By awareness programs, at least we could have prevented diseases whenever possible and where not possible to minimize complications and maximize quality of life.

Acknowledgement

Author is thankful to the reviewers for their useful suggestions and comments for improvements to their original paper. This work is subported by the Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University.

References

[1] Palaniappan, R. U. M., Ramanujam, S. and Chang, Y. (2007). "Leptospirosis: pathogenesis, immunity and Diagnosis", **Current Opinion in Infectious Diseases**. 20(3), 284–292.

[2] Michel, V., Branger, C. and Andre-Fontaine, G. (2002). "Epidemiology of Leptospirosis", **Revista Cubana de Medicina Tropical**. 54(1), 7-10.

[3] Triampo, W. *et al*. (2007). "A Simple Deterministic Model for B. PIMPUNCHAT ET AL", **International Journal of Biological and Medical Sciences**. 2(1), 22.

[4] Pongsuumpun, P., Miami, T. and Kongnuy, R. (2008). "Age structural transmission model for Leptospirosis". in **Proceedings of the 3rd International Symposium on Biomedical Engineering**. 411-416. 3-6. Singapore.

[5] Zaman, G., Khan, M. A., Islam, S., Chohan, M. I. and Jung, I. H. (2012). "Modeling dynamical interactions between leptospirosis infected vector and human population", **Applied Mathematical Sciences**. 6(26), 1287-1302.

[6] Schwitzer, G., Mudur, G., Henry, D. et al. (2005). "What are the roles and responsibilities of the media in disseminating health Information", **PLoS Medicine**. 2(7), 215.

[7] Shattuck, J. T. (1998). "Lecture medicine and the media", **The New England Journal and Medicine**. 339, 87-92.

[8] Cui, J., Sun, Y. and Zhu, H. (2008). "The impact of media on the spreading and control of infectious disease", **Journal of Dynamics and Differential Equations**. 20(1), 31-53.

[9] Misra, A.K., Sharma, A. and Shukla, J.B. (2011). "Modeling and analysis of effects of awareness programs by media on the spread of infectious diseases", **Mathematical and Computer Modelling**. 53, 1221-1228.

[10] Kaur, N., Ghosh, M. and Bhatia, S.S. (2014). "Modeling and Analysis of an SIRS Epidemic Model with Effect of Awareness Programs by Media", **International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering**. 8(1), 233-239.

[11] Bacciotti, A and Rosier, L. (2001). **Lyapunov Function and Stability in Control Theory**. London : Springer-Verlag.

[12] Lakshmikantham, V., Leela, S. and Martynyuk, A.A. (1989). **Stability Analysis of Non Linear Systems**. New York : Marcel Dikker Inc.

[13] Van den Driessche, P. and Watmough, J. (2002). "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission", **Mathematical Bioscience**. 180, 29-48.

[14] Alen, L.J.S. (2006). **An Intoduction to Mathematics Biology**. English : Peason/Prince Hall.